Testing General Relativity and the nonlinear dynamics of modified gravity theories[1]

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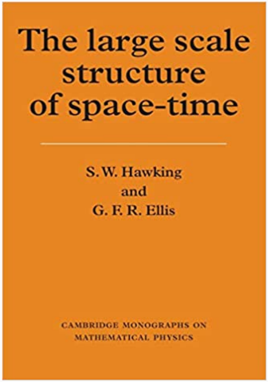
[1] 1902.01468, 1903.07543, 1911.11027

Outline

Why study modified gravity?

- model selection with LIGO/Virgo data during merger when gravity is at its strongest and most dynamical
  - Intuition: having *nonperturbative* corrections to GR could give strong constraints on potential deviations to GR
- model dark energy, dark matter, early universe smoothing and flattening mechanisms (inflation, "bouncing" models, etc) in regimes where solutions are nonlinear

Why modify General Relativity? Null Convergence Condition (NCC)



- For all null vectors k^a,

$$R_{ab}k^a k^b \geq 0 \tag{1}$$

- Plays important role in incompleteness (singularity) theorems
  - Incompleteness of FLRW cosmologies (big bang)
  - Incompleteness inside of black holes
- Standard classical matter fields cannot stably violate NCC
- Need to modify general relativity

Example of modified gravity theory used in cosmology

- Action used to violate NCC: "Galileon genesis" to precede inflation[2]

$$\mathcal{L} = R + c_1 e^{2\phi} (\partial\phi)^2 + c_2 (\partial\phi)^2 \Box\phi + c_3 (\partial\phi)^3. \qquad (2)$$

- Action used to violate NCC and have a "bouncing" cosmological solution[3]

$$\begin{aligned}\mathcal{L} =& \frac{1}{2} R + k(\phi) (\partial\phi)^2 + q(\phi) (\partial\phi)^4 - V(\phi) \\ &+ b(\phi) (\partial\phi)^2 \Box\phi + \left(f_1(\phi) + f_2(\phi) (\partial\phi)^2\right) R + \cdots \qquad (3)\end{aligned}$$

[2] Creminelli, Nicolis, Trincherini JCAP 1011 (2010) 021

[3] Ijjas, Steinhardt, Phys. Lett. B 764 (2017) pp. 289-294

Outline

Approaches to studying modified gravity theories[4]

- Study exact solutions to a particular modified gravity theory
  - "dynamical Chern Simons", "EdGB", "massive gravity", etc.
- Study effective corrections to General Relativity (Effective Field Theory)
  - Assume a set of symmetries, and then write down all terms order by order in some (set of) small expansion parameter(s) consistent with those symmetries

[4] For further discussion, see e.g.
Cayuso, Ortiz, Lehner, Phys.Rev. D96 (2017) no.8, 084043;
Allwright, Lehner, Class.Quant.Grav. 36 (2019) no.8, 084001

Nonlinear dynamics of EdGB gravity

Goal: To study the full equations of motion of a EdGB gravity
Challenge: We must then understand if theory has well posed initial value problem, obeys cosmic censorship, etc.
Reward: Could understand how modifications of GR affects the nonlinear dynamics of two black holes when they merge (and see how that affects the gravitational waveform produced during the merger)

Shift symmetric EdGB gravity

$$S_{EdGB} = \frac{1}{2} \int d^4x \sqrt{-g} \left(R - (\nabla\phi)^2 + 2\lambda\phi\mathcal{G} \right), \tag{4}$$

where $\mathcal{G}$ is the Gauss-Bonnet scalar

$$\mathcal{G} \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2. \tag{5}$$

EdGB: history

- EdGB gravity can be *motivated* using effective field theory arguments: the theory contains the leading order scalar-tensor mixing term:

$$W(\phi)\mathcal{G} \tag{6}$$

  that cannot be removed by a conformal transformation[5]
- Shift symmetric EdGB gravity: unique shift symmetric $\phi \to \phi + c$ scalar-tensor theory that does not admit **stationary** Schwarzschild black hole solutions[6]

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[5] Gross and Sloan, Nucl.Phys. B291 (1987) 41-89
[6] Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

Why EdGB gravity?

