

Geometric Visualization of Algebraic Operations

A Resource for K-12 Schools

---Client Meeting Notes---

Client: Dr. Bardzell || mjbardzell@salisbury.edu

Professor: Dr. S. Wang || spwang@salisbury.edu

Team Name: Team Fraction Abstraction

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Notes from 9/7/20 meeting with client:

(47 minutes long)

Browser can be done but is harder. Go for Executable.

Look into PC first. Multiplatform (Ipad) [optional]

Pick a language. Python Java recommended.

Different difficulty levels give user more or less info (cuts, all operations, etc)

Denominator limit (2, 3, 4, 5, 6, 7, 8, 9, 10) For now. Hard limit TBD

Game Engines are typically 3D, but we need 2D. Most likely a no-no for Game Engine. If 2D option exists maybe?

Collision: If User cuts square in half when problem is in sixths. Mouse event click and drag shaded smaller square. If Shaded square not big enough, then rectangle snaps it back.

Check for square "Sizes" to verify legal move. When 'close enough', snap in place.

Potential back and forth for operations.

Multiplication Example with only one square: 'area one' ($\frac{1}{3} * \frac{2}{5}$). Slice square in thirds vertically, then horizontally slice square into fifths. User needs to shade appropriately. User clicks one area and a third is shaded vert. User click one area and 2 fifths are shaded. OVERLAPPING AREA = ANSWER.

Addition Example with 2 squares: $\frac{1}{3} + \frac{2}{5}$. Cut and shade third on S1. 2 fifths on S2. Add needs Common denominator so make complimentary cuts on both squares. Click and drag shaded spots from one square to the other, avoiding already shaded spots.

Slicing: outlines vs draw yourself. 1st level outlines for easiness. Harder difficulty can have tic marks to represent which cuts represent what. Hover over tic = display which size. Click draws all cuts of that size for them.

Mouse Events for drag and drop and adjust colors to show what can/can't move. THIS NEEDS PRIORITY. Dr. Wang can help with advice.

No solution sets/walkthrough. Leave explanations/teaching to students. No Tutorials yay!!! User can make their "own" fractions in some mode and use random numbers.

Mixed numbers for maybe next semester. Negatives are ""Red""

Focus on Making cuts and shading and movement.

Notes from 9/14/20 meeting with client:

(40 minutes)

Division Example problem: $\frac{1}{3} / \frac{1}{2}$: Draw a box of 'area one'. Cut into thirds. Shade in one third. The divisor of $\frac{1}{2}$ asks to divide $\frac{1}{3}$ into a half although $\frac{1}{2}$ is bigger. Draw a new box. Cut into pieces of one half. The shaded area can NOT fill a cut half. Thirds and halves do not mix well, so we find a common denominator. Cut Box 1 in half horizontally. Cut Box 2 in thirds horizontally. Click and drag a piece of the shaded area from Box 1 into any of the six open boxes in Box 2. We are now in a highlighted container of box 2 that is size one half. We have filled two thirds of one half, so our answer is $\frac{2}{3}$.

(a milk analogy helps as well)

Division Example problem: $\frac{1}{2} / \frac{1}{3}$: Draw box 1 of 'area one'. Cut Box 1 into halves. Shade in how much you have. Draw Box 2 of 'area one'. Cut Box 2 into thirds. Find common denominator by cutting into thirds horizontally in Box 1. Find common denominator by cutting into halves horizontally in Box 2. Although the rectangles are in different orientations, the area of the shaded partitions are the same. Drag and drop shaded spots from Box 1 to Box 2. After 2 partitions dragged and dropped, we have filled one full container of size one third. At the end, we have a half full container of size one third. So our solution is $1 \frac{1}{2}$ or $\frac{3}{2}$.

Depending on student level, the user will have to make their own cuts. Have them hover over tics to display what fraction. After clicking the tic the cuts would be made for lower level.

(Avoid Negatives for now. Negative area will be dealt with later)

(Subtraction is Bigger minus Smaller. Could potentially be in one square)

Subtraction Example Problem in one square: $\frac{3}{5} - \frac{1}{3}$: Draw Box 1 of 'area one'. Cut Box into fifths. Shade in 3 of those fifths. Cut Box 1 horizontally into thirds. Shade in area of one third. Note: There IS overlap. [Fun idea, have a waste basket to subtract the Green from the Blue.] Remove the overlapped pieces into the "Waste Basket". The pieces not overlapped then snap to become overlapped. Then remove to the Waste Basket. Count the remaining pieces. So our solution is $\frac{4}{15}$.

