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A Bayesian Graphical Model for Matching Behaviors

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The Matching Law

The **Matching Equilibrium** holds when the ratio of investment to exploit different reward alternatives is equal to the ratio of payment obtained from them:

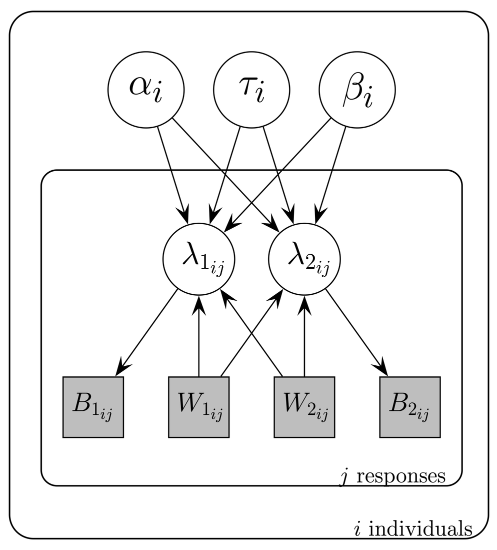
$$\frac{B_1}{B_2} = \frac{W_1}{W_2}$$

where B_i is a measure of **behavior** invested in alternative i, such as number of responses delivered in that option, and W_i is a measure of the benefits obtained from it, such as the number of **rewards**.

Empirical deviations from the strict matching equilibrium are generally well described by the **generalized** matching equation:

$$\log \frac{B_1}{B_2} = \alpha + \beta \log \frac{W_1}{W_2}$$

where $\alpha \neq 0$ is a measure of **bias** or preference towards one alternative regardless of its relative reward rate, and $\beta \neq 1$ reflects over or under **sensitivity** to the relative reward rates of both alternatives.



$$\alpha_i \sim \text{Gaussian}\left(0, \tfrac{1}{\sqrt{0.1}}\right)$$
$$\beta_i \sim \text{Gaussian}\left(0, \tfrac{1}{\sqrt{0.1}}\right)$$
$$\tau_i \sim \text{Gaussian}(2, 0.5)$$
$$\lambda_{1_{ij}} \sim \text{LogNomal}\left(\frac{\alpha_i}{2} + \frac{\beta_i}{2}\log\left(\frac{W_{1_{ij}}}{W_{2_{ij}}}\right), \tau_i\right)$$
$$\lambda_{2_{ij}} \sim \text{LogNomal}\left(-\frac{\alpha_i}{2} - \frac{\beta_i}{2}\log\left(\frac{W_{1_{ij}}}{W_{2_{ij}}}\right), \tau_i\right)$$
$$B_{1_{ij}} \sim \text{Poisson}(\lambda_{1_{ij}})$$
$$B_{2_{ij}} \sim \text{Poisson}(\lambda_{2_{ij}})$$

Time−based matching

In time−based alternatives there is a fixed probability per second of a reward being baited. The next response after baiting collects the reward.

Pigeon Experiment

Six pigeons responded in two time−based alternatives simultaneously available for approximately 130 daily sessions. In each session one alternative was more rewarding than the other, indicated by a higher baiting probability per second, although which alternative was richer changed across days. The target data include the total number of responses and rewards obtained from each alternative in each experimental session.