There are very few modified gravity theories that[7]

1. Are consistent with General Relativity in regimes where it is well tested
2. Predict observable deviations in the dynamical, strong field regime relevant to black hole mergers
3. Possess a classically well-posed initial value problem

[7] Lehner and Pretorius, Ann.Rev.Astron.Astrophys. 52 (2014) 661-694

Why EdGB gravity?

1. Are consistent with General Relativity in regimes where it is well tested
   - EdGB gravity not highly constrained by, e.g. binary pulsar tests[8]
2. Predict observable deviations in the dynamical, strong field regime relevant to black hole mergers
   - EdGB has scalarized black hole solutions[9], so it *may* predict large deviations from GR
3. Possess a classically well-posed initial value problem
   - Equations of motion are second order[10], so there may be initial data configurations for which this is true for EdGB gravity

[8] Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010

[9] e.g. Kanti, Mavromatos, Tamvakis, Winstanley, Phys.Rev. D54 (1996) 5049-5058; Sotiriou, Zhou Phys.Rev. D90 (2014) 124063

[10] e.g. Zwiebach, Phys.Lett. 156B (1985) 315-317

Equations of motion for EdGB gravity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^{\alpha}\nabla_{\gamma}\phi\right)\delta^{\beta}{}_{(\mu}g_{\nu)\delta}$$
$$-\nabla_{\mu}\phi\nabla_{\nu}\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 = 0, \quad (7)$$
$$\Box\phi + \lambda\mathcal{G} = 0. \quad (8)$$

- Equations of motion have maximum of two time derivatives acting on each field, so theory is Ostrogradsky stable
- Ostrogradsky stability *not* sufficient for theory to have mathematically sensible solutions

Outline

Coordinates

We consider EdGB gravity in spherical symmetry, and will present results from simulations that used two different coordinates:

- Schwarzschild-like coordinates

$$
\begin{aligned}
ds^2 = &- e^{2A(t,r)} dt^2 + e^{2B(t,r)} dr^2 \\
&+ r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) .
\end{aligned}
\tag{9}
$$

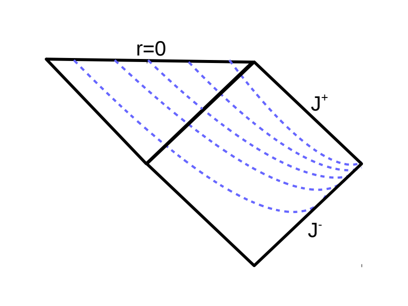
- Painlevé-Gullstrand-like coordinates

$$
\begin{aligned}
ds^2 = &- \alpha(t,r)^2 dt^2 + \left(dr + \alpha(t,r)\zeta(t,r) dt\right)^2 \\
&+ r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) .
\end{aligned}
\tag{10}
$$

Painlevé-Gullstrand (PG) coordinates: properties

- Horizon penetrating
- Not singularity avoiding
- spatially flat: ${}^{(3)}R_{ijkl} = 0$

$$\alpha = 1, \qquad \zeta = \sqrt{\frac{2m}{r}}. \tag{11}$$



Equations of motion in PG coordinates for EdGB gravity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^{\alpha}\nabla_{\gamma}\phi\right)\delta^{\beta}{}_{(\mu}g_{\nu)\delta}$$

$$-\nabla_{\mu}\phi\nabla_{\nu}\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^{2} = 0, \qquad (12)$$

$$\Box\phi + \lambda\mathcal{G} = 0. \qquad (13)$$

- Through taking algebraic combinations of the equations of motion, can define wave-like equation for ϕ (with no time derivatives acting on α and ζ).
- Hamiltonian and momentum constraints give ordinary differential equations (in r) for metric fields α and ζ

