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Summary Sheet

How to Face Opioid Crisis?

Summary

Since the opioid crisis occurred, it has caused severely negative impact on the American society. Thus we aim to analyze the propagation mechanism of opioid and then propose effective measures.

At the beginning, we simulate the spread of opioid via a cellular-automaton model. We select one city in each county as a representative and compare the simulation results with the preselected 20 cities which have high frequency of drug use incidents. Then we define the state of each cell by using continuous positive integers. Each integer represent a drug-addiction level. Analysing the consequence after several iterations, we draw the conclusion that opioid using spread radically from center to its surroundings. Moreover, there are higher spread intensities on borders of two states as well as the metropolises of each state. Also, we find 16 counties as possible locations where specific opioid use might have started in each of the five states by generating several initial points through our model. Plus, we build an improved SI model to simulate the spread of opioid. And we finally find the threshold level of opioid spreading according to the dynamical system theory.

Next, we take socio-economic factors into consideration. In order to deal with the massive database, we divide all factors into 15 categories. By using factor analyze, we extract key factors in each category. After that, we make correlation analysis to determine the impact of various factors on drug abuse incidents in each county. By weighting coefficients of each category, we can get every county's comprehensive score which indicates the correlation between socio-economic factors and drug incidents. As a result, we find 8 main factors that have tight relationships with drug abuse.Therefore, we improve our initial model by changing the drug spread intensity of each county based on its score.

Finally, we make a sensitivity analysis to our model and come up with several effective measures to cope with crisis according to the correlation between social factors and drug incidents.

Contents

1 Introduction **1**
1.1 Problem Background . . . 1
1.2 Our work . . . 1
1.3 Assumptions . . . 2
1.4 Notations . . . 2

2 Opioid Propagation Model **3**
2.1 Potential Sources of Opioid-using . . . 3
2.2 Opioid Propagation Model Based on Cellular Automaton . . . 3
2.3 Iteration Rule For Cell's State . . . 4
2.4 Threshold Determination: Improved SI Model . . . 5
2.5 Modified I . . . 6
2.6 Spread Pattern of the Opioid . . . 7
2.6.1 Initial Locations of Opioid-using . . . 7
2.6.2 Specific Concerns of the U.S. Government . . . 8

3 Analysis of Socio-economic Factors **9**
3.1 Factor Analysis Model . . . 9
3.1.1 KMO Test (*Aiser-Meyer-Olkin*) . . . 10
3.1.2 Factor extraction . . . 10
3.1.3 Factor Rotation . . . 11
3.1.4 Value of Each Common Factor in Every Sample . . . 11
3.2 Correlation Analysis . . . 11
3.3 Correlation result analysis . . . 13
3.4 Change of β . . . 14
3.5 Modified Opioid-spreading Model . . . 14
3.6 Change of the threshold . . . 15

4 Sensitivity Analysis For Opioid Propagation Model **15**

5 Sensitivity Analysis For Factor Analysis Model **17**

6 Strengths and Weaknesses **18**
6.1 Strengths . . . 18
6.2 Weaknesses . . . 18

7 Memo to the chief administrator **19**

References **20**

1 Introduction

1.1 Problem Background

Opioids are chemical substances with morphine-like effects, which are normally acted as efficient alleviators in medicine.They reduce pain by binding to opioid receptors in the central and peripheral nervous system as well as inhibiting the release of substance P. Nevertheless, the side effects(like opioid use disorder) of opioids can't be ignored. Also, the frequent and escalating recreational use of opioids can easily contribute to addiction and even death.

The current opioid crisis of the United States generated in the 1990s. During that period, the overuse and addiction of synthetic opioids like Oxycontin was a common phenomenon. As time goes by, strong synthetic opioids such as heroin and fentanyl further worsen the crisis. As a result, the opioid crisis causes a life expectancy decrease in some population groups, especially in white males. It's reported that the number of these deaths have reached 60,000 a year, and it might continues to increase in the future if there are no effective measures.

Besides, the prevalence of opioid use disorder has been increasing as well. Although, Medication-Assisted Treatment (MAT),such as methadone, buprenorphine, naloxone, has been widely used for treatment, complete healing still seems impossible currently. At the same time,opioid crisis poses great threat to the U.S. economy. For instance, some typical positions may can't be full filled due to the large-scale spread of the crisis, and heavy medical expenses will be a huge burden on both people and governments.

In response to the crisis, we are supposed to take corresponding measures. At the beginning, we use Cellular automata model to study the spread and characteristics of the reported synthetic opioid and heroin incidents. After that, we test the relationship between several socio-economic factors and the spread of opioid. After analyzing the consequence, we raise several mearsures to solve the problem.

1.2 Our work

Firstly, we are asked to find possible locations where specific opioid use might have started and then describe the spread and characteristics of the reported opioid incidents. In this part, we first select several potential drug-abuse counties according to the number of drug-abuse incidents. Then we set up a **Opiod Propagation Model** to simulate the propagation of opioids and we build an **Improved SI Epidemiological Model** to determine the threshold levels.

Then we take socio-economic factors into consideration to modify our model, and then find some strategies for countering the opioid crisis. In detail, we use factor analysis model to extract main factors in 15 categories. Then we use these main factors to calculate the correlation between each category of factors and the frequency of drug-using cases. After that, we modify our initial opioid-spread model by giving each county different drug-abuse level.

At last, we come up with several reasonable advice so as to counter the opioid crisis and use our model to test them.

