

# A General Formulation for Safely Exploiting Weakly Supervised Data

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# What is this paper about

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Weakly supervised data is one important machine learning data

It suffers one serious issue

- The usage of weakly supervised data may even degenerate performance, which means, it could be outperformed by its supervised counterpart using only a small number of labeled data

## Contribution of this work

In this work, we consider to learn a safe prediction for weakly supervised learning, where **safe means it will not be worse than its supervised counterpart**. We propose a general formulation and give theoretical analysis. The experiments also show quite promising results

- Introduction
- Proposed Approach
- Experiments
- Conclusion

# What is Weakly Supervised Learning

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- Weakly supervised learning use the data that **does not require a large amount of precise label information**
- For Example:
  - Label Noise Learning [Fr'enay and Verleysen 2014]
  - Semi-Supervised Learning [Chapelle et al. 2006])]
  - Domain Adaptation [Pan and Yang 2010]
  - ...

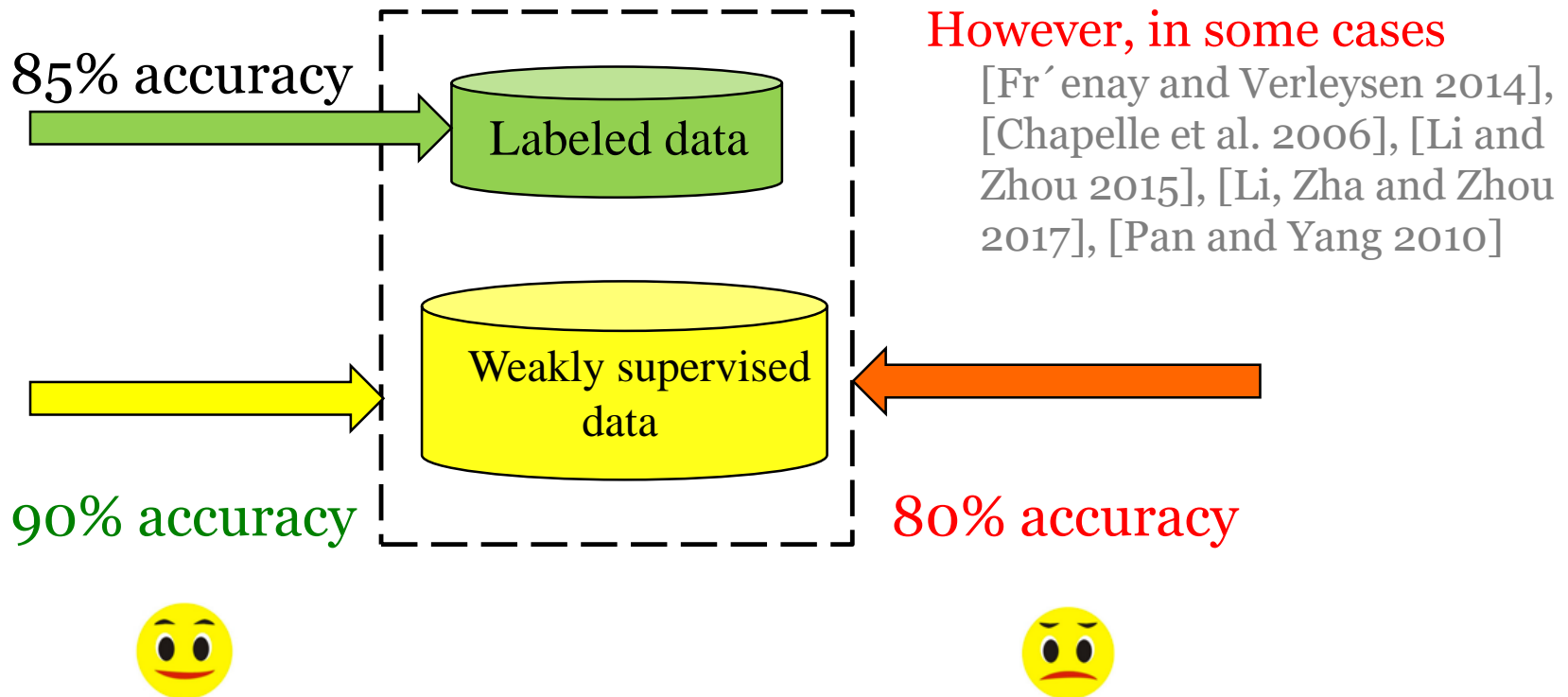
# Examples

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- Label Noise Learning
  - We have only a small number of high-quality labeled data and a lot of noisy labeled data
- Semi-Supervised Learning
  - We have only limited labeled data and need to leverage a number of unlabeled data
- Domain Adaptation
  - Label information in target domain is not sufficient and we need to exploit further label information from other domains

# Weakly supervised learning is not safe

It is often expected that weakly supervised data can help improve performance since more data are used. However, it sometimes fails.



