Solution: Using Eq. (1–8), we obtain

$$H = \Sigma P_j \log_2\left(\frac{1}{P_j}\right)$$

$$= \frac{1}{\log_{10}(2)}\left[10(0.099)\log_{10}\left(\frac{1}{0.099}\right) + 2(0.005)\log_{10}\left(\frac{1}{0.005}\right)\right]$$

or

$$H = 3.38\,\text{bits/key}$$

Using Eq. (1–9), where $T = 1/(2\,\text{keys/s}) = 0.5\,\text{s/key}$, yields

$$R = \frac{H}{T} = \frac{3.38}{0.5} = 6.76\,\text{bits/s}$$

SA1–3 Maximum Telephone Line Data Rate A computer user plans to buy a higher-speed modem for sending data over his or her analog telephone line. The telephone line has a signal-to-noise ratio (SNR) of 25 dB and passes audio frequencies over the range from 300 to 3,200 Hz. Calculate the maximum data rate that could be sent over the telephone line when there are no errors at the receiving end.

Solution: In terms of a power ratio, the SNR is $S/N = 10^{(25/10)} = 316.2$ (see dB in Chapter 2), and the bandwidth is $B = 3{,}200 - 300 = 2{,}900\,\text{Hz}$. Using Eq. (1–10), we get

$$R = B\log_2\left(1 + \frac{S}{N}\right) = 2{,}900\,[\log_{10}(1 + 316.2)]/\log_{10}(2),$$

or

$$R = 24{,}097\,\text{bits/s}$$

Consequently, a 28.8-kbit/s modem signal would not work on this telephone line; however, a 14.4-kbit/s modem signal should transmit data without error.

PROBLEMS

1–1 Assume that the Earth's terrain is relatively flat but that there is a mountain that has a height of 800 feet above the average terrain. How far away could a 100-ft cell tower be located from this mountain and yet provide cell phone coverage to a person on top of this mountain?

1–2 A high-power FM station of frequency 96.9 MHz has an antenna height of 1000 ft. If the signal is to be received 55 miles from the station, how high does a prospective listener need to mount his or her antenna in this fringe area?

★ 1–3 Using geometry, prove that Eq. (1–6) is correct.

1–4 A terrestrial microwave system is being designed. The transmitting and receiving antennas are to be placed at the top of equal-height towers, with one tower at the transmitting site and one at the receiving site. The distance between the transmitting and receiving sites is 25 miles. Calculate the minimum tower height required for an LOS transmission path.

1–5 A cellular telephone cell site has an antenna located at the top of a 100-ft tower. A typical cellular telephone user has his or her antenna located 4 ft above the ground. What is the LOS radius of coverage for this cell site to a distant user?

★ **1–6** A digital source emits -1.0-and 0.0-V levels with a probability of 0.2 each and $+3.0$- and $+4.0$-V levels with a probability of 0.3 each. Evaluate the average information of the source.

1–7 Prove that base 10 logarithms may be converted to base 2 logarithms by using the identity $\log_2(x) = [1/\log_{10}(2)]\log_{10}(x)$.

1–8 If all the messages emitted by a source are equally likely (i.e., $P_j = P$), show that Eq. (1–8) reduces to $H = \log_2(1/P)$.

★ **1–9** For a binary source:

(a) Show that the entropy H is a maximum when the probability of sending a binary 1 is equal to the probability of sending a binary 0.

(b) Find the value of maximum entropy.

★ **1–10** A single-digit, seven-segment liquid crystal display (LCD) emits a 0 with a probability of 0.25; a 1 and a 2 with a probability of 0.15 each; 3, 4, 5, 6, 7, and 8 with a probability of 0.07 each; and a 9 with a probability of 0.03. Find the average information for this source.

1–11 **(a)** A binary source sends a binary 1 with a probability of 0.3. Evaluate the average information for the source.

(b) For a binary source, find the probability for sending a binary 1 and a binary 0, such that the average source information will be maximized.

★ **1–12** A numerical keypad has the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Assume that the probability of sending any one digit is the same as that for sending any of the other digits. Calculate how often the buttons must be pressed in order to send out information at the rate of 3 bits/s.

1–13 Refer to Example 1–3 and assume that words, each 12 digits in length, are sent over a system and that each digit can take on one of two possible values. Half of the possible words have a probability of being transmitted that is $(\frac{1}{2})^{13}$ for each word. The other half have probabilities equal to $3(\frac{1}{2})^{13}$ Find the entropy for this source.

1–14 Evaluate the channel capacity for a teleprinter channel that has a 300-Hz bandwidth and an SNR of 30 dB.

★ **1–15** Assume that a computer terminal has 110 characters (on its keyboard) and that each character is sent by using binary words.

(a) What are the number of bits needed to represent each character?

(b) How fast can the characters be sent (characters/s) over a telephone line channel having a bandwidth of 3.2 kHz and an SNR of 20 dB?

(c) What is the information content of each character if each is equally likely to be sent?

1–16 An analog telephone line has an SNR of 45 dB and passes audio frequencies over the range of 300 to 3,200 Hz. A modem is to be designed to transmit and receive data simultaneously (i.e., full duplex) over this line without errors.

(a) If the frequency range 300 to 1,200 Hz is used for the transmitted signal, what is the maximum transmitted data rate?

(b) If the frequency range 1,500 to 3,200 Hz is used for the signal being simultaneously received, what is the maximum received data rate?

(c) If the whole frequency range of 300 to 3,200 Hz is used simultaneously for transmitting and receiving (by the use of a hybrid circuit as described in Chapter 8, Fig. 8–4), what are the maximum transmitting and receiving data rates?

1–17 Using the definitions for terms associated with convolutional coding, draw a block diagram for a convolutional coder that has rate $R = \frac{2}{3}$ and constraint length $K = 3$.

★ **1–18** For the convolutional encoder shown in Fig. P1–18, compute the output coded data when the input data is $x = [10111]$. (The first input bit is the leftmost element of the **x** row vector.)

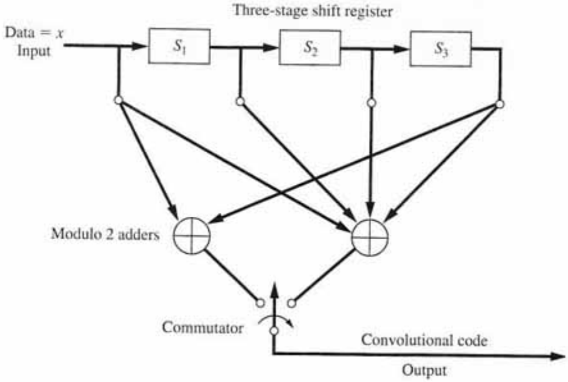

Figure P1–18

b. Given the RC low-pass filter transfer function

$$H(f) = \frac{1}{1 + j(f/f_0)}$$

where $f_0 = 1$ Hz, use the inverse fast Fourier transform (IFFT) of MATLAB to calculate the impulse response $h(t)$.

