Technische Universität München

Assignment #1: Warm up

Group 2

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Note: For higher resolution graphs please see the printed versions with the matlab code

1 Task

Compute the first and the second derivatives of f(x) analytically.

$$f(x) = \sin(nx)\,; n = 2$$

1rst derivative: $f'(x) = n * \cos(nx)$

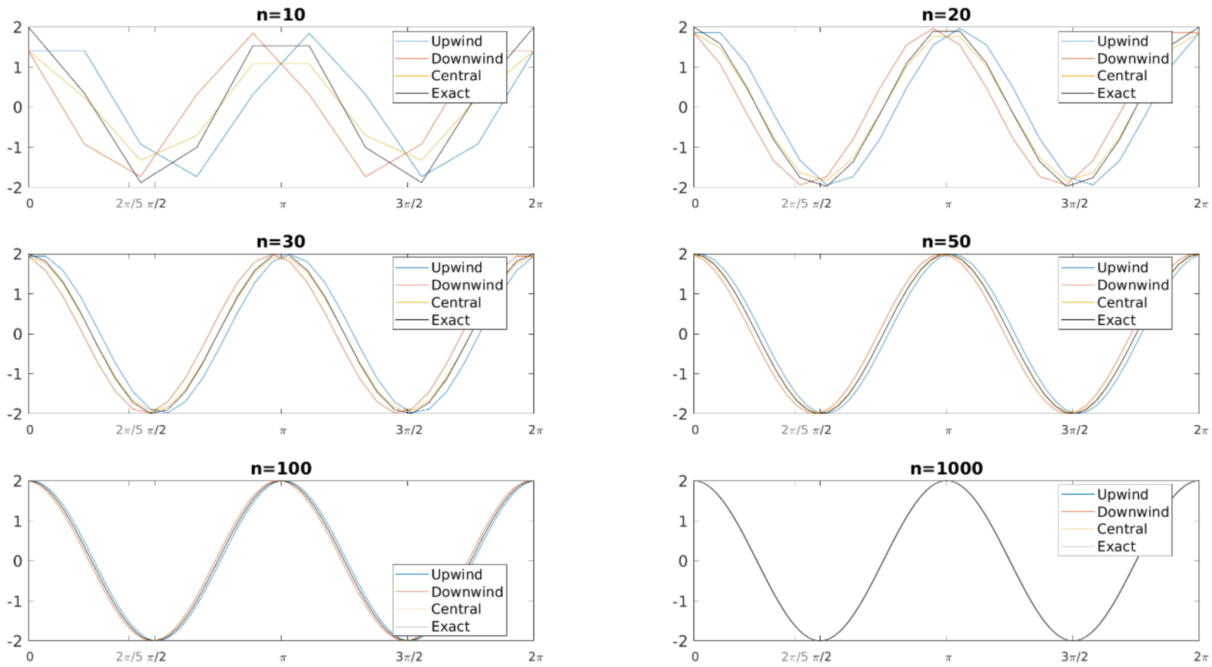
2nd derivative $f''(x) = -n^2 * \sin(nx)$

2 Task

Compute the first derivative of f (x), namely ∂f /∂x, numerically on x ∊ [0, 2π]. Assume periodic boundaries. Use central, upwind and downwind difference schemes. Choose different resolutions h. Comment on what you observe.

Boundary: $[0; 2\pi]$

Resolutions: [10,20,30,50,100,1000]



When computing the first derivative with the backward difference scheme (upwind), we observe a shift to the right, because we calculate the slope with the difference of f_i and f_{i-1}, as the formula below describes. Which means the peak of the approximated function won't be at π for a sinus-function but shifted to the right depending on the resolution. This is best observed for low resolutions as for a number n = 10.

$$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} \cong \frac{f_i - f_{i-1}}{h}$$

Formula 1: Backward Difference Scheme (Upwind)

For the forward difference scheme, we observe the opposite. There is a shift to the left due to the formula.

$$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} \cong \frac{f_{i+1} - f_i}{h}$$

formula 2: Forward Difference Scheme (Downwind)

The central difference scheme appears symmetrical to the analytical function. What we observe here is an underestimation of the amplitude. The highest deviation of the amplitude can be seen for a small number n. As we increase the resolution this error decreases.