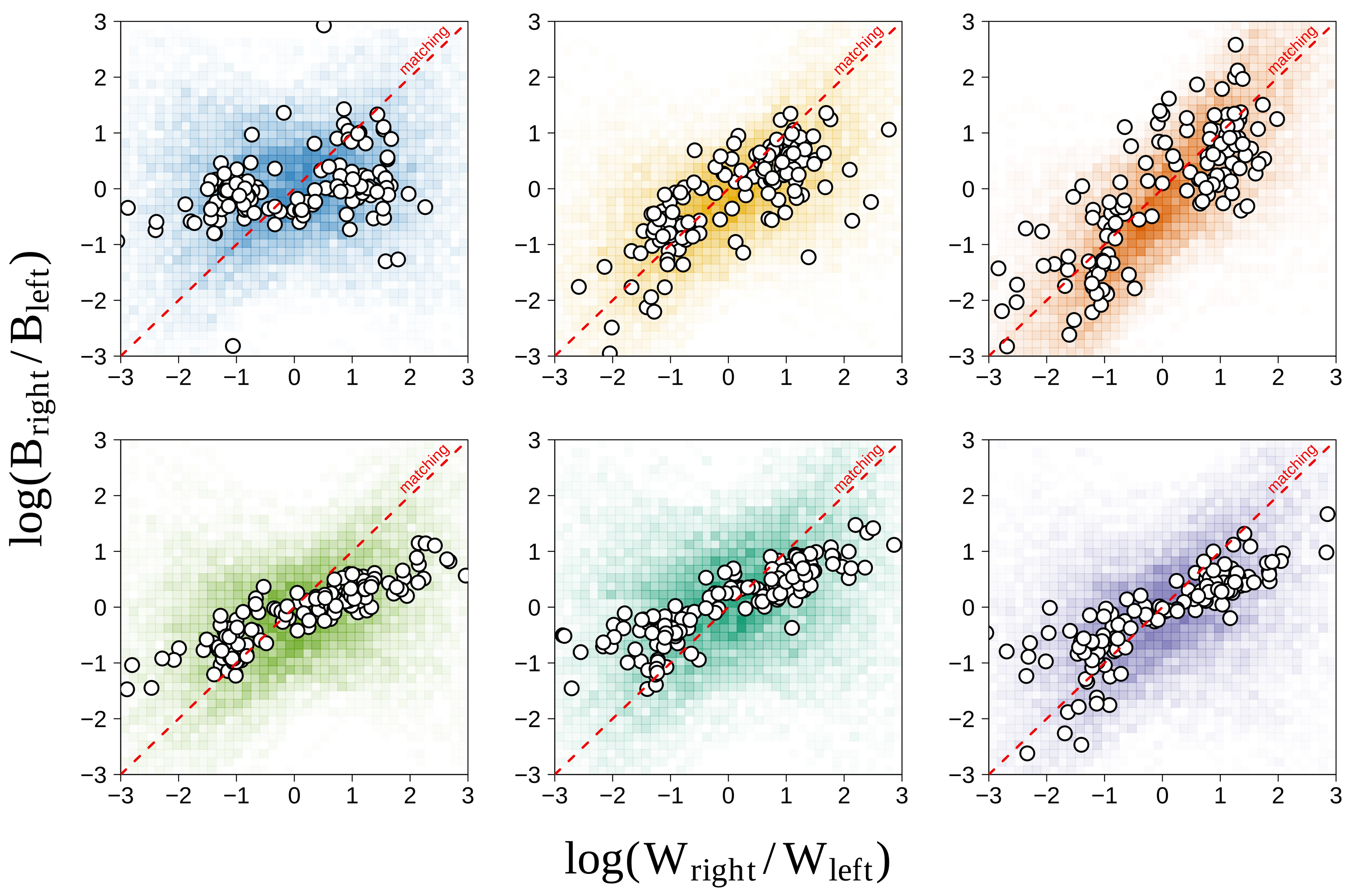
Response−based matching

In response−based alternatives, each response has a fixed probability of being rewarded, independent of time or previous responses and rewards.