Solution. (a) From Eq. (2–30), the ICFT is

$$w(t) = \int_{-\infty}^{\infty} W(f)e^{j2\pi ft}\,df$$

Referring to the discussion leading up to Eq. (2–184), the ICFT is approximated by

$$w(k\Delta t) \approx \Sigma W(n\Delta f)e^{j2\pi n\Delta fk\Delta t}\Delta f$$

But $\Delta t = T/N, \Delta f = 1/T$, and $f_s = 1/\Delta t$, so

$$w(k\Delta t) \approx N\left[\frac{1}{N}\Sigma W(n\Delta f)e^{j(2\pi/N)nk}\right]\Delta f$$

Using the definition of the IDFT as given by Eq. (2–177), we find that the ICFT is related to the IDFT by

$$w(k\Delta t) \approx f_s x(k) \tag{2–207}$$

where $x(k)$ is the kth element of the N-element IDFT vector. As indicated in the discussion leading up to Eq. (2–184), the elements of the **X** vector are chosen so that the first $N/2$ elements are samples of the positive frequency components of $W(f)$, where $f = n\Delta f$, and the second $N/2$ elements are samples of the negative frequency components of $W(f)$.

(b) Run file *SA 2_9.m* for a plot of $h(t)$ as computed by using the IFFT and Eq. (2–207). Compare this IFFT-computed $h(t)$ with the analytical $h(t)$ that is shown in Fig. 2–15b, where $\tau_0 = RC = 1/(2\pi f_0)$.

PROBLEMS

★ **2–1** For a sinusoidal waveform with a peak value of A and a frequency of f_0, use the time average operator to show that the RMS value for this waveform is $A/\sqrt{2}$.

2–2 A function generator produces the periodic voltage waveform shown in Fig. P2–2.
(a) Find the value for the DC voltage.
(b) Find the value for the RMS voltage.
(c) If this voltage waveform is applied across a 100-Ω load, what is the power dissipated in the load?

2–3 The voltage across a load is given by $v(t) = A_0 \cos \omega_0 t$, and the current through the load is a square wave,

$$i(t) = I_0 \sum_{n=-\infty}^{\infty}\left[\Pi\left(\frac{t - nT_0}{T_0/2}\right) - \Pi\left(\frac{t - nT_0 - (T_0/2)}{T_0/2}\right)\right]$$

where $\omega_0 = 2\pi/T_0$, $T_0 = 1$ sec, $A_0 = 10$ V, and $I_0 = 5$ mA.

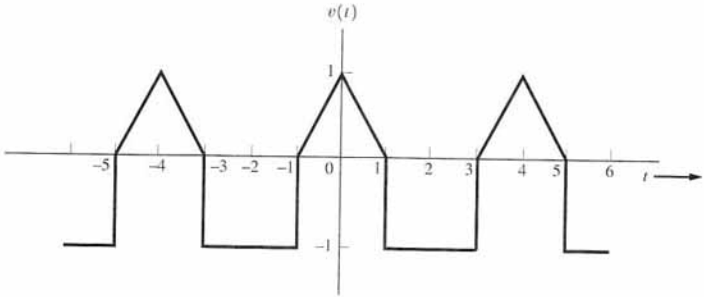

Figure P2–2

(a) Find the expression for the instantaneous power and sketch this result as a function of time.
(b) Find the value of the average power.

★ **2–4** The voltage across a 50-Ω resistive load is the positive portion of a cosine wave. That is,

$$v(t) = \begin{cases} 10 \cos \omega_0 t, & |t - nT_0| < T_0/4 \\ 0, & t \text{ elsewhere} \end{cases}$$

where n is any integer.
(a) Sketch the voltage and current waveforms.
(b) Evaluate the DC values for the voltage and current.
(c) Find the RMS values for the voltage and current.
(d) Find the total average power dissipated in the load.

2–5 For Prob. 2–4, find the energy dissipated in the load during a 1-hr interval.

2–6 Determine whether each of the following signals is an energy signal or a power signal, and evaluate the normalized energy or power, as appropriate:
(a) $w(t) = \Pi(t/T_0)$.
(b) $w(t) = \Pi(t/T_0) \cos \omega_0 t$.
(c) $w(t) = \cos^2 \omega_0 t$.

2–7 An average-reading power meter is connected to the output circuit of a transmitter. The transmitter output is fed into a 50-Ω resistive load, and the wattmeter reads 25 W.
(a) What is the power in dBm units?
(b) What is the power in dBk units?
(c) What is the value in dBmV units?

2–8 A cable company provides a 0 dBmV signal at 843 MHz via a 75-Ω coaxial cable to a cable modem which is located in a residence. This modem uses this downstream signal to produce the downloaded data signal for the Internet subscriber's computer via an ethernet connection. (The uplink signal that is sent by the modem is on 33.5 MHz.) What is the power of the 843 MHz signal that is supplied to the modem if the modem acts as a 75-Ω resistive load to this downstream signal?

2–9 Assume that a waveform with a known RMS value V_{rms} is applied across a 1,000-Ω load. Derive a formula that can be used to compute the dBm value from V_{rms}.

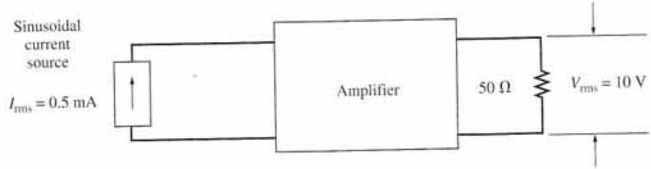

Figure P2–10

★ **2–10** An amplifier is connected to a 50-Ω load and driven by a sinusoidal current source, as shown in Fig. P2–10. The output resistance of the amplifier is 10 Ω and the input resistance is 2 kΩ. Evaluate the true decibel gain of this circuit.

★ **2–11** The voltage (RMS) across the 300-Ω antenna input terminals of an FM receiver is 3.5 μV.
(a) Find the input power (watts).
(b) Evaluate the input power as measured in decibels below 1 mW (dBm).
(c) What would be the input voltage (in microvolts) for the same input power if the input resistance were 75 Ω instead of 300 Ω?

2–12 A 103.5 MHz FM signal from an antenna is applied to a FM receiver via a coaxial cable. This signal has a power level of -100 dBm at this receiver's 75-Ω antenna input. What is the rms voltage of this FM signal at the antenna input?