- Introduction
- **Proposed Approach**
- Experiments
- Conclusion

# The Basic Setup

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- Suppose we have a set of weakly supervised learning predictions  $\{y_i\}_{i=1}^n$
- These base predictions can be obtained in various ways, e.g., by different type of algorithms
- Moreover, we can easily train a supervised method with the use of only limited labeled data and let  $y_0$  denote the prediction

**The goal:** to learn a safe prediction  $g((y_1, \dots, y_n), y_0)$ , which often outperform, and will not be worse than  $y_0$



# A Direct Approach

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Suppose we know the ground-truth  $\mathbf{y}^*$ , we can directly maximize the performance gain

$$\max_{\mathbf{y} \in \mathbb{H}^u} \ell(\mathbf{y}_0, \mathbf{y}^*) - \ell(\mathbf{y}, \mathbf{y}^*)$$

Trained on high-quality labeled data only

Ground Truth

# However

Obviously,  $y^*$  is unknown

We can construct  $y^*$  with  $\{y_i\}_{i=1}^n$ , and consider the worst case for the requirement of safeness.

SafeW

$$\max_{\mathbf{y} \in \mathbb{H}^u} \min_{\alpha \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i) - \ell(\mathbf{y}, \sum_{i=1}^n \alpha_i \mathbf{y}_i)$$

Worst case consideration [Li and Zhou, ICML2011/TPAMI2015;  
Balsubramani and Freund, COLT2015]

## Three Questions about the formulation

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$$\max_{\mathbf{y} \in \mathbb{H}^u} \min_{\alpha \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i) - \ell(\mathbf{y}, \sum_{i=1}^n \alpha_i \mathbf{y}_i)$$

- Is this formulation reasonable?
- How to setup the set of weights  $\mathcal{M}$ ?
- How to solve it efficiently?

# Is this formulation reasonable?

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$$\max_{\mathbf{y} \in \mathbb{H}^u} \min_{\boldsymbol{\alpha} \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i) - \ell(\mathbf{y}, \sum_{i=1}^n \alpha_i \mathbf{y}_i)$$

Theoretical analysis:

**Theorem 1.** *Suppose the ground-truth  $\mathbf{y}^*$  can be constructed by the base learners, i.e.,  $\mathbf{y}^* \in \{\mathbf{y} \mid \sum_{i=1}^b \alpha_i \mathbf{y}_i, \boldsymbol{\alpha} \in \mathcal{M}\}$ . Let  $\hat{\mathbf{y}}$  and  $\hat{\boldsymbol{\alpha}}$  be the optimal solution to Eq.(1), we then have  $\ell(\hat{\mathbf{y}}, \mathbf{y}^*) \leq \ell(\mathbf{y}_0, \mathbf{y}^*)$  and  $\hat{\mathbf{y}}$  has already achieved the maximal performance gain against  $\mathbf{y}_0$ .*

## If the assumption is not satisfied?

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If  $\ell(\cdot, \cdot)$  is  $\eta$ -Lipschitz, i.e.,  $|\ell(y_1, y_2) - \ell(y_1, y_3)| \leq \eta \|y_2 - y_3\|_1$

Let  $\beta^* = \arg \min_{\beta \in \mathcal{M}} \ell(\sum_{i=1}^n \beta_i \mathbf{y}_i, \mathbf{y}^*)$  and  $\epsilon = \mathbf{y}^* - \sum_{i=1}^n \beta_i^* \mathbf{y}_i$ .

We have,

**Theorem 4.** *The performance gain of  $\hat{\mathbf{y}}$  against  $\mathbf{y}_0$ , i.e.,  $\ell(\mathbf{y}_0, \mathbf{y}^*) - \ell(\hat{\mathbf{y}}, \mathbf{y}^*)$ , has a lower-bound  $-2\eta \|\epsilon\|_1$ .*

## Setup $\mathcal{M}$

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For regression  $C_{ij} = (y_i - u_i)^\top (y_j - u_j)$ , for classification  $C_{ij} = y_i^\top y_j$

We prove that  $\mathbf{C}\alpha$  indicates the performance of the base learner

And we can think the base learner has a lower-bound performance  
[Balsubramani and Freund 2015]