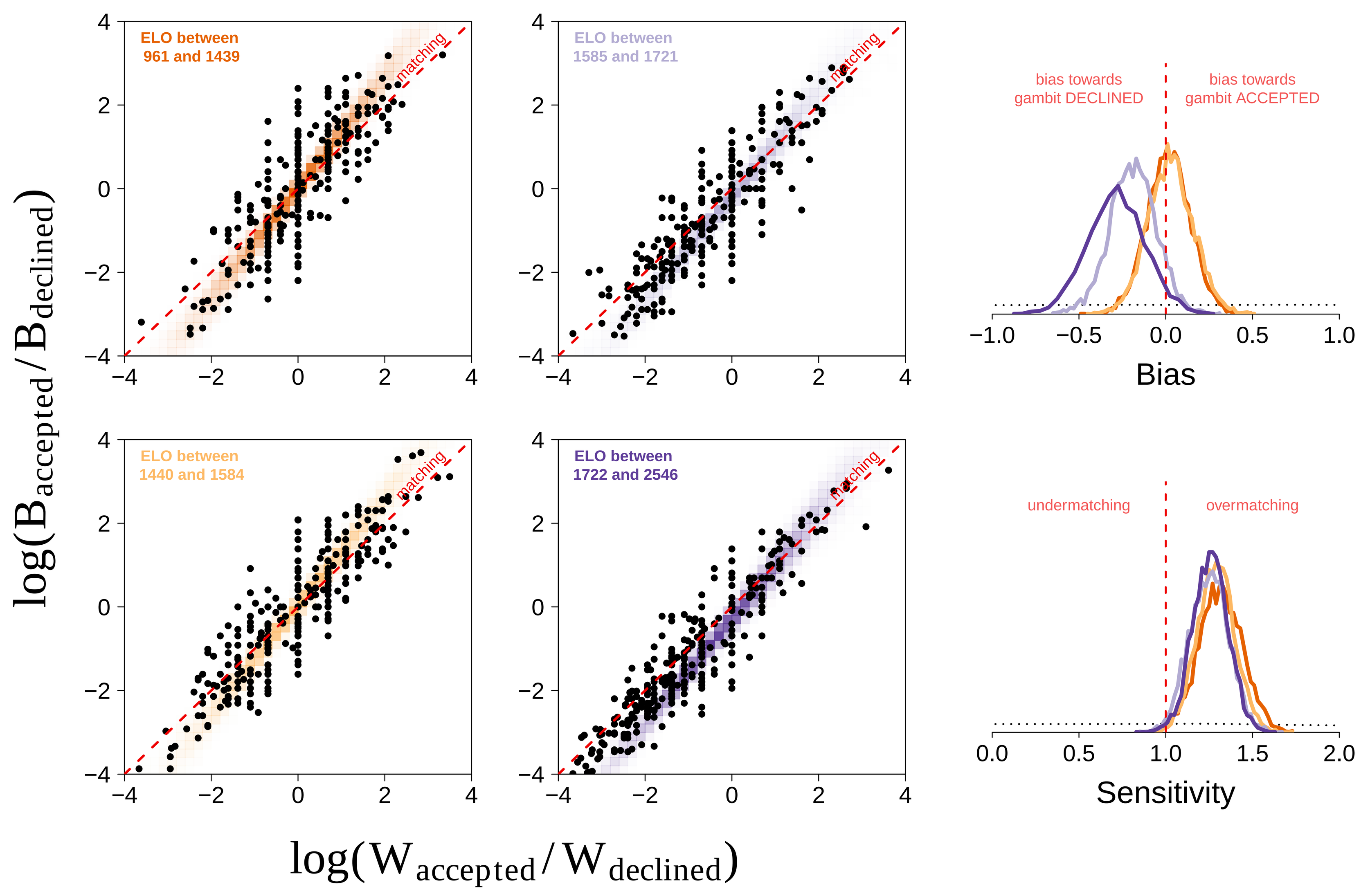
Chess Dataset

We analyzed the decisions made by the player controlling the Black pieces in response to the Queen's Gambit. To conduct this analysis, we downloaded the Lichess dataset, which contains over 4 billion total games, and filtered for games that featured the Queen's Gambit. When facing the Gambit, Black has two options: accepting (QGA) it or declining (QGD) it. The target data per player include the number of each of those decisions and the corresponding won games.

