Transmission of Negative Interest Rates: Reversal or Amplification?

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Abstract

Negative monetary policy rates have been introduced in various advanced economies since the mid 2010s. Previous studies have shown that banks are hesitant to set negative deposit rates, implying losses in deposit taking that erode equity and eventually have a negative impact on the lending of capital constrained banks. I show that when banks are not constrained by their equity, equilibrium loan rates are lower under negative interest rates in the presence of a deposit ZLB (D-ZLB) than in its absence. Thus, policy rate cuts in negative territory might stimulate the economy even more than in positive territory, provided that sufficiently many banks are not capital constrained. In a calibrated dynamic model, the effect is large and dominates the effect due to equity erosion, with the D-ZLB increasing aggregate loan supply by on average 4% when policy rates are negative.

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1 Introduction

Negative monetary policy rates, long thought to be impossible in practice, have become a reality in many advanced economies since the mid 2010s. The ECB's monetary policy rate, for example, was zero or negative for a decade, with a trough of -0.5%. Banks play a central role in the transmission of monetary policy, and it is of prime interest for central banks to thoroughly study the impact of negative interest rate policy (NIRP) on banks' lending.

A recurrent finding of empirical studies is banks' hesitance to set negative deposit rates, which suggest the existence of a deposit zero lower bound (*D-ZLB*).[1] With such a bound, conventional wisdom suggests that negative interest rate policy can be less expansionary than conventional monetary policy (Ulate, 2021) or even contractionary (Abadi et al., 2023). As banks receive negative rates on their reserves from the central bank, but still have to offer a zero interest rate to their customers, they make losses in their deposit-taking. This in turn erodes their equity and eventually forces them to reduce lending to comply with regulatory capital requirements.

However, empirical evidence regarding the impact of the D-ZLB on lending is mixed: Some studies find that banks more exposed to negative policy rates increase loan supply vis-a-vis less exposed banks (Hong and Kandrac, 2021; Demiralp et al., 2021; Bottero et al., 2022; Schelling and Towbin, 2022), others find the opposite result (Heider et al., 2019; Basten and Mariathasan, 2023; Eggertsson et al., 2024).[2] But then, how do negative interest

[1] This has been documented in different contexts by Eggertsson et al. (2024); Heider et al. (2019); Hong and Kandrac (2021) and Basten and Mariathasan (2023).

[2] Measures of exposure to negative interest rates differ in the empirical literature. Hong and Kandrac (2021) use the cross-sectional variation in bank's share prices around the central bank's announcement of NIRP. Bottero et al. (2022) use the ex-ante interbank position and liquid assets. Heider et al. (2019) use the amount of deposits relative to total assets. Demiralp et al. (2021) use both central bank reserves in excess of regulatory requirements and retail deposits relative to total assets. Basten and Mariathasan (2023) exploit tiered remuneration of reserves to construct the share of a bank's reserves affected by NIRP. Schelling and Towbin (2022) use both the measures of Heider

rates affect lending by commercial banks? This paper develops a quantitative model in which two different forces interact in opposite directions, making the resulting dominant force contingent on parameters and, thus, providing a possible explanation to the divergent findings in the empirical literature.

Specifically, this paper identifies and quantifies the importance of a novel *amplification channel* that affects banks under negative interest rates when (a) they face a D-ZLB, (b) their probability of failure is not zero, and (c) they are not equity-constrained. Through this channel, banks' supply of risky loans increases more strongly after a policy rate cut when their D-ZLB is binding. This mechanism breaks down when banks' capacity to lend is constrained by their equity, in which case the equity erosion channel emphasized by previous theoretical contributions (Ulate, 2021; Abadi et al., 2023) entirely determines their lending, making it fall as policy rates further decline into negative territory. When the channel identified in this paper dominates, policy rate cuts into negative (or more negative) interest rates might stimulate the economy even more than in positive territory. The dominance of this effect requires that sufficiently many banks are not capital constrained, meaning that they have (or are able to raise) the equity needed to meet capital requirements when they wish to increase their lending.

The amplification channel is related to limited-liability risk-taking distortions that are first established in a stylized static model (henceforth referred to as illustrating model), in which banks are financed by insured deposits, invest in risky loans, and also invest in a safe asset remunerated at the policy rate. Since the bank is exclusively funded with deposits in the illustrating model, that is, there are no capital requirements, the operation of the equity erosion channel emphasized by the previous literature is shut down in this illustrating model

et al. (2019) and Basten and Mariathasan (2023). Eggertsson et al. (2024), unlike the other studies cited above, conduct an event study with daily data, and do not use a differences-in-differences approach.

– equity and capital requirements will be introduced in the quantitative model discussed below.

Banks are local monopolists in loan markets, monopolistic competitors in deposit markets (as in e.g. Abadi et al., 2023), and are subject to a lower bound on deposit rates. An exogenously distributed fraction of the risky loans defaults. A bank fails itself whenever the fraction of defaulting loans is large enough to make its residual net worth negative.

In this framework, I show analytically that policy rate cuts are expansionary, unless the representative bank fails with certainty.[3] Further, when the bank's default probability is also non-zero and the D-ZLB is binding, the D-ZLB induces the bank to charge a lower loan rate for a given policy rate. Under the same conditions, and everything else equal, policy rate changes have larger effects on loan rates. The amplification channel is therefore only operative when banks have a non-trivial probability of failure.

The key difference to previous models studying bank lending under negative interest rates – in which the amplification effect emphasized here did not arise – is that loans are risky, and the bank itself fails when a sufficiently high proportion of its loan portfolio defaults. The amplification effect arises because the losses in deposit taking due to the D-ZLB lead the representative bank to fail for loan default rates for which it otherwise would not have failed. Since the bank is protected by limited liability, it's payoff is zero when it fails. Hence, by making the bank fail for lower loan default rates, the D-ZLB shifts losses from the bank to the deposit insurance agency, effectively increasing the conditional-on-bank-solvency profitability of loans for the bank. Thus, the bank charges a lower loan rate when the D-ZLB is binding than in the alternative scenario when it is not. The transmission

[3] The bank may fail with certainty if losses in deposit taking become so high that profits from lending can never exceed them, such that the bank is indifferent between any loan rate. A similar point was raised by Repullo (2020b).

of monetary policy changes are thus amplified when the D-ZLB is binding, due to the higher (lower) losses in deposit taking when the policy rate is lowered (raised).[4]

After establishing the novel amplification effect, I assess it's quantitative relevance relative to the equity erosion channel emphasized by, among others, Ulate (2021) and Abadi et al. (2023). The equity erosion channel requires both capital requirements and equity dynamics to be salient: losses in deposit taking erode equity over time until the scarcity of banks' equity forces them to reduce lending to comply with capital requirements. Hence, I develop a dynamic, quantitative extension of the illustrating model in which banks are financed by equity, as well as insured deposits, in order to comply with a regulatory capital requirement (which requires the bank to have a minimum amount of equity funding per unit of lending). The asset side is kept identical to the illustrating model: the bank invests in risky loans and a safe asset remunerated at the monetary policy rate. The latter is exogenous and evolves according to a Markov chain.

As before, in the quantitative model banks have market power in loan and deposit markets, and face a D-ZLB. Loan default risk is modeled following the Vasicek (2002) single risk factor model. Banks are infinitely lived, but are shut down forever if their net-worth becomes negative. This may happen either because of loan defaults, losses in deposit taking or a combination of both.

I assume that banks can raise additional equity at a convex cost, while paying out dividends is costless. Equity dynamics are therefore fully endogenous here, whereas Abadi et al. (2023) assume a constant dividend payout ratio and exogenous equity injections. Thus, in contrast to the illustrating model, in the quantitative model banks may restrict lending when their equity falls low because it is too expensive to raise sufficient equity to

[4]When the D-ZLB is not binding, the deposit rate R_D is a constant fraction of the policy rate R, such that the deposit spread $R - R_D$ (and thus the bank's profit in deposit taking) is increasing with R. However, the change in the deposit spread is systematically larger once the D-ZLB binds, since then the transmission into deposit rates of changes in R breaks down.

support the loan supply level that they would choose if having plenty of available equity – this gives rise to the equity erosion channel. Apart from their loan pricing decisions, banks are hence able to react to losses from deposits under negative interest rates by adjusting capitalization through dividend policy and equity injections. These decisions impact on loan supply and default probabilities beyond the more mechanical equity erosion effect of the losses in deposit taking explored in previous studies (e.g. Abadi et al. 2023).

The model is calibrated to Germany.[5] The quantitative analysis compares two scenarios: one in which banks are subject to a D-ZLB as observed in the real world (e.g. Heider et al., 2019) and one in which banks can raise deposits at negative deposit rates. Through the lens of the model, the only difference between NIRP and conventional monetary policy is the breakdown of the transmission into deposit rates under NIRP. Hence, the comparison of the two scenarios (D-ZLB vis-a-vis no D-ZLB) serves to assess how the effects of negative interest rate policy differ from conventional monetary policy.

The main conclusions of the quantitative analysis are four. First, banks that are sufficiently capitalized, such that their capital requirement is slack and they would thus not supply higher loan volumes even if they had more capital, supply significantly higher loan volumes under NIRP when subject to a D-ZLB: the loan rate they charge decreases by 10 basis points due to the D-ZLB. Second, 28.6% of banks supply lower loan volumes during the average spell of negative interest rates, as they are capital-constrained and have to restrict lending due to the equity-eroding effects of losses in deposit taking. However, the first effect dominates in the quantitative results. I find that the aggregate loan supply is on average 4% higher due to the D-ZLB when policy rates are negative. Third, bank default

[5]In Germany, banks have a particularly high deposit-to-loan ratio, whereas in the Euro Area the average deposit-to-loan ratio is below 1 — in the baseline model this would imply that risky banks have access to non-deposit financing at the riskless interest rate. Since this is inconsistent, and to keep the model parsimonious, the baseline model is calibrated to Germany. As a robustness check, I build an extension of the model in which debt is fairly priced. This extension is calibrated to the Euro Area. The results are similar.

probabilities are 28 basis points higher under NIRP and a D-ZLB, up from a baseline level of 49 bps. Fourth, the impact of the D-ZLB on loan supply under negative interest rates is hump-shaped over time: the amplification is strongest initially, and is weakened by the equity erosion channel over time. For a cut to -0.21%, the equity erosion channel finally dominates after 9 years, whereas for a cut to -0.5% it dominates after already four years. Nevertheless, both policy rate cuts lead to higher aggregate loan supply even after 50 years.

Taken together, the results suggest that negative interest rate policy is even more effective than conventional monetary policy at increasing credit supply on average. However, due to capital constraints being binding for some banks, its effects on the cross-section of banks are heterogeneous. The fraction of banks that decrease loan supply due to equity erosion is initially growing over time, such that the stimulating effect of NIRP on bank lending becomes weaker over time. Further, NIRP has a sizable cost in terms of financial stability, due to the large increase in bank default probabilities.

This paper contributes to the literature on the pass-through of (negative) monetary policy rates into banks' lending rates. Previous theoretical models, such as Repullo (2020b); Ulate (2021); Eggertsson et al. (2024); Onofri et al. (2023) and Abadi et al. (2023), have highlighted that the transmission of negative policy rates into loan rates is impaired due to losses in deposit taking under negative interest rates eroding equity in the presence of a regulatory capital constraints. In these models, this does not necessarily imply a reduction in bank lending. This is because profits from lending (Ulate, 2021) or reduced funding costs from alternative sources, including bank bonds (Onofri et al., 2023; Eggertsson et al., 2024), can prevent a deterioration in bank equity over time. In models in which this is the case, e.g. in Ulate (2021), the models predict that negative interest rate policy is less expansionary than conventional monetary policy. Neither of these theoretical contributions considers the interplay of monetary policy and bank risk. This paper adds to the literature by showing that in this context a novel amplification channel arises through which risky banks with deposit rates stuck at zero increase their (unconstrained) loan supply more than they would have in absence of the D-ZLB after a policy rate cut. The paper also quantifies the importance of this channel vis-a-vis the equity erosion channel emphasized by previous contributions.

As discussed above, the empirical literature has found mixed evidence regarding the pass-through of NIRP into lending rates. This paper contributes to understanding these divergent findings. Concretely, the amplification channel rationalizes empirical findings of increased loan supply of banks more exposed to negative interest rates (Hong and Kandrac, 2021; Demiralp et al., 2021; Bottero et al., 2022; Schelling and Towbin, 2022). Through the lens of previous models of negative interest rates that focus on different aspects of the equity erosion channel (Repullo, 2020b; Ulate, 2021; Eggertsson et al., 2024; Abadi et al., 2023), increased lending of banks more exposed to NIRP as documented in different contexts is difficult to rationalize. While in Abadi et al. (2023), Onofri et al. (2023) and Eggertsson et al. (2024) monetary policy cuts can increase bank equity, at least in the short run, these effects do not operate differently under negative interest rate policy than under conventional monetary policy.[6] Hence, such explanations would arguably be at odds with placebo-exercises confirming abnormal effects of NIPR (Hong and Kandrac, 2021; Heider et al., 2019).

This paper also contributes to the literature on the financial stability implications of negative interest rate policy. The empirical literature has consistently found evidence for an increase in bank riskiness (Nucera et al., 2017; Hong and Kandrac, 2021; Basten and Mariathasan, 2023; Schelling and Towbin, 2022; Heider et al., 2019). Further, evidence for portfolio reallocation towards riskier assets is found by Bottero et al. (2022); Basten

[6]In Abadi et al. (2023) such an increase can arise due to capital gains in fixed-rate long-term assets. In Onofri et al. (2023) and Eggertsson et al. (2024), it can arise because of a reduction in the cost of non-deposit funding.

and Mariathasan (2023); Schelling and Towbin (2022); Heider et al. (2019). However, the financial stability implications of negative interest rates have not been studied through the lens of a quantitative model. The amplification channel emphasized here arises in the model because of an increase in the bank's default probability, and implies a reallocation of assets from the safe asset to risky loans. Hence, the model provides a framework that jointly explains the empirical findings of increased bank default probabilities, a portfolio-reallocation towards riskier assets and an increase in bank lending.[7]

While the focus of this paper is highlighting the amplification channel, the model is not inconsistent with contractionary effects of negative interest rate policy (Basten and Mariathasan, 2023; Eggertsson et al., 2024; Heider et al., 2019). Through the lens of the model, such effects could reflect (a) portfolio reallocation away from the particular type of loans studied in a given empirical contribution (e.g. away from syndicated loans in Heider et al., 2019), or (b) a sufficiently large fraction of banks constrained by their capital requirement.[8]

The rest of the paper is organized as follows: Section 2 presents a stylized model to study the loan supply of banks unconstrained by capital requirements under negative interest rates. Section 3 presents the quantitative model. The model's calibration is described in Section 3.2. Section 3.3 presents the results of the numerical analysis. Section 4 discusses the results of both the stylized and the quantitative model, and relates them to the empircal

[7]Regarding increased bank default probabilities, the mechanism exposed here is complementary to existing explanations of increased risk taking linked to limited liability (Repullo, 2004; Dell'Ariccia et al., 2014).

[8]To be more precise, the diverging findings for the EA of Heider et al. (2019) (lower loan supply *in the syndicated loan market* of more exposed banks) and Demiralp et al. (2021) (higher loan supply *overall* of more exposed banks) for example might point to a portfolio reallocation away from syndicated loans, rather than effects due to capital requirements. Eggertsson et al. (2024) focus on lending to households, hence it cannot be ruled out that their result of a decrease in bank lending to households reflects a portfolio reallocation, rather than a decrease in total lending. Schelling and Towbin (2022) argue that the diverging findings for Switzerland in their paper and Basten and Mariathasan (2023) possibly reflect heterogeneous demand effects, which cannot be controlled for in the latter study.

literature. Section 5 concludes. Lastly, the Appendices contain additional tables and figures, as well as proofs of all lemmas, propositions and corollaries.

2 The Amplification Mechanism in a Simple Model

This section illustrates in the context of a simple static partial equilibrium model how the presence of a D-ZLB can amplify (rather then dampen) the impact of interest rate cuts on bank lending. In this model, the presence of the D-ZLB implies that banks cannot fix gross deposit rates below some $\underline{R}_D$ (so $\underline{R}_D = 1$ corresponds to to the typical case in which net deposit rates cannot be negative)). While Section 2.1 establishes the novel channels that operate in the quantitative model of Section 3 formally, Section 2.2 discusses and illustrates some additional results regarding monetary policy transmission with a numerical example.

2.1 Static Model

Consider an individual bank that operates between two dates $t = 0, 1$. At $t = 0$ the bank is financed by insured deposits D remunerated at a gross rate R_D, invests in risky loans L that pay a gross loan rate R_L and may also invest in a safe asset S remunerated at the policy rate R.

The bank is a monopolist in a local loan market where it faces a downward sloping iso-elastic loan demand function of the form $L(R_L) = AR_L^{-\epsilon}$, $\epsilon_L > 1$. At $t = 1$, a share ω of its loans default, in which case the corresponding payoff is zero. The random variable ω is assumed to have full support in $[0, 1]$ and it's pdf and cdf will be denoted by $f(\omega)$ and $F(\omega)$, respectively. To isolate the implications of the mechanism that I aim to illustrate, the bank faces no capital requirement.

As in Abadi et al. (2023), I assume that the bank competes in deposit markets in a monopolistic fashion with a unit continuum of identical other banks. Thus, the bank is facing an upward sloping deposit function $D(R_D) = \left(\frac{R_D}{\mathcal{R}_D}\right)^{-\epsilon_D} \bar{D}$, with $\epsilon_D < -1$ and $\bar{D} > 0$, where $\mathcal{R}_D$ is the CES index of deposit rates, which is taken as given by each individual bank. Reflecting the D-ZLB, banks cannot offer deposit rates below $\underline{R}_D$.

