

More Thorough Error vs Residual Analysis

Clamped upper results to help visualise areas of highest accuracy.

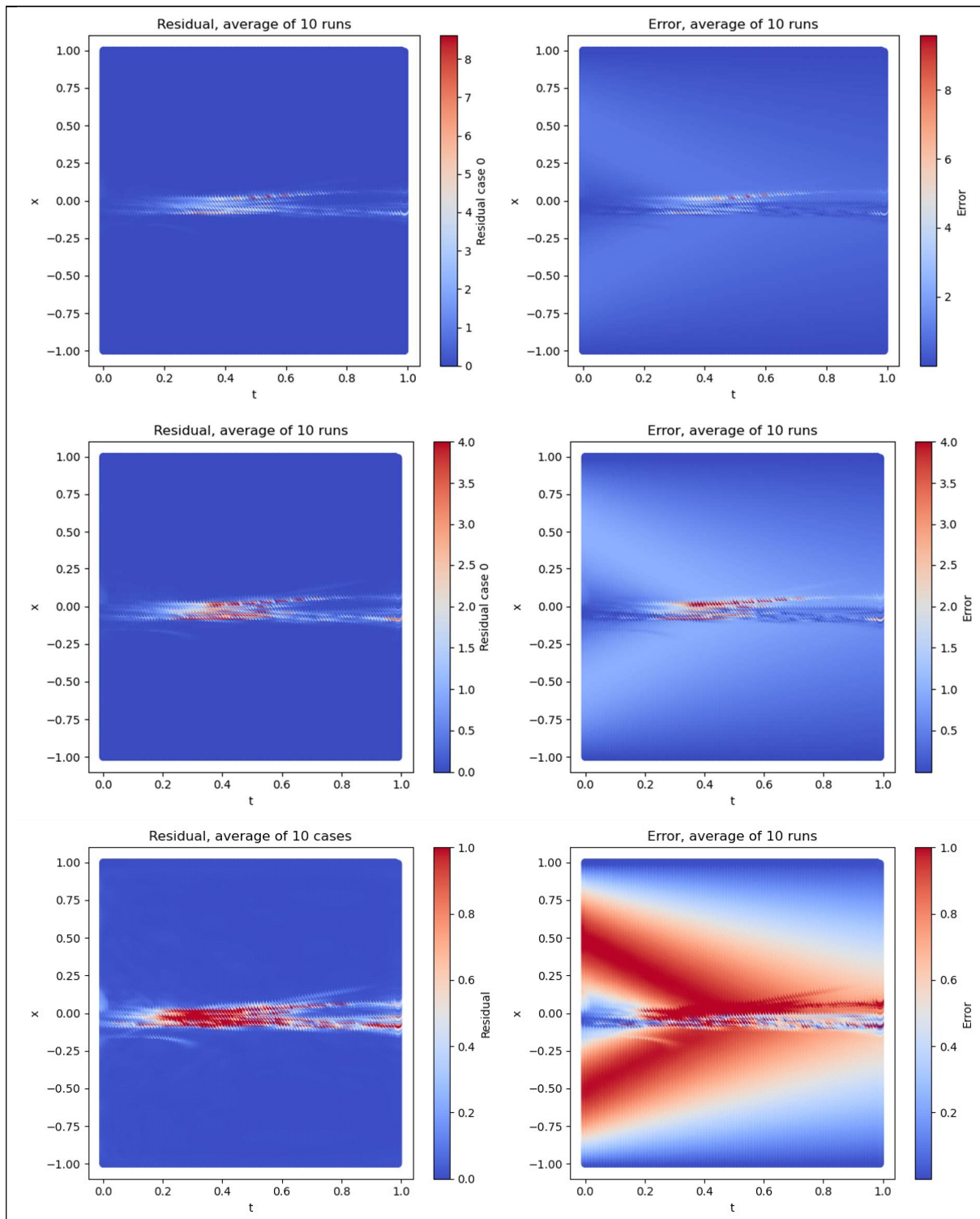


Figure 1 – Comparison of PDE residual (the cost function) vs actual error in U , averaged over 10 runs.