(Test subtraction with one or two squares with children.)

(Have dialogue for making a mistake)

(Have a hint bubble for dialogue on what to do) ***Client likes this idea

Either a software tutorial or the teacher would show.

Subtraction Example Problem with two squares: $\frac{3}{5} - \frac{1}{3}$: Draw Box 1 of 'area one'. Cut into fifths. Shade what you have. Draw Box 2 of 'area one'. Cut into thirds horizontally. Shade what there is. Cut Box 1 into thirds horizontally. Cut Box 2 into fifths horizontally. Combine two spots of the same size then it will be thrown into the Waste Basket. Kids would like that sort of animation for box cancellation. Either into trash or they FIZZLE OUT That's so cool.

We could make it open source for pip install, but it would be better for it to be independent to make it an exe without python.

Notes from 9/21/20 meeting with client:

(20 Minutes)

Confirmed that python executable can be created and done without python on receiving computer.

Push for having one square for implementation for now.

Multiplication example with two squares: $1\frac{1}{2} * \frac{1}{3}$: Draw Square 1 filled in fully and fill in Square 2 filled in half. Cut Square 1 into thirds horizontally and same with Square 2. Shade in the overlap of $\frac{1}{3}$ then shade $\frac{1}{3}$ of Square 2 and note the overlap. Issue found: $\frac{1}{3} + \frac{1}{3}$. Come back and cut Square one in half vertically to have overlapped squares be the same size. $2/6 + 1/3 = \frac{1}{2}$.

Deal with less than two squares. This would be something harder, intended for upper level.

Division and Subtraction does not need 2 Squares per se. Same with Multiplication.

Higher level is not the size of the number but how much information needs to be moved or modified. Make high level kids make cuts and shade in.

Short Term: Cut up squares and move. When and how it snaps.

Draw squares first. Cut drag and drop. Figure out Cutting. The user can put a point and drop down to hav it be a measure. Drag and drop physics.

Notes from 9/28/20 meeting with client:

(15 Minutes)

Showcased rudimentary rectangle creation, mouse events, and cut creation.

Asked about holding and maintaining cuts, not implemented yet.

How are sub rectangles going to be implemented? We plan on reducing the real rectangle with division.

Defining sub rectangles will be tough.

How will we keep track of rectangles, using an array?

Multiplication can be done in one square with no drag and drop with fractions less than one.

Start on piece pick up and movement. Huge priority wanted by client.

Have a "valid" area. Have rotation for snapping into place as well.

The physics will be hard to deal with and should be addressed.

Notes from 10/5/20 meeting with client:

Showcased rectangle creation and cut creation. Rudimentary click and drag.

Need to isolate for one at a time. Need to have a "legal move" checker.

Dr. Bardzell says the cutting and moving physics is one of the main challenges.

Throw a boolean value for inactive then when click.

Start making a second rectangle to have click and drag after legal move checking.

Try looking at Towers of Hanoi, a simple game. Note how the drag and snap into place is coded.

Main priority click and drag, menu and other UI aspects will be dealt with later.

Second rectangle and check for rotation. Have a program/function do a 90' rotation for putting a rectangle down in a new spot. Only need a snap and drop no animation needed.

Think about implementing sounds instead of moving animations, etc.

Notes from 10/12/20 meeting with client:

Showed Dr. Bardzell a working example of the drag and drop rectangle process.

Need to work on what happens when a student makes wrong cuts. Have the size fit inequality be shown and it teaches them when they are wrong.

Need scale of different fraction cuts for initial cuts. Long term think of being able to show what proportion of fraction cut ($\frac{1}{4}$ $\frac{1}{2}$)

Asked about Rescheduling Next week to Noon Monday (10/19)

Notes from 10/19/20 meeting with client:

Showcased working rectangle snapback with rotation.

Showcased variable cutting. We need to be able to scale in order to deal with larger fraction denominators. We want to be able to show kids the relative size of rectangles in regards to the denominator of a fraction.

