Physics 711 (Electromagnetism)
Assignment 2: Total 30
Handed Out: 2 March 2022
Return Date: 9 March 2022

This assignment will count towards your continuous assessment mark. After return of the assignments on 9 March, it will be graded and returned to you as soon as possible. The assignment can be returned in any electronic way convenient to you. A memo will be made available as soon as all assignments have been received. It covers section 2.1 of the class notes. All equation numbers refer to those of the notes.

Exercise 1 [5]

Give a physical explanation of why the retarded potentials of eq. (2.1.4) contain the electric potential and current densities at the earlier (retarded) time

$$t_r = t - \frac{|\vec{r}' - \vec{r}|}{c}.$$

Hint: Think in terms of the time it takes an electromagnetic signal to travel between two points.

Exercise 2 [5]+[10]

a) Show that for any function $f(x)$

$$\frac{\partial}{\partial x_i} f\left(|\vec{r} - \vec{r}'|\right) = -\frac{\partial}{\partial x_i'} f\left(|\vec{r} - \vec{r}'|\right)$$

[5]

b) This problem is technically more difficult and therefore counts as bonus points: Use (a) to show that the retarded potentials of eq. (2.1.4) satisfy the Lorentz gauge fixing condition (1.6.5):

$$\partial_\mu A^\mu \equiv \frac{\partial A^\mu}{\partial x^\mu} = \vec{\nabla} \cdot \vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0$$

Hints: You may assume that the charge and current densities vanish at infinity. Use Gauss' theorem and the continuity equation in the form

$$\frac{\partial \rho\left(\vec{r}', t_r\right)}{\partial t_r} + \vec{\nabla}' \cdot \vec{J}\left(\vec{r}', t_r\right) = 0$$

[10]

Exercise 3 [20]

a) Show that the current density (total charge that crosses a surface/unit time/unit area) associated with charges moving at velocity $\vec{v}$ and density $\rho(\vec{r},t)$ is given by $\vec{J}(\vec{r},t) = \rho(\vec{r},t)\vec{v}$.

Hint: Consider the total charge that flows through an area ΔA in a time interval Δt.

[5]

b) An infinitesimal thin ring of radius b is centred at the origin in the x-y plane, has charge density/unit length λ and rotates around the z-axis with angular velocity ω. The charge density in cylindrical coordinates is therefore given by $\rho(\vec{r},t) = \lambda\delta(r-b)\delta(z)$, where $\delta(x)$ is the Dirac-delta function. Compute the exact retarded potentials at any point on the z-axis.

[15]

Hint:. Use cylindrical coordinates.