

# The classical evolution of binary black hole systems in scalar-tensor theories<sup>1</sup>

## Gravity Seminar, University of Southampton

Justin Ripley

with William E. East

DAMTP, University of Cambridge

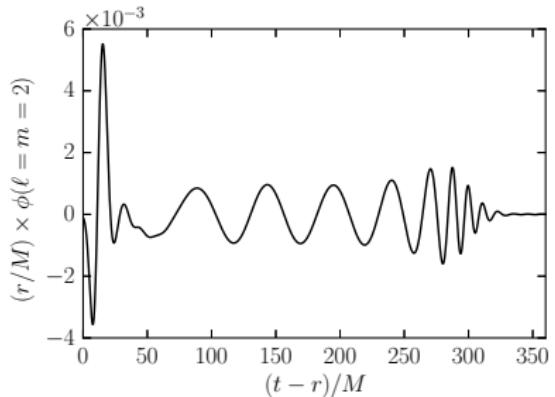
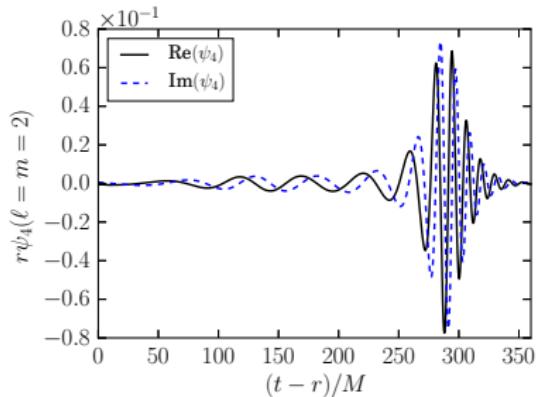
February 2, 2021

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<sup>1</sup>Mostly based on arXiv:2011.03547

# Outline and Summary

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} (R + X - V(\phi) + \alpha(\phi)X^2 + \beta(\phi)\mathcal{G}),$$
$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \quad \mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}$$



Goals: understand why we choose to study the above theory, and understand how we made these plots!

# Outline and Summary

- ▶ Why study scalar-tensor gravity theories?
- ▶ Generating gravitational waveforms for scalar-tensor gravity theories
- ▶ Technical/mathematical advances that made this possible (if there is time/interest)

# Planck units

- We will use (*reduced*) *Planck units*:  $8\pi G = c = \hbar = k_B = 1$
- Everything can be phrased in terms of the *geometrized dimension*  $L$
- Energy scale, etc. are multiples of:
  - Planck energy:  $E_p = l_p c^4 / G \sim 10^{16} \text{ ergs} \sim 10^{19} \text{ GeV}$
  - Planck length:  $l_p = (G\hbar/c^3)^{1/2} \sim 10^{-33} \text{ cm}$
  - Planck time:  $t_p = l_p/c \sim 10^{-44} \text{ s}$
  - Planck mass:  $m_p = l_p c^2 / G \sim 10^{-5} \text{ g}$
  - Planck temperature  $E_p/k_B \sim 10^{32} \text{ K}$

# Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

# Scalar-tensor (Horndeski) gravity

Theories that have a tensor ( $g_{\mu\nu}$ ) field and scalar ( $\phi$ ) field, and have second order equations of motion

$$S = \int d^4x \sqrt{-g} (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5),$$

$$\mathcal{L}_1 \equiv \frac{1}{2}R + X - V(\phi),$$

$$\mathcal{L}_2 \equiv G_2(\phi, X),$$

$$\mathcal{L}_3 \equiv G_3(\phi, X) \square \phi,$$

$$\mathcal{L}_4 \equiv G_4(\phi, X) R + \partial_X G_4(\phi, X) \delta_{\alpha\beta}^{\mu\nu} \nabla^\alpha \nabla_\mu \phi \nabla^\beta \nabla_\nu \phi,$$

$$\mathcal{L}_5 \equiv G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} \partial_X G_5(\phi, X) \delta_{\alpha\beta\gamma}^{\mu\nu\rho} \nabla_\mu \nabla^\alpha \phi \nabla_\nu \nabla^\beta \phi \nabla_\rho \nabla^\gamma \phi,$$

$$X \equiv -\frac{1}{2} (\nabla \phi)^2,$$

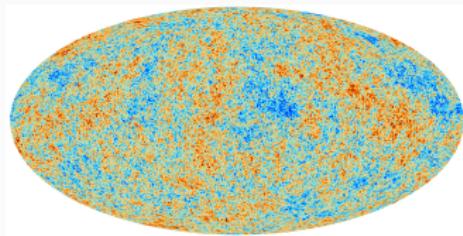
# Why study scalar-tensor gravity?

- ▶ Find a complete theory of quantum gravity
- ▶ Model the dynamics of the very early universe
- ▶ Model the dynamics of the late universe
- ▶ Test GR for sake of basic science

# Find a complete theory of quantum gravity

- ▶ GR is *nonrenormalizable*: the gravitational coupling constant,  $G$ , has units of  $(M_P)^2$  ( $M_P$  is the Planck mass.)
- ▶ Nonrenormalizability hints that GR could/'should' be modified at energies around the Planck scale  $l_p \sim 10^{-33} cm$

# Cosmology and GR



- ▶ At the largest scales the universe is approximately:
  1. homogeneous
  2. isotropic
  3. expanding
  4. Spatial sections are geometrically flat ( ${}^{(3)}R_{ijkl} = 0$ )
- ▶ Friedman-Lemaître-Robertson-Walker (FLRW) solutions to the Einstein Equations
- ▶ With suitable matter contributions and a cosmological constant, the FLRW solutions match observational cosmological data extremely well

# Late universe and GR

- To model the recent/late time expansion of the universe, need to add a *cosmological constant*  $\Lambda$  to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = T_{\mu\nu}.$$

- Is there a physical mechanism that sets the value of the cosmological constant, or is it a new fundamental constant of nature?

