SIR Models

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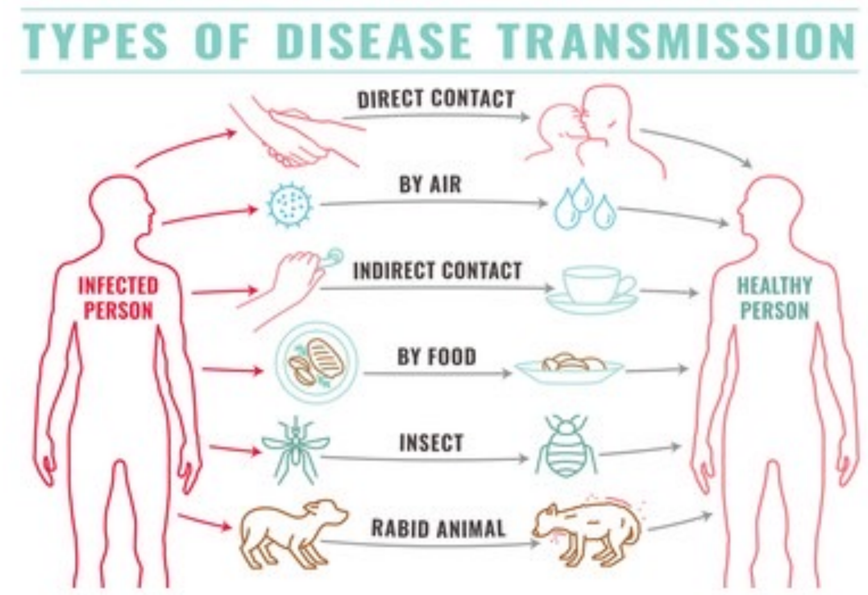
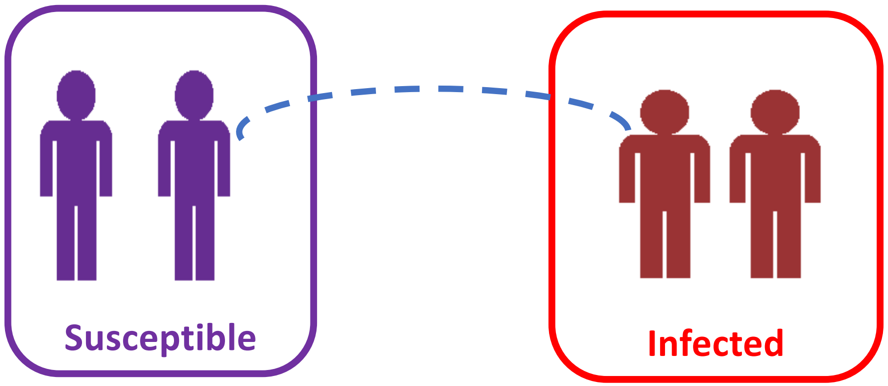
Lecture Outline

- Understanding disease dynamics
- Compartmental diagram
- Susceptible-Infectious Modeling
- Formulate SIR Model

How is disease transmitted?



Force of Infection



Force of Infection

FOI quantifies how much transmission there is.

FOI tells us about the rate at which individuals get infected in the population

The higher the FOI the earlier an individual is likely to get infected.

Force of Infection

- *Force of infection*, λ is the per capita rate at which susceptible individuals contract the infection. Rate at which new infecteds are produced = λX where X=number of susceptibles.
- Intuitively, $\lambda \propto$ number of infectious individuals.
- Kinds of transmission:
  - Frequency-dependent or 'mass action transmission': $\lambda = \beta Y/N$, where Y=# of infectious,$N =$ total population size, β= product of contact rates and transmission probability. Assumption: # of contacts is independent of N.
  - Density-dependent transmission: $\lambda = \beta Y$. Assumption: as the density of individuals increases, so does contact rate.
- If we assume host population size stays constant, let $S = X/N$, $I = Y/N$.