Other numerical tools

- Solve PDE/ODE system with (second order) finite difference methods
- Spatial compactification ($x = L$ is spatial infinity)

$$r(x) \equiv \frac{x}{1 - x/L} \tag{14}$$

- Modified Berger-Oliger style fixed mesh refinement to solve system of hyperbolic and elliptic (ode) equations[11]

[11] Pretorius and Choptuik, J.Comput.Phys. 218 (2006) 246-274

Outline

Schwarzschild initial data

- Schwarzschild initial data
- Curvature coupling with black hole mass m

$$C \equiv \frac{\lambda}{m^2}. \tag{15}$$

- Compare to “decoupled” scalar field solution: time independent solution of Schwarzschild background with scalar field obeying

$$\Box\phi + \lambda\mathcal{G} = 0. \tag{16}$$

Growth of scalar hair

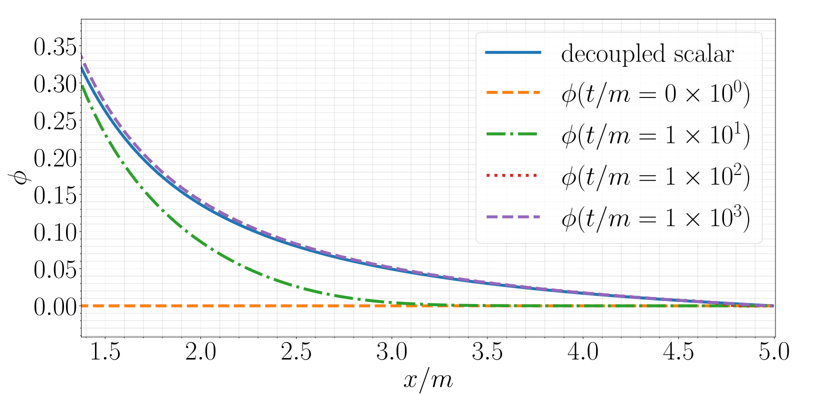


Figure: $C = 0.16$. The horizon (MOTS) is located at $x_h/m \approx 1.48$, and spatial infinity is at $x/m = 5$.

Growth of scalar hair: comparison to “decoupled limit”

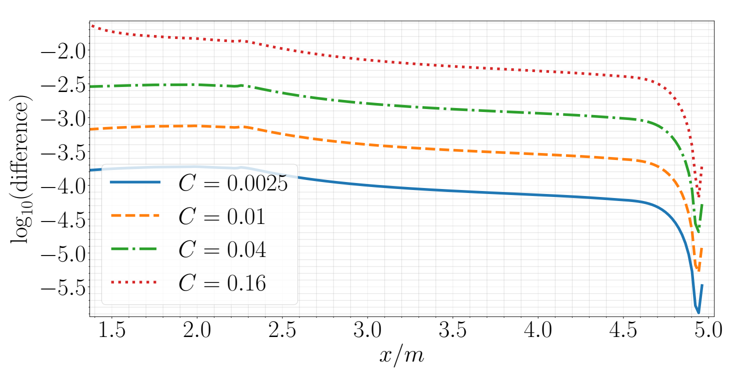
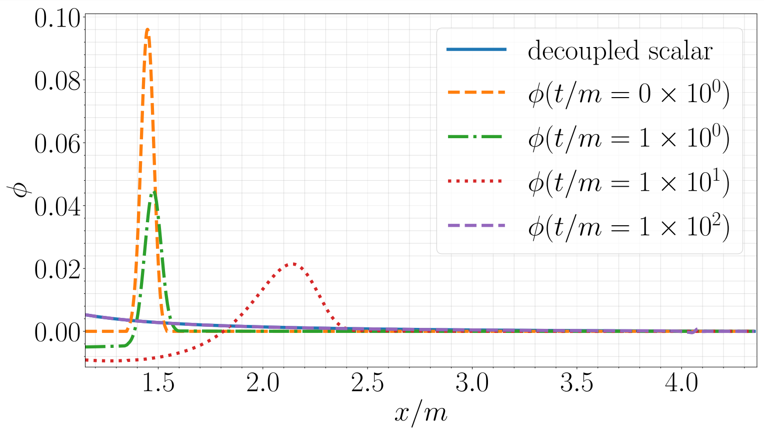