1.3 Assumptions

- The state of a cell changes step by step.
- All districts have the same basic supervision level to drug addicts, and the practical level is proportional to a current county's drug-addiction level.
- Drug users are more likely to be found near drug abuse areas.
- City with large number of drug users are more likely to be the source of drug-using.
- Non-drug users may be addicted to drugs due to curiosity, academic pressure, family background, etc., even if there are no drug users around.

1.4 Notations

The primary notations used in this paper are listed in **Table 1**. There can be some other notations to be described in other parts of the paper.

Table 1: Notations

Symbol	Definition
α	The State of a Cell at a Certain Time
β	The Spread Intensity of Drug Abuse
K	The Probability of a Cell Shifting to a Lower Level State
U	The Epsilon Neiborhood
N	The Number of Counties
L	The Level of Supervision to Drug Addicts in a county
F	The Frequency of Drug-using Incidents in a County

2 Opioid Propagation Model

2.1 Potential Sources of Opioid-using

Firstly,we screened out the places where the number of drug-abuse cases reports exceeds the overall average per year. Then we sum up number of cases in each selected place from 2010 to 2017.That's because some counties only have high frequency of incidents in a few years, which may be the result of mobility factors like population migration. So we have to consider all years together to reduce the accidental error. Next, we choose several counties that have high total frequency of incidents to be potential sources of Opioid-using. Here we assume that the more people who use drugs, the more likely this city is to be the source of drugs, which meets the reality.

2.2 Opioid Propagation Model Based on Cellular Automaton

The cellular automaton is a dynamic model which is discrete in both time and space. And the basic unit of a CA system is a cell which scattered in a Lattice Grid. At every point in time, all cells will change their states depending only on the current states of their neighboring units. By using simple local rules and discrete methods, the CA model is able to describe complex global continuous systems and is widely used to solve propagation problems. In our article, we build an opioid propagation model based on CA model to describe the spread as well as characteristics of the reported opioid incidents efficiently. In general, we use cells to represent given counties in five states and determine the neighborhood of each cell in line with the relative position between the counties. Then we simulate the propagation of opioids by changing every cell's state in a typical time series. After a certain number of iterations,we can get the final results.

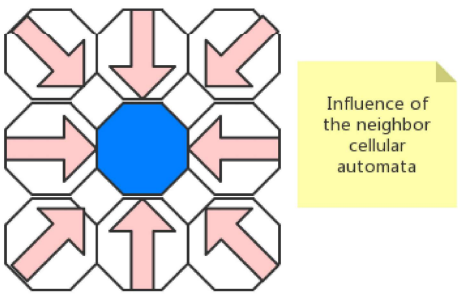

Figure 1: Cellular Automaton model

Firstly, we determine the state of each cell ($\alpha_{i,j}^t$) at time t. To simplify the problem, we use consecutive positive integers to represent increasing drug-addiction levels. $\alpha_{i,j}^t$ can be 0,1,2, ..., i_max.To be exact, when the value of $\alpha_{i,j}^t$ is zero, it means a county is not undergoing drug abuse incidents currently, and the increasing value of $\alpha_{i,j}^t$ indicates the higher frequency drug abuse in this county.

2.3 Iteration Rule For Cell's State

Initially, we simulate an environment without drug use. That is to say, states for all of the cells in the lattice grid is 0 at t_0 .Then by using the NFLIS data provided, we are able to determine the probability distribution of drug abuse incidents in each regions. According to this result, we randomly As we mentioned above, non-drug users may be addicted to drugs due to factors like curiosity, academic pressure, family background, etc., even if there are no drug users around. Therefore, we can generate several sources of the propagation.

In reality, drugs usually spread out from neighboring areas to further regions. To analogize this propagation mode, we use non-drug-addiction probability of the neighborhoods $P(A_{q_i})$ to calculate further drug-addiction probability of $\alpha_{i,j}$ .q_i denotes an adjacent area, $q_1, \ldots, q_m \in U(\alpha_{i,j}^t)$). We assume drug treatments can help reduce drug-addiction probability, and the degree of supervision is proportional to a current county's drug-addiction level. Plus, L is the basic supervision degree as is mentioned in figure 1.$K(A_{q_i})$ shows a county's probability of reduction in infection under some degree of treatments.

$$\alpha_{i,j}^0 = 0 \tag{1}$$

We define an anti-addiction factor $\bar{I}$ so as to get $P(A_{q_i})$. $\alpha_{p,q}^t$ is in the de-centered neighborhood of $\alpha_{i,j}^t$. If the neighbor cell is not undergoing drug-abuse incidents currently, it will not influence the state of $\alpha_{i,j}^t$. Whereas, if this cell is undergoing drug-abuse incidents currently, it will impact the state of $\alpha_{i,j}^t$ with the spread intensity of drug abuse β.

$$\bar{I}(\alpha_{p,q}^t) = \begin{cases} 1, & if\ \alpha_{p,q}^t - 0 \\ \\ 1-\beta, & if\ \alpha_{p,q}^t > 0 \end{cases} \tag{2}$$

$$P(A_{q_i}) = \bar{I}(\alpha_{p,q}^t)L \tag{3}$$

- If no drug treatments are taken in the county,

$$\begin{cases} P(\alpha_{i,j}^t) = 1 - \prod_{i=1}^m P(A_{q_i}) \\ \\ \alpha_{i,j}^t = \alpha_{i,j}^{t-1} + 1 \end{cases} \tag{4}$$