Compartmental Modeling

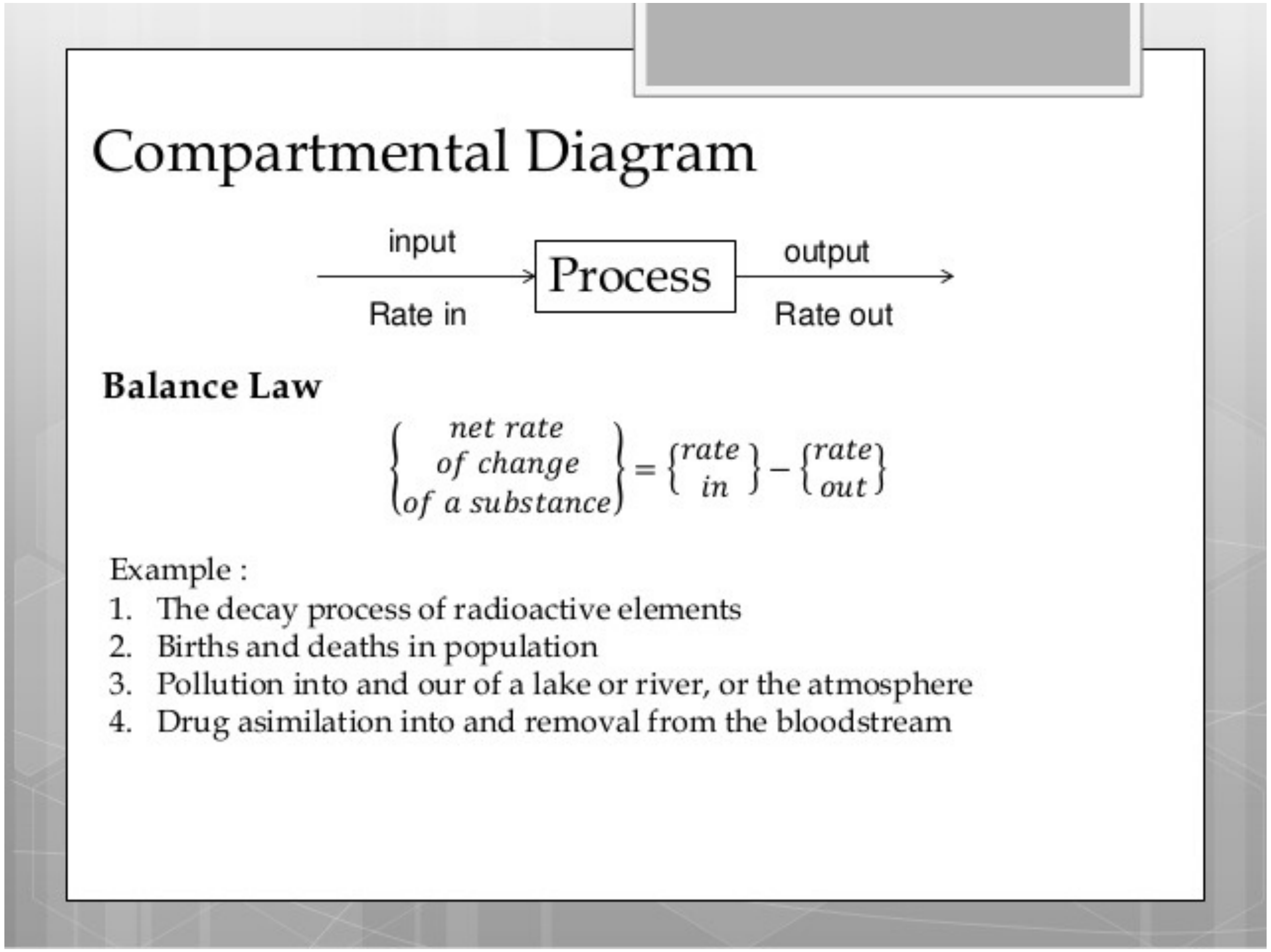
The technique is an extremely natural and valuable means with which to formulate processes.