# Late universe and GR

- If you want to have “super-accelerated” expansion, where expansion happens *faster* than is possible with a cosmological constant (i.e. when the effective equation of state  $w < -1$ ), then typically you need to modify gravity with higher derivative terms<sup>2</sup>

## THE GALILEON AS A LOCAL MODIFICATION OF GRAVITY

Alberto Nicolis<sup>a</sup>, Riccardo Rattazzi<sup>b</sup>, Enrico Trincherini<sup>c</sup>

<sup>a</sup> Department of Physics and ISCAP,  
Columbia University, New York, NY 10027, USA

<sup>b</sup> Institut de Théorie des Phénomènes Physiques, EPFL,  
CH1015 Lausanne, Switzerland

<sup>c</sup> Scuola Normale Superiore,  
Piazza dei Cavalieri 7, 56126 Pisa, Italy

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<sup>2</sup>e.g. Phys.Rev.D 79 (2009) 064036 arXiv:0811.2197 [hep-th]

# Early universe cosmology and GR: basic questions

- What mechanism set the initial conditions for the universe?<sup>3</sup>
- FLRW cosmologies are *geodesically incomplete*: what preceded the ‘big bang’?

Generalized G-inflation: Inflation with the most general second-order field equations

Tsutomu KOBAYASHI<sup>a,b</sup>, Masahide YAMAGUCHI<sup>c</sup>, and Jun'ichi YOKOYAMA<sup>d,e</sup>

<sup>a</sup>Hakubi Center, Kyoto University, Kyoto 606-8302, Japan

<sup>b</sup>Department of Physics, Kyoto University, Kyoto 606-8302, Japan

## Galilean Genesis: an alternative to inflation

Paolo Creminelli<sup>a</sup>, Alberto Nicolis<sup>b</sup> and Enrico Trincherini<sup>c,d</sup>

<sup>a</sup> Abdus Salam International Centre for Theoretical Physics  
Strada Costiera 11, 34151, Trieste, Italy

Fully stable cosmological solutions with a non-singular classical  
bounce



Anna Ijjas<sup>a,b</sup>, Paul J. Steinhardt<sup>a,b</sup>

<sup>a</sup>Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544 USA

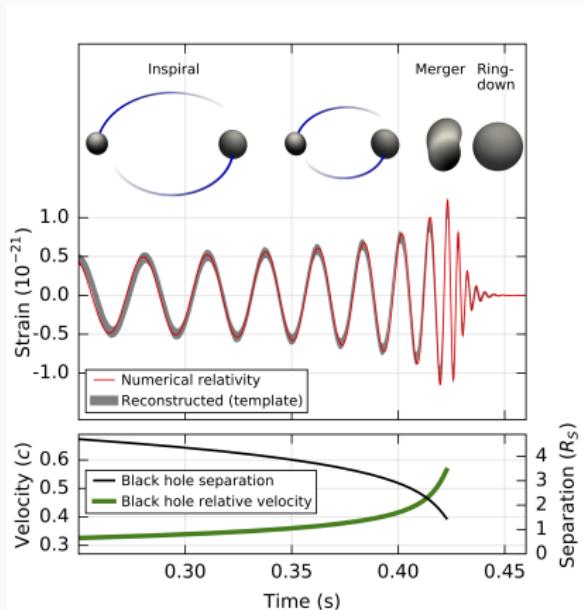
<sup>b</sup>Department of Physics, Princeton University, Princeton, NJ 08544 USA

ARTICLE INFO

ABSTRACT

<sup>3</sup> references to above papers: Prog.Theor.Phys. 126 (2011) 511-529,  
arXiv:1105.5723; JCAP 11 (2010) 021, arXiv:1107.0027; Phys.Lett.B 764  
(2017) 289-294, arXiv:1609.01253

# Test GR for the sake of basic science: gravitational waves



- Gravitational potential of earth  $\sim 10^{-9}$
- Employ *matched filtering* to extract gravitational wave signals: need to accurately model the physics!

# Test GR with gravitational waves: the need for accurate source modeling

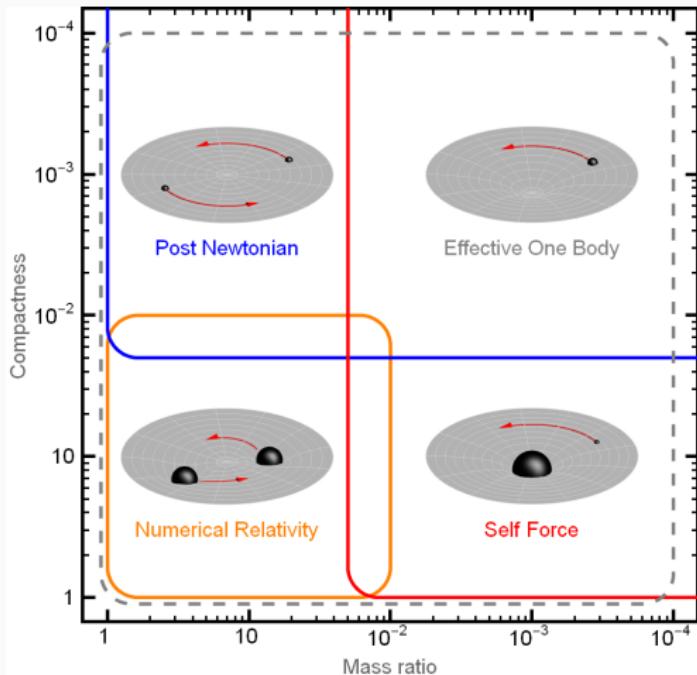


Figure: [https://en.wikipedia.org/wiki/Two-body\\_problem\\_in\\_general\\_relativity](https://en.wikipedia.org/wiki/Two-body_problem_in_general_relativity)