Idea: Upper left corner to show if they are close to or next to with the fraction equivalent. How to make them think of relative size, we want to show students how $\frac{1}{4}$ is less than $\frac{1}{2}$? Maybe a collision area for where users hover over. Focus on a low cap on the denominator (Max of 6 or $\frac{1}{6}$) to determine where and what size cut. When the user finds one third, click and then draw all cuts for one third. Constrain all collision checkers to one half of the rectangle.

Reschedule our meeting for 12:15, if early send

Notes from 10/26/20 meeting with client:

Showed our new fraction cutting process (with all cuts displayed) and Dr. Bardzell approves.

Look into fraction operations, this is where we begin to implement the different problem types. The multiplication steps will be as follows:

- 1) Make vertical cuts
- 2) Shade vertical rectangles
- 3) Make horizontal cuts
- 4) Shade remaining rectangles

For example, if we color vertical rectangles blue, and horizontal rectangles red, the overlap (in purple) would be the answer to the multiplication problem.

We will alter the state manager and maybe create different state managers for each type of fraction arithmetic problem.

We will add a shading state in order to keep things clean.

In the future we may have a way to reduce fractions and deal with mixed numbers.

Notes from 11/2/20 meeting with client:

Showcased the multiplication process and examples, as well as physics with product determination and reduction.

Dr. Bardzell noted that the coloring/shading could be more user friendly, especially for use by younger students.

One suggestion is to have the user fully color vertically and then fully color horizontal, then the user can select where the overlap is. We need to make it clear to the user, which rectangles are part of vertical shading, horizontal shading, or the overlap. This will give the user the best possible learning experience.

Dr. Bardzell recommends division for the next operation. This will likely be the hardest but it will incorporate all features: 2 rectangles, cutting, shading, and moving

Low priority on problem handlers, focus on colors and shading to be better. Most of the UI work will come later once we have a structure for the basic processes.

Notes from 11/9/20 meeting with client:

Showcased a color picker that allows the user to select their rectangle from a palette of colors before applying.

Showcased a version of vertical hatching (shading with multiple lines). Once one color is picked, the second color used is limited based on the first color selected. It can be a little confusing if the lines for shading look similar to the lines used to cut the rectangle.

Client suggests that we use solid for the initial shading, then use the cross-hatching to represent the overlapping answer. Could make colors lighter/fainter to better distinguish between cutting lines. Maybe more lines with less pixel width and spacing? We plan to try both and then compare.

Showcased division. [one third divided by one half] Shading with solid colors then? Make sure different colors for dividend and divisor.

Bugs with coloring and shading that need addressing (limit user color changing?)

[one half divided by one third] Division will need to work with mixed fractions and with answers that are greater than 1.

Work on: Shading, Division with answers of mixed fractions, and color picking, as well as various bug fixes

Notes from 11/16/20 meeting with client:

Showcasing working division with new shading, color hatching answers, and answers that are improper.

We want to display a darkish grey color once a rectangle is moved from its position. Needs to change color to show "lack" of rectangle for the user to better visualize

Showcased multiplication and cross hatch shading. Client likes multiplication.

Our client still thinking about how to implement addition, it will be last.

Subtraction we might want to play and try options. "Wastebasket" idea. Once cuts and shades are done horizontally. The non cross hatched H shade rectangle must be dragged onto a non cross hatched V Shade rectangle to cross hatch. Might want to think of timing of wastebasket (afterwards of that, replace palate with waste basket.)

Remaining non trashed rectangles are numerator for answer. Might need an 'undo' throwaway. Do we let users throw away non shaded rectangles?

We will program a sample subtraction problem this week for the client to see and possibly make changes accordingly.

Settings where answer can be given or typed? ----LATER ON

Iron out pointer collision errors in division drag and drop

Notes from 11/24/20 meeting with client:

Showed various bug fixes including drag and drop disappear

Showed basic subtraction layout

Client suggested that there may be too many colors on the screen for at least subtraction problem, this makes it difficult for the user (especially younger students) to focus on the math

Client suggests making some other color improvements (background, space behind rectangles)

Notes from 12/3/20 meeting with client:

Showcased various bug fixes with drag and drop

Showcased shading where original colors are kept in crosshatch

Showcased various bug fixes in shading, as well as crosshatched lines showing when drag and drop.

Drag and drop shading fixed

Have user type the answer? Add additional answer state. User could answer either answer or reduced answer. Have 2 boxes with a fraction bar in between for user answer.

