Robotics A

Final Project Report

ROBOTWORKS

José Luis RIVER BLANCO

Juan Antonio FLORES REYNA

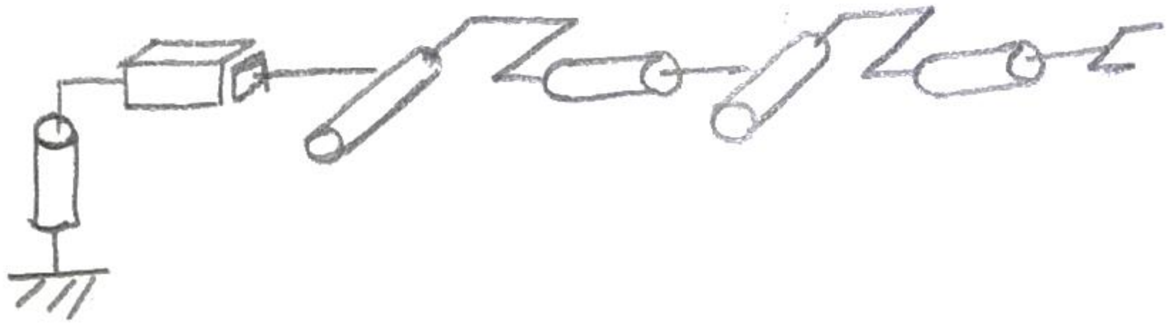
Delivery date: June 14th, 2021

Professor: Alejandro GONZALEZ DE ALBA

INDEX

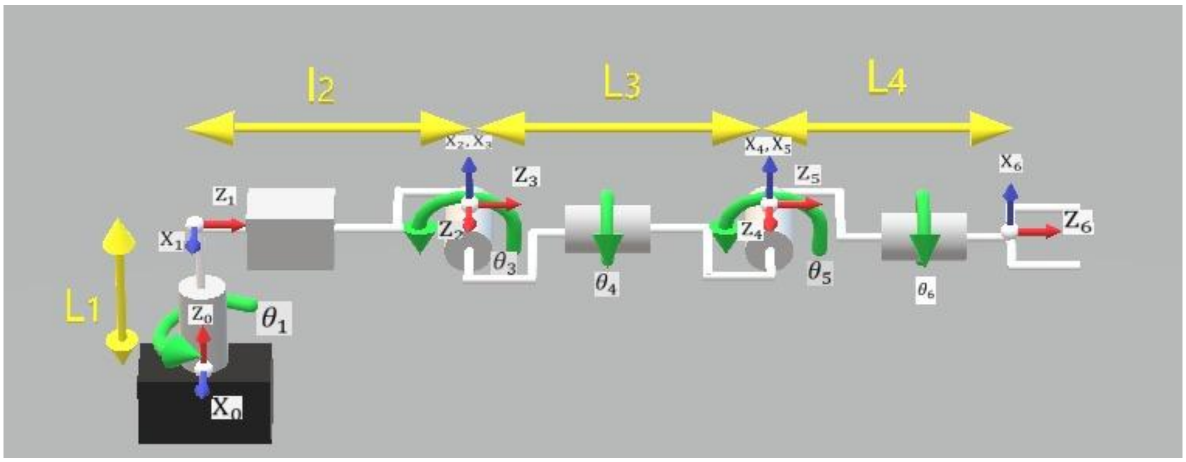
Assigned Robot

The robot model assigned by the professor for this project is the following.



Frame Assignment

For the previous robot structure, the proposed frame assignment is.



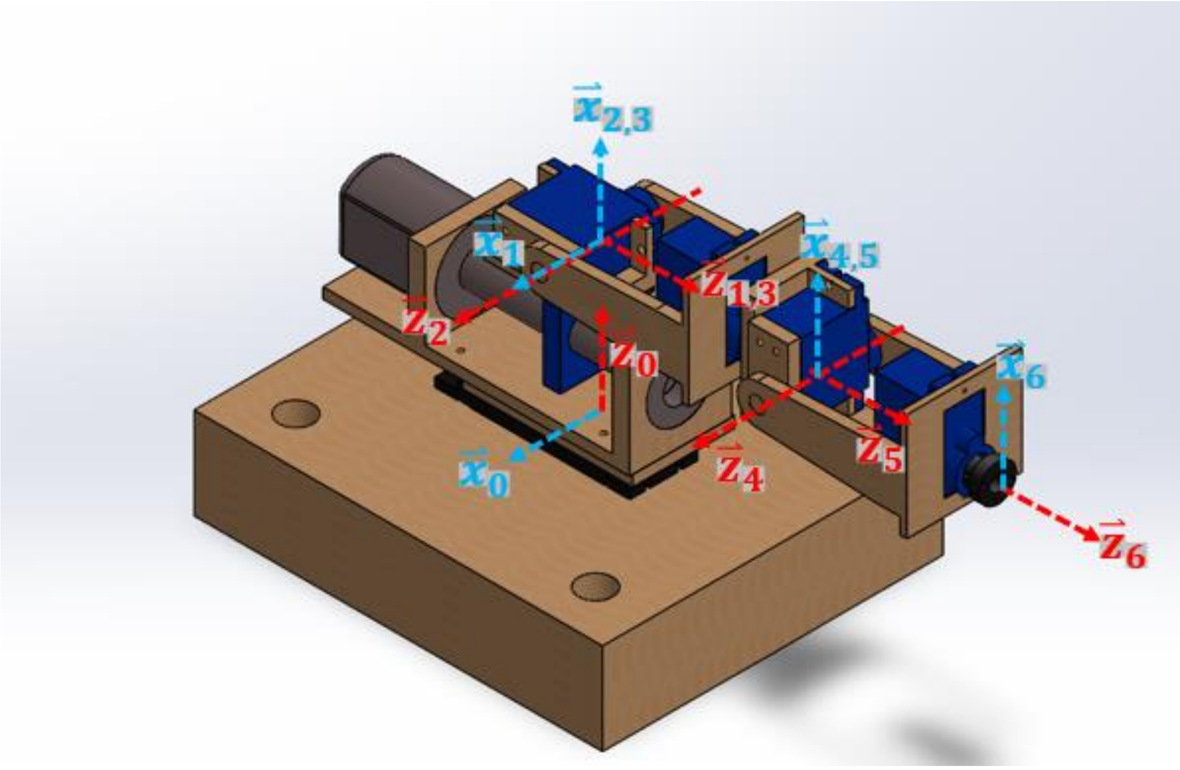
Where the rotation axis of each theta is the correspondent Z axis.

This schematic picture shows the initial position of the robot where all joint variables are equal to zero.

The prismatic joint variable l_2 is not "visually" zero for this representation, but in the CAD model the initial position respects $l_2 = 0$, so the origins 1,2 and 3 are coincident.

Once we move the prismatic joint ($l_2 \neq 0$) the origin 2 and 3 are not longer coincident to origin 1.

The next picture shows the same reference frame assignment, but this time on the CAD model. Note that the reference frames origins 1, 2 and 3 are coincident.



A more detailed description about this CAD design is given on the "Robot's Structure" section.

DH Parameters

Following the rules for establish the DH parameters from reference frames, we can get.

i	θ_i	d_i	α_i	a_i	q
1	θ_1^*	L_1	$-\pi/2$	0	0
2	$-\pi/2$	l_2^*	$-\pi/2$	0	0
3	θ_3^*	0	$\pi/2$	0	0
4	θ_4^*	L_3	$-\pi/2$	0	0
5	θ_5^*	0	$\pi/2$	0	0
6	θ_6^*	L_4	0	0	0

DGM

The Denavit-Hatenberg method has been used for solving DGM. The complete procedure can be found in annex1. Anyway, the transformations matrices ${}^{i-1}_{i}T$ gives.

$$ {}^{0}_{1}T = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}_{4}T = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{5}_{6}T = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The annex shows also the ${}^{0}_{i}T$ matrices that will help us later to calculate the IDM.

MATLAB DGM Implementation

For the implementation we have created a general DGM function which takes the DH parameters as inputs (including σ) and gives the transformation matrix ${}^{0}_{n}T$ as output.

```
function T0n = GENDGM(sigma,a,alpha,d,theta,q)
```

This has helped us to solve DGM even when we had to make some modifications on the DH parameters (at the beginning of the semester).

The complete implementation can be found in "*GENDGM.m*"

IGM

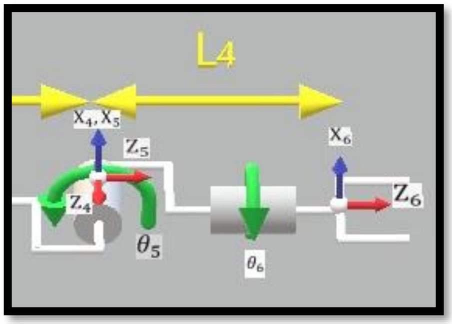
For solving IGM we need to decouple the 6dorf robot.

Decoupled Problem 1 (Solving for q1, q2 and q3)

Given the end effector's orientation matrix and the position vector as inputs.

$$ {}^{0}_{6}P = \begin{bmatrix} a_{06} \\ b_{06} \\ c_{06} \end{bmatrix} \qquad {}^{0}_{6}R = \begin{bmatrix} {}^{0}_{6}\vec{s} & {}^{0}_{6}\vec{n} & {}^{0}_{6}\vec{a} \end{bmatrix} $$

We can obtain the position vector ${}^0_5P$ as follows.

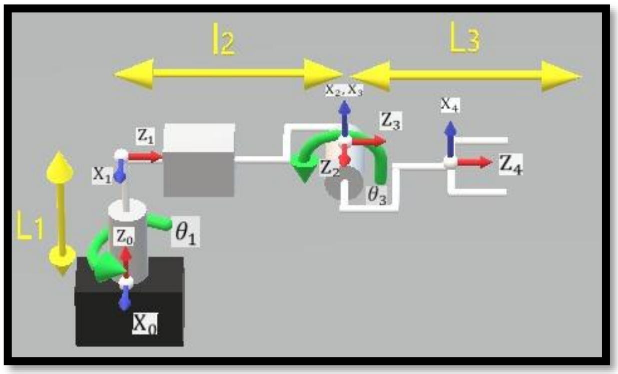


$${}^0_5P = {}^0_6P - {}^0_6\vec{a} \cdot L_4$$

So $${}^0_5P = \begin{bmatrix} a_{06} - {}^0_6a_xL_4 \\ b_{06} - {}^0_6a_yL_4 \\ c_{06} - {}^0_6a_zL_4 \end{bmatrix}$$

Note that the position vector ${}^0_5P$ is the same as ${}^0_4P$.

Knowing ${}^0_4P$, we can solve a decoupled problem as shown.



It is important to note that θ_4 does not affect the position of frame 4.

So, we are going to be interested on the positioning of the robot with this first decoupled problem.

Furthermore, the frames are the same as before.