# Guiding principles

Can we find a classical field theory that

1. Has a mathematically sensible interpretation?
2. Matches all current observations?
3. Addresses a current problem in physics?
  - 3.1 Renormalizable (or leading order interactions of a sensible quantum theory of gravity)?
  - 3.2 Incompleteness of early universe or black holes (and so admits NCC violating solutions)?
4. Can be tested/constrained with new observations?

# Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

# sEFT gravity

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} (R + X - V(\phi) + \alpha(\phi)X^2 + \beta(\phi)\mathcal{G}),$$

where

$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi,$$

$\mathcal{G}$ : the *Gauss-Bonnet scalar*

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}.$$

# Why sEFT gravity?

1. Has a mathematically sensible interpretation?
  - ▶ Yes, provided the modified gravity corrections are “small”<sup>4</sup>
2. Matches all current observations?
  - ▶ Yes, provided we do not use this theory to model the late universe ESGB gravity not highly constrained by, e.g. binary pulsar tests<sup>5</sup>

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<sup>4</sup>e.g. JLR & Pretorius, Class.Quant.Grav. 36 (2019) 13, 134001, Kovacs et. al. Phys.Rev.D 101 (2020) 12, 1240030

<sup>5</sup>e.g. Baker et. al. Phys.Rev.Lett. 119 (2017) 25, 251301, Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010

# Why sEFT gravity?

1. Addresses a current problem in physics?
  - ▶ Theory captures leading order scalar-tensor parity invariant interactions, so captures the leading order corrections from many UV complete theories of gravity<sup>6</sup>
2. Can be tested/constrained with new observations?
  - ▶ Many versions of the theory have ‘scalarized’ black hole solutions, so will be strongly constrained by gravitational wave observations<sup>7</sup>

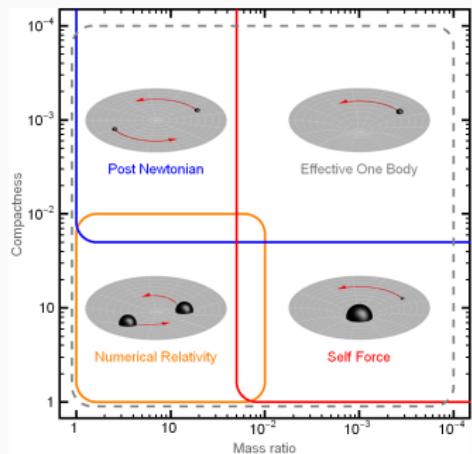
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<sup>6</sup>e.g. Weinberg, Phys.Rev.D 77 (2008) 123541

<sup>7</sup>e.g. Kanti et. al. Phys.Rev.D 54 (1996) 5049-5058

# Approaches to studying modified gravity theories<sup>9</sup>

- ▶ Order reduction approach to solve the equations of motion of a modified gravity theory <sup>8</sup>
- ▶ **Study exact (nonperturbative) solutions to particular modified gravity theories: useful for understanding physics in strong field, dynamical regime**



<sup>8</sup>e.g. Okounkova et al., Class.Quant.Grav. 36 (2019) 5, 054001;  
Okounkova et. al., Phys.Rev.D 99 (2019) 4, 044019

<sup>9</sup>e.g. Cayuso, Ortiz, Lehner, Phys.Rev. D96 (2017) no.8, 084043; Allwright, Lehner, Class.Quant.Grav. 36 (2019) no.8, 084001

# Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

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# Shift symmetric ESGB gravity

- ▶ “Shift symmetric Einstein scalar Gauss-Bonnet” (ESGB) gravity

$$S_{ESGB} = \frac{1}{2} \int d^4x \sqrt{-g} (R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + 2\lambda \phi \mathcal{G}),$$

# Shift symmetric ESGB gravity

- ▶ “Shift symmetric Einstein scalar Gauss-Bonnet” (ESGB) gravity

$$S_{ESGB} = \frac{1}{2} \int d^4x \sqrt{-g} (R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + 2\lambda \phi \mathcal{G}),$$

- ▶ This theory does not admit **stationary** Schwarzschild black hole solutions<sup>10</sup>; instead “hairy” scalar black holes should be end states in this theory

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$$\square\phi + \lambda\mathcal{G} = 0$$

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<sup>10</sup>Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

# Shift symmetric ESGB in a modified harmonic formulation<sup>11</sup>

- ▶ Collaboration with Will East
- ▶ Reformulate the equations of motion in *modified generalized harmonic* formulation
- ▶ Consider spinning black hole evolution (axisymmetric spacetime)
- ▶ Consider head on black hole collisions (axisymmetric spacetime)
- ▶ Consider binary black hole merger (no symmetry assumptions)