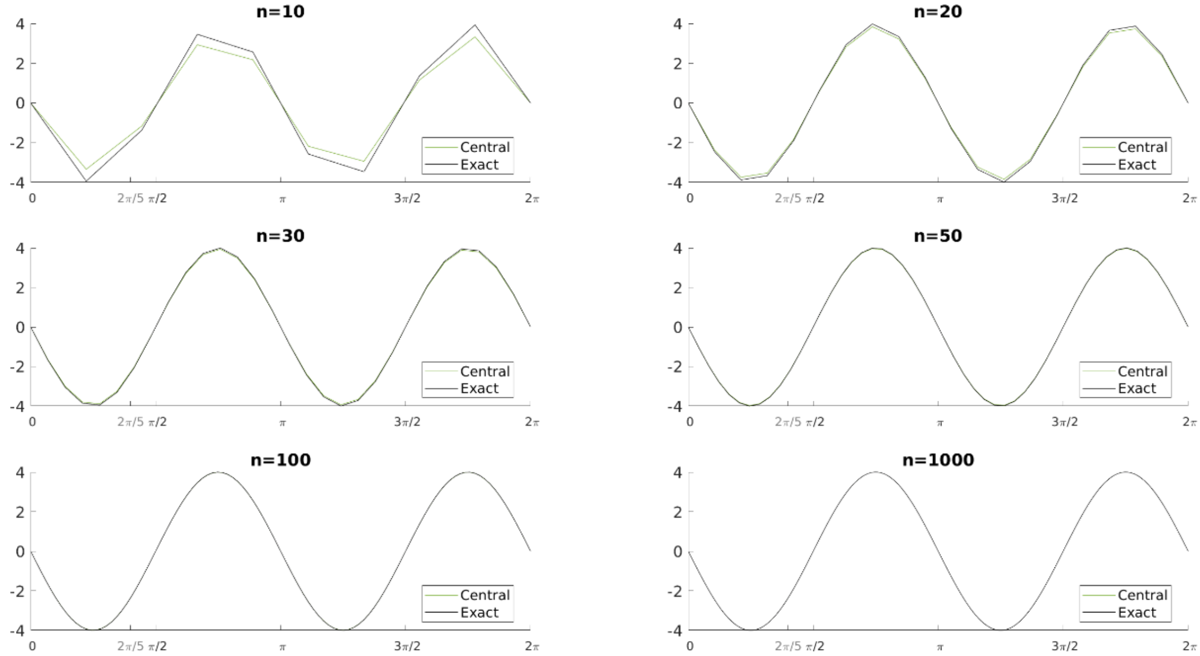
$$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} \cong \frac{f_{i+1} - f_{i-1}}{2h}$$

formula 3: Central Difference Scheme

In addition, we chose different resolutions as you can see in fig.1. We observe over all schemes a closer approximation to the analytical function with increasing resolution. This is shown in fig.1 as well.

3 **Task**

Compute the second derivative of f (x), namely ∂ 2f /∂2x, numerically on x ∈ [0, 2π]. Assume periodic boundaries. Use central difference scheme. Choose different resolutions. Comment on what you observe.