- If there are some drug treatments taken in the county,

$$\begin{cases} K(\alpha_{i,j}^t) = (\alpha_{i,j}^t)L/(i_{max}) \\ \\ \alpha_{i,j}^t = \alpha_{i,j}^{t-1} - 1 \end{cases} \tag{5}$$

2.4 Threshold Determination: Improved SI Model

In SI model, there are two possible states of nodes in the net at any time, *susceptible state* S and *infected state* I. Several propagation sources are existed at t_0, and the rest of nodes are at risk of being infected. Hereafter, these sources begin to taint their neighbors with a certain rate of infection. However, we further subdivide the drug-addiction levels as we mentioned above, which corresponds to different severities of the infection. Therefore, we use I_k to replace single I to depict various states of different drug-addiction levels. Here, S is the ratio of non-drug areas in the total areas, I_k is the percentage of drug-addiction areas which are in level k.

$$I = \sum_{k=1}^{n} I_k \tag{6}$$

$$S + I = 1 \tag{7}$$

As is shown in **Table 1**, β is the spread intensity of drug abuse, so βS is the infection ratio of one infected region at time t. Take all regions into consideration, we can conclude that the total infection ratio will be βSI. So the ratio of non-drug areas will reduce βSI. At the same time, drug treatments in drug-addiction areas can cut back LI infection ratio.

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI \\ \\ \dfrac{dI_1}{dt} = \beta SI - LI \\ \\ \vdots \\ \\ \dfrac{dI_n}{dt} = \beta SI - LI \end{cases} \tag{8}$$

$$\frac{dI}{dt} = n(\frac{L}{\beta S} - 1) \tag{9}$$

When $dI/dt = 0$, S will be $L/beta$ at this time. Actually,if β,L,n are determined, for each point on the two-dimensional plane (I, S) , we can find the distribution of its vector field based on its derivative vector $d(I, S)/dt$. The distribution is shown in **Figure 2**. Because β,L,n only have effects on Y-axis, the general direction of the phase

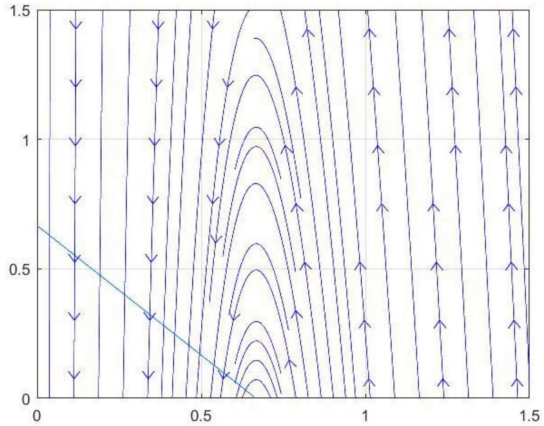

Figure 2: The distribution of vector field

diagram will not change. As **Figure 2** shows, $I = S - \beta/L$ is a dividing line, when the initial value (I_0, S_0) is below the line, we can find that

- If $S_0 - \beta/L < 0$, I will inevitably tend to 0,
- If $S_0 - \beta/L > 0$, I will increase to a large value firstly and then decrease.

Evidently,the large value of I will lead to an uncontrolled expansion of drug abuse areas. So we use β/L as our drug identification threshold level R. *According to the dynamical system theory*, we conclude that if $R < 1$, the spread of drug addiction can be in control. Otherwise, if $R > 1$, the spread of drug addiction will be out of control. Therefore, the U.S. government should have specific concerns to suppress the occurrence of this kind of situation.

2.5 Modified I

In order to better reflect the spread of drugs in recent times, we use exponential averaging algorithm to calculate the intensity of supervision after n iterations. According to exponential averaging algorithm, the weighting influence of each value decreases exponentially by time. The level of supervision$L$ is partly related to the drug use rate of a region at the previous moment, which can be depicted as

$$L(\alpha_{i,j}^t) = \mu p(\alpha_{i,j}^t) \tag{10}$$

$0 < \mu < 1$

Also,the level of supervision$L$ is related to the drug use rate of a region at current time. As is depicted in the exponential averaging algorithm($0 < \mu < 1$), the current rate of drug use is more influential.

$$\begin{aligned} L(\alpha_{i,j}^t) &= (1-k)p(\alpha_{i,j}^t) + kL(\alpha_{i,j}^t) \\ &= (1-k)p(\alpha_{i,j}^t) + k\mu p(\alpha_{i,j}^t) \end{aligned} \tag{11}$$

At first, we assume that

$$L(a_{i,j}^0) = 0 \tag{12}$$

Then we can get

$$\begin{aligned} L(a_{i,j}^t) &= (1-k)p(a_{i,j}^t) + kL(a_{i,j}^t) \\ &= (1-k)p(a_{i,j}^t) + k((1-k)p(a_{i,j}^t) + kL(a_{i,j}^t)) \\ &= (1-k)(p(a_{i,j}^t) + kp(a_{i,j}^t)) + k^2L(a_{i,j}^t) \\ &= ... \\ &= (1-k)(p(a_{i,j}^t) + kp(a_{i,j}^t) + ... + k^{t-1}p(a_{i,j}^t)) \end{aligned} \tag{13}$$

Therefore, we adjust L by consider the different influence weights at different times.