# Modified generalized harmonic (MGH) formulation<sup>12</sup>

- ▶ Specify two auxiliary Lorentzian metrics  $\hat{g}^{\mu\nu}$  and  $\tilde{g}^{\mu\nu}$  in addition to the spacetime metric  $g^{\mu\nu}$
- ▶ Specify the gauge/coordinate condition with:

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu x^\gamma = H^\gamma, \quad (1)$$

where  $H^\gamma$  is source function

- ▶ Free parameters:  $\hat{g}^{\mu\nu}$ ,  $\tilde{g}^{\mu\nu}$ ,  $H^\gamma$  (more details given at end of talk)
- ▶ Besides using the MGH formulation, we begin with GR initial data, and use standard techniques from numerical relativity

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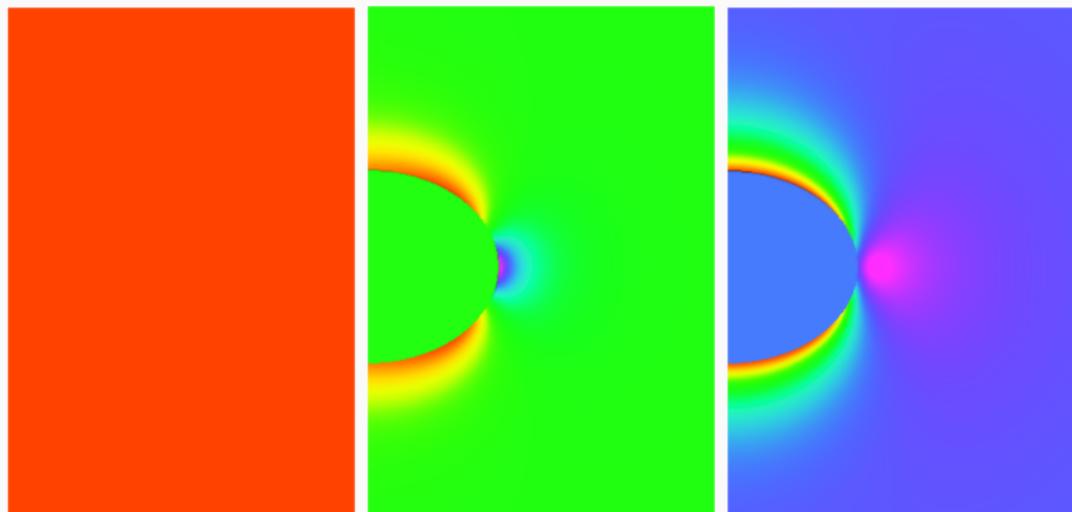
<sup>12</sup>Kovacs and Reall, Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

## Initial conditions

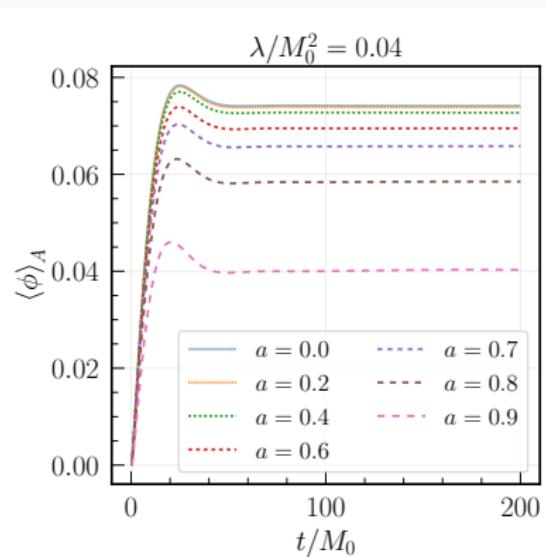
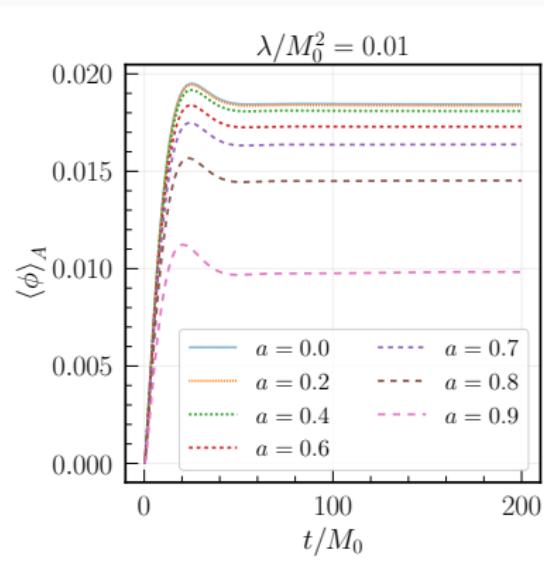
- ▶ For technical reasons, we always start with a GR solution (e.g. one spinning black hole, two boosted black holes), and then let the black holes grow scalar hair as we evolve in time
- ▶ After a finite amount of evolution, the black holes stop growing scalar hair (growth saturates)

$$S_{ESGB} = \frac{1}{2} \int d^4x \sqrt{-g} (R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2\lambda \phi \mathcal{G}),$$