Many processes may be considered as compartmental models:

The process has inputs to and outputs from a “compartment” over time

Based on the **balance law**

Compartmental Modeling



Model Assumptions

- Large populations – ignore random differences
  - Homogeneous population (well-mixed)

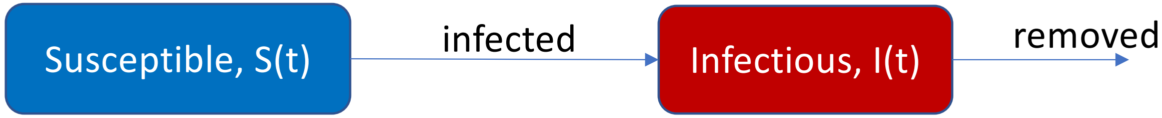
Scenario: flu epidemic at a boarding school over 30 days

- Ignore birth and death
- Disease spread by contact
- Neglect latent period
- Within time period = those recovered are immune

Natural Definitions

- Incubation period (t_{inc}) = time between infection and the appearance of visible symptoms
- Latent period (t_{lat}) = time between infection and the ability to infect someone else
- $t_{inc} > t_{lat}$

Compartmental Modeling



$$\left\{\begin{matrix} rate\ of \\ change\ in\ no. \\ of\ susceptibles \end{matrix}\right\} = 0 - \left\{\begin{matrix} rate \\ susceptibles \\ infected \end{matrix}\right\} \Rightarrow \beta S I$$

Assume the rate of susceptible getting infected by a single infectious person is directly proportional to the number of susceptible:

$$\left\{\begin{matrix} \text{rate susceptibles} \\ \text{infected by} \\ \text{one infective} \end{matrix}\right\} = \beta S(t)$$

$$\frac{dS}{dt} = -\beta S I \qquad (1)$$

$$\left\{\begin{matrix} rate\ of \\ change\ in\ no. \\ of\ infectious \end{matrix}\right\} = \left\{\begin{matrix} rate \\ susceptibles \\ infected \end{matrix}\right\} - \left\{\begin{matrix} rate \\ infectious \\ removed \end{matrix}\right\} \Rightarrow \gamma I$$

Assume the rate of infectious removed is directly proportional to the number of infectious

$$\frac{dI}{dt} = \beta S I - \gamma I \qquad (2)$$

Model Solution

$$\frac{dS}{dt} = -\beta SI \qquad (1)$$

$$\frac{dI}{dt} = \beta SI - \gamma I \qquad (2)$$

$S(0) = ?\,; I(0) = ?$ Initial Values

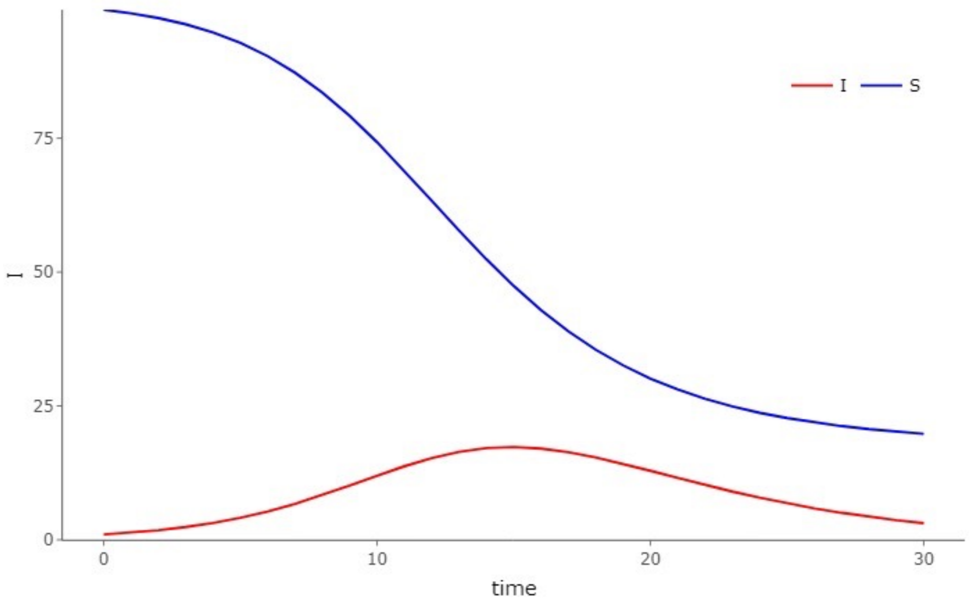
Parameters need values:
β is the transmission coefficient/probability of contact get infected
γ is the removal rate/recovered rate