Gradient Estimates using ML vs Ground Truth

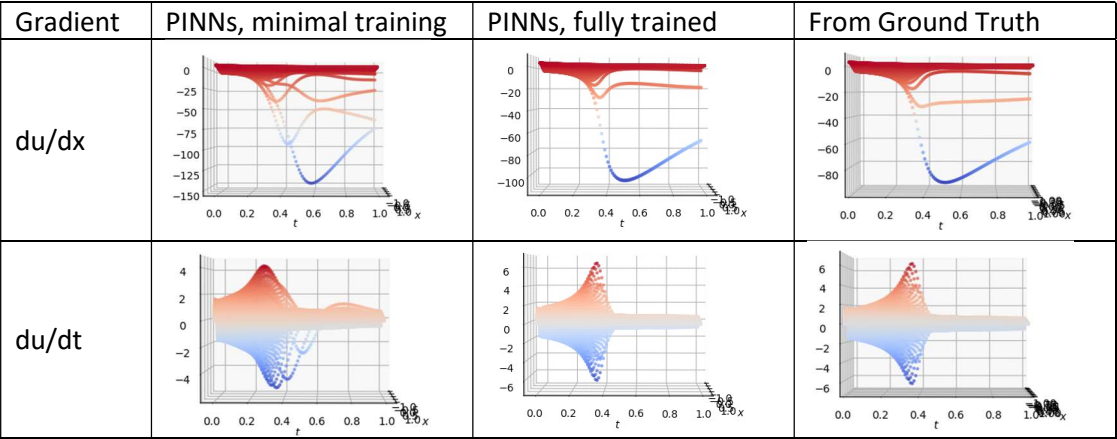


Figure 2 – Gradients obtained from PINNs vs Ground Truth

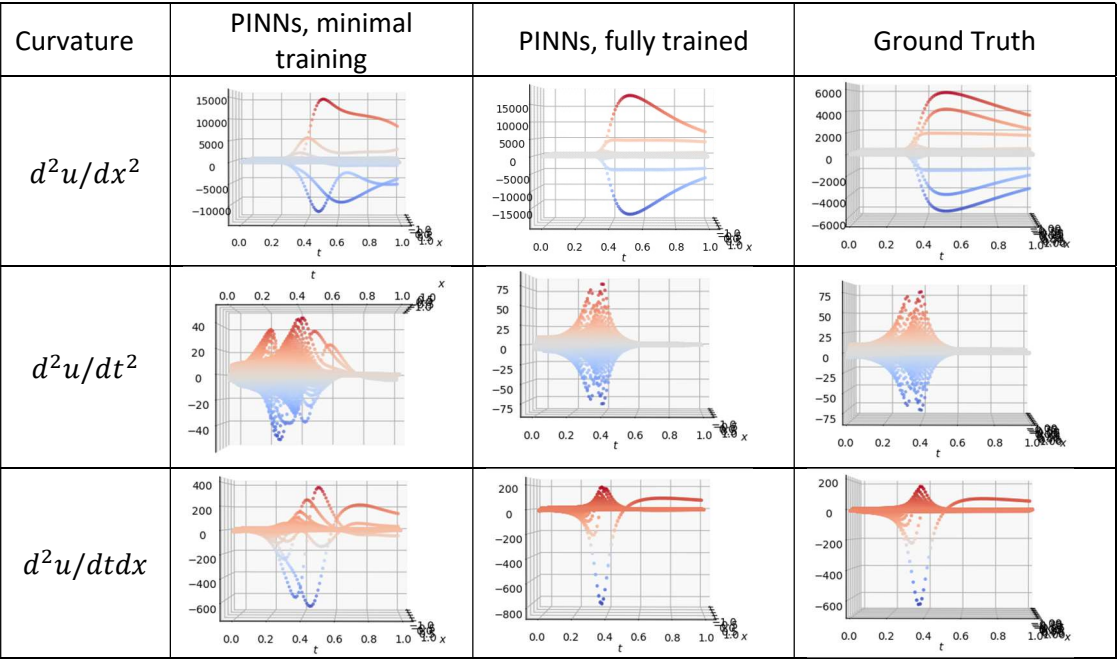


Figure 3 - Curvature obtained from PINNs vs Ground Truth

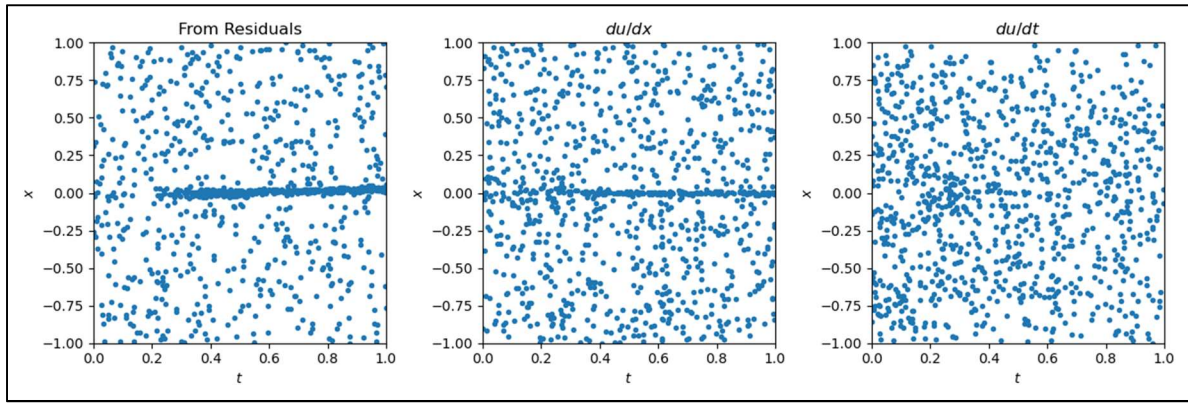