Over winter break, have a rudimentary exe for students to use. Caveat: only works on windows right now.

Make a survey for testing and a template bug reporting. Make a readme that explains what the program does, and what operations we have.

Ask teachers about subtraction in one or two boxes for testing? Survey for Teachers only, no kid survey.

No meeting next week, just send EXE and survey and readme for operations.

In summation: By next week we will have cleaned up the program and have a working exe file and README to go along with it.

Notes from 2/1/21 meeting with client:

(30 Minutes)

First client meeting of the semester!

Discussed User-Generated Problems vs Random

User generated will be better for presenting our software to teachers.

Random is good, but for presenting to new audiences could lead to big and complicated problems. We want teachers to be able to start with simple examples such as $\frac{1}{2} * \frac{1}{3}$.

We don't want to remove random generation, but list it as another option in the startup menu.

Division: $5/6 / 4/5 = 25/24$. For the one extra square here, it will be easier to visualize and understand if it is placed in another rectangle that has been divided into 24ths.

That example would be bad to present to teachers, this is one reason we want user generated problems.

Constraints for user input: User denominator cannot be more than 6, display a box that shows the limits. Hold off on improper fractions for now.

For fractions answers resulting in greater than 1, add in a second square for drag and drop to make it more clear how it fits into the final answer.

Mixed number needs to show up as well. Along with the additional square, we want the answer to be more clear as $1 \frac{1}{24}$ rather than $25/24$.

1 piece before full shaded, now all of $4/5$ is filled. Animation highlighted in gold to show 1. Now we can open up a new square cut and shaded. Display issues: Larger window or smaller squares.

Idea, move the $4/5$ s piece off, and they continue working in newly refreshed square

Error, placing a rectangle onto blank space creates cross shaded piece

User Input, Larger Display, New rectangle to be worked on, Bug fixes

After user fixes, we can start to implement better design looks for children.

Current group member responsibilities:

David: User generated input

Logan: New display window size and scaling

Jansen: Additional rectangle for fraction answers greater than 1

Notes from 2/8/21 meeting with client:

(40 minutes)

Showcased our new display with a wider screen, as well as a new background image.

Showed the toggle buttons for a user created problem.

Showed the "Create Your Problem" Screen, with working user input and validation as well as toggleable arithmetic sign (for changing operation type).

Showcased working 3rd rectangle appearing with division problems. with answer greater than 1

On the third rectangle, we need to create additional shading (the same shading as the second rectangle) as a boundary so the user can more clearly see the additional squares.

3rd rectangle should be shaded as soon as it appears on screen. Can shade a $4/5$ spot on new rectangle, but a boundary will show a "hole" for users to fill.

Collision where user tries to plant leftovers from square 1 to a white space outside of shaded boundary areas. It should snap back to origin in that case.

We do not need a third rectangle for all answers greater than 1, we only need it when there is not enough space for another shaded area in the second rectangle. For example, $\frac{1}{2} / \frac{1}{3} = \frac{3}{2}$ would only need two total rectangles.

Whereas $\frac{5}{6} / \frac{1}{6}$ would need three total rectangles for visualization.

Client showed a whiteboard example of $\frac{5}{6}$ divided by $\frac{1}{5}$. Shaded square1 $\frac{5}{6}$. Shaded square2 $\frac{1}{5}$. Cuts both accordingly. 25 shaded small rectangles of square1. In square2 after $\frac{1}{5}$ is filled, more $\frac{1}{5}$ boundaries are created and filled accordingly. 4 containers of size $\frac{1}{5}$ here filled, with only one box of size $\frac{1}{6}$ filled. Gives answer $4\frac{1}{6}$. This uses only 2 rectangles as opposed to current implementation.

The larger the numerator of the second fraction, the more likely another rectangle is needed. Have software shade in remaining sliver in square2 then shade rest in square3.

Add in mixed number representation as opposed to improper fraction for displaying. So $1\frac{2}{3}$ instead of $\frac{5}{3}$.

New exe doesn't need new collision and mixed number rep. Does need appropriate shading and boundary for answers bigger than one.

We will send the new exe to client for testing once we add the shading (boundary) for the answers > 1 .

Tasks:

Jansen: New boundary/shading for division

Logan: Replace improper fractions with mixed number representation for fractions

David: Bug fixing (for problem generator?) and other general bug fixes