Figure: Difference between the late time ($t \sim 2000m$) scalar field profile obtained from the non-linear simulations to that of the decoupled estimate , The black hole horizon (MOTS) is at $x/m \approx 1.48$, and spatial infinity is at $x/m = 5$.

Schwarzschild initial data with a perturbation



$$\phi(t,r)\big|_{t=0} = \begin{cases} \phi_0 \exp\left[-\frac{1}{(r-a)(b-r)}\right] \exp\left[-5\left(\frac{r-(a+b)/2}{a+b}\right)^2\right] & a < r < b \\ 0 & \text{otherwise} \end{cases}$$

Outline

Diagnostics: hyperbolicity of theory

- **Challenge:** must then understand if theory has well posed initial value problem, obeys cosmic censorship, etc.
- Well posed initial value problem: theory has a *strongly hyperbolic* formulation

Diagnostics: hyperbolicity

- Hyperbolicity: all the characterstic speeds in the theory are real
- Characteristic speeds: speed at which high frequency, linear perturbations travel about background solution
- Example:

$$h(x)\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} - V(x)f = 0. \tag{17}$$

Characteristic speeds are $c_\pm = \pm\sqrt{1/h(x)}$.

Diagnostics: hyperbolicity

- Hyperbolicity: all the characterstic speeds in the theory are real
- For EdGB gravity in spherical symmetry, characteristic speeds c obey equation of the form:

$$\mathcal{A}c^2 + \mathcal{B}c + \mathcal{C} = 0, \tag{18}$$

  where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ depend on metric variables, scalar field, and their derivatives.
- Three regimes: parabolic, elliptic, and hyperbolic depending on sign of the discriminant $\mathcal{D} \equiv \mathcal{B}^2 - 4\mathcal{A}\mathcal{C}$.

Results in Schwarzschild-like coordinates

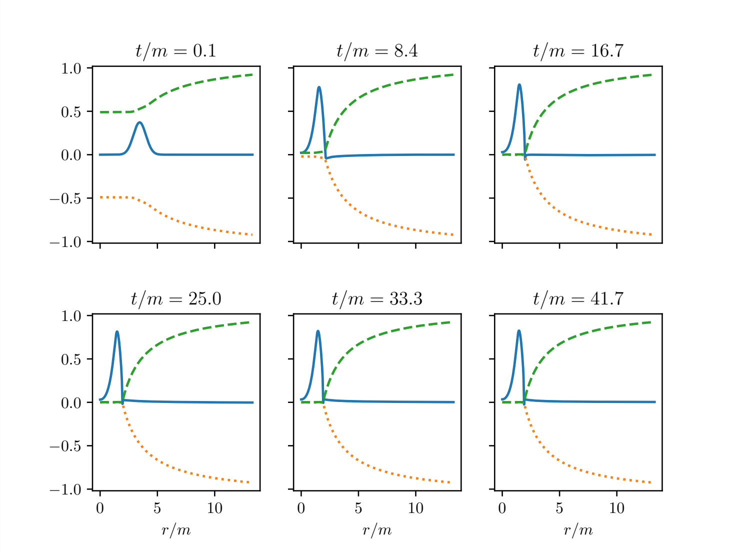
$$ds^2 = -e^{2A(t,r)}dt^2 + e^{2B(t,r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (19)$$

- Study: small initial EdGB scalar pulse, with no initial black hole [12]
- Solve equations of motion as in PG coordinate case: find evolution equation for scalar field, and constraint equations for A, B fields
- Initial data