Figure 4 – Point Resampling using different information without further tuning

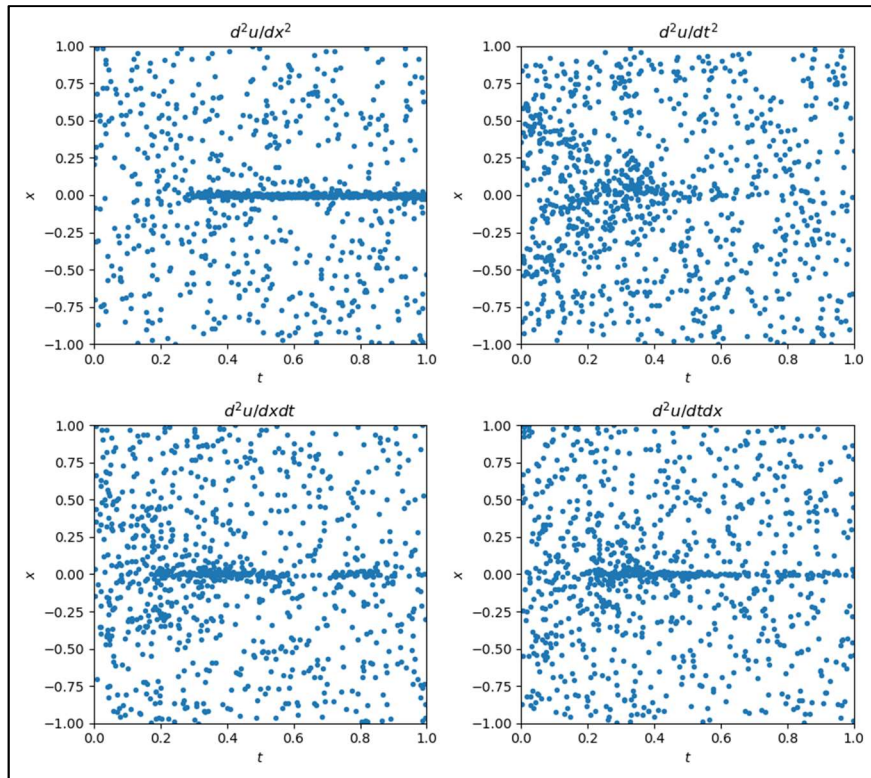


Figure 5 – Whilst $u_{xt} = u_{tx}$, the points are differently placed due to inherent randomness

Initial Loss before resampling/refining.