Initial Value Problem

- Euler Method
- Runge-Kutta Method
- Many other methods

MATLAB® Maple™ R

Model Simulation

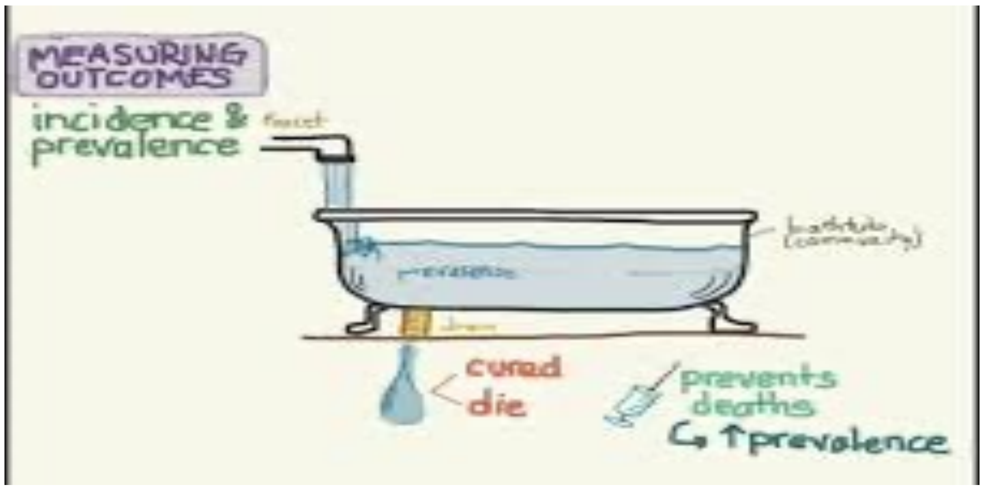


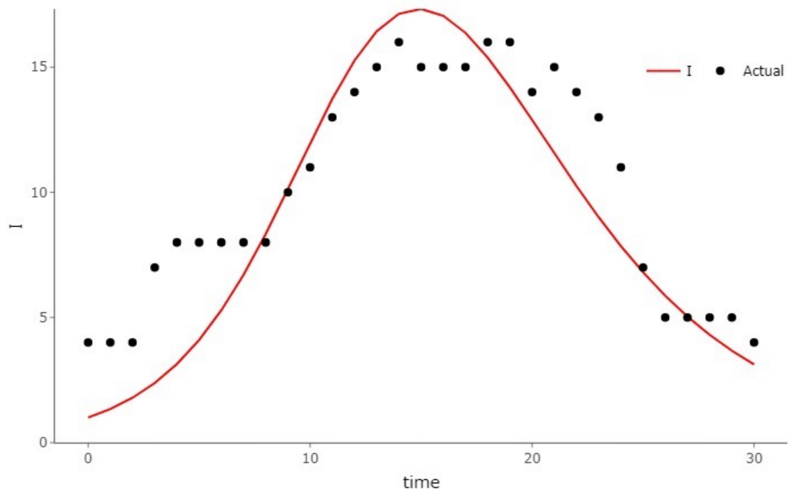
$S(0) = 99; I(0) = 1$

$\beta = 0.58; \gamma = 0.2778$

Actual data = COVID19 daily active cases

First wave: 25th Jan to 24th Feb 2020

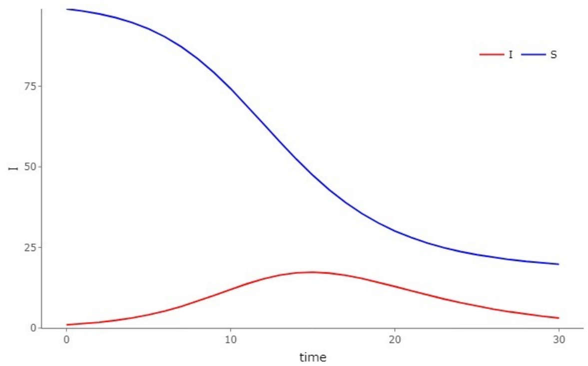




Theoretical Analysis

$$\frac{dS}{dt} = -\beta SI \qquad (1)$$