For this RPR robot the end effector is given by $\quad {}^0_4T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T \cdot {}^3_4T$

From Paul's method we can reorganize the previous equations as (see anexe1)

$${}^1_0T \cdot {}^0_4T = {}^1_2T \cdot {}^2_3T \cdot {}^3_4T$$

Where ${}^0_4P$ is known.

$$\begin{bmatrix} C\theta_1 & S\theta_1 & 0 & 0 \\ 0 & 0 & -1 & L_1 \\ -S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} . & . & . & a_{06} - {}^0_6a_xL_4 \\ . & . & . & b_{06} - {}^0_6a_yL_4 \\ . & . & . & c_{06} - {}^0_6a_zL_4 \\ . & . & . & 1 \end{bmatrix} = \begin{bmatrix} S\theta_4 & 0 & C\theta_4 & 0 \\ -C\theta_3C\theta_4 & S\theta_3 & C\theta_3S\theta_4 & -L_3S\theta_3 \\ -C\theta_4S\theta_3 & -C\theta_3 & S\theta_3S\theta_4 & l_2 + L_3C\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Developing and taking only the position vector's equations we get

$$\begin{bmatrix} C\theta_1(a_{06} - {}^0_6a_xL_4) + S\theta_1(b_{06} - {}^0_6a_yL_4) \\ -c_{06} + {}^0_6a_zL_4 + L_1 \\ -S\theta_1(a_{06} - {}^0_6a_xL_4) + C\theta_1(b_{06} - {}^0_6a_yL_4) \end{bmatrix} = \begin{bmatrix} 0 \\ -L_3S\theta_3 \\ l_2 + L_3C\theta_3 \end{bmatrix}$$

Obtaining q(1)

From row 1 we can obtain a type 2 equation:

$$X\,S\theta_1 + Y\,C\theta_1 = Z$$

With $X = b_{06} - {}^0_6a_y L_4$, $Y = a_{06} - {}^0_6a_x L_4$ and $Z = 0$

Since Z is always zero, we have only one possible solution form.

$$\theta_1 = \operatorname{atan2}\left(\frac{-Y}{X}\right)$$

Note that there is another possible solution that requires a negative length of the prismatic joint. So:

$$\theta_1' = \theta_1 + \pi$$

Obtaining q(3)

From row 2 we have again a type 2 equation:

$$c_{06} - {}^0_6a_z L_4 - L_1 = L_3 S\theta_3$$

Since Y is always zero, we have only one possible solution form.

$$S\theta_3 = Z/X$$

$$\theta_3 = \operatorname{atan2}\left(\frac{S\theta_3}{\sqrt{1-(S\theta_3)^2}}\right)$$

With $X = L_3$, $Y = 0$ and $Z = c_{06} - {}^0_6a_z L_4 - L_1$

Note that there is another possible solution (from the square root) so θ_3' is given by the negative square root.

$$\theta_3' = \operatorname{atan2}\left(\frac{S\theta_3}{-\sqrt{1-(S\theta_3)^2}}\right)$$

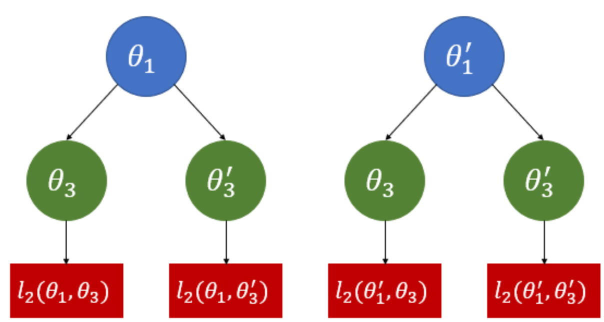
Obtaining q(2)

Taking row 3 and reorganizing for l_2 we get:

$$l_2 = -S\theta_1(a_{06} - {}^0_6a_x L_4) + C\theta_1(b_{06} - {}^0_6a_y L_4) - L_3 C\theta_3$$

l_2 is in function of θ_1 and θ_3, so each possible value may be different.

At this point we have several solutions; it is important to build each one by following the diagram below.



Decoupled Problem 2 (Solving for q4, q5 and q6)

Because we have θ_1, l_2 and θ_3; ${}^0_3T$ is known.

Using the Paul's method, we can say that:

$$ {}^4_3T\,{}^3_0T\,{}^0_6T = {}^4_6T $$

Where ${}^3_0T$ is the inverse matrix of ${}^0_3T$(known). And ${}^0_6T$ the end effector's information (known).

Noting that ${}^3_0T\,{}^0_6T = {}^3_6T$:

$$ {}^3_6R = [{}^3_6\vec{s} \quad {}^3_6\vec{n} \quad {}^3_6\vec{a}] \qquad {}^3_6P = \begin{bmatrix} a_{36} \\ b_{36} \\ c_{36} \end{bmatrix} \quad \text{(Known)} $$

We develop ${}^4_3T\,{}^3_6T$ with the matrices.

$$ {}^4_3T = \begin{bmatrix} C\theta_4 & S\theta_4 & 0 & 0 \\ 0 & 0 & -1 & L_3 \\ -S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } {}^3_6T = \begin{bmatrix} {}^3_6s_x & {}^3_6n_x & {}^3_6a_x & a_{36} \\ {}^3_6s_y & {}^3_6n_y & {}^3_6a_y & b_{36} \\ {}^3_6s_z & {}^3_6n_z & {}^3_6a_z & c_{36} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Obtaining

$$ {}^4_6T = \begin{bmatrix} C\theta_4\,{}^3_6s_x + S\theta_4\,{}^3_6s_y & C\theta_4\,{}^3_6n_x + S\theta_4\,{}^3_6n_y & C\theta_4\,{}^3_6a_x + S\theta_4\,{}^3_6a_y & C\theta_4 a_{36} + S\theta_4 b_{36} \\ -{}^3_6s_z & -{}^3_6n_z & -{}^3_6a_z & L_3 - c_{36} \\ -S\theta_4\,{}^3_6s_x + C\theta_4\,{}^3_6s_y & -S\theta_4\,{}^3_6n_x + C\theta_4\,{}^3_6n_y & -S\theta_4\,{}^3_6a_x + C\theta_4\,{}^3_6a_y & -S\theta_4 a_{36} + C\theta_4 b_{36} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Now, we need to develop the expression of ${}^4_6T$ by doing ${}^4_5T\,{}^5_6T$

$$ {}^4_5T = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^5_6T = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So,

$$ {}^4_6T = \begin{bmatrix} C\theta_5 C\theta_6 & -C\theta_5 S\theta_6 & S\theta_5 & L_4 S\theta_5 \\ S\theta_5 C\theta_6 & -S\theta_5 S\theta_6 & -C\theta_5 & -L_4 C\theta_5 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It is important to note that the matrix ${}^3_0T$ depends on the selected values of θ_1, l_2 and θ_3. So, for the next equations, they must be calculated with the corresponding values of ${}^3_6T$.

Obtaining q(5)

Taking the element (2,3) from both ${}^4_6T$ matrices, we obtain the following equation.

$$-C\theta_5 = -{}^3_6a_z$$

That represents a type 2 equation. With:

$X = 0$ $\quad Y = -1$ $\quad$ and $\quad Z = -{}^3_6a_z$

Following the IGM solutions we have

$$\cos(\theta_5) = Z/Y$$

$$\theta_5 = \text{atan2}\left(\frac{\pm\sqrt{1-\cos(\theta_5)^2}}{\cos(\theta_5)}\right)$$

Note that there are two possible solutions; Y is never zero and $-1 \le Z/Y \le 1$.

Obtaining q(4)

Taking the elements (3,4) from both ${}^4_6T$ matrices, we obtain the following equation.

$$-S\theta_4 a_{36} + C\theta_4 b_{36} = 0$$

That represents a type 2 equation. With:

$X = -a_{36}$ $\quad Y = b_{36}$ $\quad$ and $\quad Z = 0$

For this case we have three different solutions depending on the value of X and Y.

a) If $X = 0$ and $Y \neq 0$
$\cos(\theta_4) = 0$
$$\theta_4 = \text{atan2}\left(\frac{\pm\sqrt{1-\cos(\theta_4)^2}}{\cos(\theta_4)}\right) = \text{atan2}(\pm 1, 0)$$

b) If $X \neq 0$ and $Y = 0$
$\sin(\theta_4) = 0$
$$\theta_4 = \text{atan2}\left(\frac{\sin(\theta_4)}{\pm\sqrt{1-\sin(\theta_4)^2}}\right) = \text{atan2}(0, \pm 1)$$

c) If $X \neq 0$ and $Y \neq 0$
$$\theta_4 = \text{atan2}\left(\frac{-Y}{X}\right)$$
$$\theta_4' = \theta_4 + \pi$$

Obtaining q(6)

Taking the elements (3,1) and (3,2) from both ${}^4_6T$ matrices, we obtain the following system equations.

$$-S\theta_4\,{}^{3}_{6}s_x + C\theta_4\,{}^{3}_{6}s_y = S\theta_6$$
$$-S\theta_4\,{}^{3}_{6}n_x + C\theta_4\,{}^{3}_{6}n_y = C\theta_6$$

That represents a type 3 equation. With:

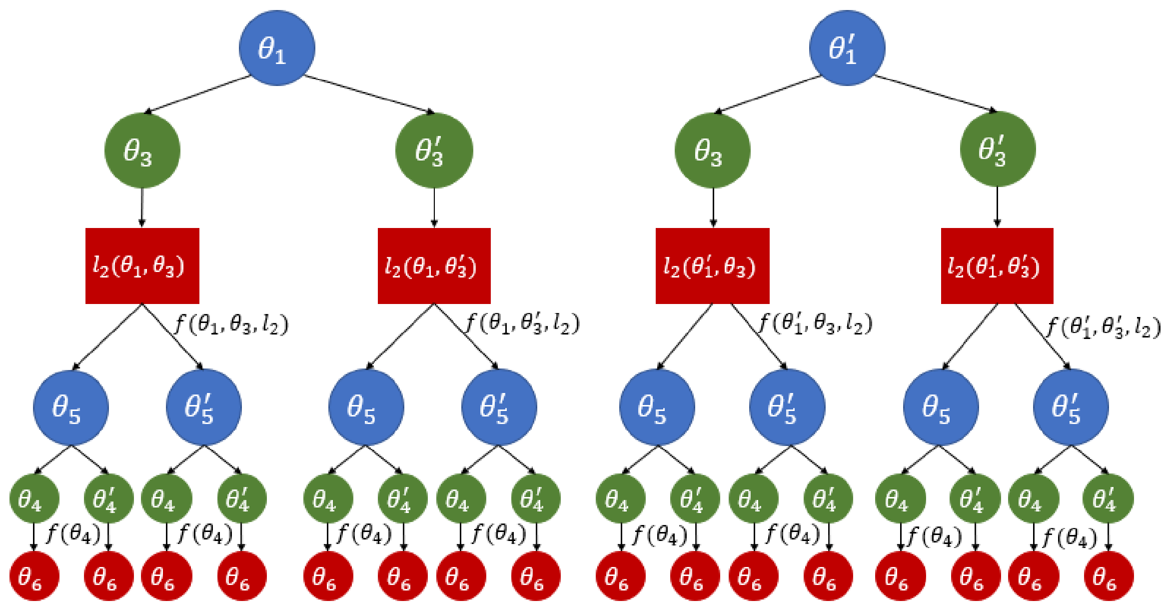
$X_1 = 1$ $\quad Y_1 = 0$ $\quad$ and $\quad Z_1 = -S\theta_4\,{}^{3}_{6}s_x + C\theta_4\,{}^{3}_{6}s_y$

$X_2 = 0$ $\quad Y_2 = 1$ $\quad$ and $\quad Z_2 = -S\theta_4\,{}^{3}_{6}n_x + C\theta_4\,{}^{3}_{6}n_y$

And its solution:

$$\theta_6 = \mathrm{atan2}\left(\frac{Z_1Y_2}{Z_2X_1}\right)$$

The final diagram of the different possible solutions becomes:



MATLAB IGM Implementation

For the implementation we have used the equations found for each joint variable.