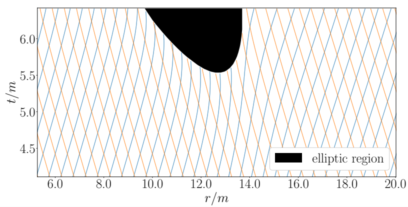
$$\phi(t,r)|_{t=0} = a_0\left(\frac{r}{w_0}\right)^2 \exp\left(-\left(\frac{r-r_0}{w_0}\right)^2\right). \quad (20)$$

[12] JLR and Frans Pretorius Phys. Rev. D 99, 084014 (2019), JLR and Frans Pretorius Class.Quant.Grav. 36 (2019) no.13, 134001

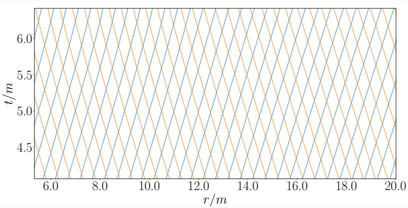
Scalar field evolution and characteristics for $\lambda R \ll 1$



EdGB gravity with $R\lambda \sim 0.1$



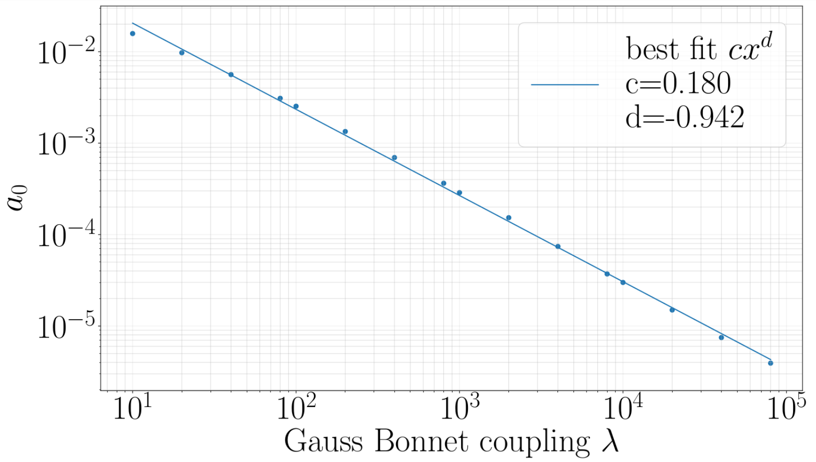
(a) EdGB characteristics



(b) Null characteristics

Figure: $a_0 = 0.02, w_0 = 8, r_0 = 20,\ \lambda = 50,\ r_{max} = 100,\ N_r = 2^{12} + 1;$
$m \sim 0.93$.

Elliptic vs hyperbolic evolution



$$\phi(t,r)|_{t=0} = a_0 \left(\frac{r}{w_0}\right)^2 \exp\left(-\left(\frac{r-r_0}{w_0}\right)^2\right). \qquad (21)$$

EdGB gravity as “mixed-type” equations

- We find EdGB equations of motion for scalar field are hyperbolic up to a given curvature scale $R \times \lambda \sim 0.1$, then equations become *elliptic*
- Mixed type PDE: solution regions where elliptic, hyperbolic
- Example: Tricomi equation

$$\partial_x^2 f + x\partial_y^2 f = 0. \tag{22}$$

- Separation line between hyperbolic and elliptic part: **sonic line**

"Mixed-type" PDE

- Terminology comes from fluid dynamics: equation of motion for steady state solutions to inviscid, compressible flow obey a mixed-type equation[13]