2–13 What is the value for the phasor that corresponds to the voltage waveform $v(t) = 15 \sin(\omega_0 t - 30°)$, where $\omega_0 = 2000\pi$?

2–14 A signal is $w(t) = 6 \sin(100\pi t - 40°) + 4 \cos(100\pi t)$. Find the corresponding phasor.

★ **2–15** Evaluate the Fourier transform of

$$w(t) = \begin{cases} e^{-\alpha t}, & t \geq 1 \\ 0, & t < 1 \end{cases}$$

2–16 Find the spectrum for the waveform $w(t) = e^{-\pi(t/T)^2}$ in terms of the parameter T. What can we say about the width of $w(t)$ and $W(f)$ as T increases?

2–17 Using the convolution property, find the spectrum for

$$w(t) = \sin 2\pi f_1 t \cos 2\pi f_2 t$$

★ **2–18** Find the spectrum (Fourier transform) of the triangle waveform

$$s(t) = \begin{cases} At, & 0 < t < T_0 \\ 0, & t \text{ elsewhere} \end{cases}$$

in terms of A and T_0.

2–19 Find the spectrum for the waveform shown in Fig. P2–19.

2–20 If $w(t)$ has the Fourier transform

$$W(f) = \frac{j2\pi f}{1 + j2\pi f}$$

find $X(f)$ for the following waveforms:
(a) $x(t) = w(2t + 2)$.
(b) $x(t) = e^{-jt} w(t - 1)$.

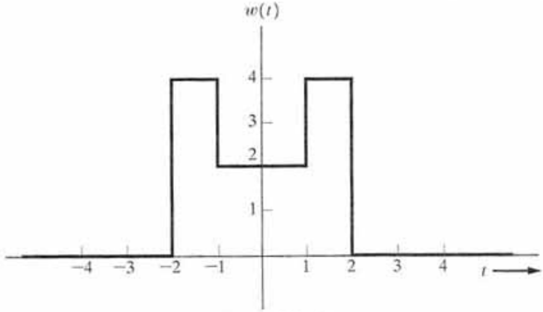

Figure P2–19

(c) $x(t) = 2\dfrac{dw(t)}{dt}$.

(d) $x(t) = w(1 - t)$.

2–21 From Eq. (2–30), find $w(t)$ for $W(f) = A\Pi(f/2B)$, and verify your answer by using the duality property.

2–22 Find the quadrature spectral functions $X(f)$ and $Y(f)$ for the damped sinusoidal waveform

$$w(t) = u(t)e^{-at}\sin\omega_0 t$$

where $u(t)$ is a unit step function, $a > 0$, and $W(f) = X(f) + jY(f)$.

2–23 Derive the spectrum of $w(t) = e^{-|t|/T}$.

★ **2–24** Find the Fourier transforms for the following waveforms. Plot the waveforms and their magnitude spectra. [*Hint:* Use Eq. (2–184).]

(a) $\Pi\left(\dfrac{t-3}{4}\right)$.

(b) 2.

(c) $\Lambda\left(\dfrac{t-5}{5}\right)$.

2–25 By using Eq. (2–184), find the approximate Fourier transform for the following waveform:

$$x(t) = \begin{cases} \sin(2\pi t/512) + \sin(70\pi t/512), & 5 < t < 75 \\ 0, & t \text{ elsewhere} \end{cases}$$

2–26 Evaluate the spectrum for the trapezoidal pulse shown in Fig. P2–26.

2–27 Show that

$$\mathcal{F}^{-1}\{\mathcal{F}[w(t)]\} = w(t)$$

[*Hint:* Use Eq. (2–33).]

2–28 Using the definition of the inverse Fourier transform, show that the value of $w(t)$ at $t = 0$ is equal to the area under $W(f)$. That is, show that

$$w(0) = \int_{-\infty}^{\infty} W(f)\,df$$

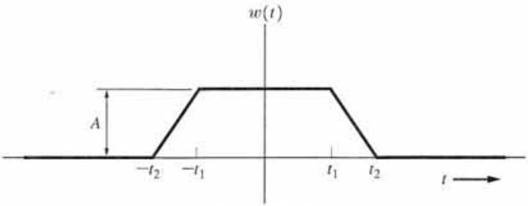

Figure P2–26

2–29 Prove that

(a) If $w(t)$ is real and an even function of t, $W(f)$ is real.

(b) If $w(t)$ is real and an odd function of t, $W(f)$ is imaginary.

2–30 Suppose that the spectrum of a waveform as a function of frequency in hertz is

$$W(f) = \frac{1}{2}\delta(f - 10) + \frac{1}{2}\delta(f + 10) + \frac{j\pi f}{2 + j2\pi f}e^{j\pi f}$$

Find the corresponding spectrum as a function of radian frequency, $W(\omega)$.

2–31 The unit impulse can also be defined as

$$\delta(t) = \lim_{\omega \to \infty}\left[Ka\left(\frac{\sin at}{at}\right)\right]$$

Find the value of K needed, and show that this definition is consistent with those given in the text. Give another example of an ordinary function such that, in the limit of some parameter, the function becomes a Dirac delta function.

★ **2–32** Use $v(t) = ae^{-at}, a > 0$, to approximate $\delta(t)$ as $a \to \infty$.

(a) Plot $v(t)$ for $a = 0.1$, 1, and 10.

(b) Plot $V(f)$ for $a = 0.1$, 1, and 10.

2–33 Show that

$$\operatorname{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$$

[*Hint:* Use Eq. (2–30) and $\int_0^\infty (\sin x)/x\,dx = \pi/2$ from Appendix A.]

2–34 Show that

$$u(t) \leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

[*Hint:* Use the linearity (superposition) theorem and the result of Prob. 2–33.]

2–35 Show that the sifting property of δ functions, Eq. (2–47), may be generalized to evaluate integrals that involve derivatives of the delta function. That is, show that

$$\int_{-\infty}^{\infty} w(x)\delta^{(n)}(x - x_0)df = (-1)^n w^{(n)}(x_0)$$

where the superscript (n) denotes the nth derivative. (*Hint:* Use integration by parts.)

2–36 Let $x(t) = \Pi\left(\frac{t - 0.05}{0.1}\right)$. Plot the spectrum of $x(t)$ using MATLAB with the help of Eqs. (2–59) and (2–60). Check your results by using the FFT and Eq. (2–184).

★ **2–37** If $w(t) = w_1(t)w_2(t)$, show that

$$W(f) = \int_{-\infty}^{\infty} W_1(\lambda)W_2(f - \lambda)\,d\lambda$$

where $W(f) = \mathcal{F}[w(t)]$.

2–38 Show that

(a) $\int_{-\infty}^{t} w(\lambda)d\lambda = w(t) * u(t)$.