The inputs are the position vector and the orientation matrix. The output is the q matrix containing all the combinations of the six joints variables, and additionally, we return a flag that says if the point is reachable.

The global values are the necessary measurements of the robot's design:

```
function [q,ok] = IGM(P06,R06)
    global L1 l2_min L3 L4 l2_max   %Constant values defined in GUI
    %% Output
    ok  = 1;
    q   = zeros(6,16);
    % q = sol1 sol2 sol3 sol4 sol5 sol6 sol8 ... sol16
    % each "sol" is a 6x1 vector
```

Along the IGM function we create some variables for respecting the notation on this document.

Knowing q1, q2 and q3 we built four different transformation matrices ${}^{3}_{6}T$ iteratively to continue with the calculation of q4, q5 and q6.

The complete script can be found in "*IGM.m*"

An additional function "*verifyIGM.m*" is used to eliminate repeated combinations of q; to eliminate non-suitable combinations comparing the resulting DGM with the original input; and to eliminate combinations whose q2 exceeds the joint limits.

The working examples (see GUI section) shows that after using "*verifyIGM.m*" we obtain 4 suitable solutions instead of 16.

Jacobian Matrix

For the calculation of the Jacobian matrix, we need to know the transformation matrices ${}^{0}_{1}T$, ${}^{0}_{2}T$, ${}^{0}_{3}T$, ${}^{0}_{4}T$, ${}^{0}_{5}T$ and ${}^{0}_{6}T$. Using MATLAB to simplify the work we defined each ${}_{k-1}^{\;\;\,0}T$ matrix.

Then, we use the form:

$$ {}^{0}_{6}J_{v}(:,k) = \sigma\ {}_{k-1}^{\;\;\,0}a + \bar{\sigma}\,{}_{k-1}^{\;\;\,0}\hat{a}\left({}^{0}_{6}P - {}_{k-1}^{\;\;\,0}P\right) $$

$$ {}^{0}_{6}J_{\omega}(:,k) = \bar{\sigma}\ {}_{k-1}^{\;\;\,0}a $$

Getting a very long expression for the Jacobian matrix. It can be seen by executing the code "*calculosJacobian.m*".

$$ {}^{0}_{6}J(1{:}6,1) = $$

$$ \begin{bmatrix} -l_2 * C\theta_1 - L_4 * (S\theta_5 * (S\theta_1 * S\theta_4 - C\theta_1 * C\theta_4 * S\theta_3) + C\theta_1 * C\theta_3 * C\theta_5) \\ L4 * (S\theta_5 * (C\theta_1 * S\theta_4 + C\theta_4 * S\theta_1 * S\theta_3) - C\theta_3 * C\theta_5 * S\theta_1) - l_2 * S\theta_1 - L_3 * C\theta_3 * S\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}_{6}^{0}J(1:6,2) = $$

$$\begin{bmatrix} -S\theta_1 \\ C\theta_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$ {}_{6}^{0}J(1:6,3) = $$

$$\begin{bmatrix} S\theta_1 * (L_3 * S\theta_3 + (L_4 * C\theta_5 * S\theta_3 + C\theta_3 * C\theta_4 * S\theta_5)) \\ -C\theta_1 * \left(L_3 * S\theta_3 + L_4 * (C\theta_5 * S\theta_3 + C\theta_3 * C\theta_4 * S\theta_5)\right) \\ L_3 * C\theta_3 + L_4 * C\theta_3 * C\theta_5 - L_4 * C\theta_4 * S\theta_3 * S\theta_5 \\ C\theta_1 \\ S\theta_1 \\ 0 \end{bmatrix}$$

$$ {}_{6}^{0}J(1:6,4) = $$

$$\begin{bmatrix} L_4 * S\theta_5 * (C\theta_1 * C\theta_4 - S\theta_1 * S\theta_3 * S\theta_4) \\ L_4 * S\theta_5 * (C\theta_4 * S\theta_1 + C\theta_1 * S\theta_3 * S\theta_4) \\ -L_4 * C\theta_3 * S\theta_4 * S_5 \\ -C\theta_1 * C\theta_3 \\ S\theta_3 \end{bmatrix}$$

$$ {}_{6}^{0}J(1:6,5) = $$

$$\begin{bmatrix} L_4 * (C\theta_1 * C\theta_5 * S\theta_4 + C\theta_3 * S\theta_1 * S\theta_5 + C\theta_4 * C\theta_5 * S\theta_1 * S\theta_3) \\ -L_4 * (C\theta_1 * C\theta_3 * S\theta_5 - C\theta_5 * S\theta_1 * S\theta_4 + C\theta_1 * C\theta_4 * C\theta_5 * S\theta_3) \\ L_4 * C\theta_3 * C\theta_4 * C\theta_5 - L_4 * S\theta_3 * S\theta_5 \\ C\theta_1 * C\theta_4 - S\theta_1 * S\theta_3 * S\theta_4 \\ C\theta_4 * S\theta_1 + C\theta_1 * S\theta_3 * S\theta_4 \\ -C\theta_3 * S\theta_4 \end{bmatrix}$$

$$ {}_{6}^{0}J(1:6,6) = $$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ S\theta_5 * (C\theta_1 * S\theta_4 + C\theta_4 * S\theta_1 * S\theta_3) - C\theta_3 * C\theta_5 * S\theta_1 \\ S\theta_5 * (S\theta_1 * S\theta_4 - C\theta_1 * C\theta_3) + C\theta_1 * C\theta_3 * C\theta_5 \\ C\theta_5 * S\theta_3 + C\theta_3 * C\theta_4 * S\theta_5 \end{bmatrix}$$

Anyway, the determinant gives the expression:

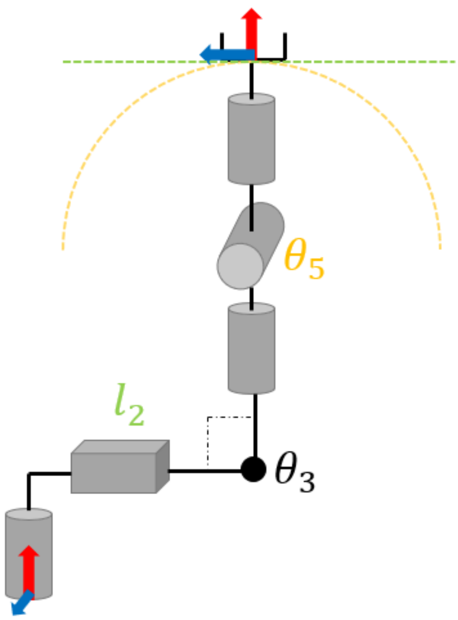
$$\det({}^{0}_{6}J) = L_3 C\theta_3 S\theta_5 (l_2 + L_3 C\theta_3)$$

Robot Singularities

The singularities can be found by equalizing the Jacobian determinant to zero. From the previous expression we have three different cases:

1. When $C\theta_3 = 0$

In this case the singularities are given by the angles $\theta_3 = \pm 90$.

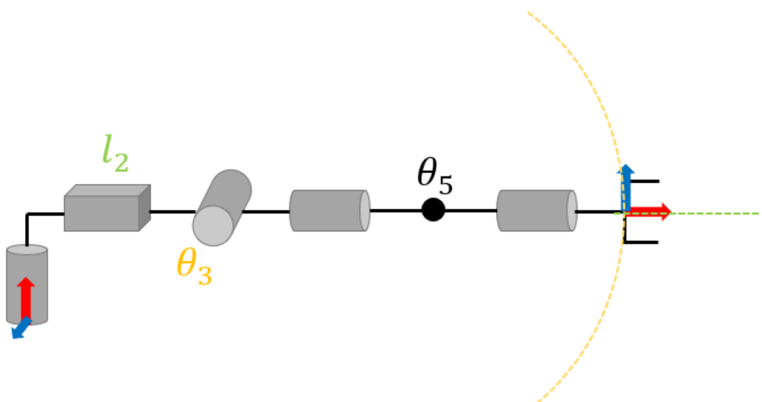


In this position ($\theta_3 = 90$), the movement of the end effector on the $\overrightarrow{z_0}$ direction is not independent. If we try to move it, the end effector will move also in another axis (either X or Y, or both).

Furthermore, the movement on the positive $\overrightarrow{z_0}$ direction is not reachable (on the negative direction for $\theta_3 = -90$).

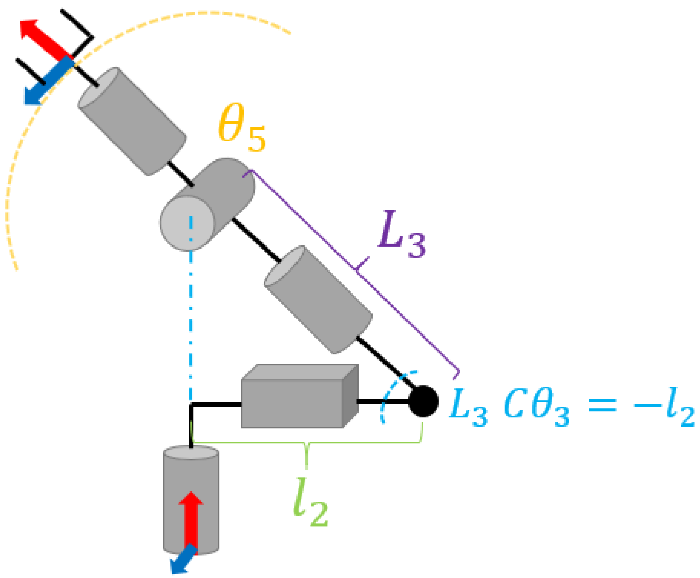
2. When $S\theta_5 = 0$

In this case the singularities are given by the angles $\theta_5 = \pm 180$ and $\theta_5 = 0$



Here we have the same phenomena as explained on the previous case, but now the movement of the end effector is not independent on the $\overrightarrow{z_1}$ direction.

3. When $l_2 + L_3 C\theta_3 = 0$



This last case is more complex and is given when the origins of the reference frames 4 and 5 are aligned with the $\overrightarrow{z_0}$ axis.

The velocity at that point is infinite with respect to the first joint since it does not affect the position of the wrist.

MATLAB Jacobian implementation

Even if we have the equations to calculate the 6x6 Jacobian matrix. We decided to code a general Jacobian matrix receiving the DH Parameters (including q) and the sigma vector as inputs.

This complete code can be found in "*Jacobian.m*" but the principal part is shown:

```
for k=1:1:6

    % Notation:     k_1 = k-1
    T0k_1    = cell2mat(T0n(k)); % We take transformation from DGM cell array
    a0k_1    = T0k_1(1:3,3);     % a is the Z components vector
    P0k_1    = T0k_1(1:3,4);     % P is the position vector

    S_a      = [ 0           -a0k_1(3)   a0k_1(2)    ;   % skew matrix
                 a0k_1(3)    0           -a0k_1(1)   ;
                 -a0k_1(2)   a0k_1(1)    0           ];

    J06_v(:,k)   = sigma(k)*a0k_1 + not(sigma(k))*S_a*(P06-P0k_1);
    J06_w(:,k)   = not(sigma(k))*a0k_1;

end

% Jacobian 6x6 matrix
J = [J06_v;J06_w];
```

RIGM

A recursive IGM function has been written to find the closest "q" vector from an initial "q_{init}". Nevertheless, it only takes into consideration the positioning of the end effector.

The function uses the previous created functions "GENDGM.m" and "Jacobian.m". It takes the desired final position and the initial "q_{init}" position as inputs and gives the "q" final configuration as output.

```
function q = RIGM(X,q_ini)
```

This script can be found in "*RIGM.m*".