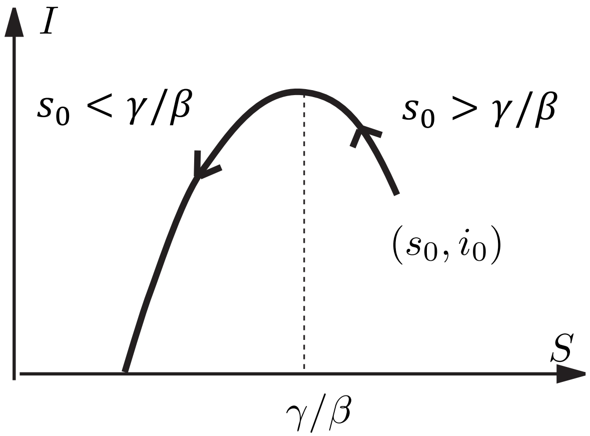
$$\frac{dI}{dt} = \beta SI - \gamma I \qquad (2)$$

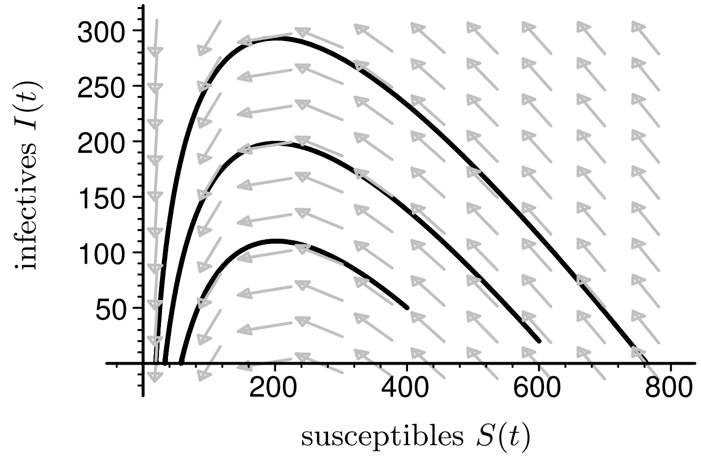


$$\frac{dI}{dS} = \frac{dI/dt}{dS/dt}.$$

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\beta S}$$

$$I = -S + \frac{\gamma}{\beta}\,\ell\text{n}(S) + K$$





For more complex models:

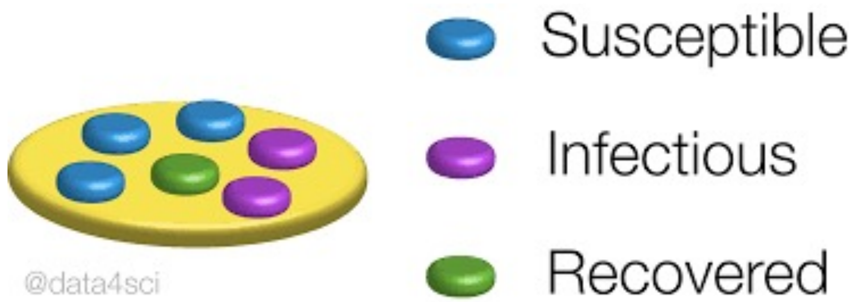
- Disease Free Equilibrium
- Next Generation Method