Each bank is managed in the interest of its risk neutral shareholders, who enjoy limited liability. When a bank defaults at $t = 1$, its assets are repossessed by a deposit insurance agency, who pays all its depositors in full. The bank maximizes its expected value at $t = 1$:

$$\max_{R_L, R_D \geq \underline{R}_D, S \geq 0} \int_0^1 max\{(1-\omega)R_L L(R_L) + RS - R_D D(R_D), 0\} dF(\omega) \tag{1}$$

subject to the balance sheet constraint:

$$L(R_L) + S = D(R_D) \tag{2}$$

To solve the bank's optimization problem, I will proceed in two steps. The constraint $S \geq 0$ is guessed to be non-binding, which can ex-post be shown to imply a parameter restriction on $\bar{D}$. Using this guess to substitute S from (2) into the objective function (1), the objective function can be written as:

$$\max_{R_L, R_D \geq \underline{R}_D} \int_0^1 max\{(R_L - R)L(R_L) + (R - R_D)D(R_D) - \omega R_L L(R_L), 0\} dF(\omega) \tag{3}$$

First the optimal deposit rate can be characterized – it is independent of the bank's choice of R_L. Let it be denoted $\hat{R}_D$. The results regarding the optimal deposit rate are summarized in the following lemma:

Lemma 1. *Assuming the bank does not always fail, the bank's optimal deposit rate is:*

$$\hat{R}_D(R) = max(R_D^*(R), \underline{R}_D) \tag{4}$$

where

$$R^*_D(R) = \frac{\epsilon_D}{\epsilon_D - 1} R \tag{5}$$

is the bank's optimal deposit rate in the absence of a D-ZLB. Moreover, $\hat{R}_D(R) = R^*_D(R)$ *for all* $R \geq R^*$ *and* $\hat{R}_D(R) = \underline{R}_D$ *for all* $R < R^*$*, where* $R^* = \frac{\epsilon_D - 1}{\epsilon_D} \underline{R}_D$*.*

By symmetry, all monopolistic competitors in the deposit market offer the same deposit rate. Hence the CES index of deposit rates is $\mathcal{R}_D = \hat{R}_D(R)$ and an individual bank's profits from deposits are given by $(R - \hat{R}_D(R))\bar{D}$.

Next, it will be shown that the deposit rate does impact the optimal loan rate, and therefore monetary policy transmission, if the D-ZLB is binding and the bank has an interior default probability. The channel works through the impact that the deposit rate has on such a probability. The bank defaults if

$$(R_L - R)L(R_L) + (R - R_D)\bar{D} - \omega R_L L(R_L) < 0,$$

that is if

$$\omega > \tilde{\omega} = \frac{(R_L - R)L(R_L) + (R - \hat{R}_D)\bar{D}}{R_L L(R_L)} \tag{6}$$

When $\tilde{\omega} > 1$ the bank never fails since ω has support in the interval $[0, 1]$ – this may happen if profits from deposits $(R - \hat{R}_D)\bar{D}$ are so large that the bank can repay $RL(R_L)$ even if all loans default. Of course, $\tilde{\omega} > 1$ cannot happen under policy rates $R < \underline{R}_D$. On the other hand, when $R < \underline{R}_D$ profits from deposits may be sufficiently negative, such that $\tilde{\omega} < 0$, and the bank always fails.

Using previous results, it is convenient to write the objective function of the bank as:

$$\Pi(R_L) = \int_0^{\tilde{\omega}} [(R_L(1 - \omega) - R)L(R_L) + (R - \hat{R}_D)D(\hat{R}_D)]dF(\omega) \tag{7}$$

where $\bar{\omega} = min(max(\tilde{\omega}, 0), 1)$, that is:

$$\bar{\omega} = \begin{cases} 0 \text{ if } \tilde{\omega} < 0 \\ \tilde{\omega} \text{ if } 0 \leq \tilde{\omega} \leq 1 \\ 1 \text{ if } \tilde{\omega} > 1 \end{cases} \tag{8}$$

Then, the following proposition about the existence of an optimal loan rate can be proven:

Proposition 1. *If there exists a R_L such that $(R_L - R)L(R_L) + (R - \hat{R}_D)D(\hat{R}_D) > 0$, the problem $\max_{R_L} \Pi(R_L)$ has a global maximum at $\hat{R}_L \in (R, \infty)$, which fulfills:*

$$(1 - \omega^{LL})\hat{R}_L = \frac{\epsilon_L}{\epsilon_L - 1} R \tag{9}$$

where ω^{LL} is the expected loan default rate in the bank's non-default region

$$\omega^{LL} = \mathbb{E}[\omega \mid \omega \leq \bar{\omega}] = \frac{1}{F(\bar{\omega})} \int_0^{\bar{\omega}} \omega dF(\omega) \tag{10}$$

The condition for existence of an optimal loan rate $\hat{R}_L \in (R, \infty)$ is always satisfied under policy rates above the lower bound on deposits $\underline{R}_D$. However, when the bank makes too high losses in deposit taking, such that profits from lending can never exceed these losses, the bank always fails. In that case, the bank is indifferent between any loan rate.

In general, it is neither guaranteed that the solution to (9) is unique, nor that it involves $\Pi''(\hat{R}_L) \neq 0$ (see Appendix C).[9]. In what follows it is assumed that parameters are such that a unique $\hat{R}_L$ solving (9) exists, and $\Pi''(\hat{R}_L) < 0$.[10]

[9]While there is a unique local maximum $\hat{R}_L$ in all my numerical simulations, the sign of $\Pi''(\hat{R}_L)$ is generally ambiguous such that multiple solutions of (9) might arise when ω^{LL} is increasing in $\hat{R}_L$. A unique local maximum involving $\Pi''(\hat{R}_L) \neq 0$ is for example guaranteed when $\omega \sim Unif[0, 1]$ and $R < \underline{R}_D$ (see Appendix C)

[10]This is required to use the implicit function theorem to characterize changes in the optimal loan rate.

Using (6) and Lemma 1, it follows that a stricter lower bound $\underline{R}_D$ on the deposit rate decreases the cutoff $\tilde{\omega}$ for any given loan rate when the lower bound is binding:

$$\left.\frac{\partial \tilde{\omega}}{\partial \underline{R}_D}\right|_{\hat{R}_L} = \begin{cases} -\dfrac{\bar{D}}{\hat{R}_L L(\hat{R}_L)} \text{ if } R < R^* \\ 0 \text{ if } R > R^* \end{cases} \tag{11}$$

which follows from (6) and Lemma 1. By the definition of $\bar{\omega}$, the D-ZLB thus increases the banks' default probability for a given loan rate, whenever the default probability is interior (as opposed to strictly zero or one).[11] This in turn decreases the conditionally expected loan default rate ω^{LL} by (10): a stricter lower bound means that (for a given loan rate) the bank defaults for loan default rates for which the bank would otherwise not have defaulted, which lowers the loan default rate ω^{LL} that the bank expects conditional on not defaulting itself.

Equation (9) suggests that whenever the D-ZLB induces such a decrease in ω^{LL}, it also affects the loan rate charged by the bank. This is summarized in the following proposition:

Proposition 2. *Assume a unique interior solution in $\hat{R}_L$ exists such that $\Pi''(\hat{R}_L) < 0$ and the bank has a non-trivial probability of failure ($\tilde{\omega} \in (0,1)$). Then a stricter deposit lower bound $\underline{R}_D$ lowers the optimal loan rate if and only if the D-ZLB constraint is binding:*

$$\frac{\partial \hat{R}_L}{\partial \underline{R}_D} \begin{cases} < 0 \textit{ if } R < R^* \\ = 0 \textit{ if } R > R^* \end{cases} \tag{12}$$

[11] Technically:

$$\left.\frac{\partial \bar{\omega}}{\partial \underline{R}_D}\right|_{\hat{R}_L} \begin{cases} = 0 \text{ if } R > R^* \text{ or } \tilde{\omega} > 1 \text{ or } \tilde{\omega} < 0 \\ \text{does not exist if } R = R^* \text{ or } \tilde{\omega} = 0 \text{ or } \tilde{\omega} = 1 \\ < 0 \text{ else} \end{cases}$$

The non-existence of the partial derivative if $\tilde{\omega} \in \{0,1\}$ is due to the non-differentiability of the function $max(x,0)$ at 0 and the function $min(x,1)$ at 1.

The proposition implies that when a bank subject to a D-ZLB has an interior default probability, a cut from any rate above R^* to any rate below R^* is more expansionary than in the alternative scenario when banks are not subject to a D-ZLB. To see this, note that the scenario in which banks are not subject to a D-ZLB corresponds to the case $\underline{R}_D \rightarrow -\infty$. For policy rates above R^*, banks charge the same loan rate in both scenarios (since the loan rate does not change with the lower bound on deposits when $R > R^*$), while for policy rates below R^* banks will charge a lower loan rate in the scenario in which they are subject to a D-ZLB (since when $R < R^*$ the loan rate decreases when the lower bound on deposits increases). In the real world scenario in which banks cannot fix negative deposit rates ($\underline{R}_D = 1$), such a cut from a policy rate above R^* to a rate below R^* may or may not be a cut into negative policy rates, since $R^* = \frac{\epsilon_D - 1}{\epsilon_D} > 1$.

However, while monetary policy cuts into policy rates below the deposit lower bound are (ceteris paribus) more expansionary, such cuts have adverse implications for bank default probabilities. It was already discussed above that $\bar{\omega}$, and thus the bank's failure probability, is decreased by the D-ZLB *for a given loan rate*. Indeed, as a corollary to Proposition 1, it can be shown that the D-ZLB decreases $\bar{\omega}$ unconditionally, when it is interior:

Corollary 1. *Assume a unique interior solution in $\hat{R}_L$ exists such that $\Pi''(\hat{R}_L) < 0$ and the bank has a non-trivial probability of failure ($\tilde{\omega} \in (0,1)$), then the D-ZLB lowers the bank's default threshold:*

$$\frac{\partial \bar{\omega}}{\partial \underline{R}_D} = \underbrace{\frac{\partial \bar{\omega}}{\partial R_D}}_{<0} \frac{\partial R_D}{\partial \underline{R}_D} + \underbrace{\frac{\partial \bar{\omega}}{\partial R_L}}_{>0} \underbrace{\frac{\partial R_L}{\partial \underline{R}_D}}_{\leq 0} \begin{cases} < 0 \text{ } if \text{ } R < R^* \\ = 0 \text{ } if \text{ } R > R^* \end{cases} \tag{13}$$

This is due to a combination of the direct impact on the bank's default probability of paying higher deposit rates, and the impact of the lower loan rate the bank offers as implied by Proposition 2.

While the results in Proposition 1 and Corollary 1 are those that operate in the quantitative model of Section 1, some additional results regarding monetary policy transmission are discussed and illustrated in the following subsection.

2.2 Monetary Policy Transmission: Numerical Example

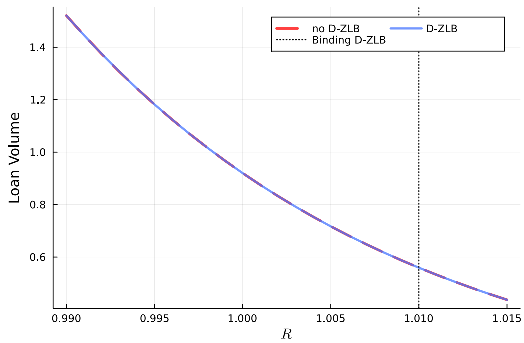
A numerical example illustrating monetary policy transmission in alternative scenarios is provided in Figure 1, which plots the loan volume $L(\hat{R}_L)$, the loan rate $\hat{R}_L$ and the bank's default probability (EDF) against the policy rate R. The figure compares banks that may not set deposit rates below 1 ($\underline{R}_D = 1$) with banks that are not subject to a lower bound on deposits, and hence always set $R_D = R_D^*$ (as defined in Lemma 1). In the left column of the figure, banks are safe (that is $Pr(\omega < \bar{\omega}) = 0$), while in the right column banks are risky (that is $Pr(\omega < \bar{\omega}) \in (0, 1)$). To this end, the numerical example assumes that the loan default distribution follows the Vasicek (2002) distribution with

$$F(\omega) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\omega) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) \tag{14}$$

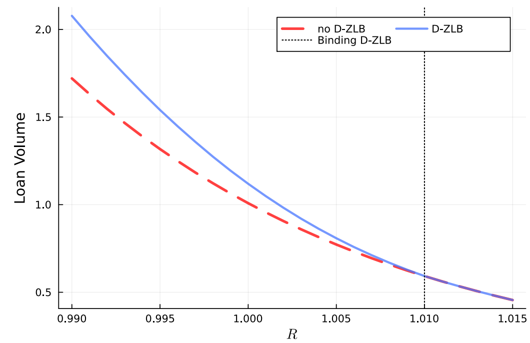
where $\Phi(\cdot)$ is the standard normal cdf, and ρ and p are parameters. The cdf $F(\omega)$ converges to a one-point distribution as $\rho \to 0$ (Vasicek, 2002), such that banks have a deterministic payoff in the limit.

The parametrization used for this numerical example is $\epsilon_D = -100$ (implying a deposit spread of 1% when the D-ZLB is not binding), $\bar{D} = 2.5$, $p = 0.01$ (i.e., a 1% loan default probability), $\epsilon_L = 50$ (implying a loan rate spread of about 3% for safe banks) and $A = 4.17$ (implying that the constraint $S > 0$ is always satisfied). For risky banks, I set $\rho = 0.2$, while for safe banks $\rho \to 0$.[12]

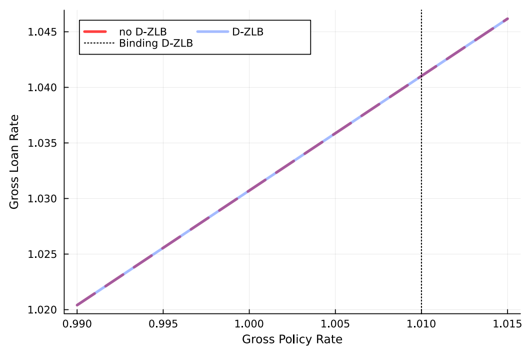
[12] The Basel II regulatory framework (Basel Committee on Banking Supervision, 2004) specifies a formula for the parameter ρ based on the loan default probability p, which would yield $\rho \approx 0.19$ for $p = 0.01$.



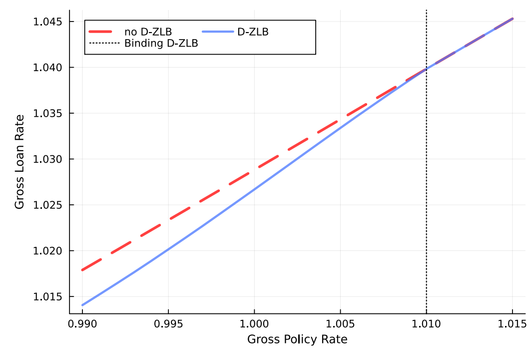
(a) *Loan Volume Safe Banks*



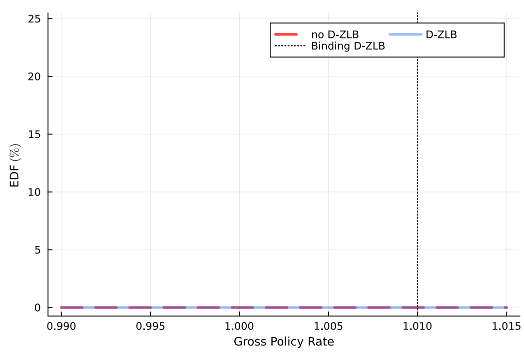
(b) *Loan Volume Risky Banks*



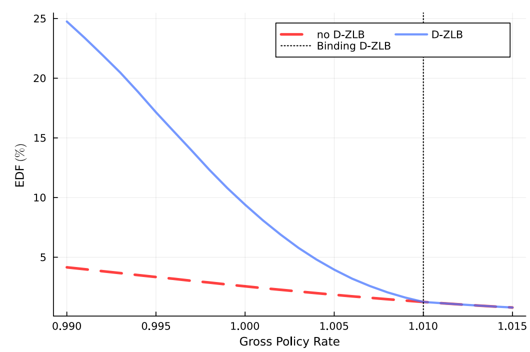
(c) *Loan Rate Safe Banks*



(d) *Loan Rate Risky Banks*



(e) *Expected Default Probability Safe Banks*



(f) *Expected Default Probability Risky Banks*

Figure 1: *Numerical Example, Stylized Model*

In the left column (safe banks), the loan default probability is 1% with probability 1, while in the right column (risky banks) the loan default probability follows a Vasicek (2002) distribution with mean 1%.

As Figure 1 shows, the loan rate is increasing in the policy rate in all scenarios. Hence, there is no reversal. Indeed, it can be shown that:

Corollary 2. *Assume a unique interior solution in $\hat{R}_L$ exists such that $\Pi''(\hat{R}_L) < 0$ and the bank does not fail with certainty, i.e. ($\tilde{\omega} \in (0, 1]$). Then, if*

$$\frac{\partial \omega^{LL}}{\partial \bar{\omega}} < \frac{\epsilon_L}{\epsilon_L - 1} \tag{15}$$

monetary policy is expansionary:

$$\frac{\partial \hat{R}_L}{\partial R} > 0 \quad \textit{for all } R \neq R^* \tag{16}$$

The condition (15) is sufficient but not necessary for monetary policy to be expansionary.[13] The first part of Corollary 2 then implies that monetary policy cuts are always expansionary – and thus there is no reversal – as long as banks do not fail with certainty (in which case the bank is indifferent between any loan rate).

The right panel of Figure 1 highlights a clear kink of monetary policy transmission at $R = R^*$ (the red vertical line). There is no such kink in the left panel. The figure thus emphasizes that the D-ZLB only amplifies monetary policy cuts into negative rates when banks have an interior default probability $Pr(\omega < \bar{\omega}) \in (0, 1)$.

The figure further shows that when banks have such an interior default probability, the loan rate in the D-ZLB scenario falls further and further below the loan rate in the scenario without D-ZLB. The reason is a fundamental difference in the transmission of monetary policy rates above and below R^*, as shall be discussed below.[14] The figure shows that the default probability of risky banks begins increasing once the D-ZLB is binding relative to the

[13]The condition is for example fulfilled for the uniform distribution.

[14]This fundamental difference is also the reason for the non-existence of the derivative $\frac{\partial \hat{R}_L}{\partial R}$ at $R = R^*$ implied by Corollary 2.

default probability prevailing in the alternative scenario in which the bank sets $R_D = R_D^*$. This reflects Corollary 1. It can be conjectured from (9)) that increased bank riskiness (everything else equal) lowers the loan rate but by the same token weakens monetary policy transmission.[15]. However, despite the sizable decrease in the default threshold shown in Figure 1, monetary policy transmission is almost linear below R^* – suggesting that the impact of the level of bank riskiness on monetary policy transmission is limited.

