Numerical Methods in Hydromechanics

Assignment #3: Finite Volume Method

Group 2

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Jo nick haul yo ass into dis zoom

1. Task

Complete the provided MatLab script according to the comments. Use upwind and central schemes for the first derivative (i.e., convective term) and central difference scheme for the second derivative (i.e., diffusive term).

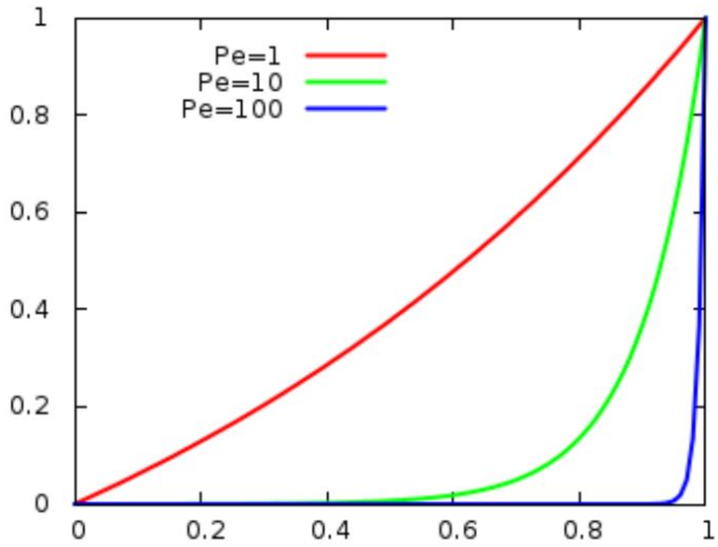
Mathematical Model: Advection-Diffusion-Equation

$$-u\frac{\partial \varphi}{\partial x}+\Gamma\frac{\partial^2 \varphi}{\partial x^2}=0$$

The higher the Pe-number is, the more the graph moves towards the blue curve below (It becomes more advective). We consider, if we assume a characteristic length L = 1, Pe-numbers of 1 < Pe < 10.

$Pe = \frac{U_0 * L}{\Gamma}$

U0 is the only parameter that changes for our simulations and is therefore seen as substitution for the Pe-number.



Derivation of the missing boundary condition (right):

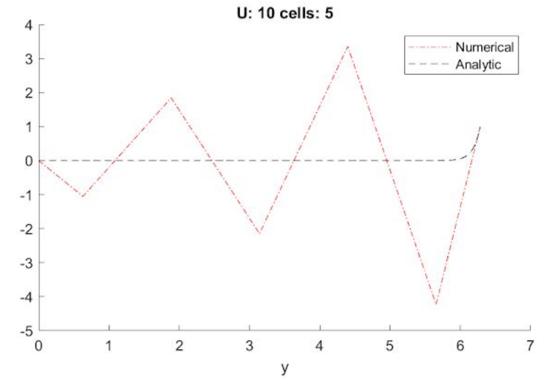
$\Phi_e = \Phi_R = 1$

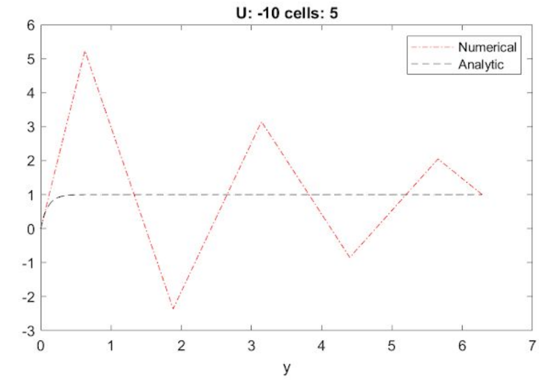
$(\frac{U_0}{2} - \frac{3\Gamma}{\Delta x})\Phi_P + (\frac{U_0}{2} + \frac{\Gamma}{\Delta x})\Phi_W = (U_0 - \frac{2\Gamma}{\Delta x})\Phi_R$

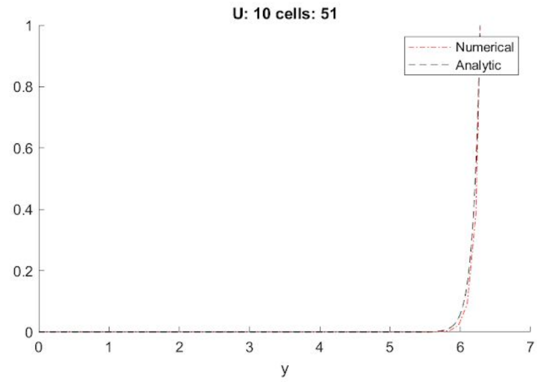
Please see attached code, that is divided into a main function ‘main_two.m’ and the sub function ‘A_D_FV.m’.

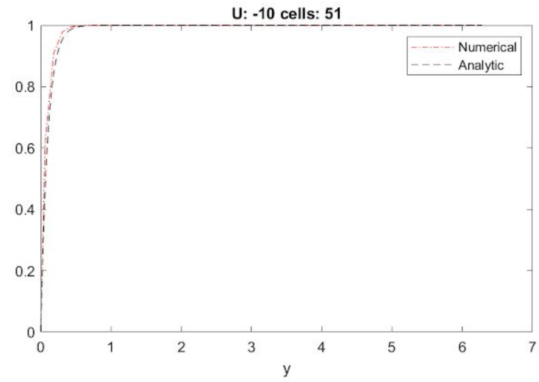
2. Task

Run the same numerical experiments asked in the last exercise, and summarise your observations.