# Scalar hair growth around spinning black holes

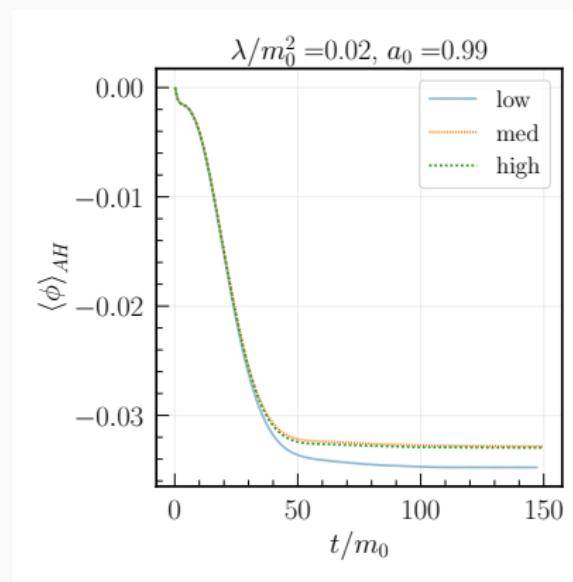


# Scalar hair growth around spinning black holes



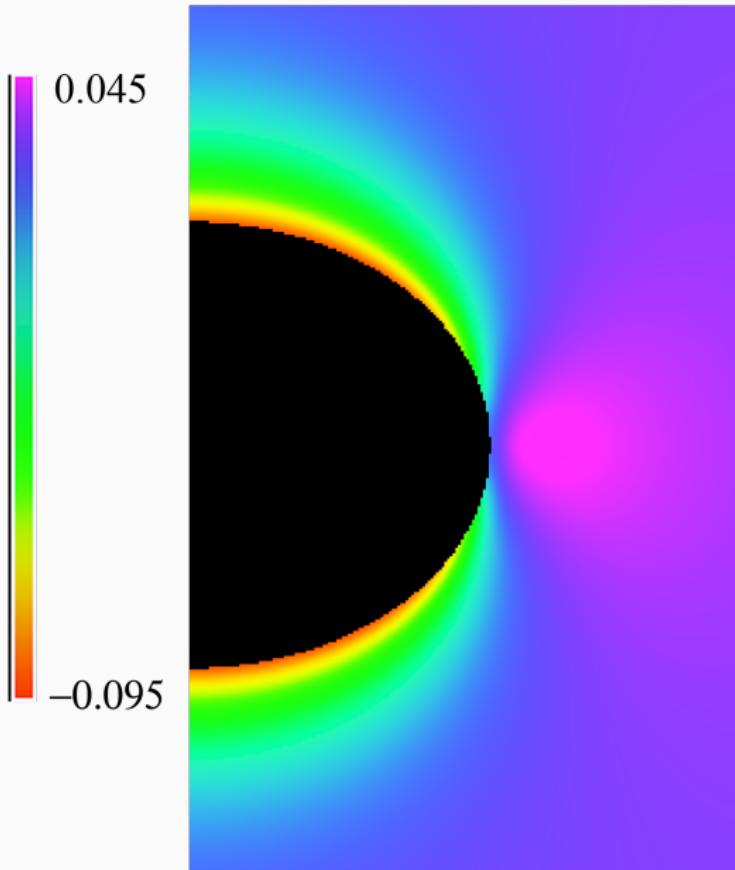
- ▶  $\langle \phi \rangle_A$ : average scalar field value on black hole horizon
- ▶  $a$ : initial dimensionless black hole spin

# Scalar hair growth around spinning black holes

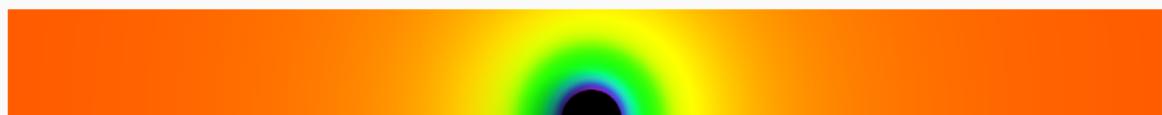
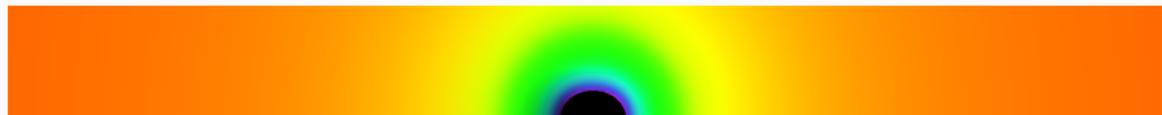
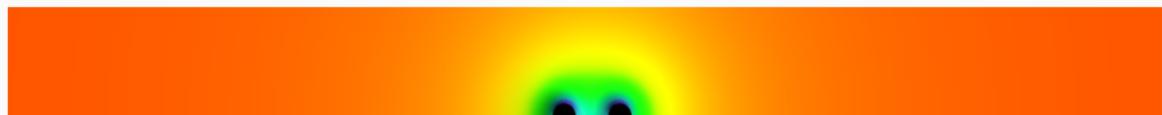


- $\langle \phi \rangle_A$ : average scalar field value on black hole horizon, at three different resolutions (convergence study)