Hence, we can setup  $\mathcal{M} = \{\alpha | \mathbf{C}\alpha \geq \delta, \mathbf{1}^\top \alpha = 1, \alpha \geq \mathbf{0}\}$

Moreover, if we have prior knowledge, we can setup  $\mathcal{M}$  more flexible

# Optimization

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Original Form:

$$\max_{\mathbf{y} \in \mathbb{H}^u} \min_{\alpha \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i) - \ell(\mathbf{y}, \sum_{i=1}^n \alpha_i \mathbf{y}_i)$$

Usually non-convex and **not easy to solve**

For regression task, we can get a convex optimization:  $\min_{\alpha \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i)$

For classification task and hinge loss, we can get a linear programming:

$$\min_{\alpha \in \mathcal{M}} \ell(\mathbf{y}_0, \sum_{i=1}^n \alpha_i \mathbf{y}_i) + \frac{1}{u} \left\| \sum_{i=1}^n \alpha_i \mathbf{y}_i \right\|_1 - 1$$

Become much easier to solve

- Introduction
- Proposed Approach
- **Experiments**
- Conclusion



# Label Noise Learning

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## □ Setup

- 8 frequently-used datasets
- 30% high-quality labeled data and 70% noisy data which their labels are random reversed with a probability  $p\%$ .