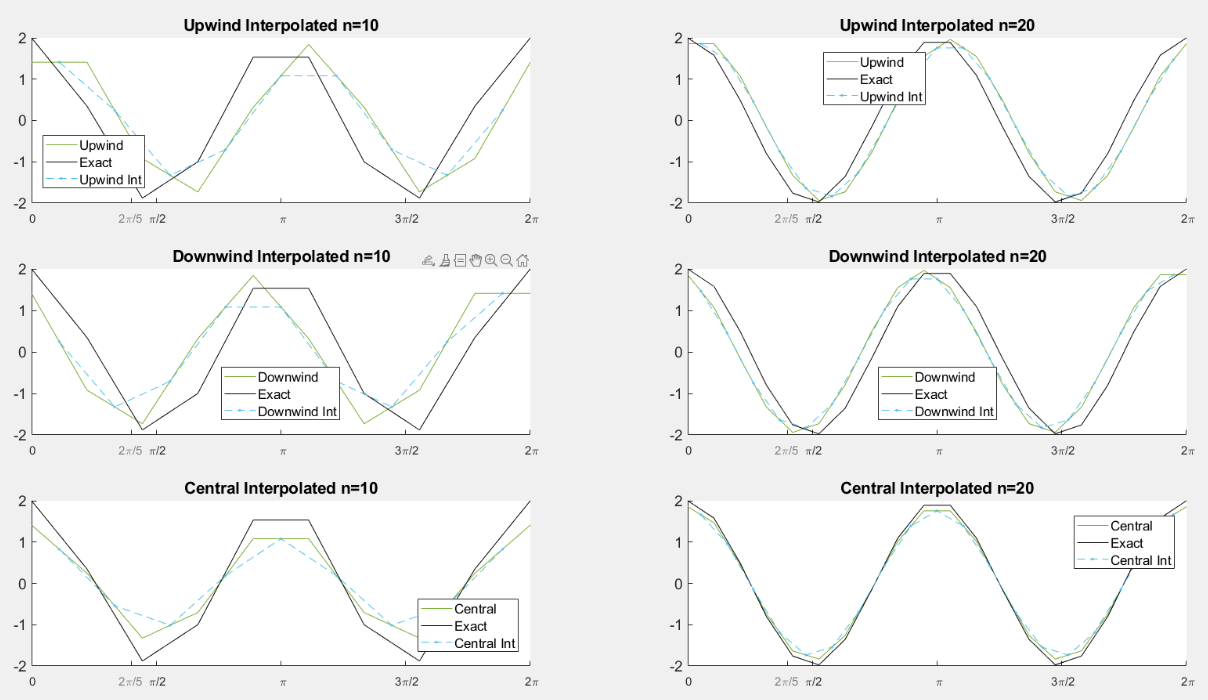


We found (see figure above) that with increasing resolution the approximation approached the analytical solution, the same as for the first derivative. Amplitude of approximations tend to be smaller such that the analytical function is consistently underestimated at all points, even more so when using interpolated values. When using interpolated points and the central difference scheme no improvement can be seen.

The error increased depending on the amplitude of the sinus curve at low amplitude the approximation is better, while at the peak the maximum difference is seen.

4 Task

Use a linear interpolation scheme to compute the function values at the middle of the grid points. Use different resolutions. Comment on what you observe.



The observations of the linear interpolation scheme are summarized in the following.