(b) $\int_{-\infty}^{t} w(\lambda)d\lambda \leftrightarrow (j2\pi f)^{-1}W(f) + \frac{1}{2}W(0)\delta(f)$.

(c) $w(t) * \delta(t - a) = w(t - a)$.

2–39 Show that

$$\frac{dw(t)}{dt} \leftrightarrow (j2\pi f)W(f)$$

[*Hint:* Use Eq. (2–26) and integrate by parts. Assume that $w(t)$ is absolutely integrable.]

2–40 As discussed in Example 2–9, show that

$$\frac{1}{T}\Pi\left(\frac{t}{T}\right) * \Pi\left(\frac{t}{T}\right) = \Lambda\left(\frac{t}{T}\right)$$

2–41 Given the waveform $w(t) = A\Pi(t/T)\sin\omega_0 t$, find the spectrum of $w(t)$ by using the multiplication theorem as discussed in Example 2–10.

★ **2–42** Evaluate the following integrals.

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin 4\lambda}{4\lambda}\,\delta(t - \lambda)\,d\lambda$.

(b) $\displaystyle\int_{-\infty}^{\infty} (\lambda^3 - 1)\,\delta(2 - \lambda)\,d\lambda$.

2–43 Prove that

$$M(f) * \delta(f - f_0) = M(f - f_0)$$

2–44 Evaluate $y(t) = w_1(t) * w_2(t)$, where

$$w_1(t) = \begin{cases} 1, & |t| < T_0 \\ 0, & t \text{ elsewhere} \end{cases}$$

and

$$w_2(t) = \begin{cases} [1 - 2|t|], & |t| < \frac{1}{2}T_0 \\ 0, & t \text{ elsewhere} \end{cases}$$

2–45 Given $w(t) = 5 + 12\cos\omega_0 t$, where $f_0 = 10$ Hz, find

(a) $R_w(\tau)$.

(b) $P_w(f)$.

2–46 Given the waveform

$$w(t) = A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2)$$

where $A_1, A_2, \omega_1, \omega_2, \theta_1$, and θ_2 are constants,

Find the autocorrelation for $w(t)$ as a function of the constants.

2–47 Referring to Prob. 2–46,
(a) Find the PSD function for $w(t)$.
(b) Sketch the PSD for the case of $\omega_1 \neq \omega_2$.
(c) Sketch the PSD for the case of $\omega_1 = \omega_2$ and $\theta_1 = \theta_2 + 90°$.
(d) Sketch the PSD for the case of $\omega_1 = \omega_2$ and $\theta_1 = \theta_2$.

★ **2–48** Given the periodic voltage waveform shown in Fig. P2–48,
(a) Find the DC value for this waveform.
(b) Find the RMS value for this waveform.
(c) Find the complex exponential Fourier series.
(d) Find the voltage spectrum for this waveform.

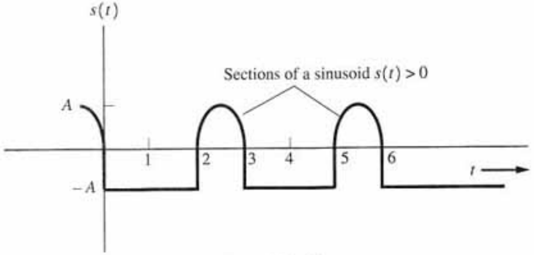

Figure P2–48

2–49 Determine if $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(-\frac{5}{2}T_2 < t < \frac{5}{2}T_2)$, where $s_1(t) = A_1 \cos(\omega_1 t + \varphi_1), s_2(t) = A_2 \cos(\omega_2 t + \varphi_2)$, and $\omega_2 = 2\pi/T_2$ for the following cases.

(a) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2$.
(b) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2 + \pi/2$.
(c) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2 + \pi$.
(d) $\omega_1 = 2\omega_2$ and $\varphi_1 = \varphi_2$.
(e) $\omega_1 = \frac{4}{5}\omega_2$ and $\varphi_1 = \varphi_2$.
(f) $\omega_1 = \pi\omega_2$ and $\varphi_1 = \varphi_2$.

★ **2–50** Let $s(t) = A_1 \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2)$. Determine the RMS value of $s(t)$ in terms of A_1 and A_2 for the following cases.
(a) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2$.
(b) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2 + \pi/2$.
(c) $\omega_1 = \omega_2$ and $\varphi_1 = \varphi_2 + \pi$.
(d) $\omega_1 = 2\omega_2$ and $\varphi_1 = \varphi_2 = 0$.
(e) $\omega_1 = 2\omega_2$ and $\varphi_1 = \varphi_2 + \pi$.

2–51 Show that

$$\sum_{k=-\infty}^{\infty} \delta(t - kT_0) \leftrightarrow f_0 \sum_{k=-\infty}^{\infty} \delta(f - nf_0)$$

where $f_0 = 1/T_0$. [*Hint:* Expand $\sum_{k=-\infty}^{\infty} \delta(t - kT_0)$ into a Fourier series and then take the Fourier transform.]

★ **2–52** Three functions are shown in Fig. P2–52.

(a) Show that these functions are orthogonal over the interval $(-4, 4)$.

(b) Find the corresponding orthonormal set of functions.

★ **2–53** Expand the waveform

$$w_1(t) = \begin{cases} 1, & 0 \le t \le 4 \\ 0, & t \text{ elsewhere} \end{cases}$$

into an orthonormal series by using the orthonormal set found in Prob. 2–52 (b).

★ **2–54** Evaluate the mean square error for the orthogonal series obtained in Prob. 2–53 by evaluating

$$\varepsilon = \int_{-4}^{4} \left[w(t) - \sum_{j=1}^{3} a_j \varphi_j(t) \right]^2 dt$$

★ **2–55** Expand the waveform

$$w_1(t) = \begin{cases} \cos(\frac{1}{4}\pi t), & -4 \le t \le 4 \\ 0, & t \text{ elsewhere} \end{cases}$$

into an orthonormal set of functions found in Prob. 2–52 (b).

★ **2–56** Evaluate the mean square error for the orthogonal series obtained in Prob. 2–55 by using the integral shown in Prob. 2–54.