To understand the reason for the fundamental difference of transmission of policy rates above and below R^*, it is useful to consider two scenarios in which a bank faces the same policy rate and has the same default probability before the policy rate cut, but is subject to a binding D-ZLB in one scenario but not in the other. An increase in the policy rate makes banks safer in both scenarios and thus pushes up loan rates further (by Eq. (9)) through changes in the deposit rate spread $R - \hat{R}_D$. By Lemma 1, these are

$$\frac{\partial(\hat{R}_D - R)}{\partial R} = \begin{cases} 1 - \frac{\epsilon_D}{\epsilon_D - 1} \\ 1 \text{ if } R < R^* \end{cases} .$$

It is clear that the change in the deposit spread, and therefore it's effect on monetary policy transmission via the change in the default probability, is particularly large in the scenario in which the D-ZLB is binding, such that $\hat{R}_D = \underline{R}_D$ and hence the deposit spread changes one-to-one with the policy rate. Formally, the following corollary can be proven:

Corollary 3. *Assume a unique interior solution in $\hat{R}_L$ exists such that $\Pi''(\hat{R}_L) < 0$ and the bank has a non-trivial probability of failure ($\tilde{\omega} \in (0,1)$). Consider two different lower bounds $\underline{R}_{D1}$, $\underline{R}_{D2}$ with $\underline{R}_{D1} > \underline{R}_{D2}$ such that the lower bound constraint would be binding under $\underline{R}_{D1}$ (that is $R^*(\underline{R}_{D1}) > R$), but not under $\underline{R}_{D2}$ (that is $R^*(\underline{R}_{D2}) < R$). Everything*

[15]The exact impact of a change in the default probability on monetary policy transmission depends on how the slope of the conditional expectation $\frac{\partial \omega^{LL}}{\partial \bar{\omega}}$ changes with $\bar{\omega}$.

else equal, a policy rate cut is more expansionary when the lower bound $\underline{R}_{D1}$ constraint is binding:

$$\left.\frac{\partial \hat{R}_L}{\partial R}\right|_{\underline{R}_{D1},R,\bar{\omega}} - \left.\frac{\partial \hat{R}_L}{\partial R}\right|_{\underline{R}_{D2},R,\bar{\omega}} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\bar{D}}{L(R_L)\left[(1-\omega^{LL}) - R_L \frac{\partial \omega^{LL}}{\partial R_L}\right]} \frac{\epsilon_D}{\epsilon_D - 1} > 0 \tag{17}$$

While the conditioning on both the policy rate and the default cutoff in Corollary 3 is counterfactual for policy rates $R \neq R^*$ in the model, it is nevertheless useful to understand the fundamental difference in the transmission of policy rates above and below the zero lower bound, as shown in Figure 1.[16]

2.3 Intuition: D-ZLB Decreases Limit Liability Subsidy

To gain intuition about these results, marginal profits of raising the loan rate are (by the Leibniz rule):

$$\Pi'(R_L) = \int_0^{\bar{\omega}} [(R_L(1-\omega) - R)L'(R_L) + (1-\omega)L(R_L)]dF(\omega) \tag{18}$$

[16] In the context of the model, keeping the default cutoff fixed can be thought of as banks receiving an exogenous compensation $\Delta = \bar{D}\left(\underline{R}_{D1} - \frac{\epsilon_D}{\epsilon_D - 1}R\right)$ from the government, such that:

$$\frac{(\underline{R}_{D1} - R)\bar{D} + RL(R_L) - \Delta}{R_L L(R_L)} = \frac{(\frac{\epsilon_D}{\epsilon_D - 1} - 1)R\bar{D} + RL(R_L)}{R_L L(R_L)}$$

and hence $\bar{\omega}$ remains the same after the change in $\underline{R}_D$.

The marginal profits can be decomposed into the marginal profits that a bank without limited liability would obtain and a marginal limited liability subsidy, in a similar fashion to Bahaj and Malherbe (2020):

$$\Pi'(R_L) = [(R_L(1-\mathbb{E}\omega) - R)L'(R_L) + (1-\mathbb{E}\omega)L(R_L)] \\ \underbrace{- \int_{\bar{\omega}}^{1} [(R_L(1-\omega) - R)L'(R_L) + (1-\omega)L(R_L)]dF(\omega)}_{\equiv MS} \tag{19}$$

where the second line of (19), which integrates over the default region, is the marginal limited liability subsidy MS. It is obvious that the default cutoff (and hence the D-ZLB) does not impact the first term, which is the marginal profit of a bank without limited liability. But through it's effect on the cutoff, the D-ZLB does impact the limited liability subsidy. It is useful to look at the change in the marginal subsidy (for a given loan rate) when the default probability decreases:

$$\left.\frac{\partial MS}{\partial \bar{\omega}}\right|_{\hat{R}_L} = [(\hat{R}_L(1-\bar{\omega}) - R)L'(\hat{R}_L) + (1-\bar{\omega})L(\hat{R}_L)]f(\bar{\omega}) > 0 \tag{20}$$

Thus, a higher default cutoff c.p. increases the marginal subsidy, since the marginal profit of raising the loan rate at the default cut-off is positive.[17] Intuitively, as in Bahaj and Malherbe (2020), a higher default cutoff $\bar{\omega}$ – i.e. a lower default probability – shifts this positive marginal profit from the deposit insurance agency to the bank's shareholders, thereby increasing the total marginal profit of lending for the bank's shareholders.

As just explained, when the bank is risky and the D-ZLB binds, a stricter lower bound c.p. increases the bank's default probability, thus shifting marginal profits at the default cutoff from the bank to the deposit insurance agency, and thereby decreasing marginal

[17]The sign of (20) follows since $\Pi'(\hat{R}_L)$ implies $sign(\hat{R}_L L'(\hat{R_L}) + L(\hat{R_L})) = sign(RL'(\hat{R_L}))$. Since $L'(\cdot) < 0$ the integrand is increasing in ω. But then for the integral to evaluate to zero, the integrand must be positive at the upper integration bound $\bar{\omega}$.

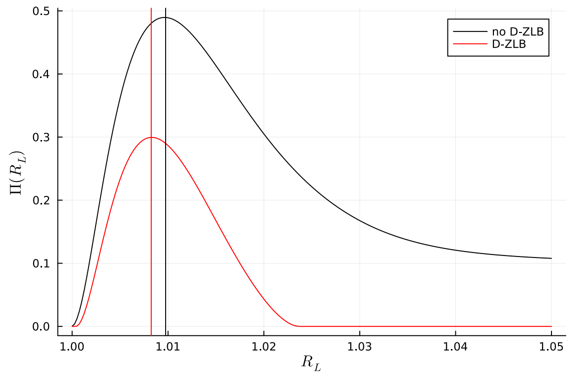


Figure 2: *Bank profits as a function of loan rates with and without a D-ZLB*

profits of lending. But a bank that has market power in lending will raise the loan rate as long as the marginal profits of doing so is positive. In consequence, as Figure 2 illustrates, the optimal loan rate is lower, since marginal profits from raising the loan rate are positive to the left of the optimal loan rate.

3 The Amplification Mechanism vis-a-vis the Equity Erosion Channel

This section presents a quantitative, dynamic model to assess the relative importance of the amplification channel emphasized in the previous section vis-a-vis the the equity erosion channel of e.g. Abadi et al. (2023). In the static model of the previous section, the equity erosion channel was mute, since banks were not subject to capital requirements. In contrast, capital requirements and equity dynamics are now introduced, allowing the model to quantify the relative importance of the erosion of equity due to losses on deposits.

3.1 Quantitative Model

Set-Up. In the spirit of Mendicino et al. (forthcoming), I assume that the economy consists of a unit continuum of islands k. On each island k there is a bank and a unit continuum of one-period firms i. Bank k can only lend to firms on its island and therefore has a local monopoly, but competes with all other banks in a monopolistic fashion in deposit markets. Banks also have access to a safe asset S remunerated at the policy rate R_t, which is common to all islands. Whenever the bank on an island fails it is immediately replaced with a new bank, which chooses its equity level endogenously, as in Corbae and D'Erasmo (2021).[18] In continuation, all model ingredients will be described in detail.

Monetary Policy. Monetary policy is taken as exogenous, and follows a two-state Markov chain. States will be denoted by $s_t \in \{P, N\}$. In state P, the central bank sets positive monetary policy rates $R(P) > 1$, while in state N the central bank lowers the monetary policy rate into negative territory $R(N) < 1$. State N will be associated with a recession caused by low firm productivity, as will be described below. States are modeled as a Markov chain, with the transition probability from state i to state j given by q_{ij}. These are estimated from EA data, as will be described in Section 3.2.

Deposit Demand. Deposit markets are identical to the stylized model of Section 2. Deposits are perfectly insured by the government, and bank k faces monopolistic competition in deposit markets, with aggregate deposit rate R_{Dt}:

$$D(R_{Dkt}, R_{Dt}) = \left(\frac{R_{Dkt}}{R_{Dt}}\right)^{-\epsilon_D} \bar{D}, \ \epsilon_D < -1$$

[18] As Corbae and D'Erasmo (2021) assume for "big banks", it is assumed that there is a cost to entering island k that is sufficiently high that no new banks enter as long as there is an incumbent bank.

Monopolistic competition in deposit markets is commonly assumed in quantitative models studying the effects of the deposit zero lower bound, e.g. Abadi et al. (2023), Eggertsson et al. (2024) and Ulate (2021). By assuming a flat deposit demand, I do not take a stance on the debate about the deposit channel of monetary policy, that is whether bank deposits are decreasing in the policy rate or not (Drechsler et al., 2017; Begenau and Stafford, 2023; Repullo, 2020a).[19]

Firms: Production & Loan Demand. Loan demand and the loan default distribution are microfounded from the problem of firms that require bank loans for production.

At time t, firms learn about their state-dependent productivity $A(s_t)$, common to all firms on all islands. The representative firm i on island k requires bank loans to acquire production factors K_{ikt}, which depreciate at rate δ. The firm is subject to a binary success shock $d_{ikt} \in \{0,1\}$. When it is unsuccessful, the firm loses a share λ of its production factors and defaults on its loan. The representative firm's revenue at time $t+1$ is then:

$$Y_{ikt+1} = (1 - d_{ikt+1})[(1-\delta)K_{ikt} + A(s_t)K^{\alpha}_{ikt}] + d_{ikt+1}(1-\lambda)K_{ikt} \quad \lambda, \alpha \in [0,1] \tag{21}$$

Bank k charges a gross interest rate $R_{Lkt} = 1 + r_{Lkt}$ for loans, and firms enjoy limited liability. Hence, the representative firm maximizes:

$$\max_{K_{ikt}} Pr(d_{ikt+1} = 0)[(1-\delta)K_{ikt} + A(s_t)K^{\alpha}_{ikt} - R_{Lkt}K_{ikt}] \tag{22}$$

[19] While Drechsler et al. (2017) present evidence for the US that bank deposits are decreasing in the Fed funds rate, Begenau and Stafford (2023) challenge their identification strategy and find a statistically weak relationship between aggregate deposits and the Fed funds rate, and an increasing relationship between deposits and the Fed funds rate for the largest 10% of banks.

The FOC is

$$(1-\delta) + \alpha A(s_t) K_{ikt}^{\alpha-1} \stackrel{!}{=} R_{Lkt} \tag{23}$$

Which implies a state-dependent loan demand of firm i on island k:

$$L_{ikt} = K_{ikt} = \left(\frac{R_{Lkt} - (1-\delta)}{\alpha A(s_t)} \right)^{\frac{1}{\alpha-1}} \tag{24}$$

and, by symmetry, total loan demand on island k is $L(R_{Lkt}, s_t) = \int_0^1 K_{ikt} dk = \left(\frac{R_{Lkt}-(1-\delta)}{\alpha A(s_t)} \right)^{\frac{1}{\alpha-1}}$. State-dependent loan-demand is a quantitatively relevant feature that allows the model to produce realistic loan-to-deposit ratios.

Loan defaults follow the Vasicek (2002) single risk factor model: there is a common shock $z_{kt} \sim N(0,1)$ to all firms on island k and an idiosyncratic shock $\epsilon_{ik} \sim N(0,1)$ that realize at t:

$$d_{ikt} = \{\varsigma + \sqrt{\rho} z_{kt} + \sqrt{1-\rho}\epsilon_{ikt} > 0\} \tag{25}$$

ς is a financial vulnerability parameter and pins down the unconditional default probability of all firms. As discussed in Gordy (2003), this setup is the model underlying the Basel capital requirements, which banks in the model are subject to. As shown in Vasicek (2002), the cdf of the default rate ω is then given by (14).

Banks. As before, banks raise insured deposits D, invest in risky loans L and a safe asset S that is remunerated at the policy rate.[20] Different to the static illustrating model in Section 2, banks are now subject to a capital requirement $\gamma L_{kt} \leq E_{kt}$, where E_{kt} denotes the equity of the representative bank k after paying out dividends and any equity injections. This is the main difference between the quantitative and the illustrating model, as it gives

[20] An extension of the model that allows for additional non-deposit funding will be considered in Section 3.6.4.

rise to an equity erosion channel as in Abadi et al. (2023) and Ulate (2021), through which banks are forced to reduce lending when losses on deposits are sufficiently large. Thus, the representative bank's balance sheet is now:

$$E_{kt} + D_{kt} = L_{kt} + S_{kt} \tag{26}$$

Every period, banks decide the amount $\nu_{kt} > 0$ to be paid out as dividends, and how much new equity I_{kt} to raise. The representative bank can raise equity at a quadratic cost $\chi_I I_{kt}^2$, similar to Corbae and D'Erasmo (2021). Dividend payouts on the other hand are frictionless. Hence, equity dynamics are completely endogenous here – this is different to both Abadi et al. (2023) and Ulate (2021), where the dividend payout ratio is fixed and exogenous.

As just discussed, a fraction ω_{kt} of the representative bank's loan portfolio defaults, with a loss given default of λ. The ex-post realized return on the loan portfolio of bank k is thus:

$$\tilde{R}_{Lkt}(\omega_{kt}) = (1 - \omega_{kt})R_{Lkt-1} + \omega_{kt}(1 - \lambda) \tag{27}$$

The bank defaults if it's loan repayments are insufficient to repay it's deposit obligations in full, i.e. if:

$$\tilde{R}_{Lkt}(\omega_{kt})L(R_{Lkt}, s_t) - R_{Dkt}\bar{D} + R(s_t)S_{kt} < 0$$

In this case, the bank is closed down forever and the bank's assets are repossessed by a deposit insurance agency, which will be discussed further below. As mentioned above, it is assumed that upon the default of the incumbent bank on island k, a new bank enters on island k immediately.

I assume that banks cannot charge gross deposit rates below $\underline{R}_D$. In the baseline model, I set $\underline{R}_D = 1$ in line with the ample empirical evidence for a zero lower bound on deposit rates (Eggertsson et al., 2024; Basten and Mariathasan, 2023; Hong and Kandrac, 2021). Since the only difference between conventional monetary policy rates and negative interest rate policy is that in the latter case the D-ZLB constraint is binding, the quantitative analysis will focus on the comparison of the baseline scenario vis-a-vis an alternative scenario in which banks are not subject to a D-ZLB. The difference between the scenarios is informative about the effects of negative interest policy vis-a-vis conventional monetary policy, as without the D-ZLB, a policy rate cut from positive to negative rates is like any other cut.

Banks are managed in the interest of their shareholders, who enjoy limited liability and discount the future at a rate $\beta(s_t)$, a function of the risk free rate $R(s_t)$. For quantitative purposes, similar to Repullo and Suarez (2012), it is assumed that bank shareholders discount the future at a rate higher than $R(s_t)$: $\beta(s_t) = \frac{1}{R(s_t)+\Delta_E}$, which reflects a constant, exogenous excess cost of equity.[21]

The bank's value function $V(s, \tilde{E}_k, R_D, \mathcal{D}_k)$ (where time subscripts are omitted) depends on the state s of the economy, the bank's level of equity before dividends or equity injections $\tilde{E}_k$, the aggregate deposit rate R_D and a binary variable $\mathcal{D}_k \in \{0, 1\}$ that indicates whether a bank has defaulted in the past ($\mathcal{D}_k = 1$) or not ($\mathcal{D}_k = 0$). By limited liability of shareholders and the assumption that a bank that has defaulted remains closed forever, the value of the bank is zero if it defaults: $V(\cdot, \cdot, \cdot, 1) = 0$, and the default indicator remains 1 forever: $\mathcal{D}_k = 1 \implies \mathcal{D}'_k = 1$.

[21] Δ_E reflects a differential cost of equity as observed in reality. As discussed in Repullo and Suarez (2012), such a cost might reflect costs of monitoring managers incurred by shareholders, a discount for lack of liquidity of equity stakes and a risk-related component. I follow Repullo and Suarez (2012) in abstracting from changes in the risk-premium component of the cost of equity due to changes in bank's leverage (Admati et al., 2010).

The problem of the bank on island k that has not defaulted in the past is then:

$$V(s, \tilde{E}_k, R_D, 0) = \max_{\nu_k \in [0, \tilde{E}_k], I_k \geq 0, R_{Lk}, R_{Dk}, S_k \geq 0} \nu_k - I_k - \chi_I I_k^2 + \beta(s)\mathbb{E}V(s', \tilde{E}_k', R_D', \mathcal{D}_k')$$

subject to $\gamma L(R_{Lk}, s) \leq \tilde{E}_k - \nu_k + I_k$

$$R_{Dk} \geq \underline{R}_D$$

$$\tilde{E}_k' = [(1-\omega_k)R_{Lk} + \omega_k(1-\lambda)]L(R_{Lk}, s) - R_{Dk}D(R_{Dk}, R_D) + R(s)S_k$$

$$\mathcal{D}_k' = \mathbf{1}(\mathcal{D}_k = 0)\mathbf{1}(\tilde{E}_k' < 0) + \mathbf{1}(\mathcal{D}_k = 1)$$

$$L(R_{Lk}, s) + S_k = D(R_{Dk}, R_D) + \tilde{E}_k - \nu_k + I_k$$

where $\mathbf{1}(\cdot)$ denotes the indicator function. This problem nests both the problem of an incumbent bank and a newly entered bank, for which $\tilde{E}_k = 0$ and $\mathcal{D} = 0$. The level of equity with which a new bank enters island k upon default of the incumbent bank is therefore determined endogenously and depends on the cost parameter χ_I.

