A Bayesian Graphical Model for Matching Law Behavior

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The Matching Law

In repeated choice scenarios, the **Matching Equilibrium** is attained when the ratio of investment to exploit different reward alternatives is equal to the ratio of payment obtained from them:

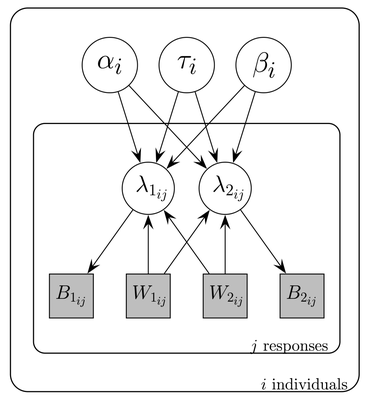
$$\frac{B_1}{B_2} = \frac{W_1}{W_2}$$

where B_i is a measure of **behavior** invested in alternative i, such as number of responses delivered to that option, and W_i is a measure of the benefits obtained from it, such as the number of **rewards**.

Empirical deviations from the strict matching equilibrium are generally well described by the **generalized** matching equation:

$$\log \frac{B_1}{B_2} = \alpha + \beta \log \frac{W_1}{W_2}$$

where $\alpha \neq 0$ is a measure of **bias** or preference towards one alternative regardless of its relative reward rate, and $\beta \neq 1$ reflects over or under **sensitivity** to the relative reward rates of both alternatives.



$$\alpha_i \sim \text{Gaussian}\left(0, \tfrac{1}{\sqrt{0.1}}\right)$$
$$\beta_i \sim \text{Gaussian}\left(0, \tfrac{1}{\sqrt{0.1}}\right)$$
$$\tau_i \sim \text{Gaussian}\left(2, 0.5\right)$$
$$\lambda_{1_{ij}} \sim \text{LogNormal}\left(\tfrac{\alpha_i}{2} + \tfrac{\beta_i}{2} \log\left(\tfrac{W_{1_{ij}}}{W_{2_{ij}}}\right), \tau_i\right)$$
$$\lambda_{2_{ij}} \sim \text{LogNormal}\left(-\tfrac{\alpha_i}{2} - \tfrac{\beta_i}{2} \log\left(\tfrac{W_{1_{ij}}}{W_{2_{ij}}}\right), \tau_i\right)$$
$$B_{1_{ij}} \sim \text{Poisson}\left(\lambda_{1_{ij}}\right)$$
$$B_{2_{ij}} \sim \text{Poisson}\left(\lambda_{2_{ij}}\right)$$

Time−based matching

In time−based alternatives there is a fixed probability per second of a reward being baited. The next response after baiting collects the reward.

Pigeon Experiment

Six pigeons responded in two time−based alternatives simultaneously available for approximately 130 daily sessions. In each session one alternative was more rewarding than the other, indicated by a higher baiting probability per second, although which alternative was richer changed across days. The target data include the total number of responses and rewards obtained from each alternative in each experimental session.