Straight Line trajectory using RIGM

To compute a straight-line trajectory, we need to define a 3D line equation given the initial point (x_i, y_i, z_i) and the final point (x_f, y_f, z_f). To do so, we use a parametric equation:

$$\begin{cases} x = x_i + t \cdot \Delta x \\ y = y_i + t \cdot \Delta y \\ z = z_i + t \cdot \Delta z \end{cases}$$

Dividing the path into equal parts we can generate a set of coordinates within the line's equation.

We consider 10 equal parts (modifiable in the code), so a step can be defined by dividing Δx into 10. And the coordinates y and z are in function of the set of x. This set of x goes from x_i to x_f with a step of $\Delta x/10$.

The parametric equation becomes:

$$\begin{cases} t(x) = (x - x_i)/\Delta x \\ y = y_i + t(x) \cdot \Delta y \\ z = z_i + t(x) \cdot \Delta z \end{cases}$$

The coordinates (x_n, y_n, z_n) are obtained iteratively and we use the RIGM to obtain each "q" vector for those coordinates. The function "*StraightLine3D.m*" computes the complete procedure from an initial and final "q" as inputs, and returns the set of q, x, y and z within the line's equation.

```
function [q,x,y,z] = StraightLine3D(q_ini, q_end)
```

IDM

The computation of the IDM will help us to determine the joint torques needed to maintain the robot in a certain position, this considers the weight of the links, each center of mass and the applied force at the end effector.

The Inverse Dynamic Model for a static position is given by the equation:

$$\Gamma = Q(q) + J^T f_e$$

Where:

- J is the 6x6 Jacobian matrix.
- f_e the 6x1 applied force vector on the end effector.
- $Q(q)$ a 6x1 vector in function of the desired position.

At the same time, we obtain $Q(q)$ from the equation:

$$Q_k = \frac{\partial P}{\partial q_k}$$

Where:

$$P = \sum_{k=1}^{n} m_k g^T r_k$$

For our case, the gravity is pointing on the $-\overrightarrow{z_0}$ direction, so:

$$g^T = [0 \quad 0 \quad -G]$$

$$G = 9.81\, m/s^2$$

And the r_k is given by:

$$r_k = {}^0_kT\,{}^k_kr(1:3)$$

Note that ${}^k_kr$ are the coordinates of the center of mass of the link "k" with respect to the reference frame "k". The ${}^0_kT$ matrices can be found in annex1.

- For $k = 1$

$${}^0_1T\,{}^1_1r = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \rightarrow {}^0_1T\,{}^1_1r(1:3) = r_1 = \begin{bmatrix} \cdot \\ \cdot \\ -y_1 + L_1 \end{bmatrix}$$

Then

$$m_1 g^T r_1 = m_1 [0 \quad 0 \quad -G] \begin{bmatrix} \cdot \\ \cdot \\ -y_1 + L_1 \end{bmatrix} = -m_1 G(-y_1 + L_1)$$

- For $k = 2$

$${}^0_2T\,{}^2_2r = \begin{bmatrix} 0 & S\theta_1 & C\theta_1 & -l_2S\theta_1 \\ 0 & -C\theta_1 & S\theta_1 & l_2C\theta_1 \\ 1 & 0 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} \rightarrow {}^0_2T\,{}^2_2r(1:3) = r_2 = \begin{bmatrix} \vdots \\ x_2 + L_1 \end{bmatrix}$$

Then

$$m_2g^Tr_2 = m_2\begin{bmatrix} 0 & 0 & -G \end{bmatrix}\begin{bmatrix} \vdots \\ x_2 + L_1 \end{bmatrix} = -m_2G(x_2 + L_1)$$

- For $k = 3$

$${}^0_3T\,{}^3_3r = \begin{bmatrix} S\theta_1S\theta_3 & C\theta_1 & -C\theta_3S\theta_1 & -l_2S\theta_1 \\ -C\theta_1S\theta_3 & S\theta_1 & C\theta_1C\theta_3 & l_2C\theta_1 \\ C\theta_3 & 0 & S\theta_3 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix}$$

$${}^0_3T\,{}^3_3r(1:3) = r_3 = \begin{bmatrix} \vdots \\ C\theta_3x_3 + S\theta_3z_3 + L1 \end{bmatrix}$$

Then

$$m_3g^Tr_3 = m_3\begin{bmatrix} 0 & 0 & -G \end{bmatrix}\begin{bmatrix} \vdots \\ x_2 + L_1 \end{bmatrix} = -m_3G(C\theta_3x_3 + S\theta_3z_3 + L1)$$

- For $k = 4$

$${}^0_4T\,{}^4_4r = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ C\theta_3C\theta_4 & -S\theta_3 & -C\theta_3S\theta_4 & L_1 + L_3S\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_4 \\ y_4 \\ z_4 \\ 1 \end{bmatrix}$$

$${}^0_4T\,{}^4_4r(1:3) = r_4 = \begin{bmatrix} \vdots \\ C\theta_3C\theta_4x_4 - S\theta_3y_4 - C\theta_3S\theta_4z_4 + L1 + L_3S\theta_3 \end{bmatrix}$$

Then

$$m_4g^Tr_4 = m_4\begin{bmatrix} 0 & 0 & -G \end{bmatrix}\begin{bmatrix} \vdots \\ C\theta_3C\theta_4x_4 - S\theta_3y_4 - C\theta_3S\theta_4z_4 + L1 + L_3S\theta_3 \end{bmatrix}$$

$$m_4g^Tr_4 = -m_4G(C\theta_3C\theta_4x_4 - S\theta_3y_4 - C\theta_3S\theta_4z_4 + L1 + L_3S\theta_3)$$

- For $k = 5$

$${}^0_5T\,{}^5_5r = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ C\theta_3C\theta_4C\theta_5 - S\theta_3S\theta_5 & -C\theta_3S\theta_4 & C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5 & L_1 + L_3S\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_5 \\ y_5 \\ z_5 \\ 1 \end{bmatrix}$$

$$ {}^0_5T\,{}^5_5r(1:3) = r_5 $$

$$ = \begin{bmatrix} \vdots \\ x_5(C\theta_3C\theta_4C\theta_5 - S\theta_3S\theta_5) - C\theta_3S\theta_4y_5 + z_5(C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5) + L_1 + L_3S\theta_3 \end{bmatrix} $$

Then

$$ m_5g^Tr_5 = -m_5G\big(x_5(C\theta_3C\theta_4C\theta_5 - S\theta_3S\theta_5) - C\theta_3S\theta_4y_5 + z_5(C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5) + L_1 + L_3S\theta_3\big) $$

- For $k = 6$

$$ {}^0_6T\,{}^6_6r = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ -C\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3S\theta_4S\theta_6 & S\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3C\theta_6S\theta_4 & u & v \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_6 \\ y_6 \\ z_6 \\ 1 \end{bmatrix} $$

Where:

$$ u = C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5 $$

$$ v = L_1 + L_4(C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5) + L_3S\theta_3 $$

$$ {}^0_6T\,{}^6_6r(1:3) = r_6 $$

$$ = \begin{bmatrix} \vdots \\ x_6[-C\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3S\theta_4S\theta_6] + y_6[S\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3C\theta_6S\theta_4] + uz_6 + v \end{bmatrix} $$

Then

$$ \begin{aligned} m_6g^Tr_6 = -m_6G\big(&x_6[-C\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3S\theta_4S\theta_6] \\ &+ y_6[S\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3C\theta_6S\theta_4] + z_6[C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5] \\ &+ L_1 + L_4(C\theta_5S\theta_3 + C\theta_3C\theta_4S\theta_5) + L_3S\theta_3\big) \end{aligned} $$

Adding all the obtained terms we would have:

$$ P = \sum_{k=1}^{n} m_kg^Tr_k $$

Finally, we derivate P (partial) to obtain each row of Q.

$$ Q_1 = \frac{\partial P}{\partial q_1} = 0 $$

Since P is not in function of θ_1.

$Q_2 = \frac{\partial P}{\partial q_2} = 0$

Since P is not in function of l_2.

$$
\begin{aligned}
Q_3 = \frac{\partial P}{\partial q_3} = &-m_3G(-S\theta_3x_3 + C\theta_3z_3) - m_4G(-S\theta_3C\theta_4x_4 - C\theta_3y_4 + S\theta_3S\theta_4z_4 + L_3C\theta_3) \\
&- m_5G\big(x_5(-S\theta_3C\theta_4C\theta_5 - C\theta_3S\theta_5) + S\theta_3S\theta_4y_5 + z_5(C\theta_5C\theta_3 - S\theta_3C\theta_4S\theta_5) \\
&+ L_3C\theta_3\big) \\
&- m_6G\big(x_6[-C\theta_6(C\theta_3S\theta_5 + S\theta_3C\theta_4C\theta_5) + S\theta_3S\theta_4S\theta_6] \\
&+ y_6[S\theta_6(C\theta_3S\theta_5 + S\theta_3C\theta_4C\theta_5) + S\theta_3C\theta_6S\theta_4] + z_6[C\theta_5C\theta_3 - S\theta_3C\theta_4S\theta_5] \\
&+ L_4(C\theta_5C\theta_3 - S\theta_3C\theta_4S\theta_5) + L_3C\theta_3\big)
\end{aligned}
$$

$$
\begin{aligned}
Q_4 = \frac{\partial P}{\partial q_4} = &-m_4G(-C\theta_3S\theta_4x_4 - C\theta_3C\theta_4z_4) \\
&- m_5G\left(x_5(-C\theta_3S\theta_4C\theta_5) - C\theta_3C\theta_4y_5 + z_5(-C\theta_3S\theta_4S\theta_5)\right) \\
&- m_6G\Big(x_6[-C\theta_6(C\theta_3S\theta_4C\theta_5) - C\theta_3C\theta_4S\theta_6] \\
&+ y_6[S\theta_6(C\theta_3S\theta_4C\theta_5) - C\theta_3C\theta_6C\theta_4] + z_6[-C\theta_3S\theta_4S\theta_5] + L_4(-C\theta_3S\theta_4S\theta_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
Q_5 = \frac{\partial P}{\partial q_5} = &-m_5G\big(x_5(-C\theta_3C\theta_4S\theta_5 - S\theta_3C\theta_5) + z_5(-S\theta_5S\theta_3 + C\theta_3C\theta_4C\theta_5)\big) \\
&- m_6G\Big(x_6[-C\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3S\theta_4S\theta_6] \\
&+ y_6[S\theta_6(S\theta_3C\theta_5 + C\theta_3C\theta_4S\theta_5)] + z_6[-S\theta_5S\theta_3 + C\theta_3C\theta_4C\theta_5] \\
&+ L_4(-S\theta_5S\theta_3 + C\theta_3C\theta_4C\theta_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
Q_6 = \frac{\partial P}{\partial q_6} = &-m_6G\big(x_6[S\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) - C\theta_3S\theta_4C\theta_6] \\
&+ y_6[C\theta_6(S\theta_3S\theta_5 - C\theta_3C\theta_4C\theta_5) + C\theta_3S\theta_6S\theta_4]\big)
\end{aligned}
$$

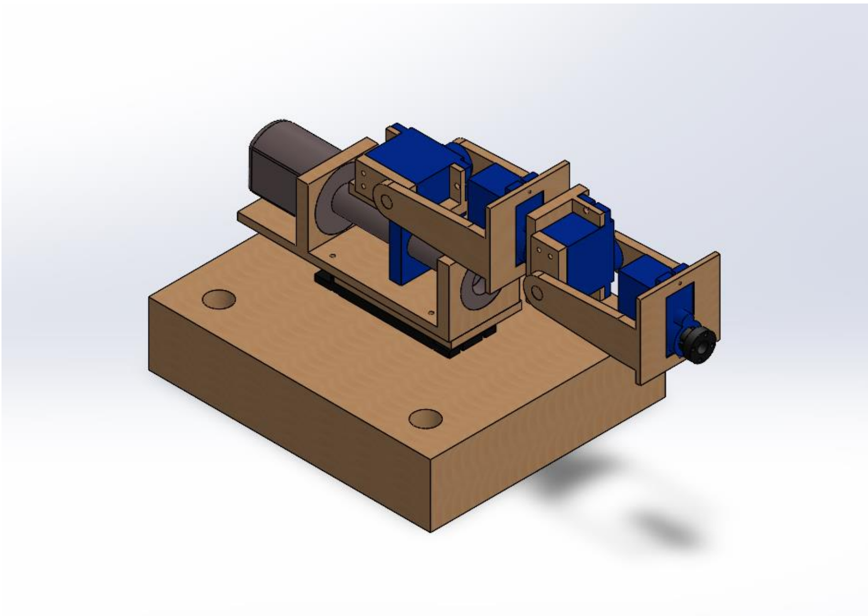
We remember that the vector force applied to the end effector is an input. And the Jacobian script has already been written. So, the torque needed is:

$$\Gamma = [Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6]^T + J^T f_e$$

The implementation can be found in "*IDM.m*".