As in the static model of Section 2 the representative bank sets the same deposit rate independently of the loan rate. The deposit rate FOC is:

$$D(R_{Dk}, R_D) + (R_{Dk} - R)\frac{\partial D(R_{Dk}, R_D)}{\partial R_{Dk}} = \lambda^{LB} \tag{28}$$

where the multiplier on the deposit lower bound is denoted by λ^{LB}, which will be binding in state N since $R(N) < 1$ and non-binding in state P. Thus, all banks will offer the same deposit rate, such that

$$R_{Dkt} = R_{Dt} = \begin{cases} 1 \text{ if } s_t = N \\ \frac{\epsilon_D}{\epsilon_D - 1} R_t \text{ if } s_t = P \end{cases}$$

and further $D(R_{Dk}, R_D) = \bar{D}$. In state N, banks therefore face losses on deposits, which makes them ceteris paribus (a) more likely to fail and (b) erodes their equity over time. Naturally, this impacts the likelihood that a bank will be constrained by the capital requirement, which is the channel emphasized by Abadi et al. (2023) in a model abstracting from any risk. The simple model in Section 2 predicts that when banks are risky the D-ZLB impacts the loan supply decision even for unconstrained banks. The following sections will confirm that this conclusion still holds here.

Aggregation. Given that there is a unit continuum of islands, the aggregate loan volume is:

$$L_t = \int_0^1 L_{kt} dk \tag{29}$$

and the aggregate loan rate is given by:

$$R_{Lt} = \int_0^1 R_{Lkt} dk \tag{30}$$

Deposit Insurance Agency. To allow an assessment of the cost of deposit insurance due to the D-ZLB, I explicitly model a deposit insurance agency. This agency repossesses the assets of a bankrupt bank and uses the proceeds as well as government funds T_t to repay depositors. I assume that repossession of the loan portfolio is costly, such that the DIA only receives $(1 - \mu_F)$ of the loan repayments of defaulting banks. The amount of government funds the DIA needs to repay depositors is then:

$$T_t = \int_0^1 \mathbf{1}(\omega_{kt} < \tilde{\omega}_{kt}) \left[R_{Dt}\bar{D} - R_t S_{Kt} - (1 - \mu_F) \left((1 - \omega_{kt})R_{Lkt} + \omega_{kt}(1 - \lambda)\right) L(R_{Lkt}, s_t)\right] dk. \tag{31}$$

3.2 Calibration

The model is calibrated at yearly frequency. The calibration of the baseline model is for Germany, which has a banking sector with a particularly high deposit-to-loan ratio. An alternative calibration for the Euro Area, where the average bank had a deposit-to-loan ratio well below one during the NIRP period, will be considered later on in a robustness exercise in a slightly extended model that allows for the non-deposit debt funding necessary to match such low deposit-to-loan ratios.

To calibrate the states of the economy and the transition probabilities, I use data on the deposit facility rate and the marginal lending rate set by the ECB between Q1 1999 and Q3 2023. Since the start of the ECB's Quantitative Easing (QE) program in Q1 2015, the relevant policy rate has been the deposit facility rate as the ECB has flooded the market with reserves.[22] Prior to QE the relevant policy rate was the marginal lending rate. I then split the time series of the relevant gross policy rate in two regimes: a low monetary policy rate regime (state N), with the relevant rate below 1% per annum, and a high monetary policy rate regime (state P) with the relevant rate above 1% per annum. Let the resulting time-series be denoted R_t^{obs}. The average policy rates in the two regimes are $R(P) = 1.03325$ (that is, 3.325% per annum) and $R(N) = 0.99783$ (that is, -0.217% per annum). To estimate the transition probabilities of the Markov chain:

$$Pr(R_t = R(j)|R_{t-1} = R(i)) = q_{ij} \ , i, j \in \{N, P\} \tag{32}$$

I estimate the following logit model:

$$Pr(R_t^{obs} = R(P)|R_{t-1}^{obs}) = \frac{exp(\beta_0 + \beta_1 R_t^{obs})}{1 + exp(\beta_0 + \beta_1 R_{t-1}^{obs})} \tag{33}$$

[22] While the ECB has carried out asset purchases since October 2014, a significant extension of the program was announced on 22 January 2015

and compute the fitted probabilities. The results are: $q_{PP} = 0.93$, $q_{PN} = 0.067$, $q_{NP} = 0.11$, $q_{NN} = 0.89$.[23]

To calibrate the discount factors of banks, I rely on estimates of the cost of bank equity from European Central Bank (2015) and Fernández and Mencía (2020). Given this evidence, a value of 5% per annum appears a reasonable choice.[24] I therefore set the excess cost of bank capital to $\Delta_E = 0.05$. Further, I set the depreciation rate to $\delta = 0.1$ and the capital requirement to the Basel II capital charge $\gamma = 0.08$ as is standard. I follow Repullo and Suarez (2012) in setting the loss-given-default parameter $\lambda = 0.45$. As discussed therein, this was the value calibrated for uncollateralized corporate loans in the Internal Rating Based Approach introduced in the Basel II framework. Next, I set $\epsilon_D = 100$ to imply a deposit spread of 1% per annum for banks in the positive monetary policy state, similar to the calibration of Abadi et al. (2023). The flat aggregate deposit supply is normalized to $\bar{D} = 50$, which is numerically advantageous. Next, I set the firm's unconditional default probability (given by $\Phi(\varsigma)$, where $\Phi(\cdot)$ is the cdf of the standard normal distribution) to 2.7%, based on Euro Area data reported in Mendicino et al. (forthcoming). Lastly I follow Mendicino et al. (forthcoming) in setting the repossession cost of assets of a bankrupt bank to 30%, i.e. $\mu_F = 0.3$.

[23] This coincides with the Maximum Likelihood estimator for the two-state Markov chain, given by $q_{ij} = \frac{n_{ij}}{n_{i1}+n_{i2}}$, where $n_{ij} = \sum_t \mathbf{1}\{R_t^{obs} = R(j)\}\mathbf{1}\{R_{t-1}^{obs} = R(i)\}$.

[24] European Central Bank (2015) reports a decomposition of the cost of bank equity into the real risk free interest rate, inflation expectations and the equity risk premium based on estimates of a dividend discount model and the CAPM for 33 European banks listed in the EURO STOXX index. Prior to the financial crisis, the excess cost of bank equity (relative to the risk free real interest rate) was relatively stable between 3% and 7%, with little differences in the total cost of equity between the four largest economies (Germany, France, Italy, Spain) for which country-specific estimates are reported. Using a similar methodology, Fernández and Mencía (2020) report similar levels of the cost of bank equity (in excess of the risk free rate) for October 2020 (after a covid-related spike) as in early 2007. Abstracting from market turmoils due to the financial crisis, sovereign debt crisis, Brexit and covid between 2008 and 2020, a 5% excess cost of bank equity therefore seems reasonable in the context of this model.

Moment	Data	Model
Mean Loan Spread	2.48	2.61
Mean Bank Default	0.66	0.62
L/D (State P)	1.05	1.05
L/D (State N)	0.92	0.91
Relative Bank Size (p_1/p_{50})	$< 5.0\%$	10.94 %

Table 1: *Model Fit*

Note: The loan spread is calculated as the difference of corporate lending rates as reported by the Bundesbank (2003-2023, excluding 2008/2009) and the relevant policy rate; Bank default probability from Mendicino et al. (forthcoming) for the Euro Area; Deposit-to-Loan Ratio for Germany from ECB BIS (1999Q1-2023Q3); Relative bank size: the median bank in Germany has a business volume of between 1-5 Billion Euro, the 1th percentile is between 0 and 50 Million, the relative size of the 1th percentile to the median is therefore below 5%.

The remaining parameters $\alpha, \rho, A(N), A(P)$ and χ_I are estimated using the Simulated Method of Moments. To pin down α, I target a loan rate spread of 2.48% (calculated from average loan rates in Germany to non-financial corporations published by the Bundesbank). For the correlation between firm defaults ρ, I target the mean bank default probability of the Euro Area as a whole (taken from Mendicino et al., forthcoming). The state-dependent productivity of firms $A(N), A(P)$ is pinned down by the mean deposit-to-loan ratio in both states (defined as above based on the policy rate), calculated from data on German banks' balance sheets available from the ECB. Lastly, for the cost of equity injections χ_I I target the relative size of the first percentile of banks to the median bank.

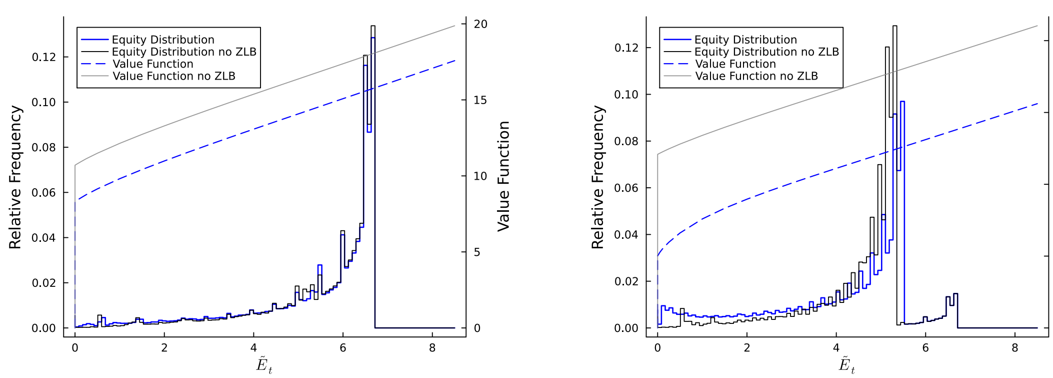
The estimated values are $A(P) = 0.204$, $A(N) = 0.1541$, $\alpha = 0.95$, $\rho = 0.263$ and $\chi_I = 10$. The data moments and model moments are summarized in Table 1.

3.3 Quantitative Results

Through the lens of the model, the only difference between negative policy rates and conventional monetary policy rates is the D-ZLB. Hence, the following discussion focusses mainly on the effects of the D-ZLB.

Figures 3 - 6 depict the value functions, and policy functions of the bank (that is, loan rates, dividends and equity injections as a function of pre-dividend equity $\tilde{E}$), alongside the distribution of $\tilde{E}$. This is based on simulating 10000 independent islands over 100 periods.

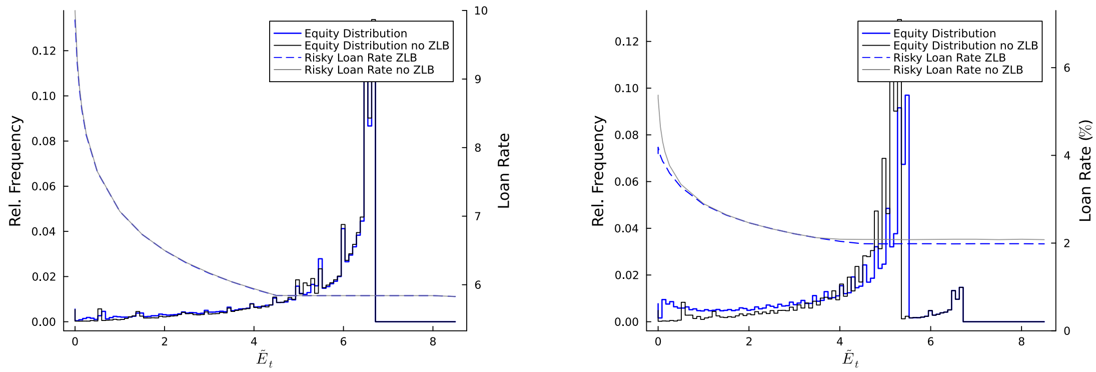
Two main observations can be drawn from the figures regarding banks' incentives: First, unconstrained banks charge lower loan rates in the presence of a binding D-ZLB. This is the amplification effect shown to arise in the simple model of Section 2. Second, when subject to a binding D-ZLB, banks pay out less dividends for any given level $\tilde{E}$ of pre-dividend equity for which the bank would have paid out non-zero dividends in the absence of the D-ZLB, and raises more equity for lower levels of $\tilde{E}$. This allows banks to maintain a higher level of loan supply than they could otherwise sustain under the D-ZLB.



(a) *State P (Policy Rate 3.25%)* **(b)** *State N (Policy Rate -0.22%)*

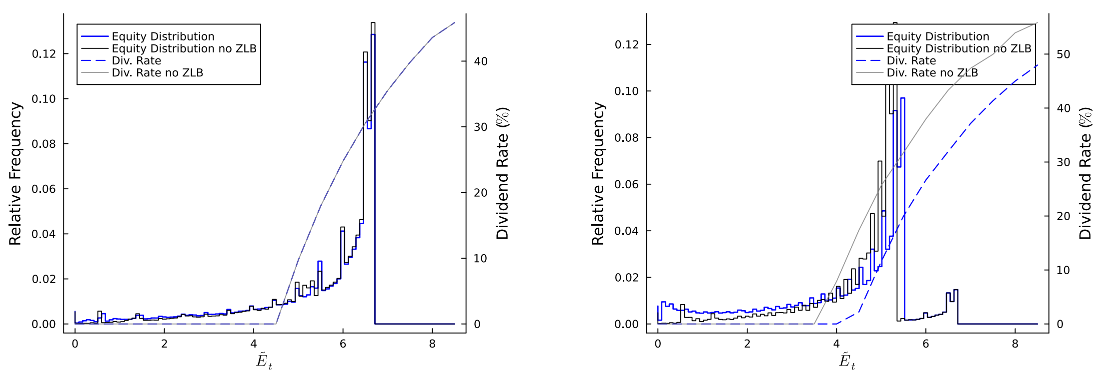
Figure 3: *Value Functions and Equity Distribution*

These adjustments in dividend payouts and equity injections, however, do not offset the effect of the losses in deposit taking that the bank incurs when the D-ZLB is binding



(a) *State P (Policy Rate 3.25%)* **(b)** *State N (Policy Rate -0.22%)*

Figure 4: *Loan Rates and Equity Distribution*

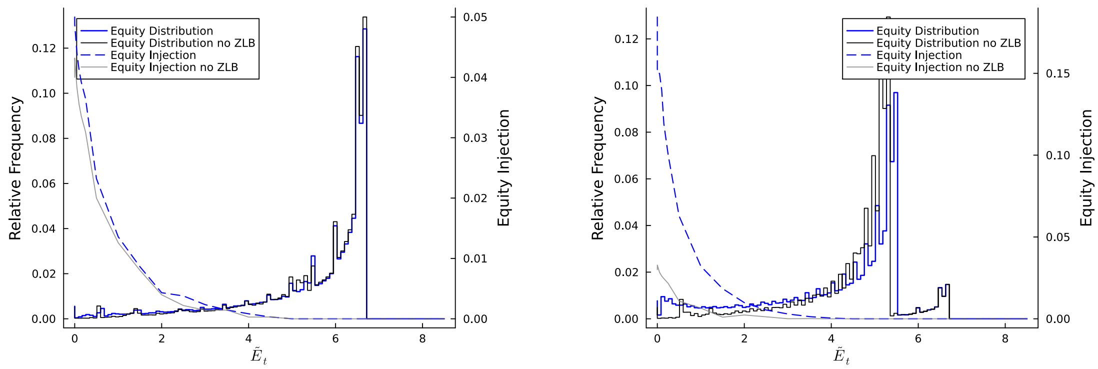


(a) *State P (Policy Rate 3.25%)* **(b)** *State N (Policy Rate -0.22%)*

Figure 5: *Dividends and Equity Distribution*

on the distribution of equity, such that the probability of banks having low equity levels, which force them to restrict lending due to a binding capital requirement, increases – in other words, the left tail of the equity distribution is thicker when banks are subject to a D-ZLB. This is the equity erosion channel of Abadi et al. (2023), but in contrast to their model dividends are chosen endogenously here.[25] Although the effect is more pronounced

[25] Abadi et al. (2023) assume a constant dividend pay-out ratio.



(a) *State P (Policy Rate 3.25%)* **(b)** *State N (Policy Rate -0.22%)*

Figure 6: *Equity Injections and Equity Distribution*

under negative policy rates (when the D-ZLB is binding), the losses in deposit taking under negative policy rates affect the distribution of equity in both states.

The results of simulating the model are summarized in Table 2. As the table shows, the amplification effect is large: the loan rate charged by unconstrained banks is 10p lower under the D-ZLB when policy rates are at -0.22%. On the other hand, the share of banks having to restrict lending to comply with capital requirements due to equity erosion, that is banks whose equity falls below the level needed to sustain the loan rate charged by well capitalized banks, increases from a level of 17.33% that would have prevailed in the absence of a D-ZLB to 39.78%. However, the amplification channel dominates, with average aggregate loan volumes being 4% higher under negative policy rates due to the D-ZLB.[26]

At the same time, however, negative interest rate policy implies a substantial deterioration of financial stability in the scenario in which banks are subject to a D-ZLB, with banks default probability increasing by 28 basis points due to the D-ZLB under negative policy rates of -0.22%, compared to the case of perfect transmission into deposit rates – in that case, the default probability is 0.49%, such that the D-ZLB effect on bank risk constitutes

[26]This is despite average loan rates being slightly higher in state N when banks are subject to a D-ZLB, an instance of Jensen's inequality.

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.2167	3.25	-0.2167
Deposit Rate (%)	2.23	0.0	2.23	-1.2
Loan Volume	52.617	45.678	53.581	43.885
Loan Rate (unconst., %)	5.83	1.98	5.84	2.08
Loan Rate (const. %)	6.37	2.45	6.36	2.48
Loan Rate (%)	5.95	2.18	5.93	2.15
Share Constrained (%)	21.9	39.78	18.92	17.33
Bankruptcy Prob. (%)	0.56	0.77	0.54	0.49
Deposit Insurance Costs	0.0907	0.1049	0.0924	0.0643
Deposit Insurance Costs (Unconditional)		0.095		0.084
Loan Volume (Unconditional)		50.514		50.643
D-ZLB Bank Risk Effect	0.02	0.28		

Table 2: *Average Results By State*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.

a very sizable 60% increase in bank default probabilities. Nevertheless, as Table 2 shows, the additional deposit insurance cost due to the D-ZLB amounts to only about 2% of the additional loan volume due to the D-ZLB in the negative interest rate state N.