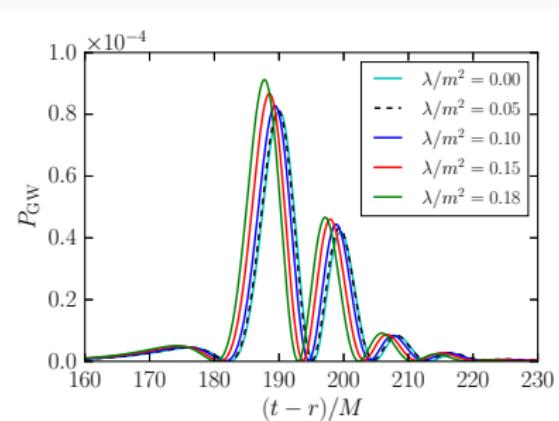
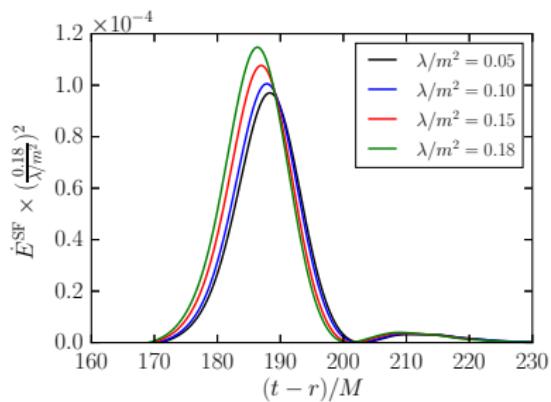
# Scalar field density around a spinning black hole