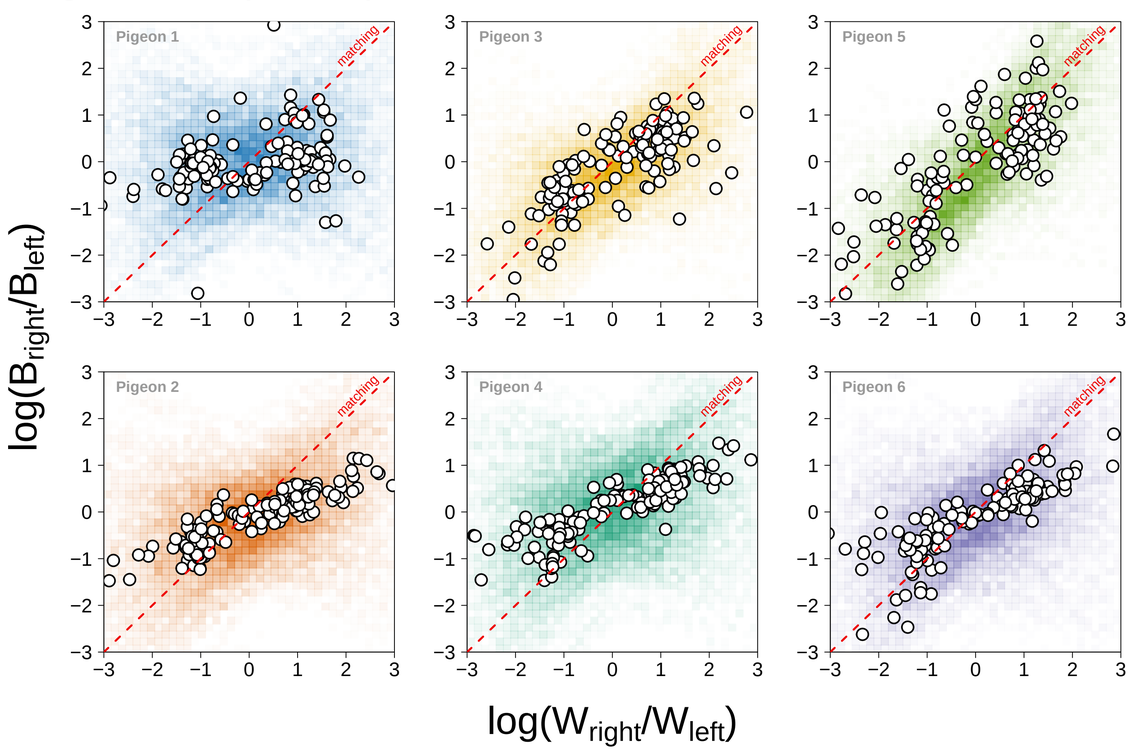
Response−based matching

In response−based alternatives each response has a fixed probability of being rewarded, independent of time or previous responses and rewards.