2.6 Spread Pattern of the Opioid

2.6.1 Initial Locations of Opioid-using

By implementing following model in MATLAB, we can identify the spread characteristics of the opioid cases in and between the five states and find specific sources of opioid using. As is shown in **Figure 3**, 16 counties in 5 states might use opioid initially.All of these counties are listed below.

- **Pennsylvania**: Philadelphia, Upper Darby, Pittsburgh
- **Ohio**: Cleveland, Akron, Youngstown, Columbus, Hamilton, Cincinnati
- **Kentucky**: Louisville, Lexington, Harlan
- **Virginia**: Harrisburg, Charlottesville, Roanoke
- **West Virginia**: Blackshurg

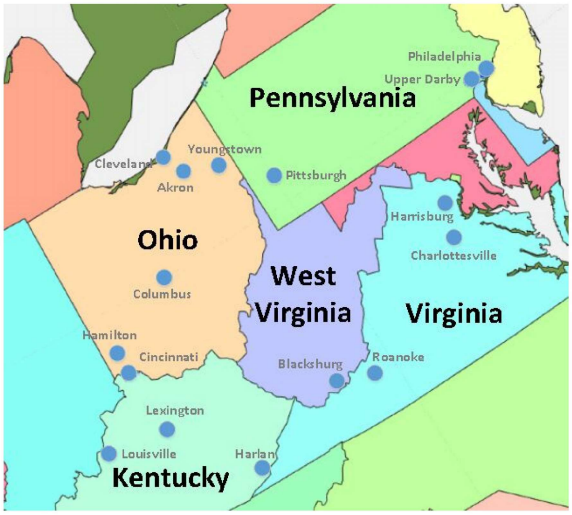

Figure 3: Sources of opioid

2.6.2 Specific Concerns of the U.S. Government

The different colors of the dots on **Figure 4** represent different degrees of opiod-use.Red represent the highest level, black represent the medium level and blue represent a low level. Observing the graph, we can find that region with opioid using spread radically from the center to the surroundings. Moreover, we discover that the spread intensity is higher at borders of two states, which may be related to the transportation convenience. Actually,there are usually more frequent traffic contacts as well as personnel flow on the border. Additionally, drug abuse incidents occur more frequently in capital and metropolitan of each state. Certainly, there are complex demographics, developed transportation networks and economy in these big cities, which increase the probability of opioid-use disorder.

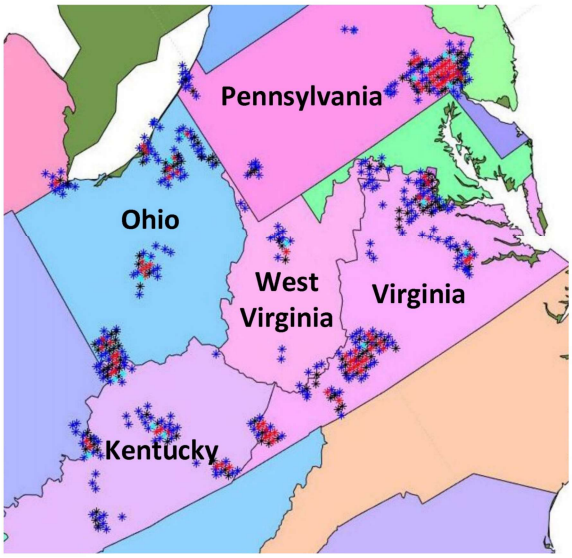

Figure 4: Spread pattern

Thus, the threshold we set in **Section 3.2** will be exceeded with the continuous spread of opioid, and the situation will be out of control when $R > 1$,which is shown in **Figure 5** .The excessive drug abuse can lead to higher crime rate which will cause social unrest. In addition, the negative impact on human body will result in higher mortality. And the lack of human resources can contribute to economic recession as well. Moreover, the toxic substances of drugs will pose threat to ecosystem. Therefore, before reaching the critical value, the U.S. government must have specific concerns to those areas of high frequency drug-addiction, such as capitals, big cities and state borders as we explained above. Among these 5 states,Kentucky, West Virginia and Virginia will face a more serious drug addiction situation. The specific areas are shown on **Figure 5**

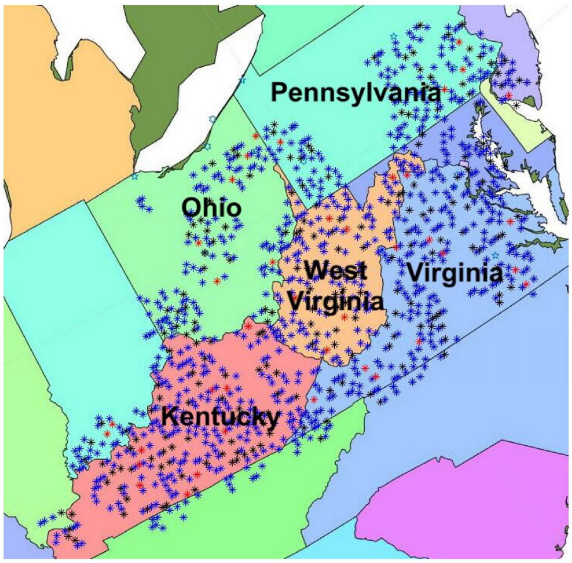

Figure 5: Situation when opioid spread is out of control

3 Analysis of Socio-economic Factors

Owing to the large amount of socio-economic indicators, we are disable to use them directly.Hence, we use the **factor analysis** to simplify the problem. The basic idea of factor analysis is to classify the closely related variables into the same category, and the correlation between different categories of variables is poor. In essence, the factor analysis reduces the number of variables by reducing the dimensionality and bringing the variables of high relevancy together.