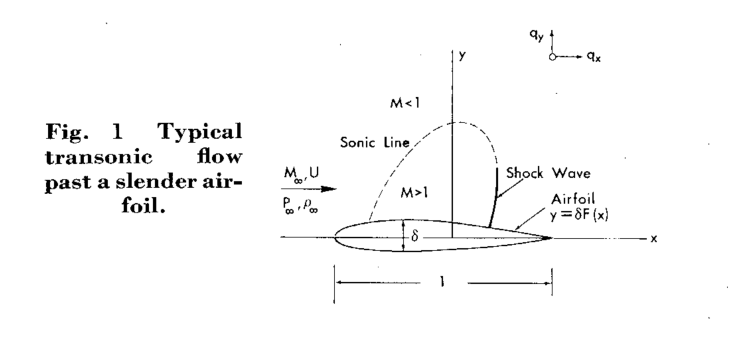


Figure: J. D. Cole and E. M. Murman. *Calculation of Plane Steady Transonic Flow*

[13] e.g. C. Morawetz, *Mathematical Approach to the Sonic Barrier*

mixed-type property vs instability

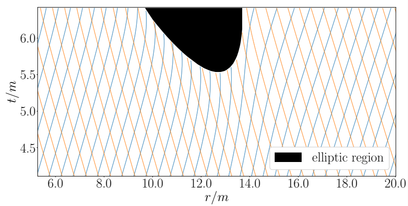


Figure: EdGB characteristics

Black region not necessarily "unstable": if a solution is unstable then we could *evolve* away from that solution. As the equations of motion are elliptic inside the black region, in order to obtain a solution one must solve the interior of the black region as a boundary value problem instead of an initial value problem.

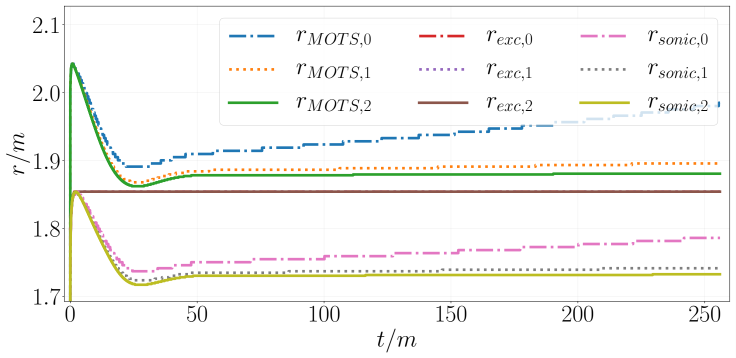
Sonic line and black holes in EdGB gravity

- Back to simulations in Painlevé-Gullstrand coordinates: begin with Schwarzschild black hole, and study subsequent evolution

Sonic line and black holes in EdGB gravity

- Sonic line forms inside EdGB black hole for any (nonzero) value of λ
- Geometry is smooth and finite up to sonic line (cannot say what geometry is past sonic line)
- For small enough λ/m^2, sonic line inside black hole horizon and elliptic region is "censored"
- For large enough λ/m^2, sonic line outside black hole horizon

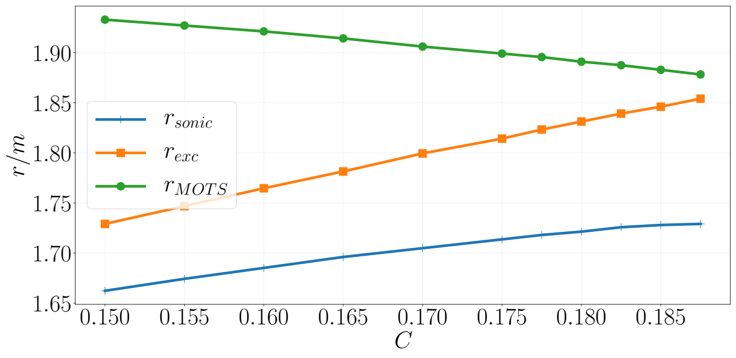
Evolution of sonic line vs. apparent horizon



EdGB black holes: elliptic region inside of black hole for small enough curvature couplings