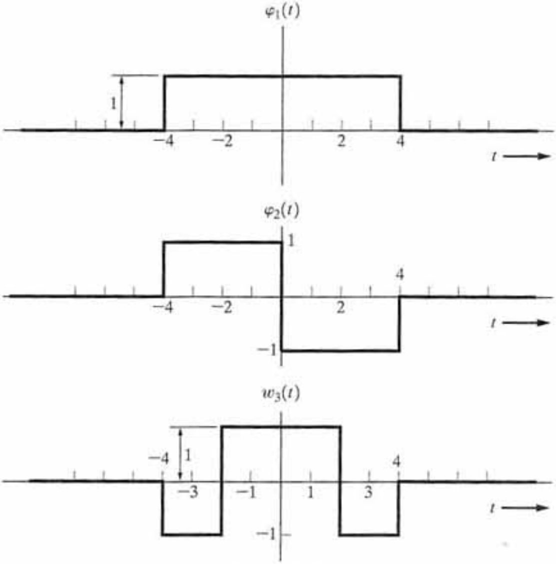

Figure P2–52

2–57 Show that the quadrature Fourier series basis functions $\cos(n\omega_0 t)$ and $\sin(n\omega_0 t)$, as given in Eq. (2–95), are orthogonal over the interval $a < t < a + T_0$, where $\omega_0 = 2\pi/T_0$.

2–58 Find expressions for the complex Fourier series coefficients that represent the waveform shown in Fig. P2–58.

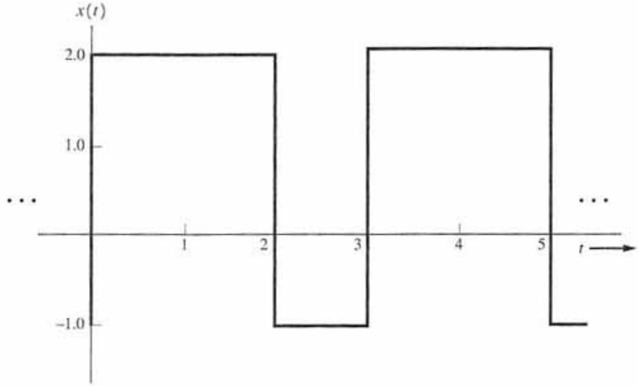

Figure P2–58

2–59 The periodic signal shown in Fig. P2–58 is passed through a linear filter having the impulse response $h(t) = e^{-\alpha t}u(t)$, where $t > 0$ and $\alpha > 0$.

(a) Find expressions for the complex Fourier series coefficients associated with the output waveform $y(t) = x(t) * h(t)$.

(b) Find an expression for the normalized power of the output, $y(t)$.

2–60 Find the complex Fourier series for the periodic waveform given in Figure P2–2.

★ **2–61** Find the complex Fourier series coefficients for the periodic rectangular waveform shown in Fig. P2–61 as a function of A, T, b, and τ_0. [*Hint:* The answer can be reduced to a $(\sin x)/x$ form multiplied by a phase-shift factor, $e^{j\theta_n(\tau_0)}$.]

2–62 For the waveform shown in Fig. P2–62, find the complex Fourier series.

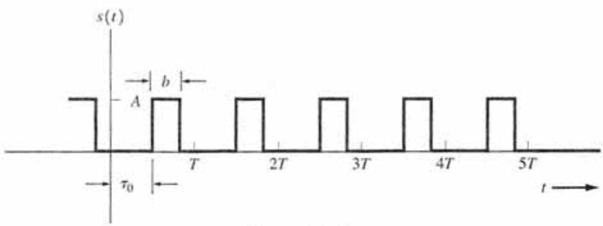

Figure P2–61

2–63 For the waveform shown in Fig. P2–62, find the quadrature Fourier series.

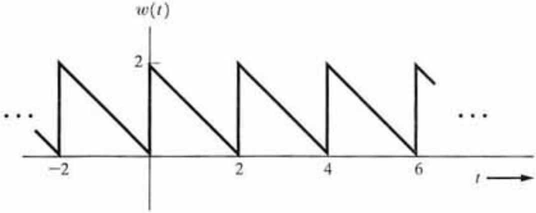

Figure P2–62

★ **2–64** Given a periodic waveform $s(t) = \sum_{n=-\infty}^{\infty} p(t - nT_0)$, where

$$p_t(t) = \begin{cases} At, & 0 < t < T \\ 0, & t \text{ elsewhere} \end{cases}$$

and $T \leq T_0$.

(a) Find the c_n Fourier series coefficients.

(b) Find the $\{x_n, y_n\}$ Fourier series coefficients.

(c) Find the $\{D_n, \varphi_n\}$ Fourier series coefficients.

2–65 Prove that the polar form of the Fourier series, Eq. (2–103), can be obtained by rearranging the terms in the complex Fourier series, Eq. (2–88).

2–66 Prove that Eq. (2–93) is correct.

2–67 Let two complex numbers c_1 and c_2 be represented by $c_1 = x_1 + jy_1$ and $c_2 = x_2 + jy_2$, where x_1, x_2, y_1, and y_2 are real numbers. Show that $\text{Re}\{\cdot\}$ is a linear operator by demonstrating that

$$\text{Re}\{c_1 + c_2\} = \text{Re}\{c_1\} + \text{Re}\{c_2\}$$

2–68 Assume that $y(t) = s_1(t) + 2s_2(t)$, where $s_1(t)$ is given by Fig. P2–48 and $s_2(t)$ is given by Fig. P2–61. Let $T = 3, b = 1.5$, and $\tau_0 = 0$. Find the complex Fourier coefficients $\{c_n\}$ for $y(t)$.

2–69 Evaluate the PSD for the waveform shown in Fig. P2–2.

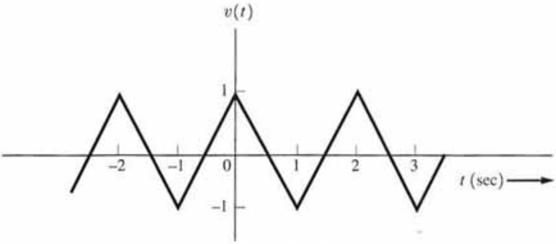

Figure P2–70

★ **2–70** Assume that $v(t)$ is a triangular waveform, as shown in Fig. P2–70.

(a) Find the complex Fourier series for $v(t)$.

(b) Calculate the normalized average power.

(c) Calculate and plot the voltage spectrum.

(d) Calculate and plot the PSD.

2–71 Let any complex number be represented by $c = x + jy$, where x and y are real numbers. If c^* denotes the complex conjugate of c, show that

(a) $\text{Re}\{c\} = \frac{1}{2}c + \frac{1}{2}c^*$.

(b) $\text{Im}\{c\} = \dfrac{1}{2j}c - \dfrac{1}{2j}c^*$.

Note that when $c = e^{jz}$, part (a) gives the definition of $\cos z$ and part (b) gives the definition of $\sin z$.

★ **2–72** Calculate and plot the PSD for the half-wave rectified sinusoid described in Prob. 2–4.

2–73 The basic definitions for sine and cosine waves are

$$\sin z_1 \triangleq \frac{e^{jz_1} - e^{-jz_1}}{2j} \quad \text{and} \quad \cos z_2 \triangleq \frac{e^{jz_2} + e^{-jz_2}}{2}.$$

Show that

(a) $\cos z_1 \cos z_2 = \frac{1}{2}\cos(z_1 - z_2) + \frac{1}{2}\cos(z_1 + z_2)$.