3.1 Factor Analysis Model

Hence, we divided the given socio-economic factors into 15 categories, they are *Households by type*, *Relationship*, *Marital status*, *Fertility*, *Grandparents*, *School Enrollment*,

Educational attainment, *Veteran Status*, *Residence*, *Place of birth*, *U.S. Citizenship status* ,*Year of entry*,*World region of birth of foreign born*, *Language* and *Ancestor* ,and we use A, B,..., O to express them. This operation can make the correlation analysis more effective. Next, we extract k common factors from each category via factor analysis.We define $(x_1, \ldots, x_p)^T$ as $\bar{x}$, $(f_1, \ldots, f_m)^T$ as $\bar{F}$, $(f_1, \ldots, \varepsilon_m)$ as $\bar{\varepsilon}$.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pm} \end{bmatrix} \tag{14}$$

$$\begin{cases} x_1 - \mu = a_{11}f_1 + a_{12}f_2 + \cdots + a_{1m}f_m + \varepsilon_1 \\ x_2 - \mu = a_{21}f_1 + a_{22}f_2 + \cdots + a_{2m}f_m + \varepsilon_2 \\ \vdots \\ x_p - \mu = a_{p1}f_1 + a_{p2}f_2 + \cdots + a_{pm}f_m + \varepsilon_p \end{cases} \tag{15}$$

$$\bar{x} - \bar{\mu} = A\bar{F} + \bar{\varepsilon} \tag{16}$$

3.1.1 KMO Test (*Aiser-Meyer-Olkin*)

We use KMO test to discover the partial correlation between variables.

$$KMO = \frac{\sum\sum_{i \neq j} r_{i,j}^2}{\sum\sum_{i \neq j} r_{i,j}^2 + \sum\sum_{i \neq j} p_{i,j}^2} \tag{17}$$

$$KMO \epsilon [0, 1]$$

- If KMO>0.7, the original factor is suitable for factor analysis
- If KMO<0.5, the original factor is unsuitable for factor analysis

3.1.2 Factor extraction

we assume $x_l = a_{l_1}f_l + a_{l_2}f_2 + \cdots + a_{l_m}f_m$.

3.1.3 Factor Rotation

In this step, we transform $\bar{F}$ orthogonally.We assume N is an orthogonal matrix,then $\bar{F}$ will turn to $N^T\bar{F}$, we call it $\bar{G}$. Meanwhile, we transform matrix A orthogonally to matrix B. Thus, **Equation 10** can change to $\bar{x} - \bar{\mu} = A\bar{N}(N^T F) + \bar{\varepsilon}$ which can also be depicted as $\bar{x} - \bar{\mu} = B\bar{G} + \bar{\varepsilon}$.Afterwards, we try to find the max value of $\sum_{j=1}^{m} b_{i,j^2}$, here $b_{i,j}$ is an element in matrix B.

3.1.4 Value of Each Common Factor in Every Sample

$$\begin{cases} f_1 = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1p}x_p \\ \\ f_2 = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2p}x_p \\ \\ \vdots \\ \\ f_m = c_{m1}x_1 + c_{m2}x_2 + \cdots + c_{mp}x_p \end{cases} \tag{18}$$

So far, we have reduced the original P variables to k variables ($k < p$) in one category. After using same pocess for the rest of the categories, we can get $f_{A_1}, \ldots, f_{A_{m_1}}, f_{B_1}, \ldots, f_{B_{m_2}}, \ldots, f_{O_1}, \ldots, f_{O_{m_1 5}}$ in the end. k_i is the number of variables for each category after simplification.

3.2 Correlation Analysis

Because $\bar{x}$ is less affected by the special factor $\bar{\varepsilon}$,we simplify **Equation 10** to

$$\bar{x} - \bar{\mu} = A\bar{F} \tag{19}$$

A is the correlation coefficient matrix of $\bar{x}$,$\bar{F}$.

According to the *numerical analysis theory* , we turn matrix A to QR via QR decomposition. Matrix Q is an orthogonal matrix composed of eigenvectors, $Q = (\lambda_1, \lambda_2, ..., \lambda_n)$, λ is the eigenvalue. Matrix P is the upper triangular matrix of non-negative diagonal elements.

$$\bar{x} - \bar{\mu} = Q(R\bar{F}) \tag{20}$$

Because $\bar{F}$ can be any matrix, we define $\bar{G} = \bar{F}$.

$$\bar{x} - \bar{\mu} = Q\bar{F} \tag{21}$$

So $\bar{F}$ is equal to $Q^T(\bar{x}-\bar{\mu})$. Thus we can get the cumulative variance contribution rate F_k, and use factors with higher rate F_k to replace the original variables approximately. We choose m factors with high rate F ,then we can the weight W of each factor in its corresponding category.

$$F_k = \frac{\sum_{j=1}^{k} \lambda_j}{\sum_{i=1}^{m} \lambda_i} \tag{22}$$

$$W_k = \frac{\lambda_k}{\sum_{i=1}^{m} \lambda_i} \tag{23}$$

Here we can get the score f of each category.

$$\begin{cases} f_A = W_{A_1} f_{A_1} + W_{A_2} f_{A_2} + \cdots + W_{A_{m_1}} f_{A_{m_1}} \\ f_B = W_{B_1} f_{B_1} + W_{B_2} f_{B_2} + \cdots + W_{B_{m_2}} f_{B_{m_2}} \\ \vdots \\ f_O = W_{O_1} f_{O_1} + W_{O_2} f_{O_2} + \cdots + W_{O_{m_15}} f_{O_{m_{15}}} \end{cases} \tag{24}$$

By analyzing scree plot in **Figure 6**, we can select factors with high eigenvalue.

