Read the instructions in the provided *Mathematica* notebook very carefully and then answer the following questions.

Simple Calculations:

Calculate the following:

A1. $\sqrt{4084441}$

A2. $(1481544)^{1/3}$
Note: *If your answer is* 493848 *then you also have to agree that* $1 + 4 \times 5 = 25$.

A3. $\frac{13\times2023}{29+31}$

A4. $\sin(\pi/3)$

A5. $\sin(60°)$
Note: *Mathematica uses radians by default. You can switch from radians to degrees by multiplying with the built-in constant* ***Degree****.*

A6. $\ln(e^7)$
Note: *In Mathematica the command* ***Log[x]*** *gives the natural logarithm of* x*, sometimes also called ln(x). The base of this logarithm is Euler's number* $e = 2.718\ldots$ *which is denoted by* ***E*** *in Mathematica. The exponential function* e^x *is implemented as* ***Exp[x]****.*

A7. $\log_{10}(10^4)$

A8. The **numerical** value of $e^{-\sin(\sqrt{7})}$.

A9. The **numerical** value of π^8 to 5 decimal places.
Hint: *Just play around with the arguments of the* ***N*** *function until you get the desired number of digits.*

A10. $|-15|$

A11. The n'th prime number, where n is your student number.

Assignments and Variables:

B1. Make the assignments $mass = 173$ and $velocity = 24$ and then calculate the value of $\frac{1}{2} \times mass \times velocity^2$. Remember to assign your final answer to B1.

Symbolic Expressions:

C1. Expand ("multiply out") $(x + y + z)^6$.

C2. Simplify the polynomial $(1 + z)^2 + (1 + z)(2 + z)$.

C3. Factorise the polynomial $(1 + z)^2 + (1 + z)(2 + z)$.

C4. Decompose the expression $x^{14}/(x^8 - 1)$ into partial fractions using **Apart**.

C5. Express $(x + 1)^2/15 + z/x^2$ as a single ratio, i.e. with a common denominator.

Lists:

D1. Construct the list $\{19, \cos(3), -3, 11\}$.

D2. Use **Table** to construct the list $\{3, 6, 9, 12, 15, 18, \ldots, 81\}$.

D3. Use the [[...]] notation to find the 17'th element of the list you constructed above.

D4. Use **Table** to construct a list of which the entries are themselves lists of the form $\{n, \cos(n/3)\}$ for $n = 0, 1, \ldots, 31$.
Note: *The first argument of **Table** determines what each element of the resulting list will look like. There is no reason why this cannot be a list itself.*

D5. Suppose the entries of the list you created above are the coordinates of points in the x-y plane. Plot these points using **ListPlot**. You can just use **D4** as the argument for **ListPlot**.

D6. Repeat the question above using **ListLinePlot**. This joins up the points.

Replacement Rules and Solving Algebraic Equations:

E1. Use replacement rules to find the value of the polynomial $x^4 + xy^2 + (y-2)^2$ at $x = 5$ and $y = -8$.
Note: *Do not assign values to x and y!*

E2. Apply the **Solve** command to the equation $x^3 - 2x^2 - 119x = 312$. The result should be a list of replacement rules for the solutions.
Note: *Remember to use ==, and not just =, when you type the equation.*

E3. Use the [[...]] notation and replacement rules to find the *value* of the third solution of the equation above. Your final answer should be a number.

E4. What is the distance of the point where the two lines $2x - 4y = 1$ and $4x + 2y = -28$ cross from the origin? *Your final answer should be a number.*
Hint: *First use **Solve[List of Equations,List of Unknowns]** to solve the two equations simultaneously. Use [[1]] to select the first (and only) solution replacement rule, and then apply the rule to an appropriate expression.*

Plotting:

F1. Generate a plot of $\sin(5\,x)/(1+x^2)$ for x ranging from $x = -10$ to $x = 10$. Set the **PlotRange** to ensure that all the features of the graph are clearly visible.

F2. Consider two particles moving in one dimension. The position of particle A is given by $x(t) = e^{-t/5}\cos(2t)$, while the position of particle B is $x(t) = e^{-t/8}$. These are typical solutions for the motion of a damped simple harmonic oscillator. Produce a plot showing the two particles' positions on the same axis system from $t = 0$ up to $t = 25$. Add sensible labels for the two axis, a plot label, and plot legends.

F3. With **RegionPlot** you can plot the set of points in the x-y plane that satisfy a given inequality. For example, the inequality $x^2 + y^2 \leq 1$ would obviously produce a disk with radius 1. Produce a plot showing the region in which $(x^2 + y^2 - 1)^3 - x^2y^3$ is negative.

F4. Produce a plot showing the set of points in the x-y plane which lie within a distance of 2 from the point $(x, y) = (1, 0)$ and at a distance greater than 2 from the point $(x, y) = (-1, 0)$.
Hint: *You can specify two inequalities using* &&.

F5. Generate a 3D plot of the function

$$\exp\left[-\sqrt{x^2+y^2}/8\right]\cos(\sqrt{x^2+y^2})$$

for x and y between -20 and 20. Set the **PlotRange** so that all the features of the graph are visible. You can make the surface smoother by increasing **PlotPoints** to 60.

F6. *Estimate*, using a plot, the largest solution to the equation $2x + 3 = 4^x$.
Hint: *On the plot, try Right-Click → Get Coordinates. Remember to assign your estimate for the approximate solution to **F6**.*

Calculus with Symbolic Expressions:

Calculate the following:

G1. $\frac{d}{dy}y^2\cos(y^8)$

G2. $\frac{d}{d\phi}\cos(1/\phi)$
Note: *You can produce ϕ with* [Esc]f[Esc]. *Look up **Greek Letters** in the help.*

G3. The derivative of $\alpha^3/(1+\alpha^4)$ to α at $\alpha = 2$.
Note: *It would obviously not work to set $\alpha = 2$ before you take the derivative. Can you replace α by 2 after taking the derivative?*

G4. A function of which the derivative to β is β^n.

G5. A function of which the derivative to x is $1/(x^2-1)^2$.

Manipulate:

H1. Suppose you invested an amount x which earns compound interest at $g\%$ per year for y years. Use **Manipulate** to produce a quick way of calculating how much your investment will be worth. Add sliders for $x \in [0, 1000]$, $g \in [0, 12]$ and $y \in [0, 25]$. Set the initial values for the sliders to $x = 100$, $g = 9.2\%$ and $y = 10$.

H2. Define the following function

$$wave(t, A, f, p) = A\sin(2\pi ft + p)$$

and generate a plot of it as a function of t for $t \in [0, 10]$ with $A = 1$, $f = 0.4$ and $p = \pi$.
Note: *Do not use **Manipulate** yet. Do not assign values to A, f, or p. Just pass these as constant arguments to your function inside **Plot**.*

H3. Generate a plot of $wave(t, A, f, p)$, again as a function of t for $t \in [0, 10]$, but where the values of $A \in [0.5, 2]$ and $f \in [0.1, 1]$ and $p \in [0, 2\pi]$ are now controlled by **Manipulate** sliders. Set **PlotRange** to a sensible **fixed** value. Its should be clear from your plot that A controls the amplitude of the wave, f the frequency, and p the phase.