In detail: (plots)

U0=10; cells=5
U0=-10; cells=5

Relatively high Pe-number combined with a low number of cells. We observe a significant oscillation. By increasing the number of cells it is observed that in between 30 and 35 grid points the wiggle vanishes and at a number close to 100 the numerical curve starts merging with the analytical.

U0=10; cells=51
U0=-10; cells=51

We use the same Pe-number, but a higher number of cells. What we observe is a good approximation and no longer any oscillation. The numerical curve of the positive value for U0 compared to the analytical solution is slightly shifted to the right.
For negative U0 values the numerical solution overestimates and for positive U0 values underestimates the analytical solution. This seems to be due to a simple reflection of values between U0 +/-.

3. Task

Compute the relative error at x = π.

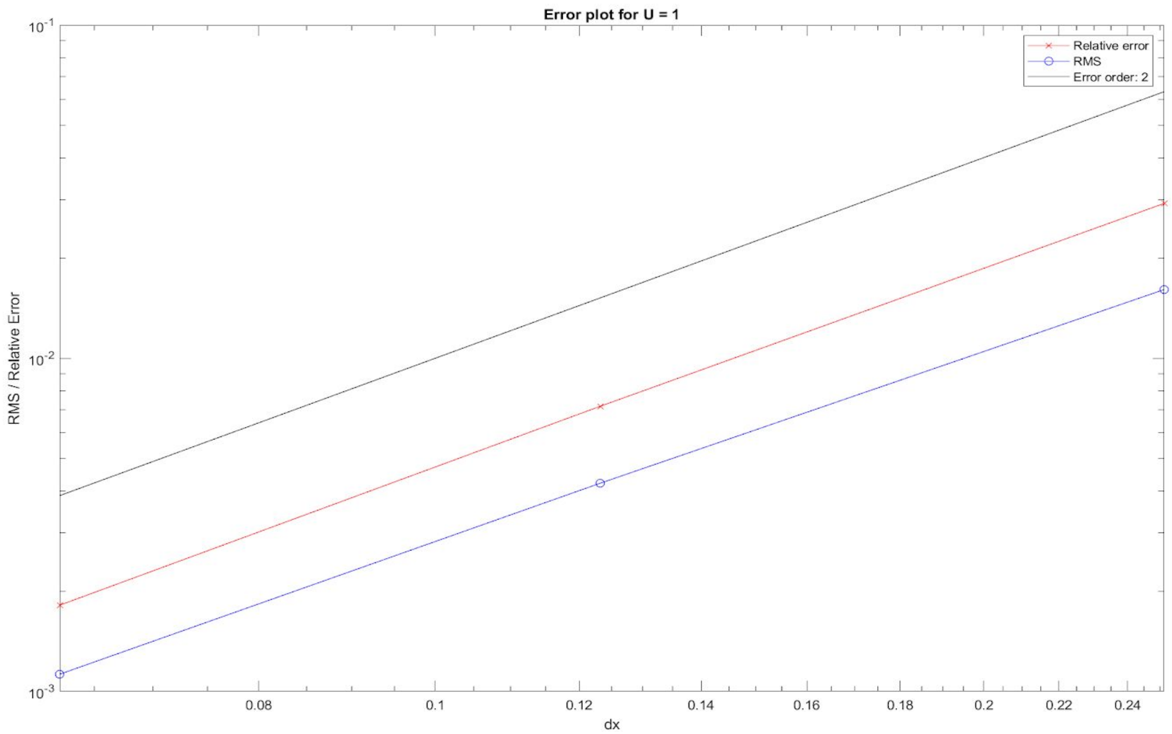
$$E_{\mathrm{rel}} = \left| \frac{\phi^{\mathrm{ex}}(x=\pi) - \phi^{\mathrm{num}}(x=\pi)}{\phi^{\mathrm{ex}}(x=\pi)} \right|$$

Analytical solution:

$$\phi^{\mathrm{ex}}(x) = \frac{e^{U_0 x/\Gamma} - 1}{e^{2\pi U_0/\Gamma} - 1}$$

Run your code with cells = [25, 51, 101] and U0 = +1.0, Γ = 1.0. Plot the relative error Erel versus the grid spacing dx in a double logarithmic scale. Comment on the order of accuracy you achieved.

As we plot the advection-diffusion-equation in numerical and analytical form, we observe for low cell values (25) already a pretty good approximation and no wiggle. That is because of the small U0, which leads to a small Peclet number and therefore a faster convergence in the numerical solution. At 101 cells we cannot distinguish between the numerical and analytical function.



4. Task

With the same parameters as above, compute the mean errors based on normalised root-mean-square.

$$E_{\text{mean}} = \frac{\sqrt{\langle(\phi^{\text{ex}} - \phi^{\text{num}})^2\rangle}}{\langle\phi^{\text{ex}}\rangle}$$

Plot Emean versus dx in a double logarithmic scale. Comment on the order of accuracy you achieved.

The loglog-plot shows that the RMS and the relative error at x = pi both have the same slope as the quadratic function, meaning that both error are accurate of order 2. In our case the amount of the RMS is smaller then the relative error for all computed grid spacings. If we would compare the RMS to a relative error at another location this behaviour however is assumed to differ. Therefore the RMS serves as a great indicator for the overall accuracy in a simulated domain, but does not give information about the error at a certain location. This might be of importance if flow behaviour at a certain position is investigated.
The results are behaving as expected, regarding the fact that we were using a central difference scheme for the first and second derivative and only the boundary conditions were evaluated using Upwind and Downwind schemes.