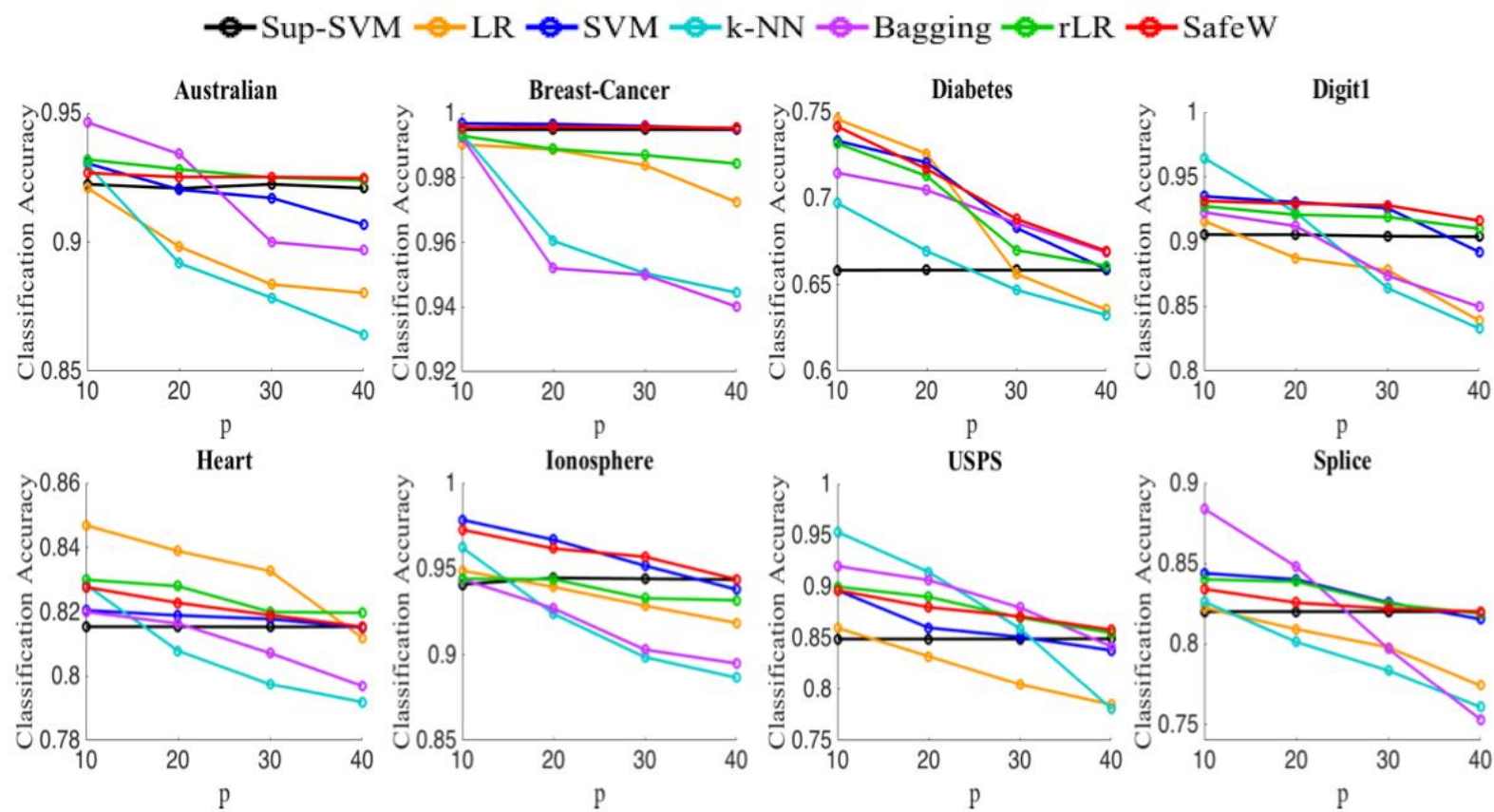
## □ Compared Methods

- Baseline
  - Sup-SVM
- 2 state-of-art noisy robust methods
  - Bagging [Fréenay and Verleysen 2014],
  - rLR [Bootkrajang and Kab'an 2012],
- 3 traditional methods
  - SVM
  - k-NN
  - Logistic Regression(LR)

## □ SafeW: adopted SVM, k-NN and LR as base learners.

# Label Noise Learning

Classification accuracy on 8 datasets with  $p$  range from 10% to 40%



# Domain Adaptation

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## ❑ Setup

- 2 benchmark datasets: 20newsgroup, landmine

## ❑ Compared Methods

- Baseline
  - Logistic Regression
- 4 transfer learning methods
  - Original
  - MIDA [Yan, Kou, and Zhang 2016]
  - TCA [Pan et al. 2011]
  - TrAdaBoost [Dai et al. 2007]

❑ SafeW: adopted Original, MIDA and TCA as base learners.

# Domain Adaptation

20newsgroup: 19,997 news and 20 groups

Dataset	Logistic Regression	Original	MIDA	TCA	TrAdaBoost	SAFEW
Comp vs Rec	.7028 $\pm$ .0091	<b>.7492 <math>\pm</math> .0135</b>	<b>.7961 <math>\pm</math> .0197</b>	<b>.7940 <math>\pm</math> .0162</b>	<b>.8077 <math>\pm</math> .0155</b>	<b>.7956 <math>\pm</math> .0170</b>
Comp vs Sci	.8225 $\pm$ .0662	.7985 $\pm$ .0194	<b>.8946 <math>\pm</math> .0188</b>	.8255 $\pm$ .0172	<b>.8583 <math>\pm</math> .0201</b>	<b>.8925 <math>\pm</math> .0212</b>
Comp vs Talk	.8423 $\pm$ .0685	.8022 $\pm$ .0182	.8231 $\pm$ .0164	.8434 $\pm$ .0110	.8247 $\pm$ .0143	.8451 $\pm$ .0158
Sci vs Talk	.7294 $\pm$ .1045	.7100 $\pm$ .0121	<b>.7456 <math>\pm</math> .0164</b>	.7022 $\pm$ .0092	.7166 $\pm$ .0213	<b>.7468 <math>\pm</math> .0153</b>
Rec vs Sci	.8006 $\pm$ .0758	.7754 $\pm$ .0161	.8033 $\pm$ .0151	<b>.8440 <math>\pm</math> .0118</b>	.8016 $\pm$ .0151	<b>.8435 <math>\pm</math> .0157</b>
Rec vs Talk	.8278 $\pm$ .0446	.8276 $\pm$ .0115	<b>.8566 <math>\pm</math> .0105</b>	<b>.8580 <math>\pm</math> .0128</b>	<b>.8415 <math>\pm</math> .0113</b>	<b>.8579 <math>\pm</math> .0105</b>
Average	.7876	.7805	.8199	.8112	.8084	.8302
Win/Tie/Loss against LR		1/2/3	4/1/1	3/2/1	3/2/1	<b>5/1/0</b>

Directly using weakly supervised data **often** degenerate performance while SafeW **does not** suffer this problem.