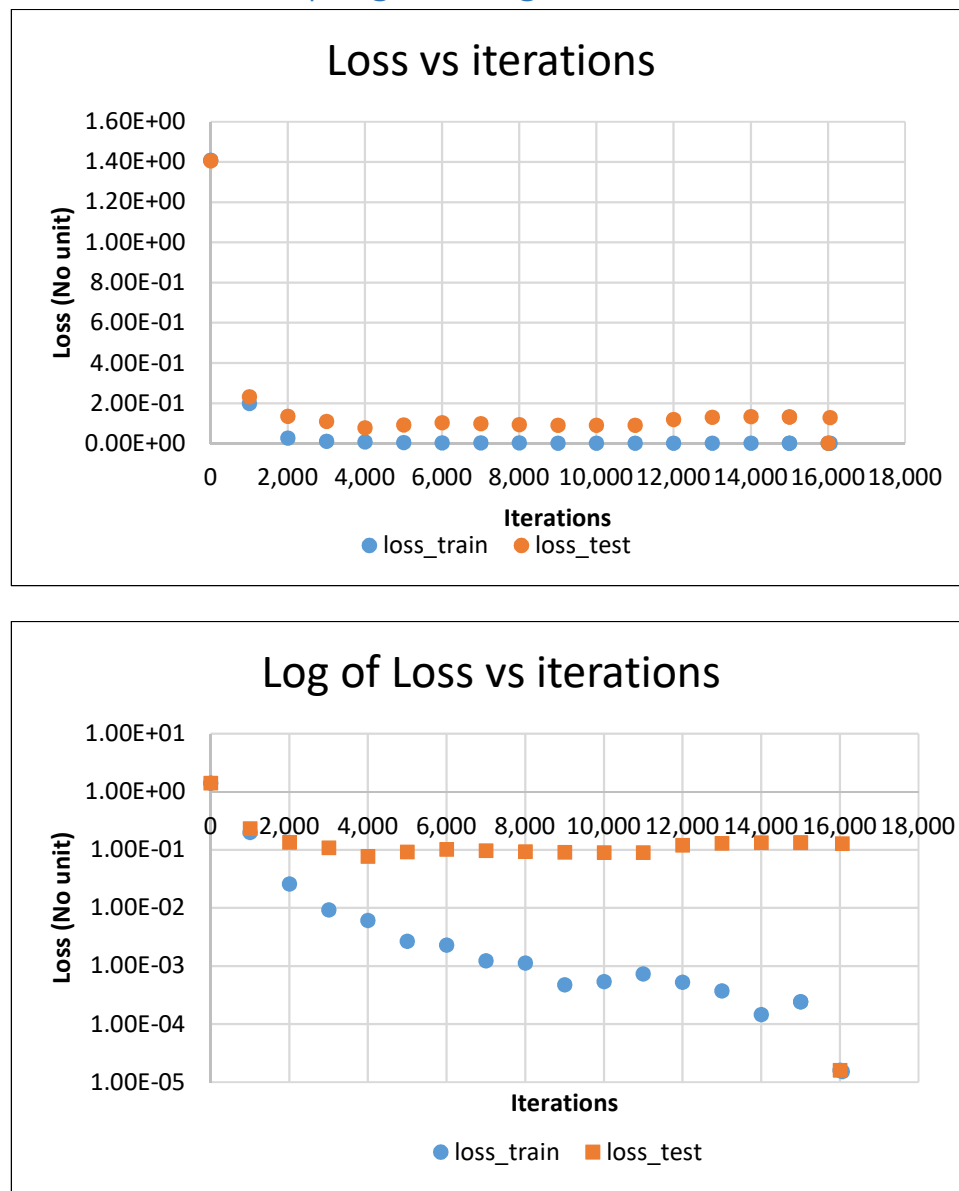


Figure 6

Resampling from gradient/curvature information (Replacing points)