ROBOT’S STRUCTURE

The robot has been designed considering the axes alignments defined at the beginning of this report. From this model we have obtained the constant lengths L_1, L_3, L_4 and the l_2 limits.



$$L_1 = 48mm$$

$$L_3 = 64mm$$

$$L_4 = 54mm$$

$$l_{2_{max}} = 22mm$$

$$l_{2_{min}} = 22mm$$

Parameters for IDM

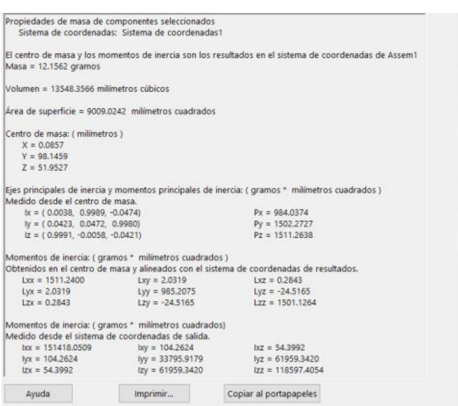
The CAD model has provided us important information about each link mass and its center of mass. We show the elements which has been considered to obtain the information:

- Joint 6

Since we do not have a gripper added to the end effector, the mass is equal to cero and the coordinates of the center of mass are coincident to the frame 6 origin.

This will change in function of the selected gripper for the desired application.

- Joint 5



Propiedades de masa de componentes seleccionados
Sistema de coordenadas: Sistema de coordenadas1

El centro de masa y los momentos de inercia son los resultados en el sistema de coordenadas de Assem1
Masa = 12.1562 gramos

Volumen = 13548.3566 milímetros cúbicos

Área de superficie = 9009.0242 milímetros cuadrados

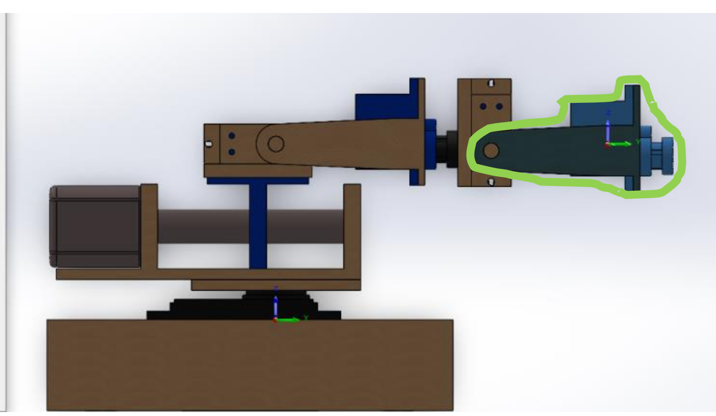
Centro de masa: (milímetros)
X = 0.0857
Y = 98.1459
Z = 51.9527

Ejes principales de inercia y momentos principales de inercia: (gramos * milímetros cuadrados)
Medido desde el centro de masa.
Ix = (0.0038, 0.9989, -0.0474) Px = 984.0374
Iy = (0.0423, 0.0472, 0.9980) Py = 1502.2727
Iz = (0.9991, -0.0058, -0.0421) Pz = 1511.2638

Momentos de inercia: (gramos * milímetros cuadrados)
Obtenidos en el centro de masa y alineados con el sistema de coordenadas de resultados.
Lxx = 1511.2400 Lxy = 2.0319 Lxz = 0.2843
Lyx = 2.0319 Lyy = 985.2075 Lyz = -24.5165
Lzx = 0.2843 Lzy = -24.5165 Lzz = 1501.1264

Momentos de inercia: (gramos * milímetros cuadrados)
Medido desde el sistema de coordenadas de salida.
Ixx = 151418.0509 Ixy = 104.2624 Ixz = 54.3992
Iyx = 104.2624 Iyy = 33795.9179 Iyz = 61959.3420
Izx = 54.3992 Izy = 61959.3420 Izz = 118597.4054

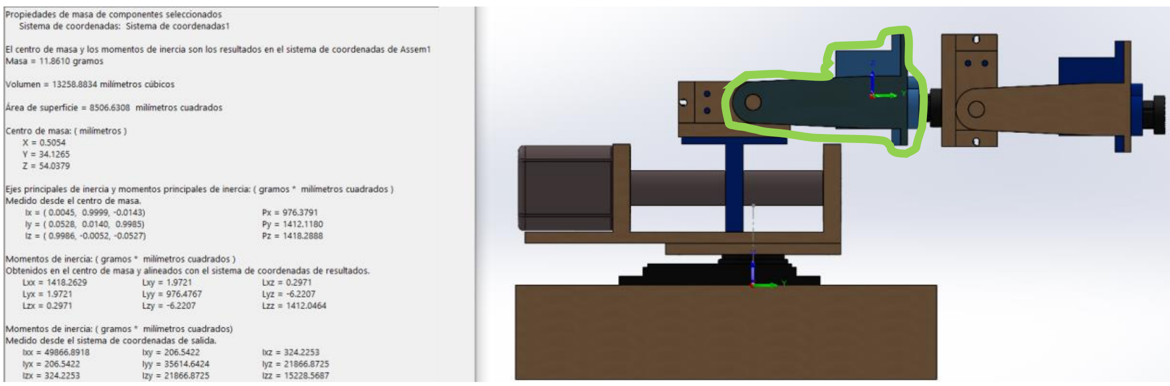
Ayuda Imprimir... Copiar al portapapeles



Note that the coordinates of the center of mas are given with respect to the global reference frame [0]. The necessary adjustments are made later to reference them to the corresponding reference frames.

- Joint 4

- Joint 3



- Joint 2 & 1

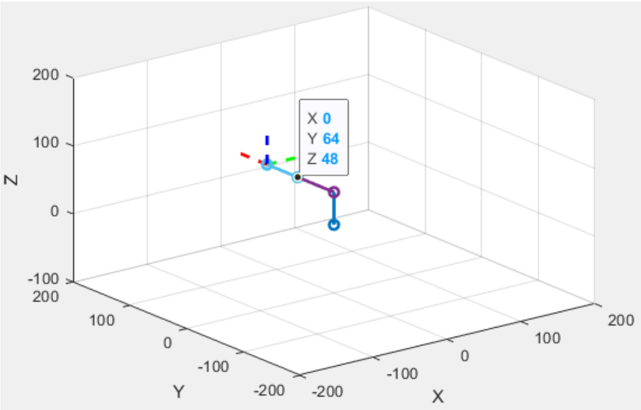
From the "IDM" section we have seen that the properties of the links 1 and 2 are not relevant to calculate the joint torques (at least for a static position).

Resume:

Joint (k)	**Mass (Kg)**	${}^{0}_{k}\boldsymbol{r}(\boldsymbol{mm})$
3	0.0118	[0.505, 34.126, 54.037]
4	0.0116	[-1.201, 62.630, 53.923]
5	0.0121	[0.085, 98.145, 51.952]

Measurements Adaptation

Let us keep in mind that the origins of the reference frames 4 and 5 are coincident. The coordinates on the initial position are:



Furthermore, the orientation of the frame 4 is:

- x4 = z0
- y4 = -y0
- z4 = x0

And for the frame 5:

- x5 = z0
- y5 = x0
- z5 = y0

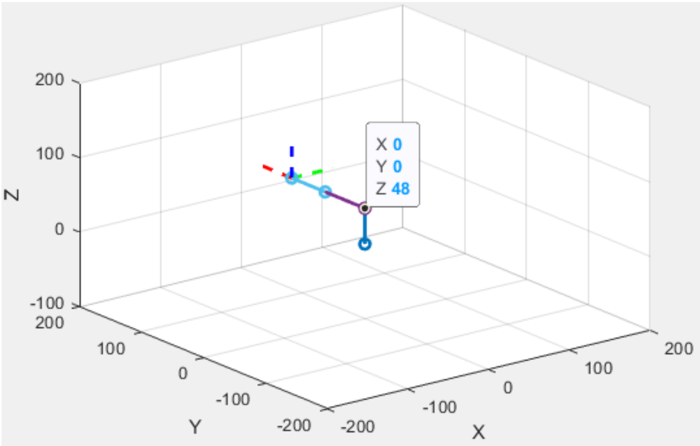
So,

$${}^{5}_{5}\mathrm{r} = [51.952 - 48 \quad 0.085 - 0 \quad 98.145 - 64]^T = [3.952 \quad 0.085 \quad 34.145]^T$$

And

$${}^{4}_{4}\mathrm{r} = [53.923 - 48 \quad 64 - 62.630 \quad -1.201 - 0]^T = [5.923 \quad 1.369 \quad -1.201]^T$$

We use the same logic for the Joint 3.



The orientation of the frame 3 is:

- x3 = z0
- y3 = x0
- z3 = y0

So,

$${}^{3}_{3}\mathrm{r} = [54.037 - 48 \quad 0.505 - 0 \quad 34.126 - 0]^T = [6.037 \quad 0.505 \quad 34.126]^T$$

Materials and components of the model

To obtain the properties of mass we had to define all the materials in SolidWorks. Since it is an own design, the materials are only the shown below:

Material	*Parts*
Plastic	Servomotors and DC motor.
ABS Plastic	Servomotor’s accessories.
Pinewood	Links and supports for servomotors.
Aluminum alloy	Screws and endless screw.

Note: The plastic’s density of the servomotors/DC motor has been selected observing the resulting weight in order to approach to the servomotor’s/DC motor’s weight.