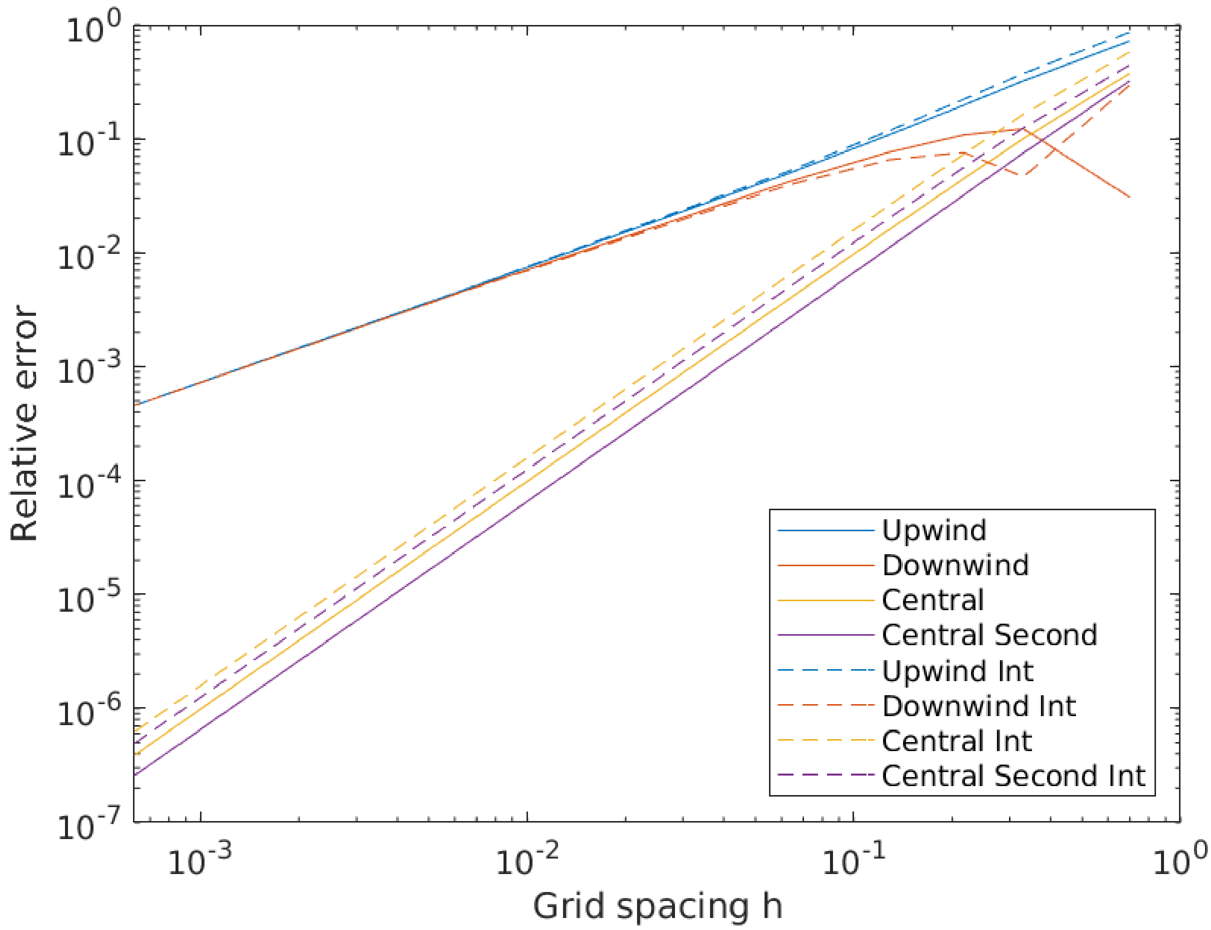
The linear interpolation of the difference schemes is calculated from the approximated function. The interpolation results in new points, which are fitted in between the points from the approximated function.

These points form another approximation of the derivative. With rising accuracy of the difference schemes the accuracy of the interpolated points increases as well, but not at a rate higher than the original approximation.

The interpolation of the Upwind and Downwind scheme can result in a lower or higher accuracy, depending on where the error is evaluated. This is not visible with the central difference scheme.

5 Task

For tasks 2, 3 and 4, plot the error versus resolution for a given position x = 2π/5. Error plots are done in logarithmic scale, i.e. the logarithm of the relative error Erel is plotted versus the logarithm of grid spacing h. Comment on the slope of the curves you obtain.



A further analysis was conducted to obtain quantitative information about the magnitude of the relative error trends and to see which impact variable gird spacings have.

For this purpose, the value of the relative error was calculated at the location of x_err=2pi/5 for all grid resolutions and comparably illustrated above.

A second linear interpolation was used in cases where neither the interpolated nor the original grid points lay on x=2pi/5. This allowed an evaluation of the error at the same x-location (x_err), as discussed with Daniel during the feedback Zoom session. The interpolated values in Task 4 were also interpolated twice to allow evaluation. This should be kept in mind in the further analysis

The plots ease to determine the order of the respective errors. As the error for the different resolutions is depicted in a double logarithmic scale the slope of the error trends can easily be calculated. The analysis yields that Upwind and Downwind errors are decreasing with the order of magnitude one (slope of one), meaning the error is decreasing linearly with smaller grid spacing. The central scheme error in contrast decreases quadratically (slope of two), which shows that the error is converging towards zero more quickly. This can be explained by the fact that central difference scheme uses information from two sides of the to be evaluated points, and Upward and Downward schemes only use one sided information, and therefore are somewhat less accurate.

Only for great grid spacings does the error behave unexpectedly, which can be seen by the non-linear form of the curve of the Downwind scheme. This behavior however varied by selecting different x_err, which gives hint that the values for great grid spacings are prone to the location of investigation.