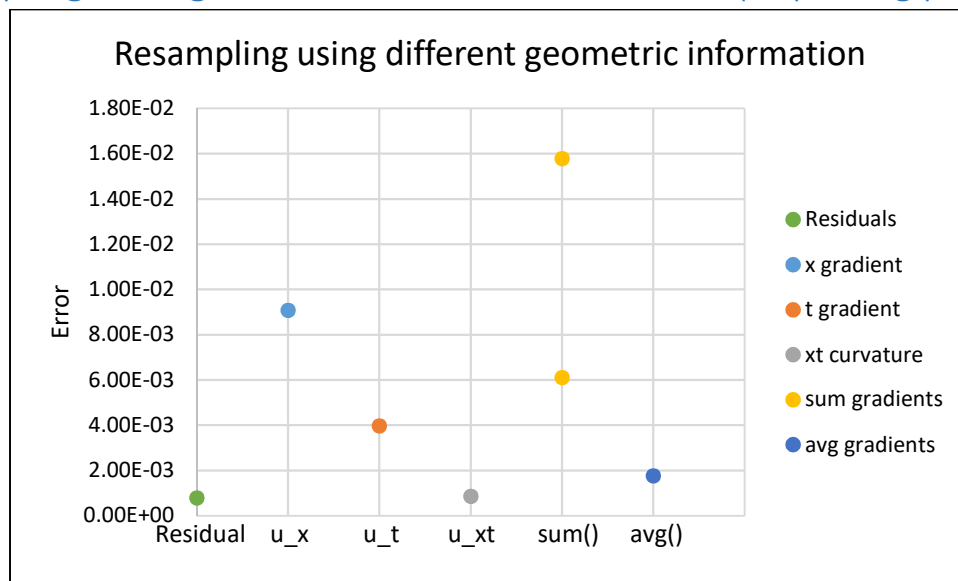


Figure 7

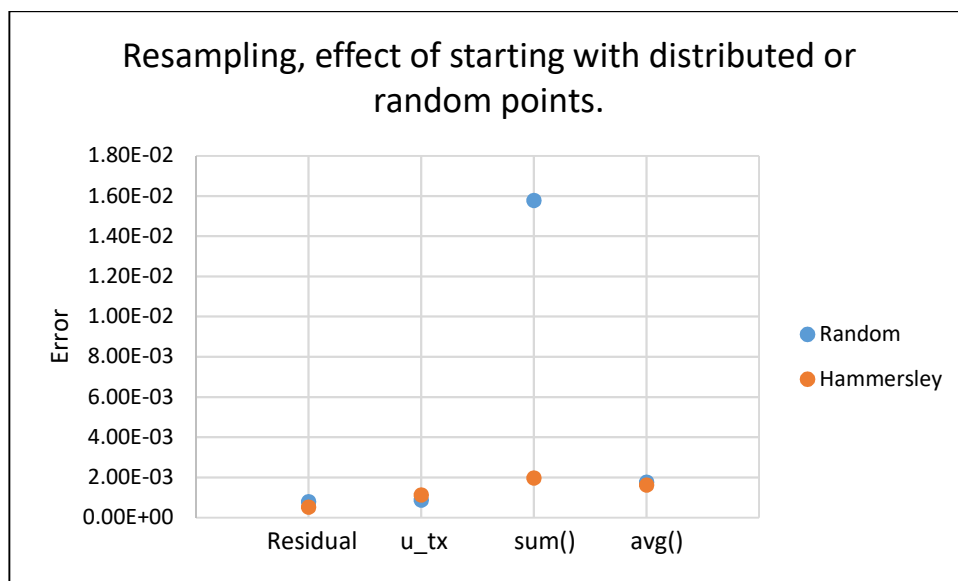


Figure 8

Refining from gradient/curvature information (Adding points)