To shed more light on the heterogeneity of the D-ZLB effect on the cross-section of loan supply, Figure 7 depicts a histogram of differences in loan supply in a given island (i.e. for a given history of loan default rates) by state. The distribution pools all periods, and hence reflects differences in loan supply for the average period during a spell of state P and state N, respectively. In state P, i.e. under positive policy rates, loan volumes are smaller under the ZLB in 9% of islands and larger in 13% (and equal in the remaining 78%). In state N, i.e. under negative policy rates, loan volumes are smaller under the ZLB in 28.6% of islands

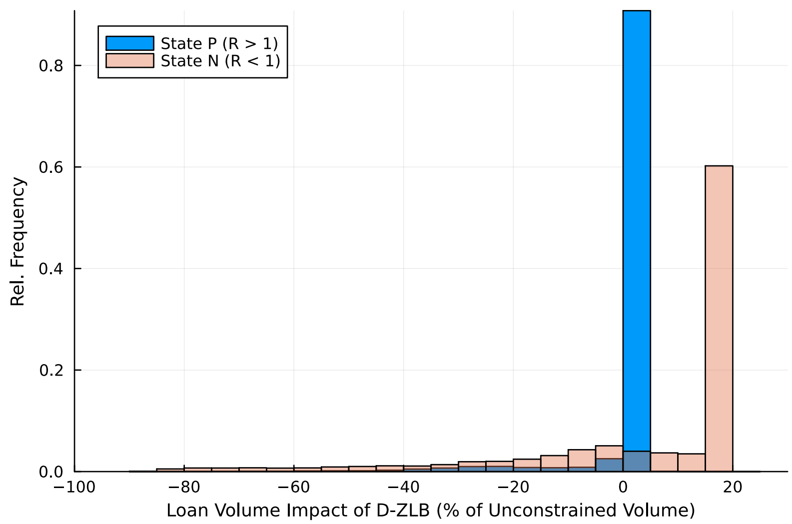


Figure 7: *Histogram Loan Supply Differences*

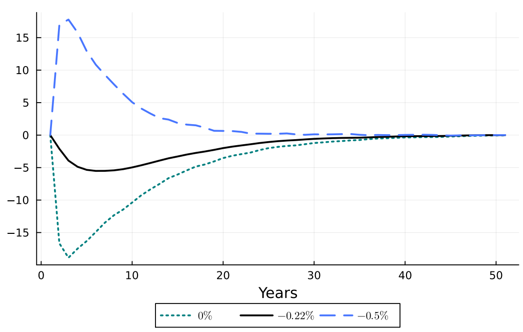
This figure depicts the distribution of differences $L_{kt}^{DZLB} - L_{kt}^{noDZLB}$ in loan supply in the scenario in which banks are subject to a D-ZLB and the scenario in which they are not subject to a D-ZLB, by aggregate state. The sequence of loan default rates ω_{kt} on a given island k is kept constant across simulations.

and larger in 71.4%. The following subsection will focus on how these differences in loan supply change over time under negative policy rates.

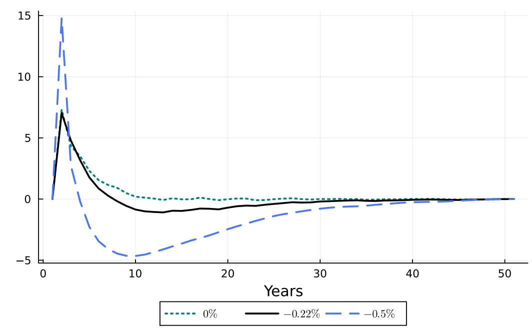
3.4 Dynamics: The D-ZLB Effect Over Time

In the model, periods of negative interest rates coincide with periods of low productivity $A(s)$. Thus far, results were presented as averages across time for the baseline monetary policy rate of -0.22% in the low productivity state. In the following, I investigate how the economy reacts over time to deeper monetary policy rate cuts when productivity drops. Specifically, I consider monetary policy rates of 0%, -0.22% (the baseline level) and -0.5%.

Figure 8 depicts impulse response functions of the economy in which banks are subject to a D-ZLB after a change from the high productivity state P to the low productivity state N for different levels of the monetary policy rate in the low productivity state. Productivity drops in period 1 and follows the Markov-chain dynamics in continuation, i.e. in every period there is an approximately 10% probability that the economy returns to the higher productivity state P. Due to the differences in equity levels of banks, the evolution depends both on the equity level of a given bank when the productivity shock hits, and of the subsequent loan default rates. The figure depicts the average, and hence aggregate, evolution.[27]



(a) *Mean Loan IRF*



(b) *Difference Loans IRFs (D-ZLB - no D-ZLB)*

Figure 8: *Evolution After Temporary Productivity Change From $A(P)$ To $A(N)$*

This figure depicts the average evolution across islands k of selected variables after a temporary change from state P to state N for varying policy rates in state N. The policy rate in state P, as well as the sequence of loan default rates ω_{kt} is kept constant across simulations.

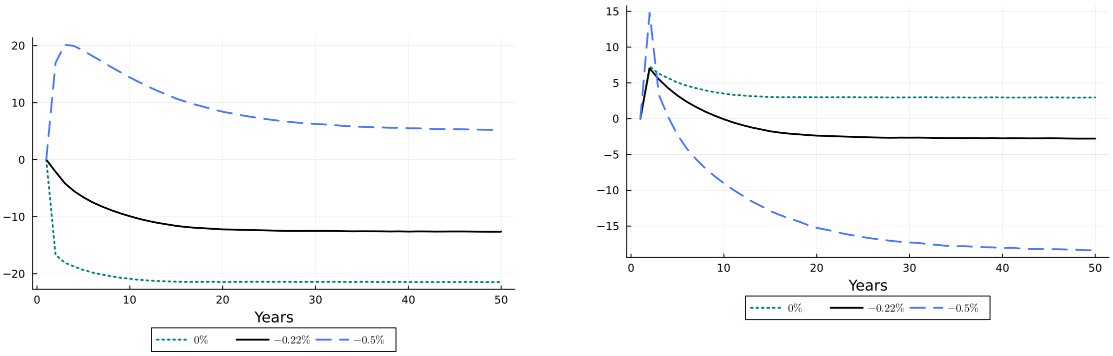
Panel (a) depicts impulse responses of loan volumes. While the productivity shock (ceteris paribus) decreases lending as it shifts down the loan demand function, all monetary policy cuts considered raise the aggregate loan volume – in other words, the monetary policy cuts considered are expansionary.

[27]Since aggregate loan supply is given by $L_t = \int_0^1 L_{kt} dk$, the aggregte loan supply coincides with the average loan supply.

Additionally, panel (b) depicts the difference between the IRFs of banks subject to a D-ZLB with those of banks that are not subject to a D-ZLB. As the panel shows, all the monetary policy cuts considered are initially more expansionary when banks are subject to a D-ZLB compared to the alternative scenario when they are not, as the increase in the loan volume is larger.

The monetary policy cut to 0% remains more expansionary on average over time when there is a D-ZLB vis-a-vis the alternative scenario without D-ZLB. The deeper monetary policy cuts to -0.22% and -0.5%, on the other hand, become less expansionary over time: the equity erosion channel begins to dominate and the monetary policy cut becomes less effective in stimulating bank lending under the D-ZLB. After how many years the effect reverses depends on the policy rate: it is the case after 7 years for the monetary policy cut to -0.22%, and already after 3 years for the monetary policy cut to -0.5%.

Next, I investigate the effects of monetary policy after a permanent change to the low productivity state. The evolution of the economy after such a permanent change is depicted in Figure 9. Two conclusions can be drawn from the figure: first, the difference in loan volumes due to the D-ZLB remains positive for one additional year. This is because the equity erosion channel still affects banks in the high productivity state P when policy rates are positive again, while the amplification effect only affects banks in state N. Second, the difference in loan volumes due to the D-ZLB becomes more negative for the cuts to -0.22% and to -0.5% than for the temporary shock, as the equity erosion effect only reaches its full strength after about 15 years for the monetary policy cut to -0.22% and even later for the cut to -0.5%. For completeness, Tables 4 and 5 in the Appendix report the average results by aggregate state s for the alternative monetary policy rates. The tables confirm that banks extend on average higher loan volumes in the presence of a D-ZLB vis-a-vis the scenario when they are not subject to a D-ZLB for policy rates of 0%, -0.22%, and -0.5%. Hence, while negative interest rate policy is on average more expansionary than



(a) *Mean Loan IRF*

(b) *Difference Loans IRFs (D-ZLB - no D-ZLB)*

Figure 9: *Evolution After Permanent Productivity Change From $A(P)$ To $A(N)$*

This figure depicts the average evolution across islands k of selected variables after a permanent change from state P to state N for varying policy rates in state N. The policy rate in state P, as well as the sequence of loan default rates ω_{kt} is kept constant across simulations.

conventional monetary policy for the negative interest rates considered, it becomes less expansionary over time.

3.5 Alternative Policy: Lower Capital Requirements Instead of Negative Interest Rates

As seen in the previous section, negative interest rates stimulate the economy at the expense of an increase in bank riskiness. A similar qualitative effect can be expected from a decrease in capital requirements in state N, i.e. in response to the negative productivity shock. In reality, such a decrease could for example be achieved by a release of countercyclical capital buffers. Can such a decrease in capital requirements achieve a similar stimulation of the economy, at a lower increase in bank riskiness? To check this, I set the policy rate in state N to $R(N) = 1$ (i.e. a 0% per annum net policy rate) and decrease capital requirements in state N to (a) $\gamma(N) = 0.07$, (b) $\gamma(N) = 0.06$ and (c) $\gamma(N) = 0.0533$. The results are presented in Table 3. None of the decreases in the capital requirement achieves a similar level of stimulation as the baseline negative policy rate of -0.22%. At the same time, the bank default probability increases beyond the baseline level when capital requirements are lowered, despite banks not facing losses in deposit taking due to the higher policy rate of 0%. Therefore, negative interest rate policy appears the better choice here overall.

Policy Rate (%)	-0.22	0.0	0.0	0.0
Capital Requirement (%)	8.0	7.0	6.0	5.33
Loan Volume (P)	52.617	51.238	50.948	50.73
Loan Volume (N)	45.678	35.585	38.684	41.196
Bankruptcy Prob. (%, P)	0.558	0.548	0.569	0.589
Bankruptcy Prob. (%, N)	0.772	0.952	1.332	1.68
Deposit Insurance Costs (P)	0.091	0.085	0.084	0.084
Deposit Insurance Costs (N)	0.105	0.11	0.172	0.235

Table 3: *Release of Capital Requirements*

3.6 Robustness

This subsection presents results from various variations of the model as robustness exercises. Each variation of the baseline model is independent from the others.

3.6.1 Higher Excess Cost of Bank Capital

I first test the sensitivity of the results to the choice of the excess cost of bank capital Δ_E, for which the available empirical evidence points to a relatively large time-series variance (European Central Bank, 2015). In the baseline calibration, I set $\Delta_E = 0.05$. Instead, I now set an 8% p.a. excess cost of bank capital, i.e. $\Delta_E = 0.08$. All other parameters are kept fixed. The results are reported in Table 7. Expectedly, the higher excess cost of capital leads to larger loan rate spreads in both states. The size of the D-ZLB effect on the loan rates is stronger than in the baseline calibration. The loan rate of unconstrained banks is 15bp lower due to the D-ZLB under negative interest rates, compared to 10bp in the baseline calibration. The effect on the average default probability under negative policy rates is also larger now in relative terms, approximately doubling from 0.37 to 0.76%. Different to the baseline calibration, aggregate loan supply is higher in the D-ZLB scenario in both states.

3.6.2 Varying Loan Default Probabilities

I next assess the role of varying loan default probabilities. In the baseline model, loan default probabilities are constant. However, loan default probabilities typically vary over the business cycle, and can be much higher in recessions (e.g. Mendicino et al., forthcoming). To gauge the importance of this for the size of D-ZLB effects, I now assume that the unconditional firm default probability $p(s_t)$ is a function of the state of the economy, and set $p(N) = 0.0485$ and $p(P) = 0.0234$. These values correspond to the average firm default probability in the Euro Area when the firm default probability is above and below the 90th percentile respectively, as reported in Mendicino et al., (forthcoming). All other parameters are kept fixed. The results are reported in Table 8. The table suggests that the higher average loan default rate in state N significantly strengthens the impact of the D-ZLB on the unconstrained loan rate. In the negative interest rate state, the unconstrained loan rate is 3.42% in absence of a D-ZLB, compared to 2.72% when banks are subject to a D-ZLB. At the same time, the D-ZLB effect on the bank default probability is much stronger: the bank default probability increases by more than 1pp in state N. Nevertheless, aggregate loan supply is higher in the D-ZLB scenario in both states.

This exercise illustrates that a change in the distribution of loan default rates (within the observed range of average loan default rates in the Euro Area) has a quantitatively important impact on the size of the D-ZLB effect.

3.6.3 Loan Default Probabilities as a Function of the Loan Rate

Up to now, loan default probabilities were assumed to be independent of loan default rates. In reality, loan rates affect loan default probabilities through at least three channels: (i) adverse selection (Stiglitz and Weiss, 1981), (ii) borrower moral hazard, with borrowers optimally increasing their own default risk when facing higher loan rates (Stiglitz and Weiss, 1981; Boyd and De Nicolo, 2005), and (iii) general equilibrium feedback effects through prices (Kiyotaki and Moore, 1997; Bernanke et al., 1999; Brunnermeier and Sannikov, 2014).

Banks internalize the impact of loan rates on the loan default probabilities, hence their optimal loan rates will be affected by such a feedback loop. To gauge the impact on the effects of the D-ZLB, I now assume that the individual borrowing firm's unconditional default probability is given by $p_t = max(a + b(R_{Lt} - 1), 1)$, following Martinez-Miera and Repullo (2010). The strength of the feedback between loan rates and the unconditional loan default probability is thus governed by b.

I set $a = 0.02$ and $b = 0.15$, such that the unconditional default probability is somewhat above (below) the baseline value of 0.027 in state P (N). The results are presented in Table 9. Additionally, the histogram of loan volume differences due to the D-ZLB is depicted in Figure 10.

In state N, i.e. under negative interest rates, the unconstrained loan rate is lower compared to the baseline in both scenarios (D-ZLB and no D-ZLB), reflecting the now decreased loan default probability in that state. The effect of the D-ZLB on the unconstrained loan rate is also lower than in the baseline (approximately 4bp), and the average loan volume only increases by around 1% in state N. Nevertheless, aggregate loan supply is higher in the D-ZLB scenario in both states.

The effect of the D-ZLB on the bank default probability is also weaker than in the baseline model. In state N, it increases by 7bp due to the D-ZLB, up from a level of 31bp.

Thus, as in the baseline model, the amplification channel still dominates over the equity erosion channel, but the D-ZLB increases bank riskiness. However, the feedback channel between loan rates and loan default rates weakens the effects.

3.6.4 Low Deposit Banks

In the baseline calibration used for the main quantitative results above, banks have abundant deposits and choose to invest a part of them at the policy rate in order to exploit market power in loan markets. While this is appropriate for the average German bank (see Table 1), it is less appropriate for the average Euro Area bank. As Table 6 shows, the average loan-to-deposit was well above 1 for Euro Area banks during the spell of negative interest rate policy conducted by the ECB. In the context of the model here, such banks would thus borrow at the policy rate ($S_{kt} < 0$). However, in reality, banks with a default probability above zero would not be able to borrow at the riskless rate $R(s_t)$. In fact, the interest rates at which commercial banks can borrow from the ECB were never negative, and accessing the facility requires posting collateral.[28] In unsecured transactions on the interbank market, on the other hand, borrowing banks with a non-zero default probability would have to compensate lenders for default risk by paying a higher interest rate (e.g. Afonso et al., 2011).

Therefore, I now assume that banks can borrow funds S^- on financial markets, such that the balance sheet of bank k is now: $L_{kt} + S_{kt} = D_{kt} + S^-_{kt} + E_{kt}$. The interest rate

[28] The ECB sets two key interest rates at which banks can borrow from the ECB against collateral: the main refinancing operation and marginal lending facility.

that lenders charge on non-deposit financing S^{-}_{kt} is denoted $\tilde{R}_{kt}$. I further assume that this debt is junior to deposits, such that the funds available to repay non-deposit lenders are given by:[29]

$$\Omega_{kt} = max(((1-\omega_k)R_{Lkt} + \omega_k(1-\lambda))L_{kt} - R_{Dt}D_{kt}, 0).$$

Hence, the expected return for lenders that provide non-deposit debt to bank k is:

$$R^S_{kt} = \tilde{R}_{kt} - \frac{\int_0^1 min\{\tilde{R}_{kt}S^{-}_{kt} - \Omega_{kt}, \tilde{R}_{kt}S^{-}_{kt}\}dF(\omega)}{S^{-}_{kt}} \tag{34}$$

I assume that lenders are risk-neutral, and may alternatively invest in the safe asset at rate R. Lenders are thus willing to lend S^{-}_{kt} to bank k if they break even in expectation:

$$R^S_{kt} = R_t \tag{35}$$

This subsection considers an alternative calibration to approximately match the average loan-to-deposit ratios of Euro Area banks. To this end, the deposit base is reduced to $\bar{D} = 35$, $A(P)$ is set to 0.2045 and $A(N)$ is set to 0.154, ρ is set to 0.258 and $\chi_I = 75$. The remaining parameters are unchanged. The corresponding model and data moments are summarized in Table 6.

The results are summarized in Table 10. A histogram of differences in lending between banks subject to a D-ZLB and those not subject to a D-ZLB is depicted in Figure 11. The amplification effect is of a similar magnitude as in the baseline model: the unconstrained loan rate is 11bp lower in the presence of a D-ZLB in the negative monetary policy state.

[29]This is in line with EU regulation. Article 108 of the EU Bank Recovery and Resolution Directive 2014/59/EU grants eligible deposits of natural persons and micro, small and medium-sized enterprises priority over claims of unsecured creditors. Eligible deposits, as defined in Article 2, No (95) of the directive, are all deposits not listed in Article 5 of the Directive 2014/49/EU, which provides exclusions for deposits associated with e.g. money laundering. Thus, for the purposes of this model, all deposits are assumed eligible.

Once again, the amplification channel dominates over the equity erosion channel in the quantitative results, with the average loan volume being 4.4% higher in the presence of a D-ZLB in state N. Average aggregate loan volumes are higher in both states. The effect on the default probability is also similar to baseline, with an increase of 26 basis points due to the D-ZLB under negative policy rates (corresponding to a 53% increase).

4 Discussion

The results from the quantitative model presented above highlight two novel results: first, monetary policy is on average *more* expansionary in negative territory in the presence of a D-ZLB, at least initially. Second, NIRP entails substantial risks for financial stability, due to sizable increases in bank default probabilities in the presence of a D-ZLB. Both results are two sides of the same medal, and arise because sufficiently capitalized banks charge lower loan rates on risky loans via a novel amplification channel established in the stylized model of Section 2, that is operative when banks have some probability of default. To the best of my knowledge, none of the papers assessing NIRP (e.g. Abadi et al., 2023; Ulate, 2021; Darracq Paries et al., 2023; Eggertsson et al., 2024) has considered risky banks, precluding this channel from operating in these papers, and precluding them from assessing financial stability risks.

Relation to the Empirical Literature. The novel amplification channel allows the model to rationalize several observations made in the empirical literature on the effect of negative interest rate policy.