(b) $\sin z_1 \sin z_2 = \frac{1}{2}\cos(z_1 - z_2) - \frac{1}{2}\cos(z_1 + z_2)$.

(c) $\sin z_1 \cos z_2 = \frac{1}{2}\sin(z_1 - z_2) + \frac{1}{2}\sin(z_1 + z_2)$.

2–74 Let two complex numbers be respectively represented by $c_1 = x_1 + jy_1$ and $c_2 = x_2 + jy_2$, where x_1, x_2, y_1, and y_2 are real numbers. Show that

$$\text{Re}\{c_1\}\,\text{Re}\{c_2\} = \tfrac{1}{2}\text{Re}\{c_1 c_2^*\} + \tfrac{1}{2}\text{Re}\{c_1 c_2\}$$

Note that this is a generalization of the $\cos z_1 \cos z_2$ identity of Prob. 2–73, where, for the $\cos z_1 \cos z_2$ identity, $c_1 = e^{jz_1}$ and $c_2 = e^{jz_2}$.

2–75 Prove that the Fourier transform is a linear operator. That is, show that

$$\mathcal{F}[ax(t) + by(t)] = a\mathcal{F}[x(t)] + b\mathcal{F}[y(t)]$$

2–76 Plot the amplitude and phase response for the transfer function

$$H(f) = \frac{j10f}{5 + jf}$$

2–77 Given the filter shown in Fig. P2–77, where $w_1(t)$ and $w_2(t)$ are voltage waveforms.

(a) Find the transfer function.

(b) Plot the magnitude and phase response.

(c) Find the power transfer function.

(d) Plot the power transfer function.

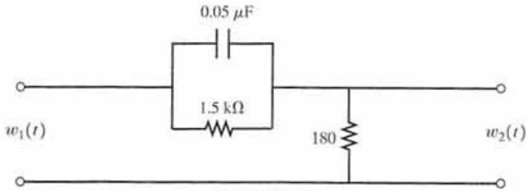

Figure P2–77

2–78 A signal with a PSD of

$$\mathcal{P}_x(f) = \frac{2}{(1/4\pi)^2 + f^2}$$

is applied to the network shown in Fig. P2–78.

(a) Find the PSD for $y(t)$.

(b) Find the average normalized power for $y(t)$.

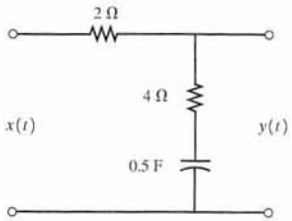

Figure P2–78

2–79 A signal $x(t)$ has a PSD

$$\mathcal{P}_x(f) = \frac{K}{[1 + (2\pi f/B)^2]^2}$$

where $K > 0$ and $B > 0$.

(a) Find the 3-dB bandwidth in terms of B.

(b) Find the equivalent noise bandwidth in terms of B.

★ **2–80** The signal $x(t) = e^{-400\pi t}u(t)$ is applied to a brick-wall low-pass filter whose transfer function is

$$H(f) = \begin{cases} 1, & |f| \le B \\ 0, & |f| \le B \end{cases}$$

Find the value of B such that the filter passes one-half the energy of $x(t)$.

2–81 Show that the average normalized power of a waveform can be found by evaluating the autocorrelation $R_w(\tau)$ at $\tau = 0$. That is, $P = R_w(0)$.

[*Hint:* See Eqs. (2–69) and (2–70).]

2–82 The signal $x(t) = 0.5 + 1.5\cos\left[\left(\frac{2}{3}\right)\pi t\right] + 0.5\sin\left[\left(\frac{2}{3}\right)\pi t\right]$ V is passed through an *RC* low-pass filter (see Fig. 2–15a) where $R = 1\,\Omega$ and $C = 1\,\text{F}$.

(a) What is the input PSD, $\mathcal{P}_x(f)$?

(b) What is the output PSD, $\mathcal{P}_y(f)$?

(c) What is the average normalized output power, P_y?

2–83 The input to the RC low-pass filter shown in Fig. 2–15a is

$$x(t) = 1.0 + 2.0\cos\omega_x t + 0.5\sin\omega_x t$$

Assume that the cutoff frequency is $f_0 = 1.5f_x$.

(a) Find the input PSD, $\mathcal{P}_x(f)$.

(b) Find the output PSD, $\mathcal{P}_y(f)$

(c) Find the normalized average power of the output, $y(t)$.

★ **2–84** Using MATLAB, plot the frequency magnitude response and phase response for the low-pass filter shown in Fig. P2–84, where $R_1 = 7.5\,\text{k}\Omega$, $R_2 = 15\,\text{k}\Omega$, and $C = 0.1\,\mu\text{F}$.

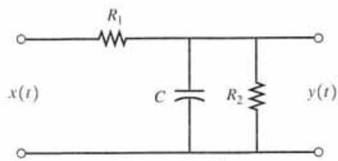

Figure P2–84

2–85 A comb filter is shown in Fig. P2–85. Let $T_d = 0.1$.

(a) Plot the magnitude of the transfer function for this filter.

(b) If the input is $x(t) = \Pi(t/T)$, where $T = 1$, plot the spectrum of the output $|Y(f)|$.

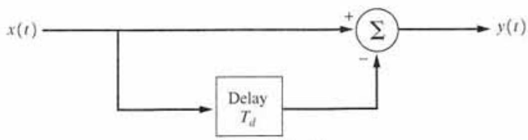

Figure P2–85

2–86 A signal $x(t) = \Pi(t - 0.5)$ passes through a filter that has the transfer function $H(f) = \Pi(f/B)$. Plot the output waveform when

(a) $B = 0.6\,\text{Hz}$.

(b) $B = 1\,\text{Hz}$.

(c) $B = 50\,\text{Hz}$.

★ **2–87** Examine the distortion effect of an *RC* low-pass filter. Assume that a unity-amplitude periodic square wave with a 50% duty cycle is present at the filter input and that the filter has a 3-dB bandwidth of 1,500 Hz. Using a computer, find and plot the output waveshape if the square wave has a frequency of

(a) 300 Hz.

(b) 500 Hz.

(c) 1,000 Hz.

(*Hint:* Represent the square wave with a Fourier series.)

2–88 Given that the PSD of a signal is flat [i.e., $P_s(f) = 1$], design an *RC* low-pass filter that will attenuate this signal by 20 dB at 15 kHz. That is, find the value for the *RC* of Fig. 2–15 such that the design specifications are satisfied.