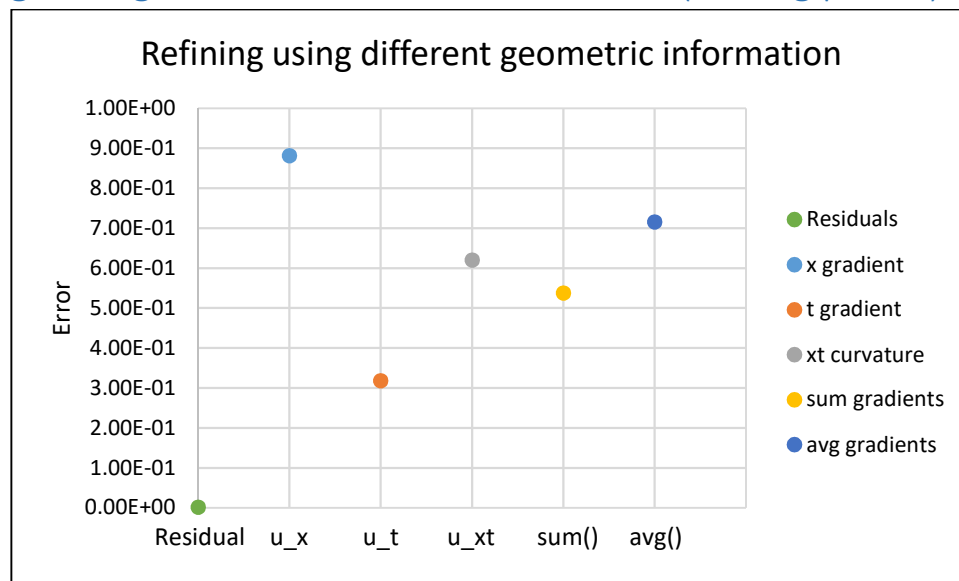


Figure 9

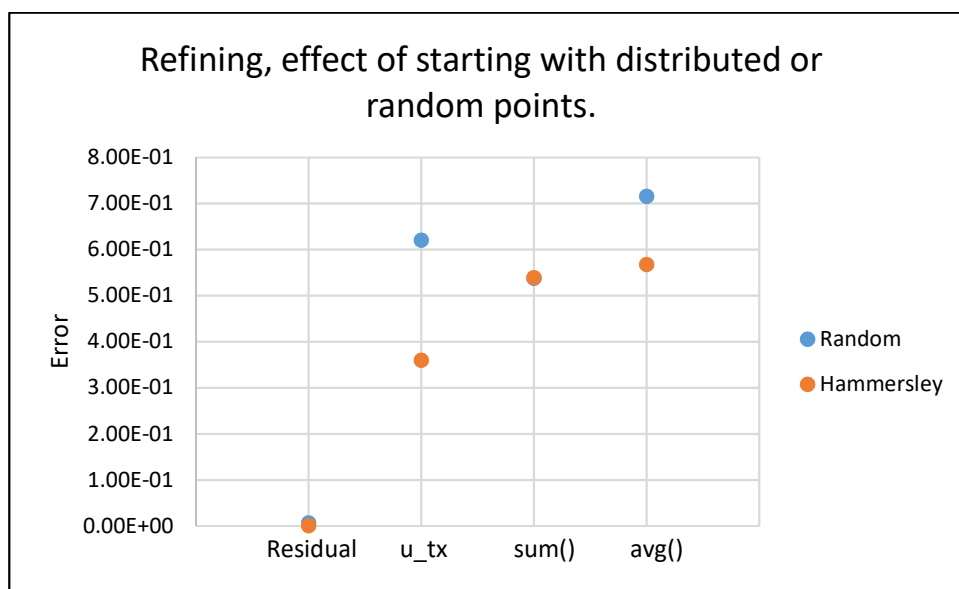