First, it offers an explanation of the finding of some authors (e.g. Demiralp et al., 2021; Hong and Kandrac, 2021; Bottero et al., 2022; Schelling and Towbin, 2022) of more exposed banks increasing their loan supply relative to less exposed banks under negative interest rates. This could potentially be explained by models abstracting from bank risk such as Ulate (2021), Abadi et al. (2023) or Eggertsson et al. (2024) if either the loan demand elasticity or the asset composition of such more exposed banks – the measures used in different studies where discussed in Footnote 2 – systematically differed vis-a-vis less exposed banks. Through the lens of the model in Abadi et al. (2023) for example, if more exposed banks had a larger share of fixed-rate long-term assets (which increase in real value after policy rate cuts) in their balance sheet compared to low deposit banks, their (market valued) equity would be predicted to increase relatively more after a policy rate cut due to the revaluation of the bond portfolio, allowing such banks to at least initially sustain a higher loan supply level.[30] But then the market value of such more exposed banks' should also increase more vis-a-vis low deposit banks upon policy rate cuts in positive territory. Thus, if this was driving the empirical findings, placebo exercises such as those for stock price changes upon monetary policy cuts in positive territory reported in Hong and Kandrac (2021) (Appendix C therein) should fail, as stock price changes reflect changes in the market value of equity.[31] But their placebo exercises confirm abnormal reactions of stock prices of high deposit banks (a common measure of exposure to negative policy rates in the literature) vis-a-vis low deposit banks upon policy cuts into negative rate, with a much smaller and generally statistically insignificant predicted impact of the share of wholesale deposits on stock price changes after policy rate cuts above zero.

Second, the portfolio rebalancing effect towards riskier assets documented in e.g. Heider et al. (2019), Bottero et al. (2022) and Basten and Mariathasan (2023) can also be explained by the model presented here. To this end, S may be interpreted as safe lending. Since the

[30]As stressed by Abadi et al. (2023), erosion of equity due to losses on deposits takes time, whereas bond revaluation effects are immediate, hence decreases in loan supply due to the equity erosion channel take time to arise.

[31]Changes in the number of shares and dividend payouts are not a concern since Hong and Kandrac (2021) consider 40 minute windows around the announcement of policy rate cuts.

aggregate deposit base $\bar{D}$ is constant, a decrease in the loan rate under a D-ZLB means that the share of risky loans in the banks' portfolio rises. Heider et al. (2019) argue that their finding is likely due to a risk-taking effect working through franchise values (as in Repullo, 2004) – the model presented here offers a different and potentially complementary explanation for this pattern.

Further, Demiralp et al. (2021) argue that their results of higher loan supply of banks more exposed to negative interest rates reflect such a portfolio-rebalancing effect. The model presented here confirms the intuition that both are related, but stresses that both effects can be two sides of the same medal and allows to understand the relationship better: in the model, the reason for the increased loan supply is not that banks want to reduce their exposure to the safe asset yielding negative interest rates – if this were the case, the same effect would arise with safe banks, but with safe banks there is separation between loan and deposit rates and no special effects of NIRP arise. Rather, as has been seen, loan supply increases because of the impact that the higher bank default risk due to the D-ZLB has on the marginal limited liability subsidy, which induces lower loan rates and hence an increased investment in risky lending vis-a-vis investment in the safe asset – a portfolio rebalancing effect.

The amplification channel identified in this paper explains these patterns (increase in bank lending of more exposed banks and portfolio-reallocation) in a way that is consistent with an increase in bank riskiness due to negative policy rates, as documented in various empirical studies (Nucera et al., 2017; Hong and Kandrac, 2021; Basten and Mariathasan, 2023; Schelling and Towbin, 2022; Heider et al., 2019) – in fact, the increase in bank riskiness due to losses in deposit taking due to the D-ZLB is what gives banks an incentive to increase lending in the first place.

At the same time, the existence of the amplification channel identified in this paper is not inconsistent with empirical studies documenting a decrease in bank lending under negative policy rates (Basten and Mariathasan, 2023; Eggertsson et al., 2024; Heider et al., 2019). As discussed in the introduction, through the lens of the model such effects could reflect (a) portfolio reallocation away from the particular type of loans studied in a given empirical contribution (e.g. away from syndicated loans in Heider et al., 2019 or away from household lending in Eggertsson et al., 2024), or (b) a sufficiently large fraction of banks constrained by their capital requirement.[32]

Policy Implications. Despite the on average higher loan supply under negative interest rate policy due to the D-ZLB in the quantitative results – reflecting the dominance of the amplification channel – these also suggest substantial heterogeneity between banks. Some banks are forced to restrict lending in the presence of a D-ZLB to fulfill the capital requirement due to losses from deposits eroding equity, as argued by Abadi et al. (2023). For this reason, monetary policy may become less expansionary on average over time, as in Ulate (2021). In the simulations of the model calibrated for Germany presented above, this happens for monetary policy rates of -0.22% (the baseline calibration) if rates are kept negative for about 9 years, and for policy rates of -0.5% after already 4 years. The simulations thus suggest that the ECB's spell of negative interest rates between 2014 and 2022 with a trough of -0.5% that was kept between 2019 and 20222 likely amplified lending (on average) for German banks beyond the effect that a conventional monetary policy cut would have had. While the simulations predict that over longer time horizons the equity erosion effect emphasized by e.g. Abadi et al. (2023) becomes stronger such that the additional stimulatory effect of negative interest rate policy fades off and loan volumes fall

[32]While Heider et al. (2019) conduct a suggestive robustness exercise with the whole loan portfolio of Euro Area observed at yearly frequency, the data quality does not permit them a regression analysis. Demiralp et al. (2021) on the other hand, using data on the whole loan portfolio of Euro Area banks, find that banks in the Euro Area more exposed to negative policy rates increased lending vis-a-vis their peers. These divergent findings suggest that a portfolio-reallocation explanation of the findings in Heider et al. (2019), consistent with the channel identified in this paper, is plausible.

below the level that would be expected for a conventional monetary policy rate cut, the results suggest that the cuts into negative policy rates still remain expansionary, with a throughout higher aggregate loan supply under a negative policy rate after a cut to both -0.22% and to -0.5% even after 50 years of such negative policy rates.

5 Conclusion

This paper has studied the effects of negative monetary policy rates on banks in the presence of a zero lower bound on deposit rates (D-ZLB). Banks' hesitance to set negative deposit rates has been a salient finding of empirical studies of the introduction of negative policy rates in various advanced economies since the mid 2010s. This paper highlights for the first time that this lack of transmission into deposit rates changes the loan supply decision of banks not constrained by capital requirements when banks are risky. This is because in that case, equilibrium loan rates are affected by banks' default probability, which in turn is affected by deposit rates. I show in a simple model that such unconstrained banks charge lower loan rates on risky loans: an amplification effect of negative interest rates. However, this comes at a cost of financial stability, with bank default probabilities increasing due to the losses from deposits. It is important to note that this novel amplification effect for *unconstrained* banks is not in contradiction with the reversal result of Abadi et al. (2023), which is due to banks being constrained by capital requirements. The novel mechanism can however explain empirical findings of banks more exposed to negative interest rates increasing their credit supply compared to less exposed banks (e.g. Hong and Kandrac, 2021; Demiralp et al., 2021; Bottero et al., 2022).

After establishing these result formally in a stylized model, the paper proceeds to quantify the relative importance of the various effects of negative monetary policy rates, namely the amplification effect for unconstrained banks, the financial stability effect and the effect on binding capital requirements highlighted by previous contributions (Ulate, 2021; Abadi et al., 2023), in a quantitative model of the banking industry. The model is designed to quantify the effects of a lower bound on deposits on loan supply and financial stability – noting that in the absence of such a D-ZLB, there is no difference between conventional and negative interest rate policy in the model.

In the quantitative model, calibrated for Germany, the amplification effect is large and dominates the other effects, such that loan volumes are on average about 4% higher under negative interest rates in the presence of a D-ZLB. At the same time, negative interest rate policy implies a substantial deterioration of financial stability under a D-ZLB, with bank default probabilities increasing by 28 basis points under negative policy rates of -0.22% – constituting a very sizable 60% increase.

References

Abadi, J., Brunnermeier, M., and Koby, Y. (2023). The Reversal Interest Rate. *American Economic Review*, 113(8):2084–2120.

Admati, A. R., DeMarzo, P. M., Hellwig, M., and Pfleiderer, P. (2010). Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity Is Not Expensive. Technical report, Preprints of the Max Planck Institute for Research on Collective Goods.

Afonso, G., Kovner, A., and Schoar, A. (2011). Stressed, not frozen: The federal funds market in the financial crisis. *The Journal of Finance*, 66(4):1109–1139.

Bahaj, S. and Malherbe, F. (2020). The Forced Safety Effect: How Higher Capital Requirements Can Increase Bank Lending. *The Journal of Finance*, 75(6):3013–3053.

Basel Committee on Banking Supervision (2004). International Convergence of Capital Measurement and Capital Standards: A Revised Framework. Report, Bank for International Settlements.

Basten, C. and Mariathasan, M. (2023). Interest Rate Pass-Through And Bank Risk-Taking Under Negative-Rate Policies With Tiered Remuneration of Central Bank Reserves. *Journal of Financial Stability*, 68:101160.

Begenau, J. and Stafford, E. (2023). Uniform rate setting and the deposit channel. *Available at SSRN 4136858*.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of Macroeconomics*, 1:1341–1393.

Bottero, M., Minoiu, C., Peydró, J.-L., Polo, A., Presbitero, A. F., and Sette, E. (2022). Expansionary Yet Different: Credit Supply and Real Effects of Negative Interest Rate Policy. *Journal of Financial Economics*, 146(2):754–778.

Boyd, J. H. and De Nicolo, G. (2005). The Theory of Bank Risk Taking and Competition Revisited. *The Journal of Finance*, 60(3):1329–1343.

Brunnermeier, M. K. and Sannikov, Y. (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104(2):379–421.

Corbae, D. and D'Erasmo, P. (2021). Capital buffers in a quantitative model of banking industry dynamics. *Econometrica*, 89(6):2975–3023.

Darracq Paries, M., Kok, C., and Rottner, M. (2023). Reversal Interest Rate and Macroprudential Policy. *European Economic Review*, 159:104572.

Dell'Ariccia, G., Laeven, L., and Marquez, R. (2014). Real Interest Rates, Leverage, and Bank Risk-Taking. *Journal of Economic Theory*, 149:65–99.

Demiralp, S., Eisenschmidt, J., and Vlassopoulos, T. (2021). Negative Interest Rates, Excess Liquidity and Bank Business Models: Banks' Reaction to Unconventional Monetary Policy in the Euro Area. *European Economic Review*, 136:103745.

Drechsler, I., Savov, A., and Schnabl, P. (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics*, 132(4):1819–1876.

Eggertsson, G. B., Juelsrud, R. E., Summers, L. H., and Wold, E. G. (2024). Negative Nominal Interest Rates and the Bank Lending Channel. *Review of Economic Studies*, 91(4):2201–2275.

European Central Bank (2015). Financial Stability Review. May 2015.

Fernández, L. G. and Mencía, J. (2020). Recent Developments in the Cost of Bank Equity in Europe. *Economic Bulletin*, (4):1–11.

Gordy, M. B. (2003). A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules. *Journal of Financial Intermediation*, 12(3):199–232.

Heider, F., Saidi, F., and Schepens, G. (2019). Life Below Zero: Bank Lending under Negative Policy Rates. *The Review of Financial Studies*, 32(10):3728–3761.

Hong, G. H. and Kandrac, J. (2021). Pushed Past the Limit? How Japanese Banks Reacted to Negative Rates. *Journal of Money, Credit and Banking*.

Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2):211–248.

Martinez-Miera, D. and Repullo, R. (2010). Does Competition Reduce the Risk of Bank Failure? *The Review of Financial Studies*, 23(10):3638–3664.

Nucera, F., Lucas, A., Schaumburg, J., and Schwaab, B. (2017). Do Negative Interest Rates Make Banks Less Safe? *Economics Letters*, 159:112–115.

Onofri, M., Peersman, G., and Smets, F. (2023). The Effectiveness of a Negative Interest Rate Policy. *Journal of Monetary Economics*, 140:16–33.

Repullo, R. (2004). Capital Requirements, Market Power, and Risk-Taking in Banking. *Journal of Financial Intermediation*, 13(2):156–182.

Repullo, R. (2020a). The deposits channel of monetary policy: A critical review.

Repullo, R. (2020b). The Reversal Interest Rate. A Critical Review. *CEMFI Working Papers*, (wp2020_2021).

Repullo, R. and Suarez, J. (2012). The Procyclical Effects of Bank Capital Regulation. *The Review of Financial Studies*, 26(2):452–490.

Schelling, T. and Towbin, P. (2022). What Lies Beneath—Negative Interest Rates And Bank Lending. *Journal of Financial Intermediation*, 51:100969.

Stiglitz, J. E. and Weiss, A. (1981). Credit Rationing in Markets with Imperfect Information. *The American Economic Review*, 71(3):393–410.

Ulate, M. (2021). Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates. *American Economic Review*, 111(1):1–40.

Vasicek, O. (2002). The Distribution of Loan Portfolio Value. *Risk*, 15(12):160–162.

Appendices

A Additional Tables and Figures

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.0	3.25	-0.0
Deposit Rate (%)	2.23	0.0	2.23	-0.99
Loan Volume	50.171	32.832	52.066	28.743
Loan Rate (unconst., %)	5.84	2.22	5.84	2.35
Loan Rate (const. %)	6.74	2.89	6.36	2.78
Loan Rate (%)	6.0	2.35	5.95	2.4
Share Constrained (%)	18.45	19.91	23.16	11.19
Bankruptcy Prob. (%)	0.32	0.37	0.52	0.4
Deposit Insurance Costs	0.0528	0.0378	0.086	0.0342
Deposit Insurance Costs (Unconditional)		0.048		0.07
Loan Volume (Unconditional)		44.917		44.999
D-ZLB Bank Risk Effect	-0.2	-0.04		

Table 4: *Average Results By State ($R(N) = 0$)*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.5	3.25	-0.5
Deposit Rate (%)	2.23	0.0	2.23	-1.49
Loan Volume	56.461	72.651	51.995	71.238
Loan Rate (unconst., %)	5.77	1.66	5.84	1.73
Loan Rate (const. %)	6.34	2.13	6.72	2.37
Loan Rate (%)	5.9	1.93	5.96	1.89
Share Constrained (%)	20.76	58.56	13.09	24.65
Bankruptcy Prob. (%)	0.58	1.17	0.39	0.31
Deposit Insurance Costs	0.098	0.172	0.0665	0.0737
Deposit Insurance Costs (Unconditional)		0.12		0.069
Loan Volume (Unconditional)		61.367		57.826
D-ZLB Bank Risk Effect	0.19	0.86		

Table 5: *Average Results By State* ($R(N) = -0.005$)

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.

Moment	Data	Model
Mean Loan Spread	2.55	2.55
Mean Bank Default	0.66	0.7
L/D (State P)	1.3	1.31
L/D (State N)	1.1	1.12
Relative Bank Size	2.0	5.89

Table 6: *Moments & Targets (Low Deposit Model)*

Note: The loan spread is calculated as the difference of corporate lending rates as reported by the ECB (2003-2023, excluding 2008/2009) and the relevant policy rate; Bank default probability from Mendicino et al. (forthcoming) for the Euro Area; Deposit-to-Loan Ratio for the Euro Area from ECB BIS (1999Q1-2023Q3); Relative bank size for Germany, due to lack of EA data: the median bank in Germany has a business volume of between 1-5 Billion Euro, the 1th percentile is between 0 and 50 Million, the relative size of the 1th percentile to the median is therefore below 5%.

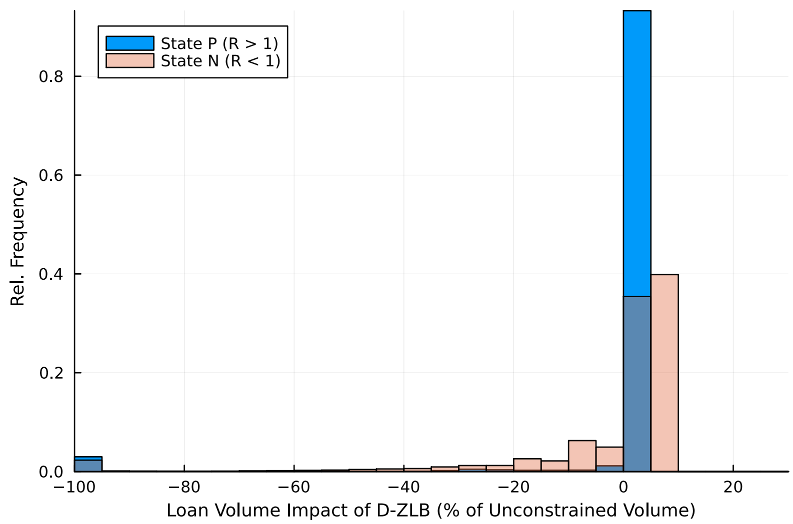


Figure 10: *Histogram Loan Supply Differences when Loan Default Probabilities are a Function of the Loan Rate (Section 3.6.3))*

This figure depicts the distribution of differences $L_{kt}^{DZLB} - L_{kt}^{noDZLB}$ in loan supply in the scenario in which banks are subject to a D-ZLB and the scenario in which they are not subject to a D-ZLB, by aggregate state. The sequence of loan default rates ω_{kt} on a given island k is kept constant across simulations.

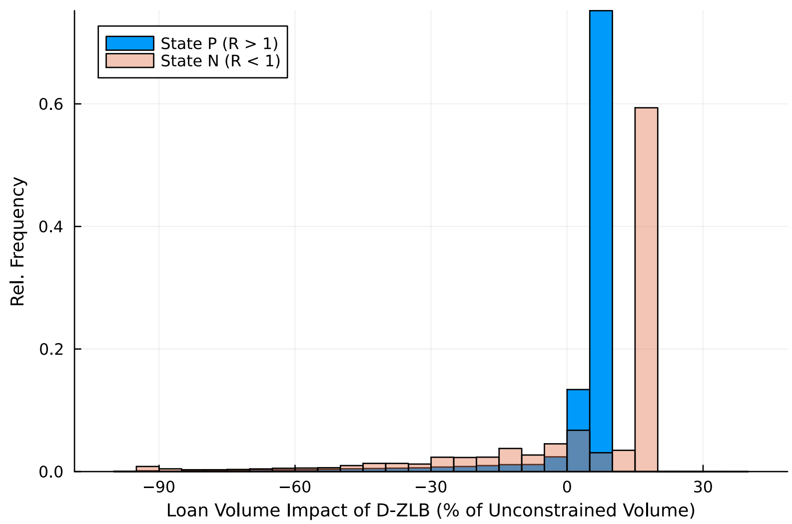


Figure 11: *Histogram Loan Supply Differences (Low Deposit Model of Section 3.6.4)*

This figure depicts the distribution of differences $L_{kt}^{DZLB} - L_{kt}^{noDZLB}$ in loan supply in the scenario in which banks are subject to a D-ZLB and the scenario in which they are not subject to a D-ZLB, by aggregate state. The sequence of loan default rates ω_{kt} on a given island k is kept constant across simulations.