# Head on black hole collisions

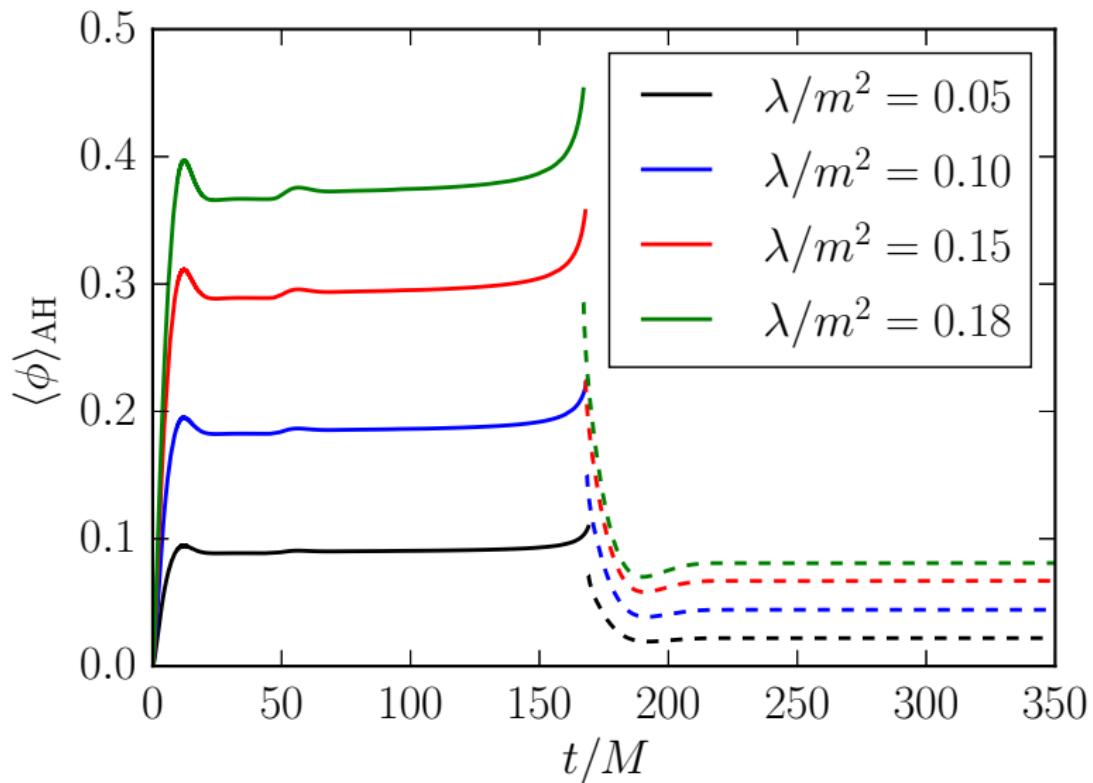


# Head on black hole collisions: gravitational and scalar radiation

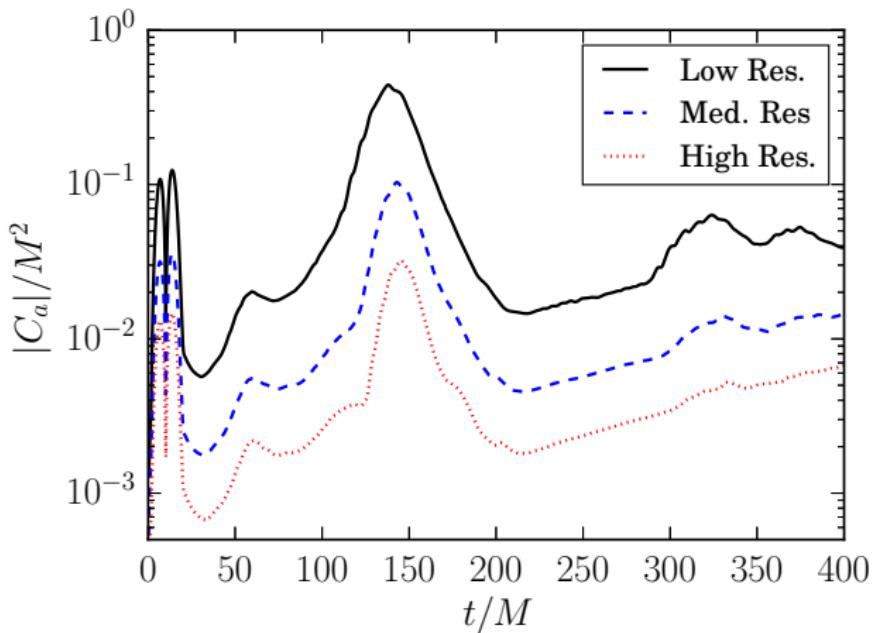


Flux of scalar field vs flux of gravitational waves

# Head on black hole collisions: scalar field on horizon



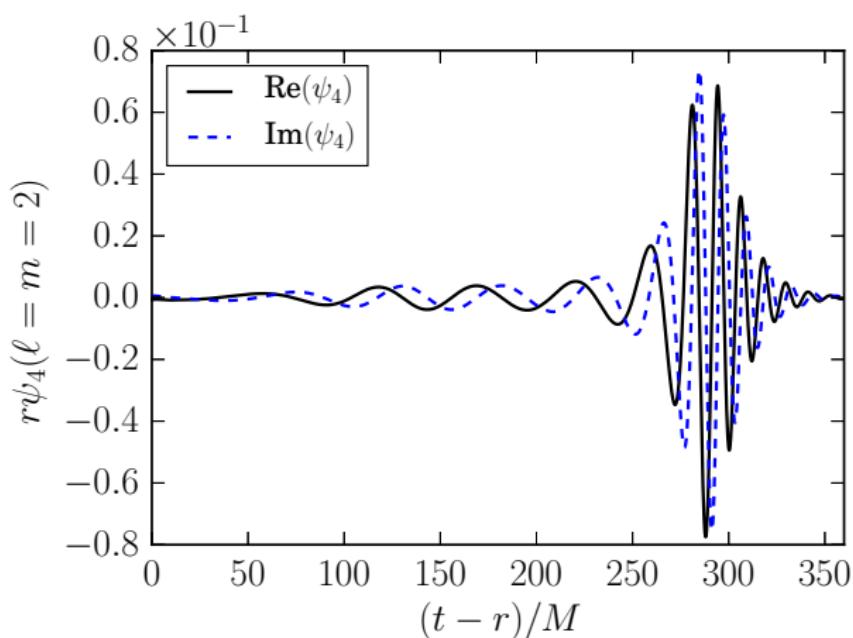
# Head on black hole collisions: convergence



Convergence of “constraint violation”:

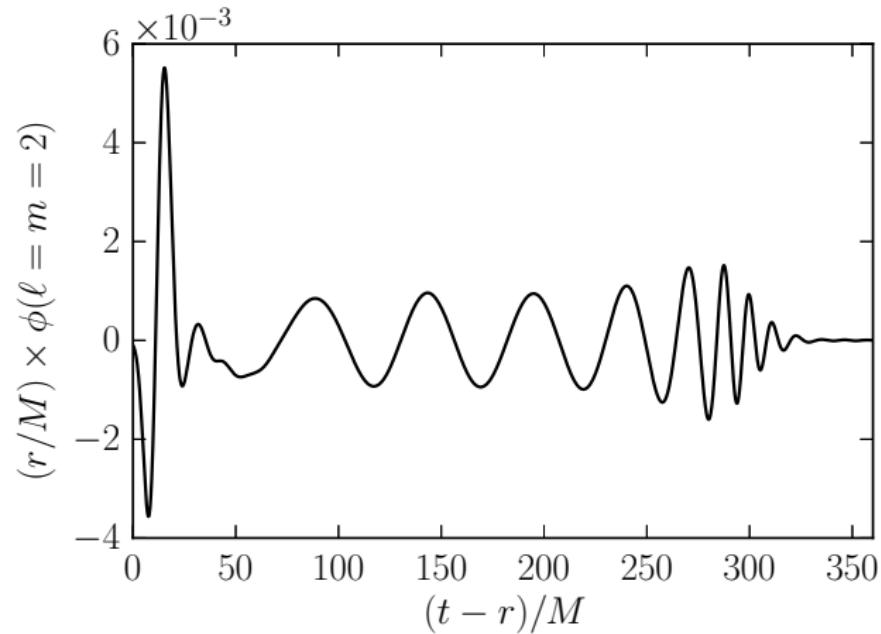
$$C^\alpha \equiv H^\alpha + \tilde{g}^{\mu\nu} \Gamma_{\mu\nu}^\alpha$$

# Binary black hole collisions



Gravitational wave strain from two ESGB binary black holes merging

# Binary black hole collisions



Radiated scalar waves

# What was the main challenge? Finding a well-posed initial value formulation for the theory

- sEFT gravity has a well-posed initial value problem in generic spacetimes, provided the modified gravity corrections are “small”, when one specifies their coordinate according to a *modified generalized harmonic* (MGH) condition<sup>13</sup>:

$$H^\gamma + \Gamma_{\alpha\beta}^\gamma \tilde{g}^{\alpha\beta} = 0. \quad (3)$$

- $H^\gamma$ : free function one can choose
- $\tilde{g}^{\alpha\beta}$ : “auxiliary” metric one can choose (*not* the “physical” metric  $g^{\alpha\beta}$ )
- In contrast to “generalized harmonic” formulation<sup>14</sup>:  
$$H^\gamma + \Gamma_{\alpha\beta}^\gamma g^{\alpha\beta} = 0$$

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<sup>13</sup>Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

<sup>14</sup>e.g. Pretorius, Class.Quant.Grav. 22 (2005) 425-452

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Review: scalar-tensor gravity theories