# Domain Adaptation

Landmine: 29 domain, 9 features

Dataset	Logistic Regression	Original	MIDA	TCA	TrAdaBoost	SAFEW
Domain 20	.9215 $\pm$ .0173	.9237 $\pm$ .0034	<b>.9265 <math>\pm</math> .0039</b>	.9255 $\pm$ .0045	.9183 $\pm$ .0029	<b>.9271 <math>\pm</math> .0035</b>
Domain 21	.9360 $\pm$ .0095	.9310 $\pm$ .0047	.9384 $\pm$ .0045	.9304 $\pm$ .0051	.9261 $\pm$ .0033	.9396 $\pm$ .0038
Domain 22	.9594 $\pm$ .0051	.9555 $\pm$ .0038	.9506 $\pm$ .0065	<b>.9650 <math>\pm</math> .0017</b>	.9095 $\pm$ .0026	<b>.9648 <math>\pm</math> .0016</b>
Domain 23	.9361 $\pm$ .0095	.9310 $\pm$ .0041	<b>.9424 <math>\pm</math> .0045</b>	.9314 $\pm$ .0051	<b>.9627 <math>\pm</math> .0043</b>	<b>.9426 <math>\pm</math> .0038</b>
Domain 24	.9535 $\pm$ .0052	.9524 $\pm$ .0029	.9447 $\pm$ .0025	.9432 $\pm$ .0029	.9535 $\pm$ .0034	.9550 $\pm$ .0024
Average	.9413	.9387	.9405	.9391	.9340	.9458
Win/Tie/Loss against LR		0/3/2	2/1/2	1/1/3	1/2/2	<b>3/2/0</b>

We have similar observations.

# Semi-Supervised Learning

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## □ Setup

- 8 commonly used regression datasets
- 10 labeled data are chosen for each dataset

## □ Compared Methods

- Baseline
  - 1NN
- Semi-Supervised Regressor
  - Self-kNN [Yarowsky 1995]
  - Self-LS [Hastie, Tibshirani, and Friedman 2001]
- Ensemble methods
  - Average
  - Safer [Li, Zha and Zhou 2017]

□ For Average, Safer and SafeW: adopted Self-kNN(Euclidean), Self-kNN(Cosine) and Self-LS as base learners.

# Semi-Supervised Learning

## Mean Square Error on 8 datasets

Dataset	1NN	Self- $k$ NN(Euclidean)	Self- $k$ NN(Cosine)	Self-LS	Average	Safer	SAFEW
abalone	.020 $\pm$ .010	<b>.014 <math>\pm</math> .005</b>	<b>.014 <math>\pm</math> .003</b>	<b>.013 <math>\pm</math> .004</b>	<b>.012 <math>\pm</math> .003</b>	<b>.013 <math>\pm</math> .005</b>	<b>.013 <math>\pm</math> .005</b>
bodyfat	.019 $\pm$ .005	.018 $\pm$ .006	.019 $\pm$ .005	.041 $\pm$ .013	.023 $\pm$ .009	.018 $\pm$ .007	.017 $\pm$ .005
cadata	.083 $\pm$ .029	<b>.063 <math>\pm</math> .012</b>	<b>.058 <math>\pm</math> .009</b>	<b>.056 <math>\pm</math> .007</b>	<b>.057 <math>\pm</math> .009</b>	<b>.060 <math>\pm</math> .013</b>	<b>.057 <math>\pm</math> .005</b>
cpusmall	.024 $\pm$ .012	.027 $\pm$ .011	.028 $\pm$ .009	.025 $\pm$ .010	.024 $\pm$ .005	.025 $\pm$ .011	.024 $\pm$ .009
housing	.039 $\pm$ .010	.036 $\pm$ .009	.033 $\pm$ .006	.036 $\pm$ .009	<b>.034 <math>\pm</math> .008</b>	<b>.034 <math>\pm</math> .009</b>	<b>.033 <math>\pm</math> .005</b>
mg	.051 $\pm$ .009	<b>.039 <math>\pm</math> .006</b>	<b>.038 <math>\pm</math> .006</b>	<b>.035 <math>\pm</math> .015</b>	<b>.038 <math>\pm</math> .014</b>	<b>.038 <math>\pm</math> .006</b>	<b>.038 <math>\pm</math> .006</b>
mpg	.022 $\pm$ .007	.020 $\pm$ .006	.018 $\pm$ .006	.021 $\pm$ .008	.020 $\pm$ .006	<b>.019 <math>\pm</math> .004</b>	<b>.018 <math>\pm</math> .004</b>
pyrim	.023 $\pm$ .006	<b>.021 <math>\pm</math> .005</b>	.022 $\pm$ .005	.052 $\pm$ .014	<b>.020 <math>\pm</math> .007</b>	<b>.020 <math>\pm</math> .006</b>	<b>.020 <math>\pm</math> .006</b>
Ave. Mse.	.035	.030	.029	.035	.029	.030	.028
Win/Tie/Loss against 1NN		4/3/1	3/4/1	3/3/2	5/2/1	6/2/0	6/2/0

SafeW also obtain safe predictions

- Introduction
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# Conclusion

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- We propose a general formulation for safely exploiting weakly supervised data
- It has three advantages
  - Has safeness guarantee for commonly used loss functions in both regression and classification tasks
  - Can setup the weight of base learner flexibly
  - Can be solved globally in an efficient manner

Thanks!