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.2167	3.25	-0.2167
Deposit Rate (%)	2.23	0.0	2.23	-1.2
Loan Volume	38.927	31.367	39.474	27.565
Loan Rate (unconst., %)	6.09	2.22	6.09	2.37
Loan Rate (const. %)	6.52	2.7	6.9	2.73
Loan Rate (%)	6.17	2.4	6.22	2.4
Share Constrained (%)	16.73	35.79	14.49	9.29
Bankruptcy Prob. (%)	0.48	0.76	0.45	0.37
Deposit Insurance Costs	0.0557	0.0708	0.0567	0.0308
Deposit Insurance Costs (Unconditional)		0.06		0.049
Loan Volume (Unconditional)		36.636		35.865
D-ZLB Bank Risk Effect	0.02	0.39		

Table 7: *Average Results By State (8% Excess Cost of Bank Capital))*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained *is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.*

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.2167	3.25	-0.2167
Deposit Rate (%)	2.23	0.0	2.23	-1.2
Loan Volume	62.444	14.208	61.369	6.255
Loan Rate (unconst., %)	5.63	2.72	5.63	3.34
Loan Rate (const. %)	6.15	3.27	6.16	3.74
Loan Rate (%)	5.85	2.86	5.87	3.35
Share Constrained (%)	35.4	29.14	37.24	1.29
Bankruptcy Prob. (%)	0.48	1.45	0.38	0.2
Deposit Insurance Costs	0.0815	0.0534	0.0795	0.0031
Deposit Insurance Costs (Unconditional)		0.073		0.056
Loan Volume (Unconditional)		47.827		44.668
D-ZLB Bank Risk Effect	0.1	1.26		

Table 8: *Average Results By State (Varying Firm Default Probability p, Section 3.6.2)*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained *is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.*

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.2167	3.25	-0.2167
Deposit Rate (%)	2.23	0.0	2.23	-1.2
Loan Volume	54.39	74.08	53.37	73.17
Loan Rate (unconst., %)	5.82	1.72	5.84	1.76
Loan Rate (const. %)	8.96	2.23	9.53	2.18
Loan Rate (%)	7.15	1.83	7.23	1.83
Share Constrained (%)	16.1	47.06	14.55	25.99
Bankruptcy Prob. (%)	0.48	0.38	0.49	0.31
Deposit Insurance Costs	0.08	0.08	0.08	0.07
Deposit Insurance Costs (Unconditional)		0.08		0.08
Loan Volume (Unconditional)		60.35		59.37
D-ZLB Bank Risk Effect	-0.01	0.07		

Table 9: *Average Results By State when Loan Default Probabilities are a Function of the Loan Rate (Section 3.6.3)*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.

	D-ZLB		No D-ZLB	
Policy Rate (%)	3.25	-0.2167	3.25	-0.2167
Deposit Rate (%)	2.23	0.0	2.23	-1.2
Loan Volume	45.91	39.19	44.23	37.54
Loan Rate (unconst.)	5.82	1.95	5.88	2.06
Loan Rate (const.)	6.39	2.47	6.37	2.39
Loan Rate	5.98	2.18	5.98	2.13
Share Constrained	26.09	41.81	20.06	20.36
Bankruptcy Prob. (Percent)	0.63	0.78	0.6	0.52
Deposit Insurance Costs	0.0979	0.0941	0.0918	0.0637
Deposit Insurance Costs (Unconditional)		0.0959		0.0771
Loan Volume (Unconditional)		42.38		40.72
D-ZLB Bank Risk Effect	0.03	0.26		

Table 10: *Average Results By State (Low Deposit Model of Section 3.6.4)*

The table reports averages over a simulation of 100 years and 10000 islands with independent shocks z_{kt}. Loan rate (const.) and Loan rate (unconst.) are the loan rates charged by, respectively, banks that restrict lending due to low equity levels, and those that have sufficient equity. Share Constrained is the share of banks that restrict lending due to low equity levels. The ZLB Bank Risk Effect is the change in the bankruptcy probability due to the deposit ZLB.

B Proofs

Proof of Lemma 1. The objective function of the bank was given by:

$$\max_{R_L, R_D \geq \underline{R}_D} \int_0^1 max\{(R_L - R)L(R_L) + (R - R_D)D(R_D) - \omega R_L L(R_L), 0\} dF(\omega) \quad \text{(B.1)}$$

To replace the $max(\cdot)$ function within the interval, note that the bank will default (and thus receive zero) if $\omega > \tilde{\omega}$, where

$$\tilde{\omega}(R_L, \hat{R}_D(R, \underline{R}_D), R, \underline{R}_D) = \frac{(R_L - R)L(R_L) + (R - \hat{R}_D)\bar{D}}{R_L L(R_L)}$$

Given the support of $\omega \in [0, 1]$, the upper bound of integration is:

$$\bar{\omega} = \begin{cases} 0 \text{ if } \tilde{\omega} < 0 \\ \tilde{\omega} \text{ if } 0 \leq \tilde{\omega} \leq 1 \\ 1 \text{ if } \tilde{\omega} > 1 \end{cases} \tag{B.2}$$

The objective of the bank as a function of R_D for any given R_L can then be written as:

$$\mathcal{L}(R_D) = \int_0^{\bar{\omega}} [(R_L(1 - \omega) - R)L(R_L) - (R_D - R)D(R_D)]dF(\omega) \tag{B.3}$$

If $\tilde{\omega} \in [0, 1]$, the integrand evaluated at the upper bound is zero by definition of $\bar{\omega}$. If, on the other hand, $\tilde{\omega} > 1$ or $\tilde{\omega} < 0$, the upper bound is unaffected by marginal changes in the deposit rate. Thus, it follows from Leibniz Rule:

$$\mathcal{L}'(R_D) = F(\bar{\omega}) \left[-D(R_D) - (R_D - R)D'(R_D) \right] \tag{B.4}$$

where $D'(R_D) = (-\epsilon_D)\frac{D(R_D)}{R_D}$ with $\epsilon_D < -1$, such that:

$$\mathcal{L}'(R_D) = F(\bar{\omega})(-\epsilon_D)\frac{D(R_D)}{R_D} \left[R - \frac{\epsilon_D - 1}{\epsilon_D} R_D \right] \tag{B.5}$$

We have $\frac{(-\epsilon_D)D(R_D)}{R_D} > 0$ for any $R_D > 0$. Further, if the bank does not always fail we have $F(\bar{\omega}) > 0$, in which case:

$$\mathcal{L}'(R_D^*) \stackrel{!}{=} 0 \iff R_D^* = \frac{\epsilon_D}{\epsilon_D - 1} R \tag{B.6}$$

Then any $R_D > R_D^*$ would feature $\mathcal{L}' < 0$ since $\epsilon_D < -1$. Conversely, any $R_D < R_D^*$ would feature $\mathcal{L}' > 0$, implying that the bank could reach higher profits by decreasing or increasing, respectively, R_D. So R_D^* is the unique maximizer of (B.3). Now, if $R_D^* \leq \underline{R}_D$, then R_D^* is also the banks' optimal deposit rate in the presence of the D-ZLB. However, if $R_D^* < \underline{R}_D$, then the bank maximizes (B.1) at the corner $\underline{R}_D$, since $\mathcal{L}' < 0$ for any $R_D > R_D^*$, such that setting a deposit rate higher than $\underline{R}_D$ leads to lower profits, and the bank cannot set a deposit rate lower than $\underline{R}_D$. Thus in general:

$$\hat{R}_D = max(R_D^*, \underline{R}_D) \tag{B.7}$$

Further, $R_D^*(R)$ is a differentiable and increasing function with:

$$\frac{\partial R_D^*(R)}{\partial R} = \frac{\epsilon_D}{\epsilon_D - 1} > 0 \tag{B.8}$$

and for any given $\underline{R}_D$, there exists $R^* = \frac{\epsilon_D - 1}{\epsilon_D} \underline{R}_D$ s.t. $R_D^*(R) > \underline{R}_D$ for all $R > R^*$ and $R_D^*(R) < \underline{R}_D$ for all $R < R^*$. ■

Proof of Proposition 1. First consider the possible corner solutions. It is never optimal for the bank to set loan rates lower than R: by definition $\Pi(R) \geq \Pi(R_L)$ for all $R_L < R$, since for all such R_L, profits from lending $[(1-\omega)R_L - R)]L(R_L)$ are negative for all ω. If the bank does not make losses in deposit taking $(R - \hat{R}_D)D(\hat{R}_D) \geq 0$, the inequality is strict: $\Pi(R) > \Pi(R_L)$. Now consider the other corner, i.e. an infinite loan rate. When setting an infinite loan rate (and thus extending zero loans), loan demand implies $\lim_{R_L \to \infty} R_L L(R_L) = \lim_{R_L \to \infty} R_L^{-\epsilon_L+1} = 0$ since $\epsilon_L > 1$. But then clearly $\lim_{R_L \to \infty} \Pi(R_L) = max((R - \hat{R}_D)D(\hat{R}_D), 0)$.

Now focus on the case in which the bank does not make losses in deposits, i.e. $(R - \hat{R}_D)D(\hat{R}_D) \geq 0$. To shorten notation, define $\pi(\omega; R_L) = (R_L(1-\omega) - R)L(R_L) + (R - \hat{R}_D)D(\hat{R}_D)$, such that $\Pi(R_L) = \mathbb{E}\pi(\omega; R_L)$. Since $\max(x, 0)$ is convex, Jensen's inequality implies that $\mathbb{E}max(\pi(\omega; R_L), 0) \geq max(\mathbb{E}\pi(\omega; R_L), 0)$. But then by definition:

$$\mathbb{E}max(\pi(\omega; R_L), 0) \geq max(L(R_L) \int_0^1 (R_L(1-\omega) - R) dF(\omega) + \underbrace{(R - \hat{R}_D)D(\hat{R}_D)}_{\geq 0}, 0)$$

Hence, $\mathbb{E}max(\pi(\omega; R_L), 0) = \Pi(R_L) > (R - \hat{R}_D)D(\hat{R}_D)$ for all $R_L > \frac{R}{1-\mathbb{E}\omega}$ s.t. $L(R_L) > 0$. Since $\mathbb{E}\omega \neq 1$, the set $[\frac{R}{1-\mathbb{E}\omega}, \infty)$ is not empty. Further, we had that $\lim_{R_L \to \infty} \Pi(R_L) = max((R - \hat{R}_D)D(\hat{R}_D), 0) = (R - \hat{R}_D)D(\hat{R}_D) > \Pi(R)$. This guarantees that there is a loan rate $R_L \in (R, \infty)$ that gives a higher payoff to the bank than setting an infinite loan rate and hence extending zero loans, and also a higher payoff than setting $R_L \leq R$.

Next, consider the case $(R - \hat{R}_D)D(\hat{R}_D) < 0$. Obviously, if the bank never breaks even for any loan rate, i.e. $(R_L - R)L(R_L) + (R - \hat{R}_D)D(\hat{R}_D) \leq 0$ for all R_L, then $\Pi(R_L) = 0$ for all R_L. If that is not the case, i.e. if there exists a loan rate R_L s.t. $(R_L - R)L(R_L) + (R - \hat{R}_D)D(\hat{R}_D) > 0$, then $\max_{R_L} \Pi(R_L) > 0$ since ω has full support on $[0, 1]$. But since $(R - \hat{R}_D)D(\hat{R}_D) < 0$, we have that $\lim_{R_L \to \infty} \Pi(R_L) = max((R - \hat{R}_D)D(\hat{R}_D), 0) = 0$. Hence, unless the bank fails with certainty for any loan rate, there is a loan rate $R_L \in \mathbb{R}$ that gives a strictly positive payoff, and hence there must be an interior maximum.

Having established the conditions for existence of an interior solution, let us now turn to necessary conditions. Since $\Pi(R_L)$ is a differentiable function, it must be that at the interior maximum $\Pi'(R_L) = 0$. It was assumed that loan demand is given by

$$L(R_L) = AR_L^{-\epsilon_L}, \ \epsilon_L > 1$$

Such that

$$\frac{R_L L'(R_L)}{L(R_L)} = -\epsilon_L \tag{B.9}$$

The FOC of the maximization problem is:

$$\Pi'(R_L) = \int_0^{\tilde{\omega}} [((1-\omega)R_L - R)L'(R_L) + (1-\omega)L(R_L)]dF(\omega) \overset{!}{=} 0 \tag{B.10}$$

Define

$$\omega^{LL} = \int_0^{\tilde{\omega}} \frac{\omega}{F(\tilde{\omega})} dF(\omega)$$

such that the FOC becomes:

$$F(\tilde{\omega})[((1-\omega^{LL})R_L - R)L'(R_L) + (1-\omega^{LL})L(R_L)] = 0 \tag{B.11}$$

Factoring out $L'(R_L)$ and using (B.9) the FOC can be written as:

$$F(\tilde{\omega})L'(R_L)[((1-\omega^{LL})\left(\frac{\epsilon_L - 1}{\epsilon_L}\right) R_L - R] = 0 \tag{B.12}$$

and the optimal loan rate $\hat{R}_L$ thus fulfills:

$$(1-\omega^{LL})\hat{R}_L = \frac{\epsilon_L}{\epsilon_L - 1} R > R \tag{B.13}$$

■

Proof of Proposition 2.

The optimal loan rate $\hat{R}_L$ is characterized by Eq. (B.12). The equation defines $\hat{R}_L$ as an implicit function of R and $\underline{R}_D$:

$$\Xi(\hat{R}_L(R,\underline{R}_D), \hat{R}_D(R,\underline{R}_D), R, \underline{R}_D) = (1-\omega^{LL})\left(\frac{\epsilon_L - 1}{\epsilon_L}\right)\hat{R}_L - R = 0 \tag{B.14}$$

Let Ξ_i denote the derivative of Ξ with respect to it's i'th argument. Differentiating Ξ with respect to $\underline{R}_D$ yields:

$$\Xi_1 \frac{\partial \hat{R}_L}{\partial \underline{R}_D} + \Xi_2 \frac{\partial \hat{R}_D}{\partial \underline{R}_D} + \Xi_4 = 0 \tag{B.15}$$

Calculating the derivatives of Ξ:

$$\Xi_1 = \frac{\epsilon_L - 1}{\epsilon_L} \left[(1 - \omega^{LL}) - \hat{R}_L \frac{\partial \omega^{LL}}{\partial R_L} \right] > 0 \tag{B.16}$$

at the optimal loan rate $\hat{R}_L$. This implies that Ξ_1 must be positive – if it was negative at $\hat{R}_L$, then $\hat{R}_L$ would be a local minimum. Further, it was assumed that parameters and functional forms are such that at the (unique) local maximum $\Xi_1 \neq 0$ to ensure that the implicit function theorem can be applied. Section C establishes some sufficient conditions for this assumption to hold.

Next,

$$\Xi_2 = -\left(\frac{\epsilon_L - 1}{\epsilon_L} \right) \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial \bar{\omega}}{\partial R_D} R_L \tag{B.17}$$

where from Eq. (6):

$$\frac{\partial \bar{\omega}}{\partial R_D} = -\frac{\bar{D}}{\hat{R}_L L(\hat{R}_L)} \tag{B.18}$$

since it was assumed that $\bar{\omega} \in (0,1)$ in the statement of the proposition; and further:

$$\frac{\partial \hat{R}_D}{\partial R} = \begin{cases} 0 \text{ if } R < R^* \\ \frac{\epsilon_D}{\epsilon_D - 1} \text{ if } R > R^* \\ \text{and else does not exist} \end{cases} \tag{B.19}$$

as shown in Lemma 1. Lastly, $\Xi_4 = 0$.Hence:

$$\frac{\partial \hat{R}_L}{\partial \underline{R}_D} = -\frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial R_D}{\partial \underline{R}_D} \frac{\bar{D}}{L(\hat{R}_L)} \frac{1}{(1-\omega^{LL}) - \hat{R}_L \frac{\partial \omega^{LL}}{\partial R_L}} \begin{cases} < 0 \text{ if } R < R^* \\ \text{does not exist if } R = R^* \\ = 0 \text{ else} \end{cases} \tag{B.20}$$

■

Alternative Proof of Proposition 2.

The optimal loan rate $\hat{R}_L$ is characterized by Eq. (B.12). The equation defines $\hat{R}_L$ as an implicit function of R and $\underline{R}_D$:

$$\Xi(\hat{R}_L(R, \underline{R}_D), \hat{R}_D(R, \underline{R}_D), R, \underline{R}_D) = (1-\omega^{LL}) \left(\frac{\epsilon_L - 1}{\epsilon_L}\right) \hat{R}_L - R = 0 \tag{B.21}$$

Let Ξ_i denote the derivative of Ξ with respect to it's i'th argument. Differentiating Ξ with respect to $\underline{R}_D$ yields:

$$\Xi_1 \frac{\partial \hat{R}_L}{\partial \underline{R}_D} + \Xi_2 \frac{\partial \hat{R}_D}{\partial \underline{R}_D} + \Xi_4 = 0 \tag{B.22}$$

Calculating the derivatives of Ξ:

$$\Xi_1 = \frac{\epsilon_L - 1}{\epsilon_L} \left[(1-\omega^{LL}) - \hat{R}_L \frac{\partial \omega^{LL}}{\partial R_L} \right] > 0 \tag{B.23}$$

at the optimal loan rate $\hat{R}_L$. This implies that Ξ_1 must be positive – if it was negative at $\hat{R}_L$, then $\hat{R}_L$ would be a local minimum. Further, it was assumed that parameters and functional forms are such that at the (unique) local maximum $\Xi_1 \neq 0$ to ensure that the implicit function theorem can be applied. Section C establishes some sufficient conditions for this assumption to hold.