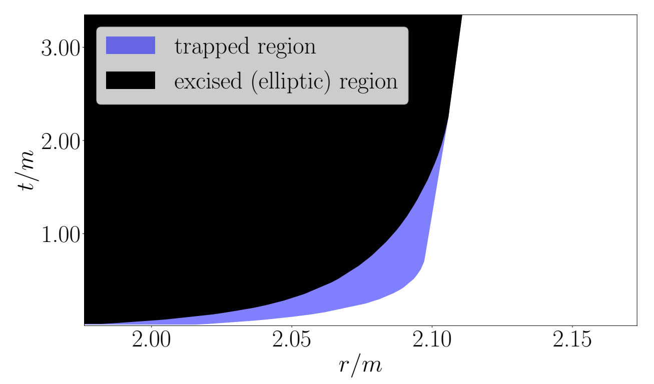
Evolution of sonic line vs. apparent horizon

Sonic point approaches black hole horizon for larger curvature couplings (estimate “extremal” $\lambda/m^2 \sim 0.23$)



"Superextremal" curvature-coupling

For large enough curvature couplings, elliptic region grows outside of black hole: have "naked" elliptic region and can on longer run simulations



"Superextremal" curvature-coupling

- Formation of naked elliptic region: breakdown of casual evolution
- No universal curvature coupling value for which have "naked" elliptic region
- With large enough gradients can trigger elliptic region formation, so triggering elliptic region formation depends on initial data [14]

[14] JLR and Frans Pretorius Phys. Rev. D 99, 084014 (2019), JLR and Frans Pretorius Class.Quant.Grav. 36 (2019) no.13, 134001

Additional work on well-posedness of modified gravity theories

- Bernard, Lehner, and Luna[15] consider spherically symmetric dynamics of

$$\mathcal{L} = (1 + G_4(\phi))\, R + (\partial\phi)^2 - V(\phi) + G_2\left(\phi, (\partial\phi)^2\right). \quad (23)$$

  see also Papallo and Reall, who study Horndeski theories in less symmetric spacetimes[16].

- Kovacs and Reall[17] have found a set of gauge conditions that may lead to well-posed evolution for small enough deviations from GR

$$\mathcal{L} = R + (\partial\phi)^2 + f_1(\phi)\,(\partial\phi)^4 - V(\phi) + f_2(\phi)\mathcal{G}. \quad (24)$$

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[15] Phys.Rev. D100 (2019) no.2, 024011
[16] Papallo, Reall, Phys.Rev. D96 (2017) no.4, 044019
[17] 2003.08398

Work in progress: study other forms of EdGB gravity

- Study more general class of EdGB gravity theories or couplings, e.g.[18]

$$\mathcal{L} = R - (\nabla\phi)^2 - \mu^2\phi^2 - 2\lambda\phi^4 + \frac{1}{8}\eta\phi^2\mathcal{G}. \qquad (25)$$

- Do these theories have problems with hyperbolicity in the strong field, dynamical regime? (preliminary answer: *yes*)

[18] Macedo et. al., Phys.Rev. D99 (2019) no.10, 104041

Future directions with other modified gravity theories

- Are there modified gravity theories that offer interesting solutions (e.g. violate Null Convergence Condition) but which are also sensible as classical field theories?
- Apply effective field theory approach to numerical relativity to search for deviations from GR around black hole binary merger

Outline

Conclusion

- Study nonlinear dynamics of EdGB black holes in spherical symmetry
- For ‘subextremal’ couplings: nonlinearly stable scalarized black holes form with an elliptic region inside the horizon
- For ‘superextremal’ couplings: elliptic region grows outside of black hole horizon, evolution problem ill-posed
- EdGB gravity violates weak/strong cosmic censorship
- potential future directions:
  - prove mixed-type property of EdGB is gauge invariant
  - Apply analysis to other varieties of EdGB gravity (e.g. $\phi^4\mathcal{G}$), or other modified gravity theories
  - Look for better behaved modified gravity theories that are *also* well motivated
  - Numerical relativity simulations of effective field theory extensions of GR