Problem Set 1

Instructions: When appropriate, answer the question by creating concise and well labeled tables or graphs. Be sure to hand-in your computer code and log files with sufficient annotation/comments.

1) Consider the following potential outcomes model:

$$Y_0 = 2 + U_0$$

$$Y_1 = 3 + U_1$$

$$V = 1 + U_V$$

with $(U_0, U_1, U_V) \sim N(0, \Sigma)$, and Σ corresponding to the following variance-covariance elements: $V(U_0) = 1$, $V(U_1) = 1$, $V(U_V) = 1$, $Cov(U_0, U_1) = 0.4$, $Cov(U_0, U_V) = 0.4$, and $Cov(U_1, U_V) = -0.5$.

a) For this model, compute the ATE.

b) Write computer code to simulate this model for 10,000 draws from the (U_0, U_1, U_V) distribution.

c) Assume $D = 1$ if $V \geq 0$, and $D = 0$ if $V < 0$. Given this model for treated and control groups, using your simulation draws, compute the ATE, ATET, and ATEU, the corresponding OLS estimates for each of these parameters, and the exact bias. (Exact bias is your estimate using your 10,000 draws/observations minus the true parameter value or your computation of the parameter from this simulation.)

d) Continuing c), plot the distribution of treatment effects (Δ), and the distribution of treatment effects for the treated and untreated (3 separate graphs).

e) Now assume D is randomly assigned. For this model, compute the OLS estimates and exact bias for each of the ATE, ATET, and ATEU parameters.

2) Now let's include X and Z variables in our model. Assume both are scalars, with distributions $X \sim Uniform[0, 1]$ and $Z \sim Uniform[-2, 2]$. Assume X and Z are independent of each other and independent of U_0, U_1, U_V.

Assume:

$$Y_0 = 2 + 0.5 * X + U_0$$

$$Y_1 = 3 + 0.25 * X + U_1$$

$$V = 1 + 1 * Z + U_V$$

with $(U_0, U_1, U_V) \sim N(0, \Sigma)$, and Σ corresponding to the following variance-covariance elements: $V(U_0) = 1$, $V(U_1) = 1$, $V(U_V) = 1$, $Cov(U_0, U_1) = 0.25$, $Cov(U_0, U_V) = -0.25$, and $Cov(U_1, U_V) = 0.5$.

a) For this model, compute the ATE.

b) Create a figure for the ATE as a function of X (ATE(X)), with ATE(X) on the vertical axis and X on the horizontal axis.

c) Write computer code to simulate this model for 10,000 draws from the (U_0, U_1, U_V, X, Z) distribution.

d) Assume $D = 1$ if $V \geq 0$, and $D = 0$ if $V < 0$. Given this model for treated and control groups, using your simulation draws, compute the ATE, the corresponding OLS estimates for this parameter, and the exact bias. (Exact bias is your estimate using your 10,000 draws/observations minus the true parameter value)

e) Continuing d), plot the distribution of treatment effects (Δ).

f) Now assume D is randomly assigned. For this model, compute the OLS estimates and exact bias for the ATE.