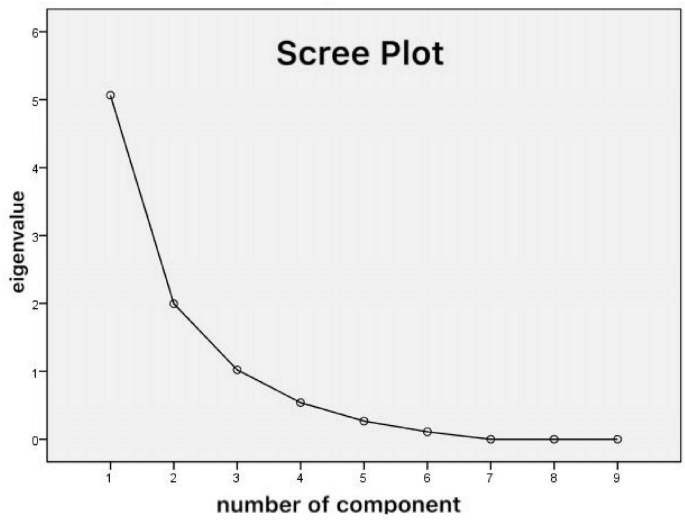

Figure 6: Scree plot

Thus we can get the correlation index $Corr(F_i, f_k)$ to determine the relationship between socio-economic indicators and drug-abuse levels clearly in county$i$.

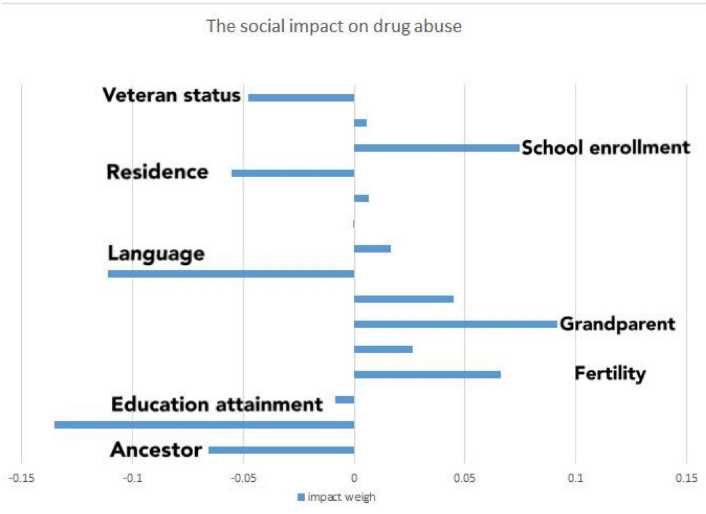

Figure 7: Social impact on drug abuse

3.3 Correlation result analysis

As is shown in **Figure 7**, *Educational attainment*, *Veteran Status*, *Fertility*, *Grandparents's Responsibility*, *School Enrollment*, *Residence*, *Language* and *Ancestor* have higher weighs among all factors.

Grandparents's Responsibility has maximum positive correlation with the drug-use incidents. Generally, due to the poor health and spoiling emotion, grandparents usually have less supervision over their grandchildren. Thus compared to those who are supervised by parents, these children are more likely to take drugs.

School Enrollment and *Educational attainment* will also greatly impact drug abuse. In fact, people with higher education level have a rich fund of knowledge, they clearly know the danger of drug abuse, so they will not take drugs normally.

Residence and *Language* are related to drug abuse as well. People who live in an area that have a lot drug addicts can be easily affected. Plus, if the language people speak is commonly use in a region, drug trafficking will be easier, which will lead to the rapid spread of drugs.

Fertility and Ancestor are another relevant factors. Gene can determine the sensitivity of people's drug addiction (Jennifer Lyden,el,2019), so some races are more likely to be addicted. As for *Fertility*, if women give birth to their children at reproductive age, these kids will be healthier usually. So they will have better immunity system to against drug addiction.

Veteran Status also influence drug abuse to some extent.

Therefore, governments should pay more attention to the above issues. Firstly, they

have to improve the current education system so that more people can get better education, which will raise people's awareness of anti-drug. Secondly, governments should strengthen the governance of drug-concentrated areas to fighting against drug trafficking. Thirdly, governments are supposed to limit the use of opioid in therapy, so as to reduce the drug-addiction probability of people who are more sensitive to opioid.