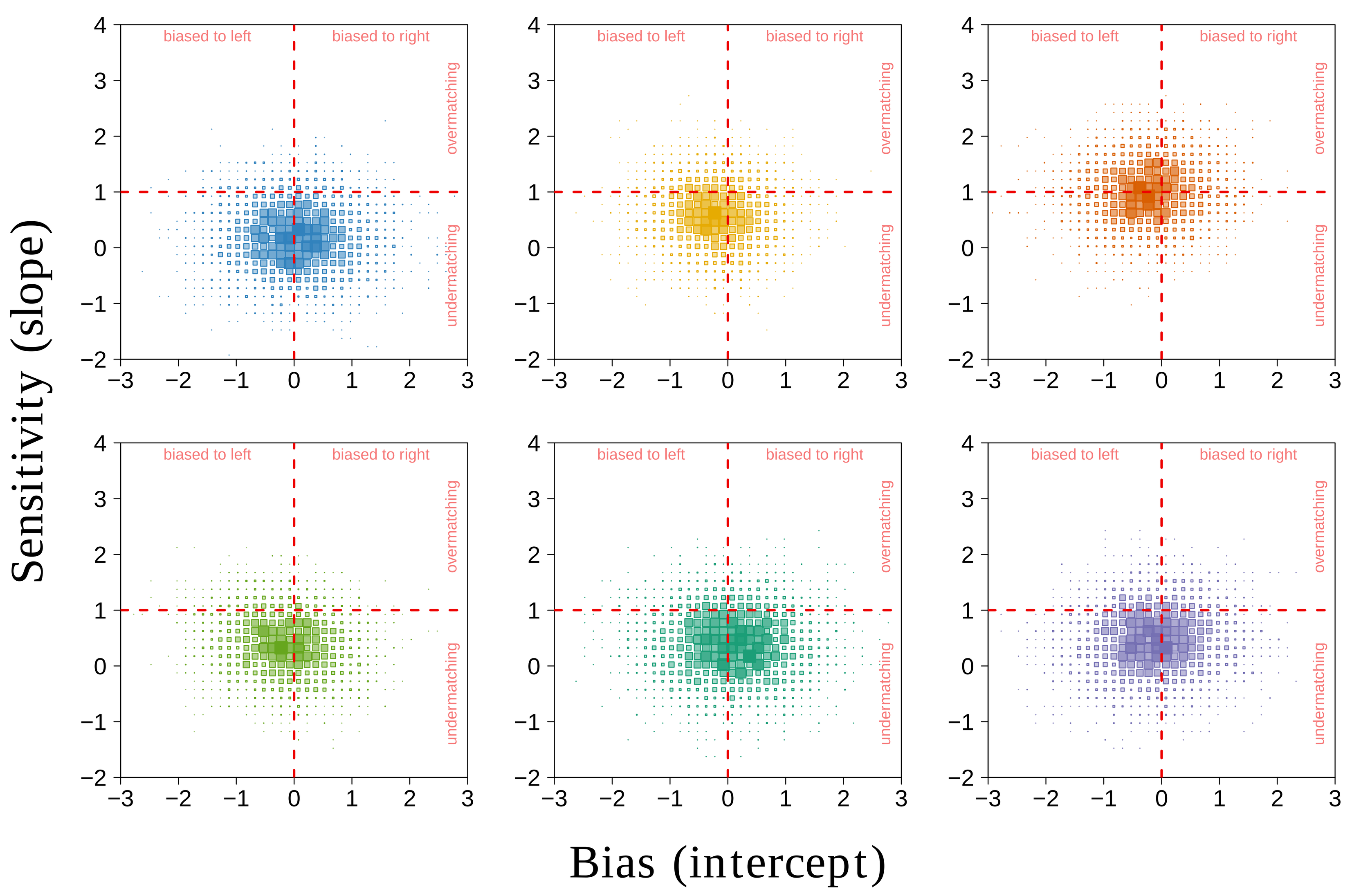
Pigeon data and posterior predictive



Grouping by player level (ELO)



Joint posterior distributions



Grouping by player experience