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Shift symmetric

Conclusion

# Conclusion

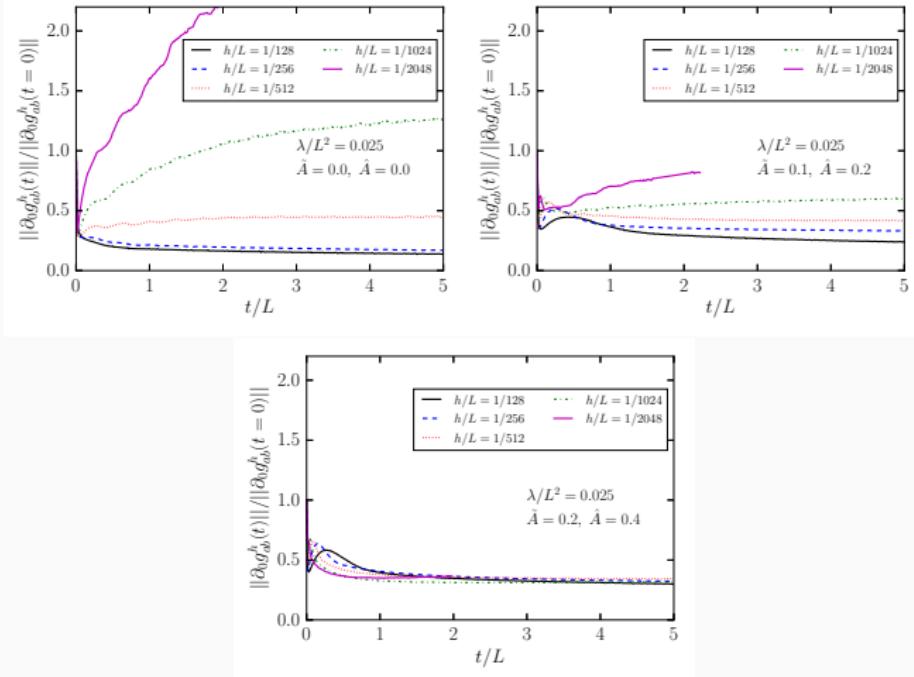
- ▶ GR is an extremely successful theory of gravity, but there are still reasons to study modified gravity theories
  - ▶ early universe: inflation, genesis, bouncing, ...
  - ▶ late universe: dark energy, ...
- ▶ Can test GR with gravitational waves
  - ▶ for that you need gravitational waveform templates to compare to data
- ▶ **Claim:** We now have the tools to produce gravitational waveforms produced during the merger of two black holes for a whole class of scalar-tensor gravity theories

# Future directions

- ▶ Further develop the MGH formulation of general relativity and scalar-tensor gravity theories
  - ▶ What are “good” choices for the auxiliary metrics?
- ▶ Binary black hole waveform catalogues for other kinds of scalar-tensor gravity theories
- ▶ Consider early universe cosmological simulations in these theories

# Backup slides

# Hyperbolicity test: Self-convergence in harmonic vs modified harmonic gauge



# Additional work on well-posedness of modified gravity theories

- ▶ Bernard, Lehner, and Luna<sup>15</sup> consider spherically symmetric dynamics of

$$\mathcal{L} = (1 + G_4(\phi)) R + (\nabla\phi)^2 - V(\phi) + G_2 \left( \phi, (\nabla\phi)^2 \right).$$

see also Papallo and Reall, who study Horndeski theories in less symmetric spacetimes<sup>16</sup>.

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<sup>15</sup>Phys. Rev. D100 (2019) no.2, 024011

<sup>16</sup>Papallo, Reall, Phys. Rev. D96 (2017) no.4, 044019

## Shift symmetric ESGB: equations of motion

$$S_{ESGB} = \frac{1}{2} \int d^4x \sqrt{-g} (R - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2\lambda \phi \mathcal{G}),$$

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 2\lambda \delta_{\alpha\beta\rho\sigma}^{\gamma\delta\kappa\lambda} R^{\rho\sigma}{}_{\kappa\lambda} (\nabla^\alpha \nabla_\gamma \phi) \delta^\beta_{(\mu} g_{\nu)\delta} \\ - \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 = 0, \\ \square \phi + \lambda \mathcal{G} = 0. \end{aligned}$$

# Order reduction approach for ESGB gravity<sup>17</sup>

Assume  $\epsilon \sim \lambda$  and  $|\epsilon| \ll 1$

$$\begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{(0)} + \epsilon g_{\mu\nu}^{(1)} + \epsilon^2 g_{\mu\nu}^{(2)} + \dots \\ \phi &= \phi^{(0)} + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots \end{aligned} \quad (4a)$$

$$\phi^{(0)} = 0, \quad (5a)$$

$$R_{\mu\nu}[g_{\alpha\beta}^{(0)}] - \frac{1}{2}g_{\mu\nu}R[g_{\alpha\beta}^{(0)}] = 0 \quad (5b)$$

$$\square\phi^{(1)} = \lambda\mathcal{G}\left[g_{\alpha\beta}^{(0)}\right], \quad (6a)$$

$$R_{\mu\nu}[g_{\alpha\beta}^{(0)}] - \frac{1}{2}g_{\mu\nu}R[g_{\alpha\beta}^{(0)}] = 0 \quad (6b)$$

$$R_{\mu\nu}[g_{\alpha\beta}^{(2)}] - \frac{1}{2}g_{\mu\nu}R[g_{\alpha\beta}^{(2)}] = \lambda \times F\left[\phi^{(1)}\right] \quad (7)$$

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<sup>17</sup>Okounkova, *Phys. Rev. D* 100 (2019)