- Servomotor MG92B (Joints 3,4,5 and 6)

Weight: 13.8g
Dimension: 22.8x12x31 mm
Stall torque: 3.1kg.cm (5.0v); 3.5kg.cm (6.0v)
Operating speed: 0.13sec/60degree (5.0v);0.08sec/60degree (6.0v)

- Servomotor MG995 (Joint 1)

Weight: 55 g
Dimension : 40.7 x 19.7 x 42.9 mm approx.
Stall torque: 8.5 kgf.cm (4.8 V); 10 kgf.cm (6 V)
Operating speed: 0.2 s/60º (4.8 V), 0.16 s/60º (6 V)

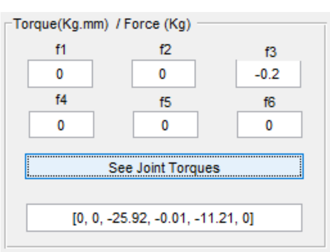
- DC Motor (Joint 2)

Weight: 10 g
Dimension : 1.56cm diameter.
Load: 0.1 mN.m to 0.98 mN.m.
Operating speed: <15000 rpm
Operating voltage: 1V to 7V

The plans of the other parts can be found in annex 2.

Required torque for the demanded load & Workspace

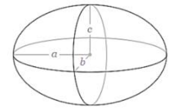
The critical position of the robot to support the 200gr load is when all the rotational joints are equal to zero and the prismatic joint is in the positive limit. Using the GUI interface (see GUI section), we get the maximum torque needed on the third joint.



2.59kg.cm is less than the maximum 3.1kg.cm provided by the servomotor. So, we can guarantee the performance with the 200gr load.

For calculate the workspace we consider no collisions, and the volume is given by two half ellipses, since “c” is not the same up and down.

$$V = \frac{4}{3}\pi a b c$$



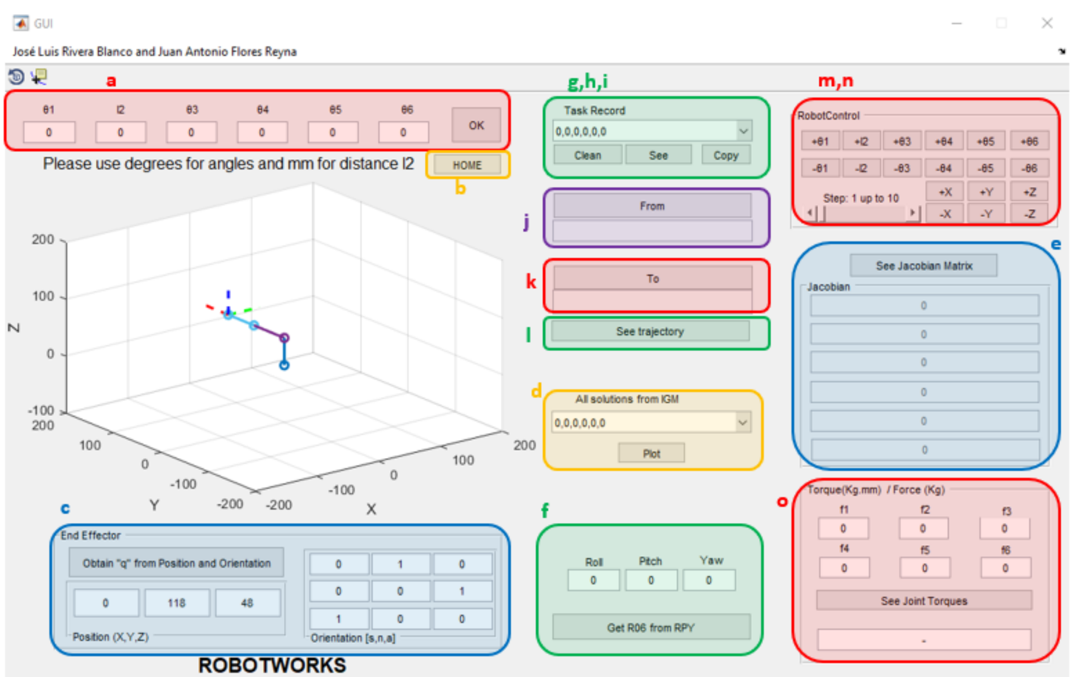
Having $a = b = 14cm, c_1 = 16.6cm, c_2 = 7cm$ we obtain $V = 9687cm^3$

GUI

The GUI interface allow us to plot the links between reference frames and the end-effectors position and orientation.

We have added the following functionalities:

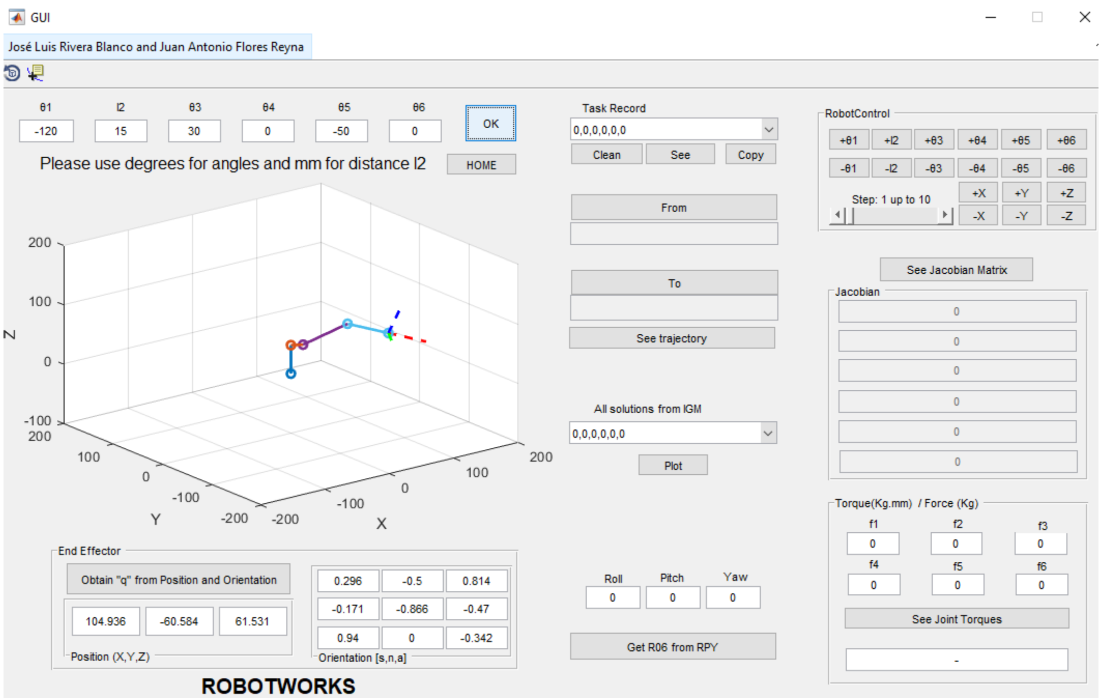
a) Plot DGM and get “T06” from “q” (OK button).
b) Return to the initial position (Home button).
c) Plot IGM and get “q” from “T06” (End Effector’s button).
d) Show other solutions from IGM (all solutions menu) and plot them (Plot button).
e) Show the Jacobian Matrix of the current “q” (See Jacobian Matrix button).
f) Set R06 from Roll Pitch Yaw representation (Roll Pitch Yaw button).
g) A Task Record Menu.
h) Show the selected task on the plot (See button).
i) Copy the selected task on the joint variables text boxes (Copy button).
j) Set the selected task in menu as the initial position in a trajectory (From button).
k) Set the selected task in menu as the final position in a trajectory (To button).
l) Move the robot to follow a straight-line trajectory (See trajectory button).
m) Incremental/Decremental control of the joints with a selected step.
n) Incremental/Decremental control of the x, y, and z positions with a selected step.
o) Get joint torques from an applied force at the end effector (See joint torques button).



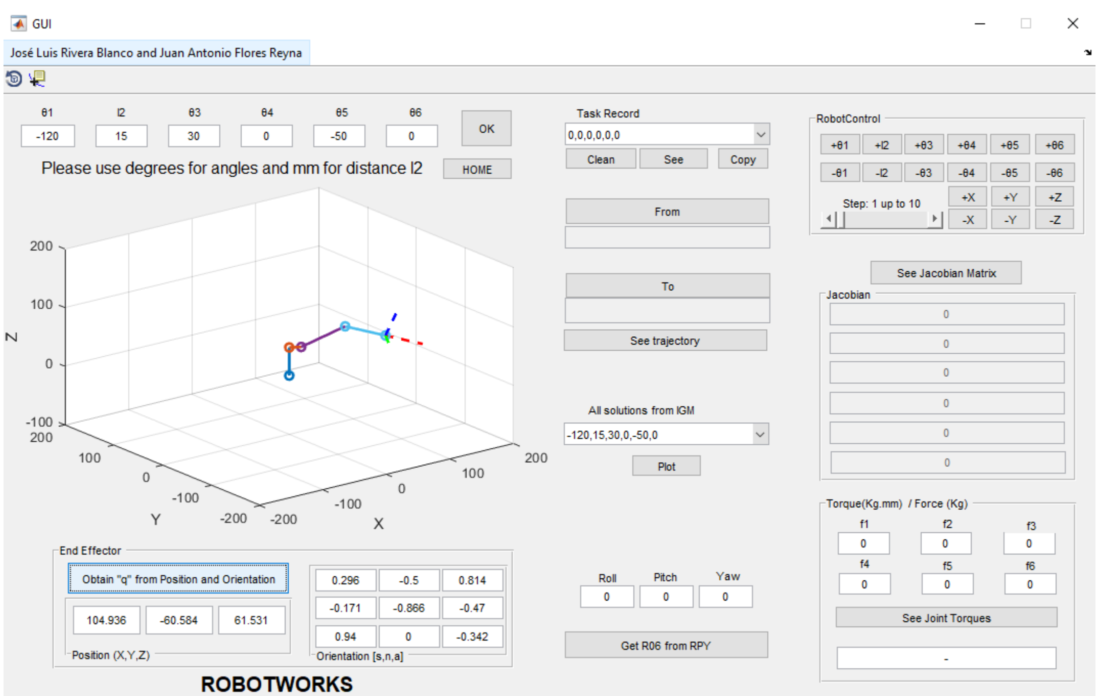
The code of this interface can be found in "*GUI.m*".

GUI Tests

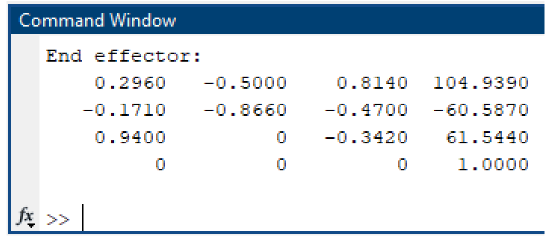
DGM. By setting joint variables and pushing the "OK" button we should observe the new links positions and orientations. Also, we get the end effector's transformation matrix.



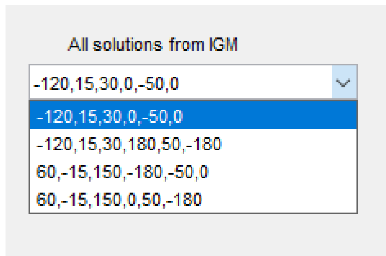
We prove the IGM by clicking the End Effector's button (the plot must be the same):



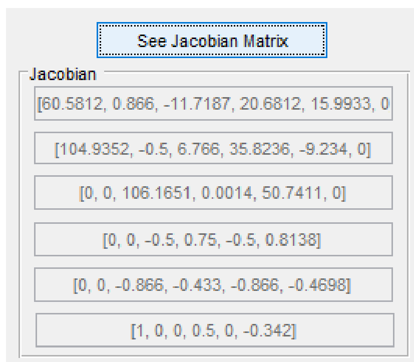
Furthermore, it prints on the Command Window the result of the DGM from the “q” vector obtained from IGM. We observe the same position and orientation with the first possible combination of “q”.