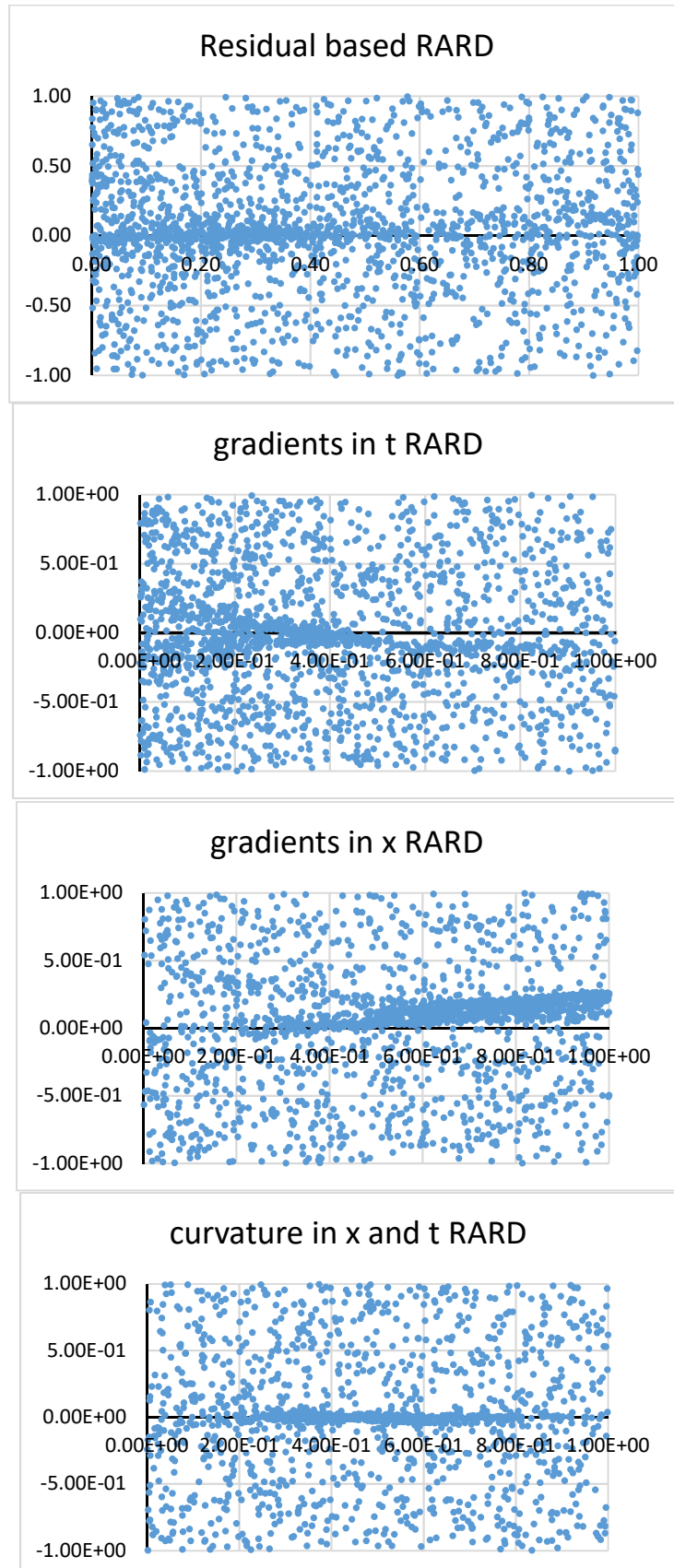
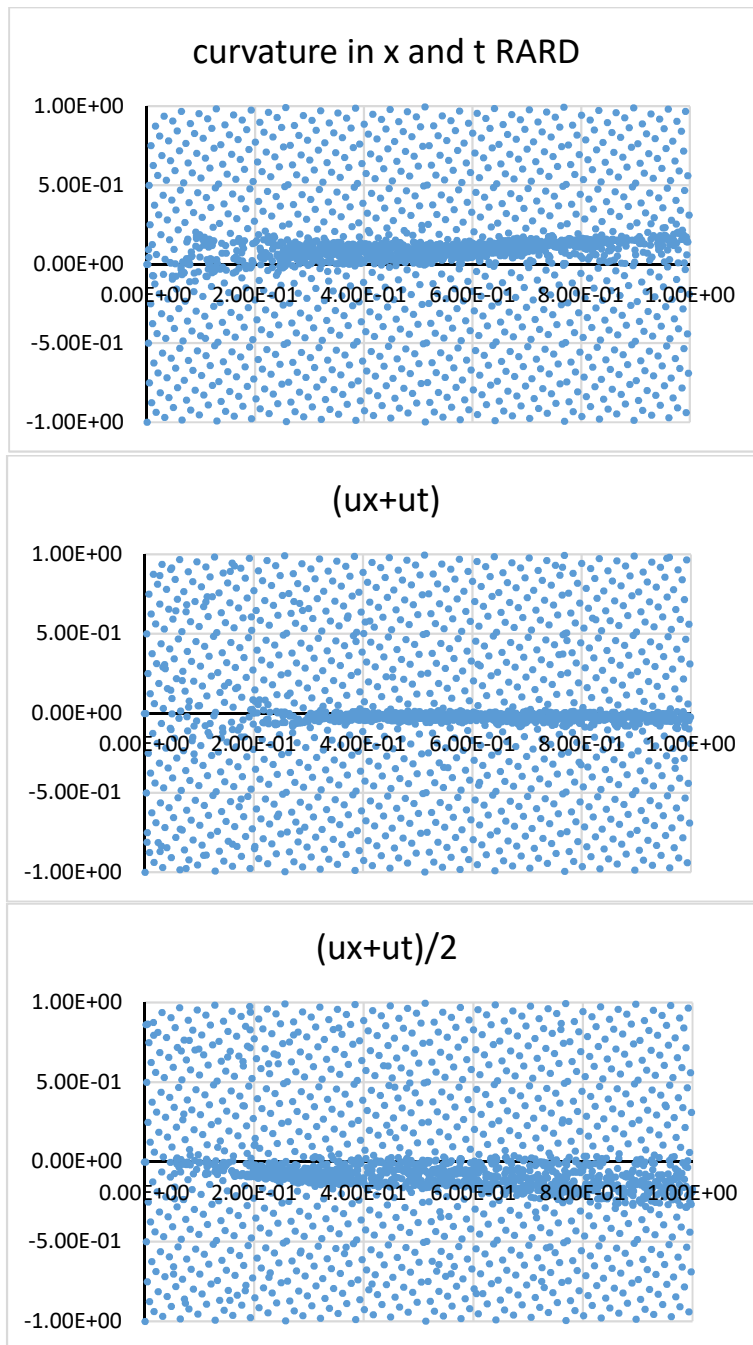