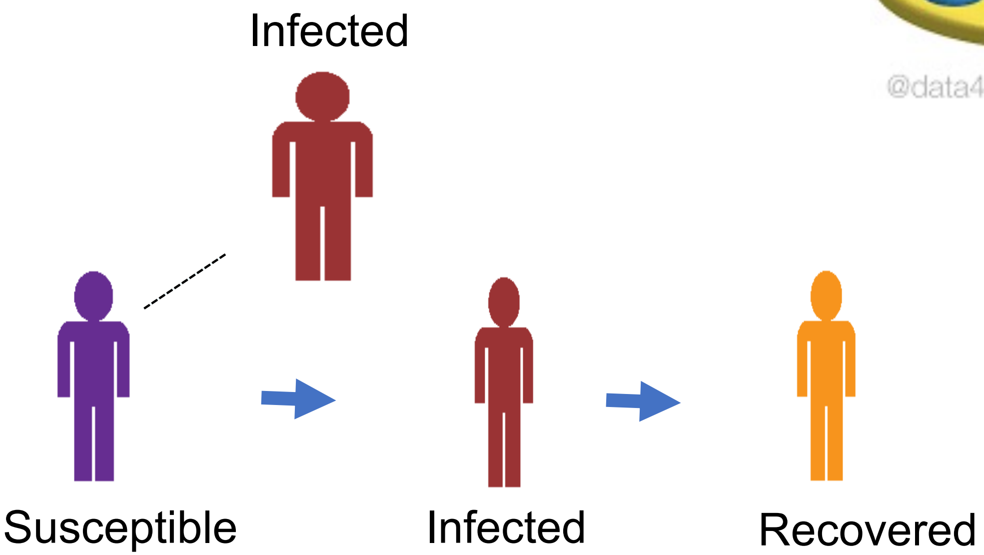
R_0 (Basic Reproduction Number)

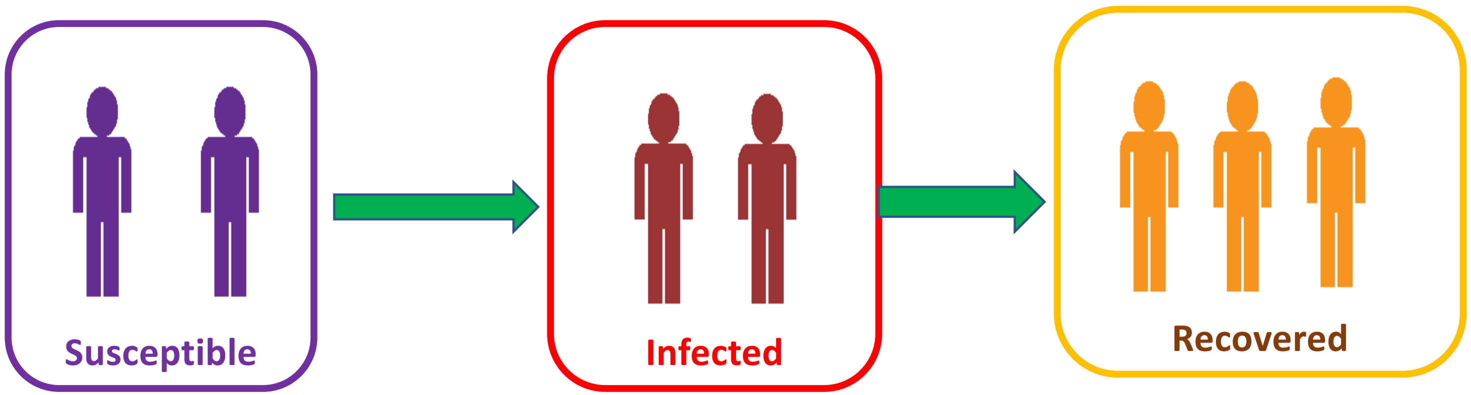
$R_0 > 1$, Infection take off
$R_0 < 1$, Infection die out

$$\therefore R_0 = S_0 \frac{\beta}{\gamma}$$

SIR Model Formulation







$$\boxed{S} \longrightarrow \boxed{I} \longrightarrow \boxed{R}$$

Figure 1.1: Compartmental diagram

$$\frac{dS}{dt} = -\beta IS,$$

$$\frac{dI}{dt} = \beta IS - \gamma I,$$

$$\frac{dR}{dt} = \gamma I.$$

three main assumptions : (i) a closed population (no births, no deaths, no migration), (ii) spatial homogeneity, (iii) disease transmission by contact between susceptible and infected

Interpretation of parameters

- Larger β imply that the disease is more easily spread
  - Also it reflects the degree to which the population interacts
  - More crowded (individual experience greater number of contacts per unit time) leads to higher β
- β usually estimated to be the **per-capita contact rate divide by the number of individuals**
- γ is estimated to be the **inverse of the difference of incubation period and latent period**