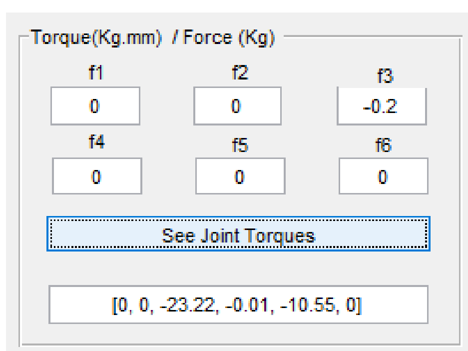
“Other Solutions” shows the other combinations. We obtain the same graph when we click the plot button.



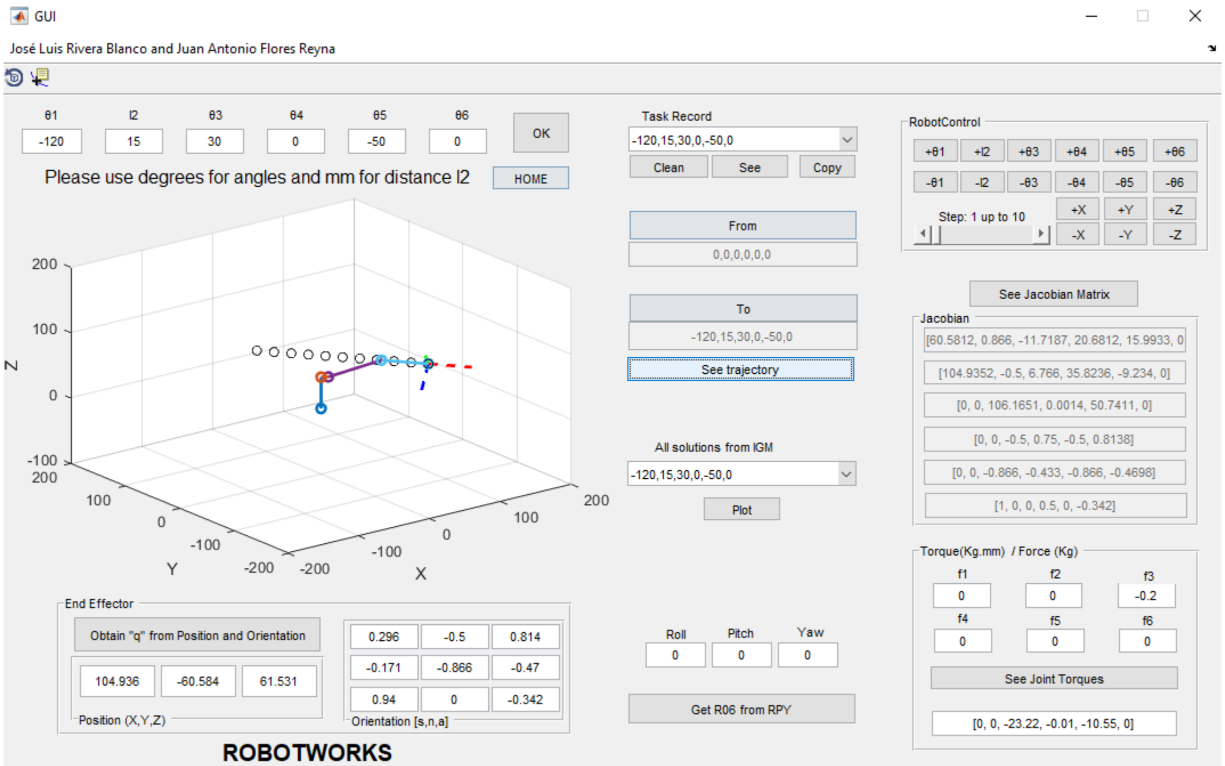
We press “See Jacobian Matrix”, and we get:



Adding a force of 0.2 Kg on the -z0 direction we get the following joint torques for the current position.



Finally, we use the task record menu to create a trajectory between the initial position and the current position:



It is possible to see the movement of the robot when executing the GUI interface since each point is plotted with the corresponding joint variables, a delay is used to perceive changes between two points.

In the same way the robot moves instantly when using the Robot Control Panel.

ANNEXES

Annex1

Given the DH Parameters

i	θ_i	d_i	α_i	a_i
1	θ_1^*	L_1	$-\pi/2$	0
2	$-\pi/2$	l_2^*	$-\pi/2$	0
3	θ_3^*	0	$\pi/2$	0
4	θ_4^*	L_3	$-\pi/2$	0
5	θ_5^*	0	$\pi/2$	0
6	θ_6^*	L_4	0	0

We can develop ${}^{i-1}_{i}T$ following the Denavit-Hatenberg method.

$$
{}^{0}_{1}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & 0 \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}_{6}T = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Doing $\quad {}^{1}_{4}T = {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot {}^{3}_{4}T$

$${}^{1}_{4}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}_{4}T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -C\theta_3 & 0 & -S\theta_3 & 0 \\ -S\theta_3 & 0 & C\theta_3 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}_{4}T = \begin{bmatrix} S\theta_4 & 0 & C\theta_4 & 0 \\ -C\theta_3 C\theta_4 & S\theta_3 & C\theta_3 S\theta_4 & -L_3 S\theta_3 \\ -C\theta_4 S\theta_3 & -C\theta_3 & S\theta_3 S\theta_4 & l_2 + L_3 C\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The values in green were used in the "decoupled problem 1" section.

The ${}^{0}_{i}T$ matrices are given by:

$${}^{0}_{2}T = {}^{0}_{1}T \cdot {}^{1}_{2}T = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & S\theta_1 & C\theta_1 & -l_2 S\theta_1 \\ 0 & -C\theta_1 & S\theta_1 & l_2 C\theta_1 \\ 1 & 0 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{3}T = {}^{0}_{2}T \cdot {}^{2}_{3}T = \begin{bmatrix} 0 & S\theta_1 & C\theta_1 & -l_2 S\theta_1 \\ 0 & -C\theta_1 & S\theta_1 & l_2 C\theta_1 \\ 1 & 0 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S\theta_1 S\theta_3 & C\theta_1 & -C\theta_3 S\theta_1 & -l_2 S\theta_1 \\ -C\theta_1 S\theta_3 & S\theta_1 & C\theta_1 C\theta_3 & l_2 C\theta_1 \\ C\theta_3 & 0 & S\theta_3 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using MATLAB:

$${}^{0}_{4}T = {}^{0}_{3}T \cdot {}^{3}_{4}T =$$

```
[ C1*S4 + C4*S1*S3,  C3*S1, C1*C4 - S1*S3*S4, - S1*l2 - C3*L3*S1]
[ S1*S4 - C1*C4*S3, -C1*C3, C4*S1 + C1*S3*S4,   C1*l2 + C1*C3*L3]
[            C3*C4,    -S3,           -C3*S4,          L1 + L3*S3]
[                0,      0,                0,                   1]
```

$${}^{0}_{5}T = {}^{0}_{4}T \cdot {}^{4}_{5}T =$$

```
[ C5*(C1*S4 + C4*S1*S3) + C3*S1*S5, C1*C4 - S1*S3*S4, S5*(C1*S4 + C4*S1*S3) - C3*C5*S1, -S1*(l2 + C3*L3)]
[ C5*(S1*S4 - C1*C4*S3) - C1*C3*S5, C4*S1 + C1*S3*S4, S5*(S1*S4 - C1*C4*S3) + C1*C3*C5,  C1*(l2 + C3*L3)]
[                C3*C4*C5 - S3*S5,           -C3*S4,                C5*S3 + C3*C4*S5,      L1 + L3*S3]
[                               0,                0,                               0,               1]
```

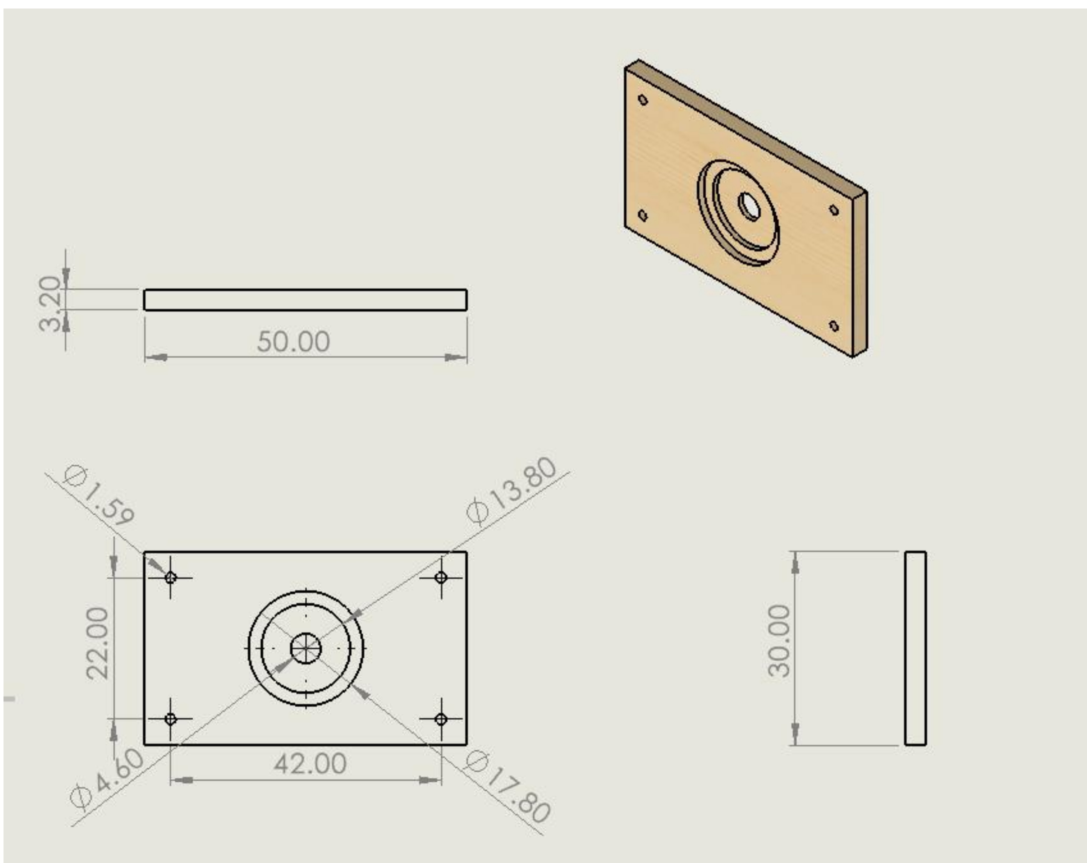
$${}^{0}_{6}T = {}^{0}_{5}T \cdot {}^{5}_{6}T =$$

```
[ S6*(C1*C4 - S1*S3*S4) + C6*(C5*(C1*S4 + C4*S1*S3) + C3*S1*S5),
[ S6*(C4*S1 + C1*S3*S4) + C6*(C5*(S1*S4 - C1*C4*S3) - C1*C3*S5),
[                          - C6*(S3*S5 - C3*C4*C5) - C3*S4*S6,
[                                                            0,
                                                              ...

 C6*(C1*C4 - S1*S3*S4) - S6*(C5*(C1*S4 + C4*S1*S3) + C3*S1*S5),
 C6*(C4*S1 + C1*S3*S4) - S6*(C5*(S1*S4 - C1*C4*S3) - C1*C3*S5),
                           S6*(S3*S5 - C3*C4*C5) - C3*C6*S4,
                                                            0,
                                                              ...

 S5*(C1*S4 + C4*S1*S3) - C3*C5*S1, L4*(S5*(C1*S4 + C4*S1*S3) - C3*C5*S1) - S1*l2 - C3*L3*S1]
 S5*(S1*S4 - C1*C4*S3) + C1*C3*C5, C1*l2 + L4*(S5*(S1*S4 - C1*C4*S3) + C1*C3*C5) + C1*C3*L3]
                 C5*S3 + C3*C4*S5,                        L1 + L4*(C5*S3 + C3*C4*S5) + L3*S3]
                                0,                                                        1]
```