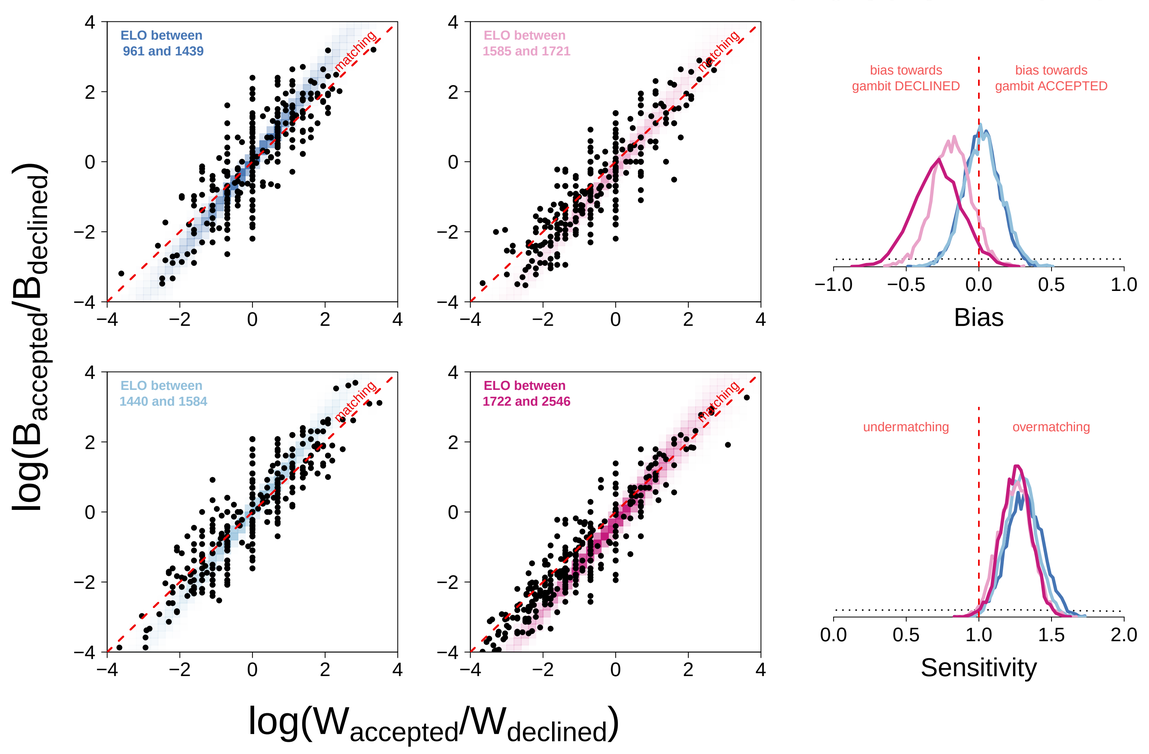
Chess Dataset

We analyzed the decisions made by the player controlling the Black pieces in response to the Queen's Gambit (QG). For this analysis, we worked with the lichess.org dataset, which contains over 4 billion games, and filtered for openings that featured the QG. When facing the QG, Black has two options: accepting it or declining it. The target data per player are the number of each of those decisions and the corresponding victories. Only one month worth of data is reported next.