2–89 The bandwidth of $g(t) = e^{-0.1t}$ is approximately 0.5 Hz; thus, the signal can be sampled with a sampling frequency of $f_s = 1$ Hz without significant aliasing. Take samples a_n over the time interval (0, 14). Use the sampling theorem, Eq. (2–158), to reconstruct the signal. Plot and compare the reconstructed signal with the original signal. Do they match? What happens when the sampling frequency is reduced?

★ **2–90** A waveform, $20 + 20\sin(500t + 30°)$, is to be sampled periodically and reproduced from these sample values.

(a) Find the maximum allowable time interval between sample values.

(b) How many sample values need to be stored in order to reproduce 1 sec of this waveform?

2–91 Using a computer program, calculate the DFT of a rectangular pulse, $\Pi(t)$. Take five samples of the pulse and pad it with 59 zeros so that a 64-point FFT algorithm can be used. Sketch the resulting magnitude spectrum. Compare this result with the actual spectrum for the pulse. Try other combinations of the number of pulse samples and zero-pads to see how the resulting FFT changes.

★ **2–92** Using the DFT, compute and plot the spectrum of $\Lambda(t)$. Check your results against those given in Fig. 2–6c.

2–93 Using the DFT, compute and plot $|W(f)|$ for the pulse shown in Fig. P2–26, where $A = 1, t_1 = 1\text{s}$, and $t_2 = 2\text{s}$.

2–94 Let a certain waveform be given by

$$w(t) = 4\sin(2\pi f_1 t + 30°) + 2\cos(2\pi f_2 t - 10°),$$

where $f_1 = 10\,\text{Hz}$ and $f_2 = 25\,\text{Hz}$,

(a) Using the DFT, compute and plot $|W(f)|$ and $\theta(f)$

(b) Let $\mathcal{P}_w(f)$ denote the PSD of $w(t)$. Using the DFT, compute and plot $\mathcal{P}_w(f)$.

(c) Check your computed results obtained in parts (a) and (b) against known correct results that you have evaluated by analytical methods.

2–95 Using the DFT, compute and plot $|S(f)|$ for the periodic signal shown in Fig. P2–48, where $A = 5$.

2–96 The transfer function of a raised cosine-rolloff filter is

$$H(f) = \begin{cases} 0.5[1 + \cos(0.5\pi f/f_0)], & |f| \le 2f_0 \\ 0, & f \text{ elsewhere} \end{cases}$$

Let $f_0 = 1\,\text{Hz}$. Using the IFFT, compute the impulse response $h(t)$ for this filter. Compare your computed results with those shown in Fig. 3–26b for the case of $r = 1$.

2–97 Given the low-pass filter shown in Fig. 2–15,

(a) Find the equivalent bandwidth in terms of R and C.

(b) Find the first zero-crossing (null) bandwidth of the filter.

(c) Find the absolute bandwidth of the filter.

2–98 Assume that the PSD of a signal is given by

$$\mathcal{P}_s(f) = \left[\frac{\sin(\pi f/B_n)}{\pi f/B_n} \right]^2$$

where B_n is the null bandwidth. Find the expression for the equivalent bandwidth in terms of the null bandwidth.

2–99 Table 2–4 gives the bandwidths of a BPSK signal in terms of six different definitions. Using these definitions and Eq. (2–201), show that the results given in the table are correct.

★ **2–100** Given a triangular pulse signal

$$s(t) = \Lambda(t/T_0)$$

(a) Find the absolute bandwidth of the signal.

(b) Find the 3-dB bandwidth in terms of T_0.

(c) Find the equivalent bandwidth in terms of T_0.

(d) Find the zero-crossing bandwidth in terms of T_0.

peak (at $f = 96{,}000$ Hz $= 2.5R$) is down by only 17.9 dB. The power spectrum is falling off as $1/f^2$, which is only 6 dB per octave. Referring to Fig. 3–48 and knowing that the envelope of the PSD is described by $(1/\pi f T_b)^2$, we find that a bandwidth of $f = 386$ kHz $= 10.1R$ (i.e., 10 times the data rate) is needed to pass the frequency components that are not attenuated by more than 30 dB. Thus, the spectrum is broad when rectangular sampling pulses are used. (This was first illustrated in Fig. 2–24.)

Consequently, for applications of digital signaling over bandlimited channels, some sort of filtered pulse shape is required to provide good out-of-band spectral attenuation and yet not introduce ISI. For example, from Eq. (3–74), a filtered pulse corresponding to an $r = 0.5$ raised cosine characteristic would have infinite attenuation at the frequency (absolute bandwidth) of $B = \frac{1}{2}(1 + r)D = (0.5)(1.5)(38{,}400) = 28{,}800$ Hz $= 0.75R$. Referring to Fig. 3–26a and using Eq. (3–69), we see that the $r = 0.5$ raised cosine spectrum would be down by 30 dB at $f = 20{,}070$ Hz $= 0.523R$ and 100 dB at $f = 22{,}217$ Hz $= 0.579R$. For the 30-dB-down bandwidth criterion, this is a bandwidth (savings) of 19 times smaller than that required for rectangular pulse signaling.

PROBLEMS

3–1 Demonstrate that the Fourier series coefficients for the switching waveform shown in Fig. 3–1b are given by Eq. (3–5b).

3–2 **(a)** Sketch the naturally sampled PAM waveform that results from sampling a 1-kHz sine wave at a 4-kHz rate.

(b) Repeat part (a) for the case of a flat-topped PAM waveform.

★ **3–3** The spectrum of an analog audio signal is shown in Fig. P3–3. The waveform is to be sampled at a 10-kHz rate with a pulse width of $\tau = 50$ μs.

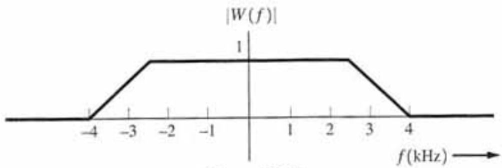

Figure P3–3

Find an expression for the spectrum of the naturally sampled PAM waveform. Sketch your result.

★ **3–4** Referring to Prob. 3–3, find an expression for the spectrum of the flat-topped PAM waveform. Sketch your result.

3–5 **(a)** Show that an analog output waveform (which is proportional to the original input analog waveform) may be recovered from a naturally sampled PAM waveform by using the demodulation technique shown in Fig. 3–4.

(b) Find the constant of proportionality, C, that is obtained with this demodulation technique, where $w(t)$ is the original waveform and $Cw(t)$ is the recovered waveform. Note that C is a function of n, where the oscillator frequency is nf_s.

3–6 Figure 3–4 illustrates how a naturally sampled PAM signal can be demodulated to recover the analog waveform by the use of a product detector. Show that the product detector can also be used to recover $w(t)$ from an instantaneously sampled PAM signal, provided that the appropriate filter $H(f)$ is used. Find the required $H(f)$ characteristic.