3.4 Change of β

Using the correlation index $Corr(F_i, f_k)$ to calculate the weight of each category, we can get the comprehensive impact of socio-economic factors to a county i's drug-abuse level, which can be depicted as

$$\varphi(F_i) = Corr(F_i, f_A)f_A + Corr(F_i, f_B)f_B + \cdots + Corr(F_i, f_O)f_O \tag{25}$$

In our former model, we assume every county has the same spread intensity of drug abuse β. After considering the socio-economic factors ,we have to modify our previous model. Owing to the different impact of each factor in different county, we use β_i to denote the spread intensity in county i.

$$\beta_i = k\varphi(F_i) \tag{26}$$

3.5 Modified Opioid-spreading Model

Under the influence of various socio-ecological factors, different counties will act to drug-use cases distinctly. Therefore, due to the change of β ,we have to renew the value of anti-addiction factor $\bar{I}$ so as to get new non-drug-addiction probability of the neighborhoods $P(A_{q_i})'$.The new iterative process is as follows:

$$\bar{I}'(\alpha_{p,q}^t) = \begin{cases} 1, & if\ \alpha_{p,q}^t = 0 \\ 1 - k\varphi(F_i), & if\ \alpha_{p,q}^t > 0 \end{cases} \tag{27}$$

$$P(A_{q_i})' = \bar{I}'(\alpha_{p,q}^t)L \tag{28}$$

- If no drug treatments are taken in the county,

$$\begin{cases} P(\alpha_{i,j}^t)' = 1 - \prod_{i=1}^m P(A_{q_i})' \\ \alpha_{i,j}^t = \alpha_{i,j}^{t-1} + 1 \end{cases} \tag{29}$$

- If there are some drug treatments taken in the county,

$$\begin{cases} K(\alpha_{i,j}^t) = (\alpha_{i,j}^t)L/(i_{max}) \\ \\ \alpha_{i,j}^t = \alpha_{i,j}^{t-1} - 1 \end{cases} \tag{30}$$

3.6 Change of the threshold

As is mentioned above, $k\varphi(F_i)$ is the new spread intensity of drug abuse in county i, so the infection ratio of county i will change to $k\varphi(F_i)S$. And the total infection ratio will be $k\varphi(F_i)SI$.

$$\begin{cases} \dfrac{dS}{dt} = -k\varphi(F_i)SI \\ \\ \dfrac{dI_1}{dt} = k\varphi(F_i)SI - LI \\ \\ \vdots \\ \\ \dfrac{dI_n}{dt} = k\varphi(F_i)SI - LI \end{cases} \tag{31}$$

$$\frac{dI}{dt} = n(\frac{L}{k\varphi(F_i)S} - 1) \tag{32}$$

The dividing line in **figure 2** will turn to $I = S - k\varphi(F_i)/L$. So we can find that

- If $S_0 - k\varphi(F_i)/L < 0$, I will inevitably tend to 0,
- If $S_0 - k\varphi(F_i)/L > 0$, I will increase to a large value firstly and then decrease.

Thence, $k\varphi(F_i)/L$ will be our new drug identification threshold level R'. If $R' < 1$, the spread of drug addiction can be in control. Otherwise, if $R' > 1$, the spread of drug addiction will be out of control.

4 Sensitivity Analysis For Opioid Propagation Model

After analyze the opioid-spreading model, we can conclude that with the increasing spread intensity β of drug abuse, there will be more drug abuse incidents. And the number of the drug abuse incidents will reach a stable value in the end. Plus,this value

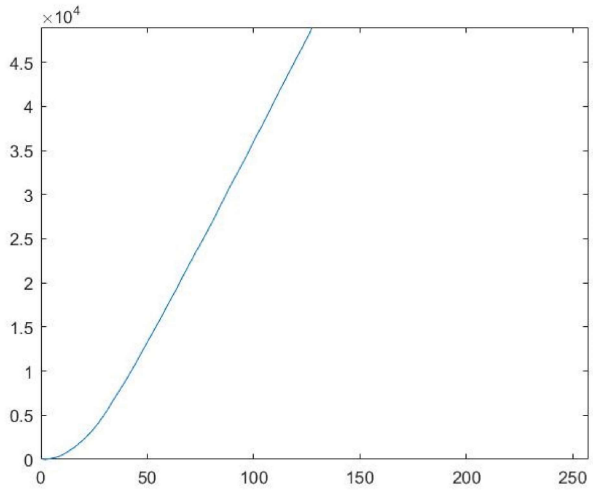

Figure 8: Sensitivity analysis consequence for Opioid Propagation Model

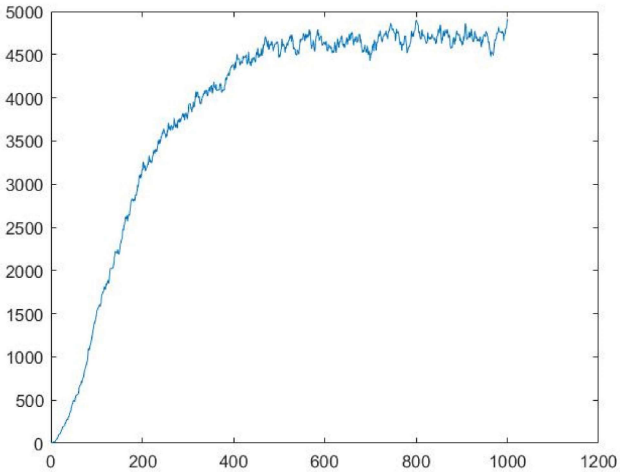

Figure 9: β is 0.2

have positive correlation with the degree of supervision L, so the stable value will fluctuate.

By tuning the parameter we can conclude that the greater the ratio between two parameter, the bigger slope it has. And the volatility is determined by the ratio of the supervision degree L to the spread intensity of drug abuse,which is also called the threshold R as we mentioned above. When the ratio increases, the fluctuation value will decrease. If $R > 1$, the y value of the curve will tend to 0. Else, the relationship between the supervision degree L and the spread intensity β is shown in the following figure.