Next,

$$\Xi_2 = -\left(\frac{\epsilon_L - 1}{\epsilon_L}\right) \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial \bar{\omega}}{\partial R_D} R_L \tag{B.24}$$

where from Eq. (6):

$$\frac{\partial \bar{\omega}}{\partial R_D} = -\frac{\bar{D}}{\hat{R}_L L(\hat{R}_L)} \tag{B.25}$$

since it was assumed that $\bar{\omega} \in (0,1)$ in the statement of the proposition; and further:

$$\frac{\partial \hat{R}_D}{\partial R} = \begin{cases} 0 \text{ if } R < R^* \\ \frac{\epsilon_D}{\epsilon_D - 1} \text{ if } R > R^* \\ \text{and else does not exist} \end{cases} \tag{B.26}$$

as shown in Lemma 1. Lastly, $\Xi_4 = 0$.Hence:

$$\frac{\partial \hat{R}_L}{\partial \underline{R}_D} = -\frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial R_D}{\partial \underline{R}_D} \frac{\bar{D}}{L(\hat{R}_L)} \frac{1}{(1-\omega^{LL}) - \hat{R}_L \frac{\partial \omega^{LL}}{\partial R_L}} \begin{cases} < 0 \text{ if } R < R^* \\ \text{does not exist if } R = R^* \\ = 0 \text{ else} \end{cases} \tag{B.27}$$

■

Proof of Corollary 1. From Proposition 2 it is straightforward to show that a bank is more likely to fail under the D-ZLB:

$$\frac{\partial \bar{\omega}}{\partial \underline{R}_D} = \underbrace{\frac{\partial \bar{\omega}}{\partial R_D}}_{<0} \frac{\partial R_D}{\partial \underline{R}_D} + \underbrace{\frac{\partial \bar{\omega}}{\partial R_L}}_{>0} \underbrace{\frac{\partial R_L}{\partial \underline{R}_D}}_{\leq 0} \begin{cases} < 0 \text{ if } R < R^* \\ \text{does not exist if } R = R^* \\ = 0 \text{ else} \end{cases} \tag{B.28}$$

which follows from Eq. (B.25), Eq. (C.5) and Proposition 2. ■

Proof of Corollary 2. Differentiating Eq. (B.21) with respect to R yields:

$$\Xi_1 \frac{\partial R_L}{\partial R} + \Xi_2 \frac{\partial \hat{R}_D}{\partial R} + \Xi_3 = 0 \tag{B.29}$$

Ξ_1 and Ξ_2 have already been computed above in the proof of Proposition 2. Lastly:

$$\Xi_3 = -\frac{\epsilon_L - 1}{\epsilon_L}\frac{\partial \omega^{LL}}{\partial R}\hat{R}_L - 1 \tag{B.30}$$

$$\frac{\partial \omega^{LL}}{\partial R} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}}\frac{S}{\hat{R}_L L(\hat{R}_L)} \tag{B.31}$$

Thus, the pass-through of the monetary policy rate into loan rates is given by:

$$\frac{\partial \hat{R}_L}{\partial R} = \frac{\frac{\partial \omega^{LL}}{\partial \bar{\omega}}\frac{\epsilon_L - 1}{\epsilon_L}\left[\frac{S}{L(\hat{R}_L)} - \frac{\bar{D}}{L(\hat{R}_L)}\frac{\partial \hat{R}_D}{\partial R}\right] + 1}{\frac{\epsilon_L - 1}{\epsilon_L}\left[(1-\omega^{LL}) - \hat{R}_L\frac{\partial \omega^{LL}}{\partial R_L}\right]} \tag{B.32}$$

which using the balance sheet identity $L + S = D$, can be written as:

$$\frac{\partial \hat{R}_L}{\partial R} = \frac{\frac{\partial \omega^{LL}}{\partial \bar{\omega}}\frac{\epsilon_L - 1}{\epsilon_L}\frac{\bar{D}}{L(\hat{R}_L)}\left(1 - \frac{\partial \hat{R}_D}{\partial R}\right) + 1 - \frac{\partial \omega^{LL}}{\partial \bar{\omega}}\frac{\epsilon_L - 1}{\epsilon_L}}{\frac{\epsilon_L - 1}{\epsilon_L}\left[(1-\omega^{LL}) - \hat{R}_L\frac{\partial \omega^{LL}}{\partial R_L}\right]} \tag{B.33}$$

where by Eq. (B.26) and $\epsilon_D < -1$ we have that $1 - \frac{\partial \hat{R}_D}{\partial R} > 0$. For the derivative $\frac{\partial \omega^{LL}}{\partial \bar{\omega}}$ note that by definition: $\omega^{LL} = \int_0^{\bar{\omega}} \omega \frac{f(\omega)}{F(\bar{\omega})} d\omega$. Hence, by the Leibniz rule:

$$\frac{\partial \omega^{LL}}{\partial \bar{\omega}} = \bar{\omega}\frac{f(\bar{\omega})}{F(\bar{\omega})} - \int_0^{\bar{\omega}} \omega \frac{f(\omega) f(\bar{\omega})}{F(\bar{\omega})^2} d\omega \tag{B.34}$$

And using the definition of ω^{LL}, this simplifies to:

$$\frac{\partial \omega^{LL}}{\partial \bar{\omega}} = (\bar{\omega} - \omega^{LL})\frac{f(\bar{\omega})}{F(\bar{\omega})} > 0 \tag{B.35}$$

Hence if $\frac{\partial \omega^{LL}}{\partial \bar{\omega}} < \frac{\epsilon_L}{\epsilon_L - 1}$ (where $\frac{\epsilon_L}{\epsilon_L - 1} > 1$), then $\frac{\partial \hat{R}_L}{\partial R} > 0$. This is a relatively weak sufficient condition, which is for example fulfilled by the uniform distribution $\omega \sim Unif[0,1]$, in which case $\frac{\partial \omega^{LL}}{\partial \bar{\omega}} = \frac{1}{2}$. ■

Proof of Corollary 3. Eq. (9) characterizes the loan rate as an (implicit) function $R_L(R, \bar{\omega})$ of the policy rate R and the default cut-off $\bar{\omega}$. The bank will therefore charge the same loan rate for a given policy rate and default cutoff, independent of the deposit lower bound. This is because (as shown in Proposition 2) the effect of the deposit lower bound on the loan rate operates entirely through the change in the default cutoff. A change in the deposit lower bound therefore does not affect the loan rate if the bank receives an additional payoff $\Delta = \bar{D}\left(\underline{R}_{D1} - \frac{\epsilon_D}{\epsilon_D - 1} R\right)$ from the government that exactly offsets the effect of the change in the deposit lower bound, such that:

$$\bar{\omega}(\underline{R}_{D1}, \Delta) = 1 - \frac{(\underline{R}_{D1} - R)\bar{D} + RL(R_L) - \Delta}{R_L L(R_L)} = 1 - \bar{\omega}(\underline{R}_{D2}, 0)\frac{(\frac{\epsilon_D}{\epsilon_D - 1} - 1)R\bar{D} + RL(R_L)}{R_L L(R_L)}$$

and hence $\hat{R}_L = R_L(R, \bar{\omega}(\underline{R}_{D1}, \Delta)) = R_L(R, \bar{\omega}(\underline{R}_{D2}, 0)$.

Further, Lemma 1 implies that

$$\frac{\partial R_D}{\partial R} \begin{cases} \frac{\epsilon_D}{\epsilon_D - 1} \text{ if } R > R^* \\ \text{does not exist if } R = R^* \\ 0 \text{ if } R < R^* \end{cases} \tag{B.36}$$

Using this, Corollary 3 follows directly from Corollary 2:

$$\begin{aligned} &\left.\frac{\partial \hat{R}_L}{\partial R}\right|_{R^*(\underline{R}_{D1}) > R, R, \bar{\omega}} - \left.\frac{\partial \hat{R}_L}{\partial R}\right|_{R^*(\underline{R}_{D2}) < R, R, \bar{\omega}} \\ &= \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\bar{D}}{L(\hat{R}_L)\left[(1 - \omega^{LL}) - \hat{R}_L \frac{\partial \omega^{LL}}{\partial R_L} |_{\hat{R}_L}\right]} \frac{\epsilon_D}{\epsilon_D - 1} > 0 \end{aligned} \tag{B.37}$$

$\blacksquare$

C Uniqueness and Conditions of Implicit Function theorem

In order for Proposition 2 to characterize transmission of monetary policy, it is required that the FOC Eq. (9) – denoted Ξ as in the previous section – admits a unique solution and the conditions of the implicit function theorem are fulfilled.

Applicability of the Implicit Function Theorem In particular, this requires (a) $\Xi(\hat{R}_L(R, \underline{R}_D), \hat{R}_D(R, \underline{R}_D), R, \underline{R}_D) = 0$, (b) Ξ_1, Ξ_2, Ξ_3 and Ξ_4 to be continuous in an open set that contains the point $(R, \underline{R}_D), \hat{R}_D(R, \underline{R}_D), R, \underline{R}_D)$ and (c) $\Xi_1 \neq 0$. Condition (a) is fulfilled by definition for $\hat{R}_L(R, \underline{R}_D)$. The conditions for the existence of such a loan rate that solves the FOC have been established in Proposition 1. The relevant partial derivatives for Condition (b) were detailed in the proof of Proposition 2. Condition (b) is fulfilled as long as $\frac{\partial \omega^{LL}}{\partial \bar{\omega}}$ is continuous. This is because sums and products of continuous functions are continuous.

In the following, I shall therefore focus on condition (c) $\Xi_1 > 0$. While I do not show that the condition $\Xi_1 > 0$ holds in general, I establish it for a specific example. Concretely, it shall be shown that if $\frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \geq 0$ and $R < \underline{R}_D$, no local maximum can involve $\Xi_1 > 0$. To see this, first note that from Eq. (B.12):

$$\Pi''(\hat{R}_L) = L'(R_L) F(\bar{\omega}) \Xi_1$$

$$\Pi''(\hat{R}_L) = L'(R_L) F(\bar{\omega}) \frac{\epsilon_L - 1}{\epsilon_L} \left[(1 - \omega^{LL}) - R_L \frac{\partial \omega^{LL}}{\partial R_L} \right]$$

Assume that $\Pi''(\hat{R}_L) = 0$. But then it can be shown that $\Pi'''(\hat{R}_L) \neq 0$, such that $\hat{R}_L$ is a saddle point not a local maximum:

$$\Pi'''(\hat{R}_L) = \left(L''(R_L)F(\bar{\omega}) + L'(R_L)f(\bar{\omega})\frac{\partial \bar{\omega}}{\partial R_L} \right) \underbrace{[(1-\omega^{LL}) - R_L \frac{\partial \omega^{LL}}{\partial R_L}]}_{=0} +$$

$$+ L'(R_L)F(\bar{\omega})\frac{\epsilon_L - 1}{\epsilon_L} \left[-2\frac{\partial \omega^{LL}}{\partial R_L} - \frac{\partial^2 \omega^{LL}}{\partial R_L^2} R_L \right] \tag{C.1}$$

As long as the bank does not always fail, $L'(R_L)F(\bar{\omega}) < 0$, hence we can focus on Ξ_{11}. From Eq. (B.23):

$$\Xi_{11} = \frac{\epsilon_L - 1}{\epsilon_L} \left[-2\frac{\partial \omega^{LL}}{\partial R_L} - \frac{\partial^2 \omega^{LL}}{\partial R_L^2} R_L \right] \tag{C.2}$$

We know that:

$$\frac{\partial \omega^{LL}}{\partial R_L} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial \bar{\omega}}{\partial R_L} \tag{C.3}$$

$$\frac{\partial \bar{\omega}}{\partial R_L} = \left[\frac{((1-\bar{\omega})R_L - R)L'(R_L) + (1-\bar{\omega})L(R_L)}{R_L L(R_L)} \right] \tag{C.4}$$

Using the assumption of iso-elastic demand, Eq. (C.4) can be written as:

$$\frac{\partial \bar{\omega}}{\partial R_L} = \frac{1}{R_L} \left[(1-\bar{\omega})(1-\epsilon_L) + \frac{R}{R_L}\epsilon_L \right] \tag{C.5}$$

Note that at the optimum: $\frac{R}{R_L} = (1-\omega^{LL})\frac{\epsilon_L - 1}{\epsilon_L}$, such that:

$$\frac{\partial \omega^{LL}}{\partial R_L} |_{R_L = \hat{R}_L} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \left[(1-\bar{\omega})(1-\epsilon_L) + (1-\omega^{LL})(\epsilon_L - 1) \right] = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} (\omega^{LL} - \bar{\omega})(1-\epsilon_L) \geq 0 \tag{C.6}$$

where the sign follows from $\epsilon_L > 1$, Eq. (B.35) and the fact that $\omega^{LL} \leq \bar{\omega}$, with strict inequality whenever $\bar{\omega} \in (0, 1)$.

Next, we have:

$$\frac{\partial^2 \omega^{LL}}{\partial R_L^2} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{\partial^2 \bar{\omega}}{\partial R_L^2} + \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \tag{C.7}$$

where:

$$\frac{\partial^2 \bar{\omega}}{\partial R_L^2} = -\frac{\partial \bar{\omega}}{\partial R_L} \left[\frac{1}{R_L}(1 - \epsilon_L) \right] - \left[\frac{(1 - \bar{\omega})(1 - \epsilon_L)}{R_L^2} + 2\frac{R}{R_L^3}\epsilon_L \right] \tag{C.8}$$

$$\implies \frac{\partial^2 \bar{\omega}}{\partial R_L^2} R_L = -\frac{\partial \bar{\omega}}{\partial R_L}(1 - \epsilon_L) - \frac{\partial \bar{\omega}}{\partial R_L} - \frac{R}{R_L^2}\epsilon_L \tag{C.9}$$

Using Eq. (C.7) and (C.9), it follows that:

$$\begin{aligned} -2\frac{\partial \omega^{LL}}{\partial R_L} - \frac{\partial^2 \omega^{LL}}{\partial R_L^2} R_L &= -2\frac{\partial \omega^{LL}}{\partial R_L} + \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \left(\frac{\partial \bar{\omega}}{\partial R_L}(1 - \epsilon_L) + \frac{\partial \bar{\omega}}{\partial R_L} + \frac{R}{R_L^2}\epsilon_L \right) - R_L \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \\ &= \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \left[\frac{\partial \bar{\omega}}{\partial R_L}(-\epsilon_L) + \frac{R}{R_L^2}\epsilon_L \right] - R_L \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \\ &= \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \left[\frac{\partial \bar{\omega}}{\partial R_L} - \frac{R}{R_L^2} \right](-\epsilon_L) - R_L \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \end{aligned} \tag{C.10}$$

Using Eq. (C.5), this simplifies to:

$$-2\frac{\partial \omega^{LL}}{\partial R_L} - \frac{\partial^2 \omega^{LL}}{\partial R_L^2} R_L = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \left[\frac{(1 - \bar{\omega})}{R_L} - \frac{R}{R_L^2} \right](-\epsilon_L)(1 - \epsilon_L) - R_L \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \tag{C.11}$$

$$= \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \frac{(R_D - R)D(R_D)}{R_L^2 L(R_L)} \epsilon_L(\epsilon_L - 1) - R_L \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \left(\frac{\partial \bar{\omega}}{\partial R_L} \right)^2 \tag{C.12}$$

Using Eq. (C.5) allows to write this as:

$$-2\frac{\partial \omega^{LL}}{\partial R_L} - \frac{\partial^2 \omega^{LL}}{\partial R_L^2} R_L = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} \epsilon_L(\epsilon_L - 1) \frac{(R_D - R)D(R_D)}{R_L^2 L(R_L)} - \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \frac{(\omega^{LL} - \bar{\omega})^2(\epsilon_L - 1)^2}{R_L} \tag{C.13}$$

Plugging this back into Eq. (C.2) yields:

$$\Xi_{11} = \frac{\partial \omega^{LL}}{\partial \bar{\omega}} (\epsilon_L - 1)^2 \frac{(R_D - R) D(R_D)}{R_L^2 L(R_L)} + \frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \frac{(\omega^{LL} - \bar{\omega})^2}{R_L} \frac{(\epsilon_L - 1)^3}{\epsilon_L} \tag{C.14}$$

Such that under negative interest rates ($R < \underline{R}_D$), the first term is positive. The second term is positive or zero if $\frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \geq 0$. Hence, if $\frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \geq 0$ and $\bar{\omega} \in (0,1)$: $\Pi''(\hat{R}_L) = 0 \implies \Pi'''(\hat{R}_L) < 0$. This contradicts that $\hat{R}_L$ is a local maximum. Hence, under policy rates $R < \underline{R}_D$ any local maximum must involve $\Pi''(\hat{R}_L) < 0$ if $\frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} \leq 0$ (and cannot involve $\Pi''(\hat{R}_L) = 0$). Similarly, under a uniform distribution ($\frac{\partial^2 \omega^{LL}}{\partial \bar{\omega}^2} = 0$): $\Pi''(\hat{R}_L) = 0 \iff R = \underline{R}_D$. Hence, $R < \underline{R}_D$ is a sufficient (but not necessary) condition for $\Pi''(\hat{R}_L) > 0$ under a uniform distribution of ω.

Uniqueness: Example Assume the conditions of Proposition 1 are fulfilled, such that some R_L exists that solves the FOC:

$$(1 - \omega^{LL}) \left(\frac{\epsilon_L - 1}{\epsilon_L} R_L \right) - R = 0$$

For the sake of providing an example, consider $\omega \sim Unif[0,1]$. Focus on the case of a bank with an interior default probability $\bar{\omega} \in (0,1)$, since this is the case which Proposition 2 focuses on. Then, the FOC is:

$$\left(1 - \frac{1}{2} \bar{\omega} \right) \left(\frac{\epsilon_L - 1}{\epsilon_L} R_L \right) = R \tag{C.15}$$

By definition of $\bar{\omega}$:

$$\left(1-\frac{1}{2}\bar{\omega}\right)R_L = R_L - \frac{1}{2}\left(1-\frac{(R_D-R)D(R_D)+RL(R_L)}{R_L L(R_L)}\right) \tag{C.16}$$

$$= \frac{1}{2}\left(R_L + (R_D-R)\frac{D(R_D)}{L(R_L)} + R\right) \tag{C.17}$$

$$= \frac{1}{2}\left(R_L + (R_D-R)D(R_D)AR_L^{\epsilon} + R\right) \tag{C.18}$$

where the last line uses the definition of $L(R_L)$. Clearly, this is increasing in R_L if $R_D - R \geq 0$ (i.e. when $R \leq \underline{R}_D$)). A sufficient conditions for a unique maximum under a uniform distribution is therefore $R \leq \underline{R}_D$.