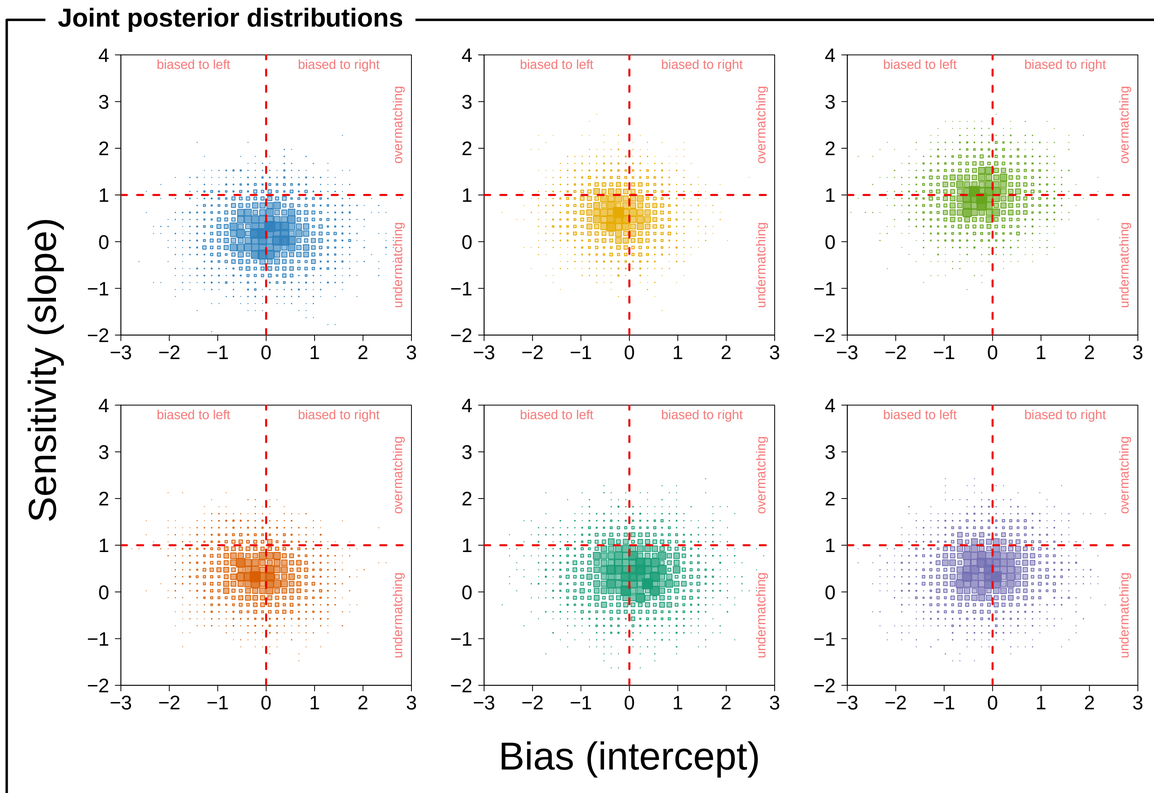
Pigeon data and posterior predictive



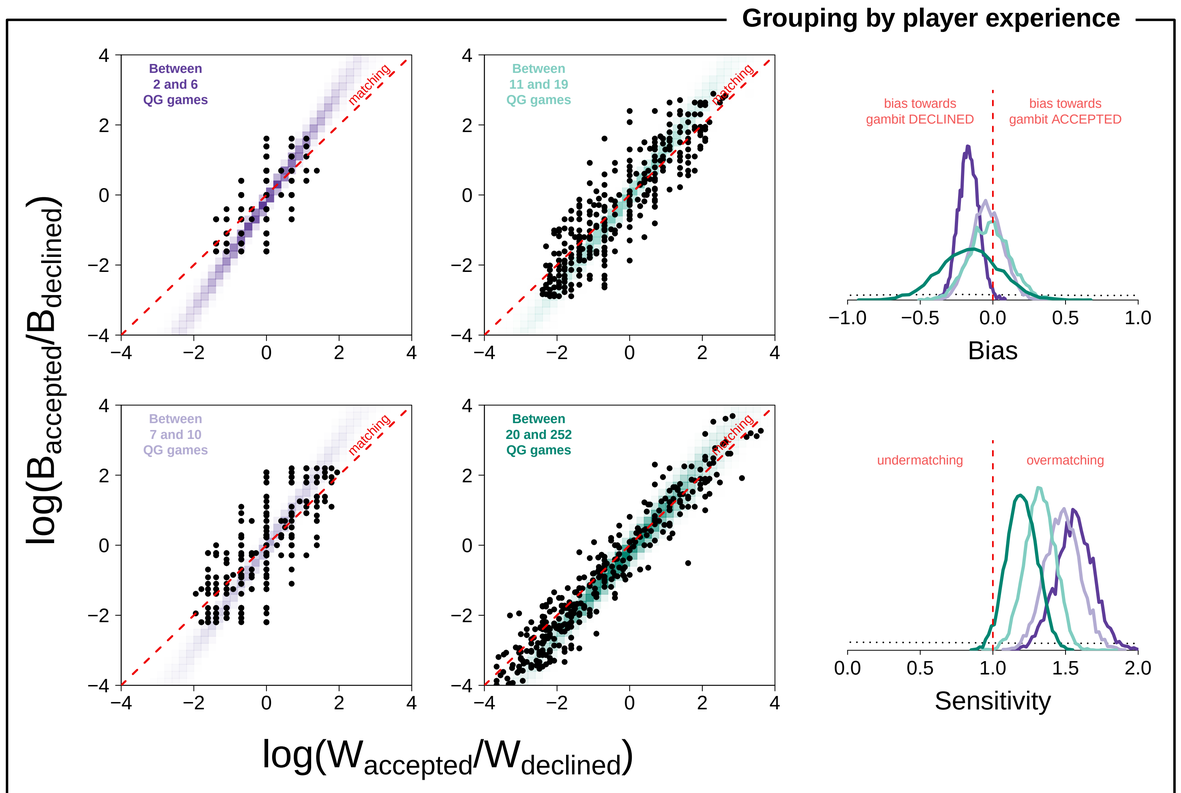
Grouping by player level (ELO)



Joint posterior distributions



Grouping by player experience



Pigeons summary

The distribution of pigeon responses among the two alternatives is influenced by the distribution of rewards obtained from them, albeit high individual variability and systematic deviations from the matching equilibrium.

Most pigeons clearly undermatch, with only one individual showing sensitivity levels consistent with strict matching. This may reflect a potentially over−abundant environment.

However, all birds are unbiased and show no systematic preference for any alternative.

General Conclusions

The matching equilibrium is a robust phenomenon that arises under different types of environmental constrains.

Nevertheless, there is considerable variability across individuals, and certain systematic relationships between the matching parameters and other grouping variables appear evident.

Future steps will include developing hiearchical extensions and explanatory models to better characterize the relationship between matching parameters and different covariates.

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Chess summary

The matching relationship reasonably describes choices by Black against the QG, although certain deviations from the equilibrium appear systematic.

Players of all levels are oversensitive to each won game. Moreover, lower rated players are unbiased, but higher rated players prefer declining the gambit beyond the matching prediction.

Inexperienced and highly experienced players are also biased towards declining, although over−sensitivity to each won game systematically diminishes as experience against the gambit accumulates.

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