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \quad \Longrightarrow \quad N = S + I + R.$$

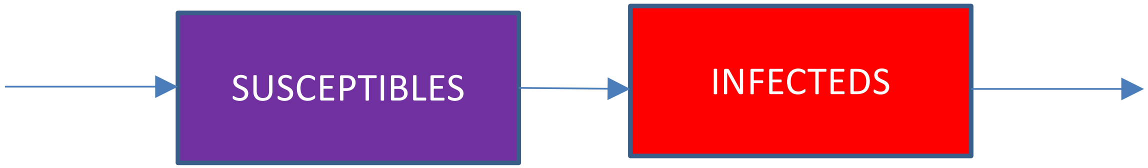
N is constant by assumption (i)

Interpretation of Parameters

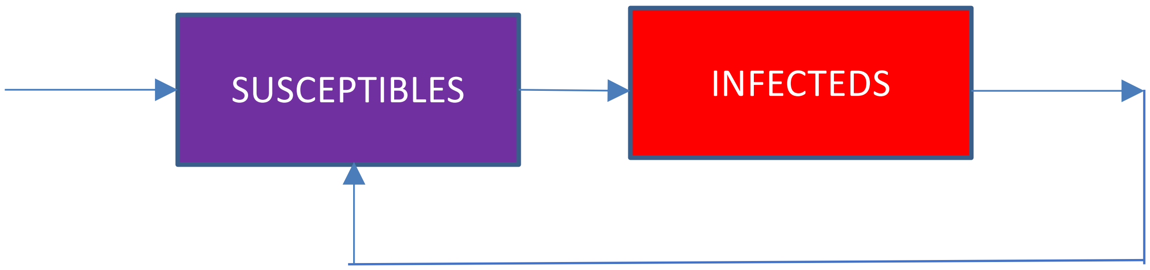
- Estimating γ
- 1/{average period of being infectious}
- $\gamma = \frac{1}{t_{inc}}$
- If t_{lat} is considered then
- $\gamma = \frac{1}{t_{inc} - t_{lat}}$

Model Variants

- Effect of births



- Disease without immunity



Questions to ask

- When a rapid increase in I is it always followed by a decrease?
- If parameters are adjusted, can we limit the increase? Or prevent it?
- Must the transmission coefficient be constant? Or can it vary? – vary with population size?

Questions to ask

- Consider at initial stage, I_0 introduced into a population of S_0
  - Will an epidemic occur? Or will it die out?

$$\frac{dI}{dt} = bSI - rI = I(bS - r)$$

  - If $S_0 < \frac{r}{b}$ then $\frac{dI}{dt} < 0$ implying infection dies out
  - **Threshold phenomenon**
  - If $S_0 > \frac{r}{b}$ then $\frac{dI}{dt} > 0$ implying epidemic occur

R_0 (Basic reproduction number)

- Can also be viewed as a threshold
- If each person infects more than one person then the infection takes off
- If not, then the infection will die out…
- Transmission rate and length of transmission

The importance of R_o

For an infectious disease with average infectious period $1/\gamma$ and transmission rate β, $R_o = \beta/\gamma$:

- For a closed population, an infectious disease can only invade if there is a threshold fraction of susceptibles greater than $1/R_o$.
- Vaccination policy: if proportion of susceptibles is reduced to below $1/R_o$, can eradicate the disease.

Effects of Parameters

- Parameters :
  - How quickly do people move from one compartment to another?
  - What drives this?

Constructing a mechanistic model

- "Fitting" to data
  - Do we have counts for the components of the model?
  - How well does the model match these?
    - Can we recreate the patterns of the data?
    - Are we "close" to the actual values of our outcome?
    - Data: cases (over time), serology= past infections
- Sometimes we estimate the parameters using this data

- Contact me for further enquiries:
- ljane@unimas.my

"No model, or set of models, can serve as a crystal ball to predict what will happen in the future, but they can shed light and provide much needed perspective on aspects of the epidemic that might be otherwise unknowable."

- JEN KATES, JOSH MICHAUD, & LARRY LEVITT
in *Coronavirus Policy Watch*

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