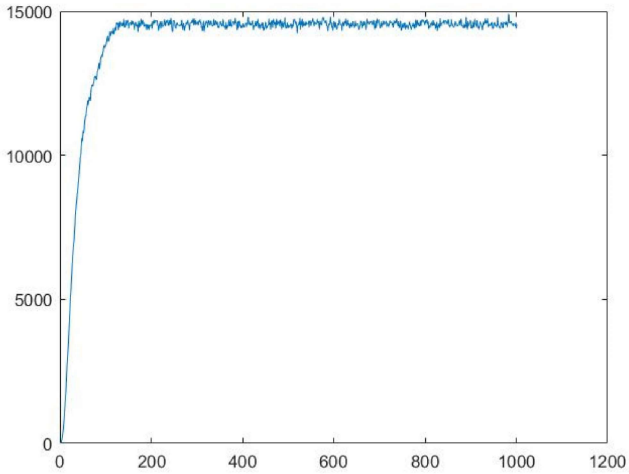

Figure 10: $beta$ is 0.8

5 Sensitivity Analysis For Factor Analysis Model

According to the simulation of model 2, We make a sensitivity analysis by tuning the score of each category we have classified. As the figure we have shown above, which is basically match the subjective conditions that education-relevant factors have big correlation with the drug crisis.

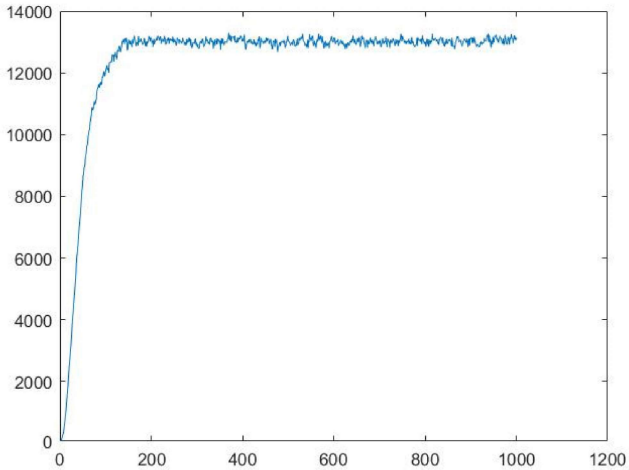

Figure 11: Before tuning the score of education attainment factors

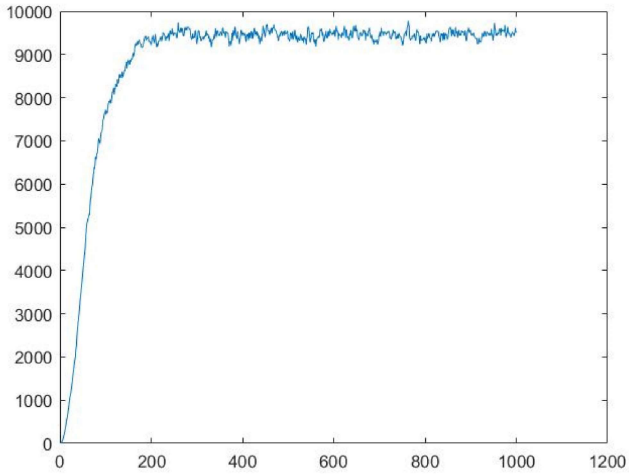

Figure 12: After tuning the score of educational attainment factor

6 Strengths and Weaknesses

6.1 Strengths

- We further refine the degree of drug addiction by giving them different levels depicted as consecutive integers.
- We use factor analysis to get relational evaluation functions so that we can quantify the relationship between drug use and various social factors.
- The model has a good visualization effect and can simulate the spread of opioid dynamically.

6.2 Weaknesses

- Each iteration of the model produces local errors, resulting in a large overall error.
- Simulation models are difficult to derive specific quantitative values.

7 Memo to the chief administrator

Dear Chief Administrator,

In order to help counter the opioid crisis, we are writing to inform you of our work. Firstly, we do research on the spread of opioid. By using CA model and the improved SI model, We identify 16 counties as the sources where opioid cases get started. Then by simulating the process of drug spreading, we find that drugs spread radically from center to its surroundings. What I want stress is that there are higher spread intensities on borders of two states as well as the some metropolises.So the government are supposed to strengthen supervision towards these places. Also, we find the threshold level of opioid spreading according to the dynamical system theory. This threshold is determined by the intensity level and the degree of treatments to drug addiction.If the threshold is exceeded, the spread of drug abuse will be out of control. Thus, the government should take measures to reduce opiod spreading level and enhance the degree of treatments.

As is known to all, the spread of drugs can be impacted by various socio-economic factors. By using factor analyze, we find 8 main factors that have tight relationships with drug abuse, like education, race, family, etc. Therefore,the governments should pay more attention to the above issues, such as improving the current education system to raise people's awareness of anti-drug, enhancing the governance of drug-concentrated areas to fighting against drug trafficking and limiting the use of opioid in therapy, so as to reduce the drug-addiction probability of people who are more sensitive to opioid. After taking these factors into consideration, we improve our model so as to reflect social impacts specifically.

Finally, we make a sensitivity analysis to our model and come up with several effective measures to cope with crisis.Therefore, governments should pay more attention to the above issues. Firstly, governments have to improve the current education system so that more people can get better education, which will raise people's awareness of anti-drug. Secondly, governments should strengthen the governance of drug-concentrated areas to fighting against drug trafficking. Thirdly, governments are supposed to limit the use of opioid in therapy, so as to reduce the drug-addiction probability of people who are more sensitive to opioid.

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