Figure 10

Point distribution resulting from refinement with different input information:





Possible Issues – Refinement with $c=0$ may not be suitable as loss changes location, gradients don't. Recover loss to look at. Tuning to have $c > 0$ could help, as well as hammersley initialisation. Perhaps increasing prop of initial points, using hammer, could help rard not make initial mistakes... which goes against its strength. Perhaps a combination of refinement and resample

Figure 11 – Sample point distributions. Note this is not average, so results not necessarily representative. See Error table overleaf

Table 1 – Companion table to Figure 11

Initial Distribution	Input information	Error
Random	Residual based error	6.40E-03
	gradients in t	3.18E-01
	gradients in x	8.81E-01
	curvature in x and t	6.20E-01
Hammer	curvature in x and t	3.60E-01
	sum x and t gradients	5.39E-01
	avg x and t gradients	5.67E-01

While sum(x and t) looks best in figure 11, can see from error that it's an exceptionally good example where the first resample presumably guessed correctly the area of the shockwave, leading to ideal positioning of refinement points. This, on average, may happen less, therefore leading to the higher error.

Main Results, 2000 training points

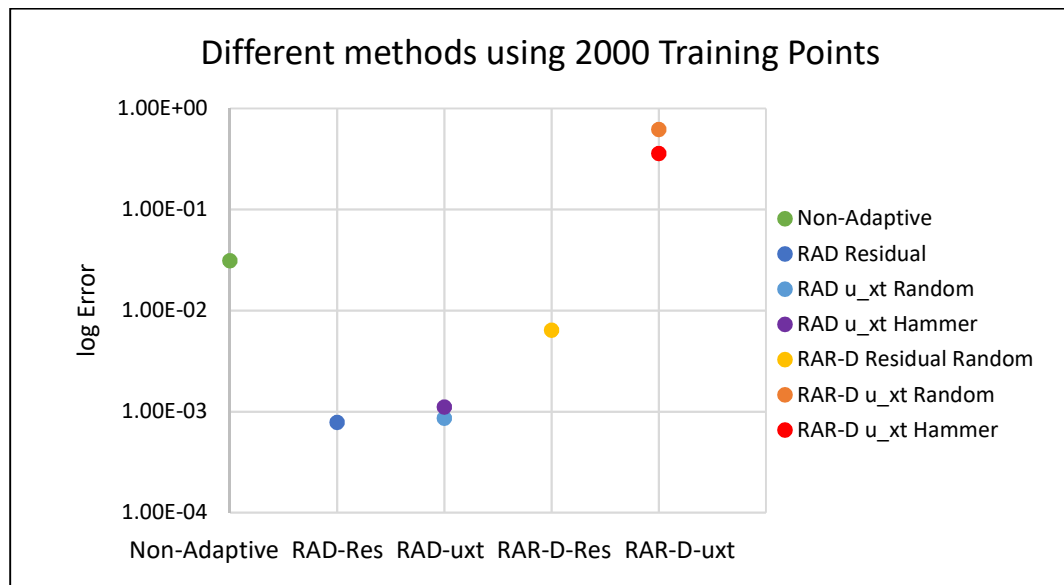


Figure 12