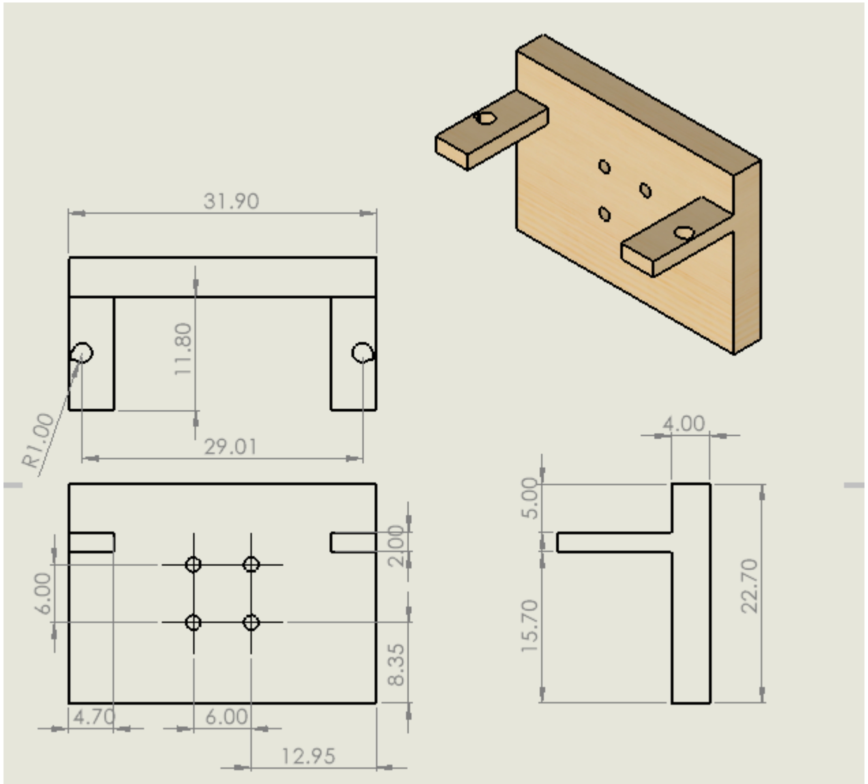
Annex2

Drawings from designed parts:

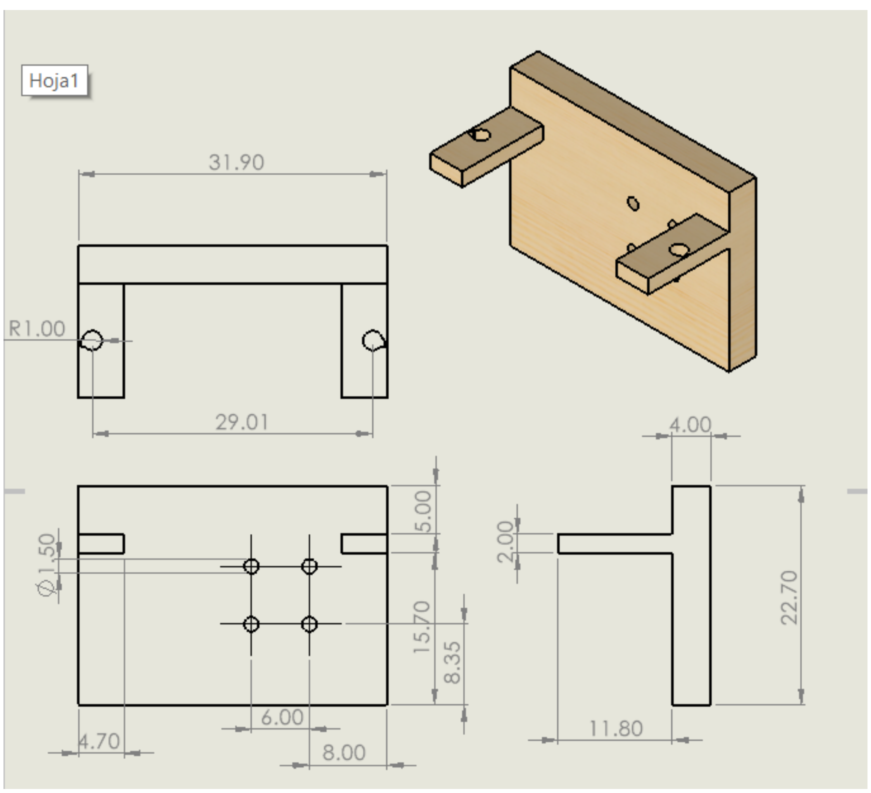
Support 1



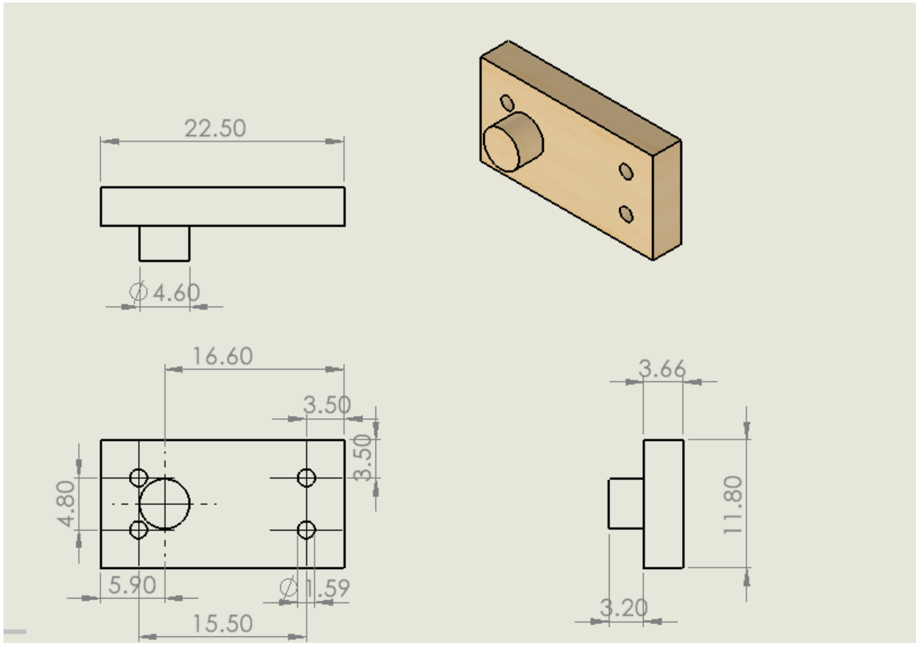
Support 2



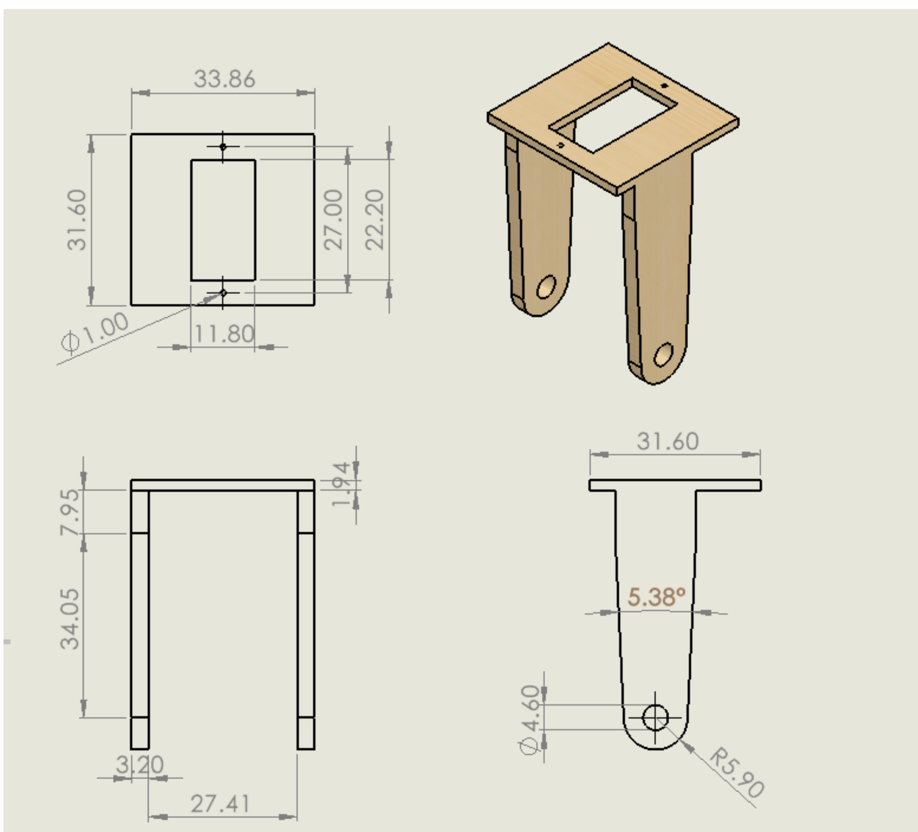
Support 3



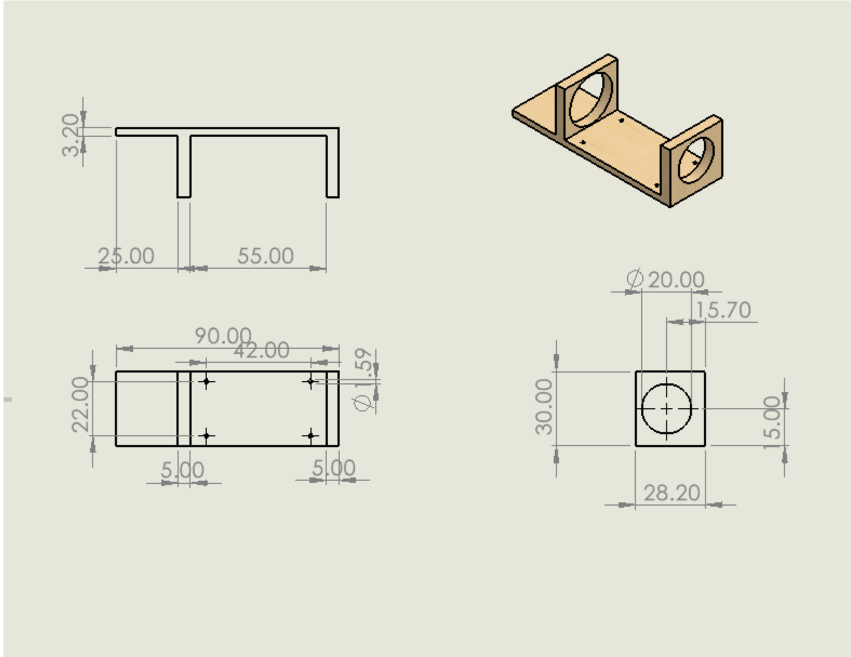
Support 4



Link for rotational joints



Support for prismatic joint



ABS Servomotor accessory