★ **3–7** Assume that an analog signal with the spectrum shown in Fig. P3–3 is to be transmitted over a PAM system that has AC coupling. Thus, a PAM pulse shape of the Manchester type, as given by Eq. (3–46a), is used. The PAM sampling frequency is 10 kHz. Find the spectrum for the Manchester encoded flat-topped PAM waveform. Sketch your result.

3–8 In a binary PCM system, if the quantizing noise is not to exceed $\pm P$ percent of the peak-to-peak analog level, show that the number of bits in each PCM word needs to be

$$n \geq [\log_2 10]\left[\log_{10}\left(\frac{50}{P}\right)\right] = 3.32 \log_{10}\left(\frac{50}{P}\right)$$

(*Hint:* Look at Fig. 3–8c.)

★ **3–9** The information in an analog voltage waveform is to be transmitted over a PCM system with a $\pm 0.1\%$ accuracy (full scale). The analog waveform has an absolute bandwidth of 100 Hz and an amplitude range of -10 to $+10$ V.

(a) Determine the minimum sampling rate needed.
(b) Determine the number of bits needed in each PCM word.
(c) Determine the minimum bit rate required in the PCM signal.
(d) Determine the minimum absolute channel bandwidth required for transmission of this PCM signal.

3–10 A 700-Mbyte CD is used to store PCM data. Suppose that a voice-frequency (VF) signal is sampled at 8 ksamples/s and the encoded PCM is to have an average SNR of at least 30 dB. How many minutes of VF conversation (i.e., PCM data) can be stored on the hard disk?

3–11 An analog signal with a bandwidth of 4.2 MHz is to be converted into binary PCM and transmitted over a channel. The peak-signal quantizing noise ratio at the receiver output must be at least 55 dB.

(a) If we assume that $P_e = 0$ and that there is no ISI, what will be the word length and the number of quantizing steps needed?
(b) What will be the equivalent bit rate?
(c) What will be the channel null bandwidth required if rectangular pulse shapes are used?

★ **3–12** Compact disk (CD) players use 16-bit PCM, including one parity bit with 8 times oversampling of the analog signal. The analog signal bandwidth is 20 kHz.

(a) What is the null bandwidth of this PCM signal?
(b) Using Eq. (3–18), find the peak (SNR) in decibels.

3–13 An audio CD is created by sampling analog stereo audio signals at a rate of 44.1 ksamples/s for each. Each sample is then encoded into a 16-bit word. What is the data rate of the resulting composite digital?

3–14 Given an audio signal with spectral components in the frequency band 300 to 3,000 Hz, assume that a sampling rate of 7 kHz will be used to generate a PCM signal. Design an appropriate PCM system as follows:

(a) Draw a block diagram of the PCM system, including the transmitter, channel, and receiver.
(b) Specify the number of uniform quantization steps needed and the channel null bandwidth required, assuming that the peak signal-to-noise ratio at the receiver output needs to be at least 30 dB and that polar NRZ signaling is used.
(c) Discuss how nonuniform quantization can be used to improve the performance of the system.

3–15 The SNRs, as given by Eqs. (3–17a) and (3–17b), assume no ISI and no bit errors due to channel noise (i.e., $P_e = 0$). How large can P_e become before Eqs. (3–17a) and (3–17b) are in error by 0.1% if $M = 4$, 8, or 16.

★ **3–16** In a PCM system, the bit error rate due to channel noise is 10^{-4}. Assume that the peak signal-to-noise ratio on the recovered analog signal needs to be at least 30 dB.

(a) Find the minimum number of quantizing steps that can be used to encode the analog signal into a PCM signal.

(b) If the original analog signal had an absolute bandwidth of 2.7 kHz, what is the null bandwidth of the PCM signal for the polar NRZ signaling case?

3–17 Referring to Fig. 3–20 for a bit synchronizer using a square-law device, draw some typical waveforms that will appear in the bit synchronizer if a Manchester encoded PCM signal is present at the input. Discuss whether you would expect this bit synchronizer to work better for the Manchester encoded PCM signal or for a polar NRZ encoded PCM signal.

3–18 **(a)** Sketch the complete $\mu = 10$ compressor characteristic that will handle input voltages over the range -5 to $+5$ V.

(b) Plot the corresponding expandor characteristic.

(c) Draw a 16-level nonuniform quantizer characteristic that corresponds to the $\mu = 10$ compression characteristic.

3–19 For a 4-bit PCM system, calculate and sketch a plot of the output SNR (in decibels) as a function of the relative input level, $20 \log (x_{\text{rms}}/V)$ for

(a) A PCM system that uses $\mu = 10$ law companding.

(b) A PCM system that uses uniform quantization (no companding).

(c) Which of these systems is better to use in practice? Why?

★ **3–20** The performance of a $\mu = 255$ law companded PCM system is to be examined when the input consists of a sine wave having a peak value of V volts. Assume that $M = 256$.

(a) Find an expression that describes the output SNR for this companded PCM system.

(b) Plot $(S/N)_{\text{out}}$ (in decibels) as a function of the relative input level, $20 \log (x_{\text{rms}}/V)$. Compare this result with that shown in Fig. 3–10.

3–21 A multilevel digital communication system sends one of 16 possible levels over the channel every 0.8 ms.

(a) What is the number of bits corresponding to each level?

(b) What is the baud rate?

(c) What is the bit rate?

3–22 In the United States, HDTV stations transmit on-the-air eight-level RF (radio frequency) signals (see Sec. 8–9). If the baud (symbol) rate for these signals is 10.76 Msymbols/s, calculate the effective bit rate for these transmitted signals.

3–23 A multilevel digital communication system is to operate at a data rate of 1.5 Mb/s.

(a) If 4-bit words are encoded into each level for transmission over the channel, what is the minimum required bandwidth for the channel?

(b) Repeat part (a) for the case of 8-bit encoding into each level.

★ **3–24** Consider a deterministic test pattern consisting of alternating binary 1's and binary 0's. Determine the magnitude spectra (not the PSD) for unipolar NRZ signaling as a function of T_b, the time needed to send one bit of data. How would the magnitude spectra change if the test pattern was changed to an alternating sequence of four 1's followed by four binary 0's?

★ **3–25** Rework Prob. 3–24 for the case of unipolar RZ signaling where the pulse width is $\tau = \frac{3}{4} T_b$.

3–26 Consider a random data pattern consisting of binary 1's and 0's, where the probability of obtaining either a binary 1 or a binary 0 is $\frac{1}{2}$. Calculate the PSD for unipolar NRZ signaling as a function of T_b, the time needed to send one bit of data.

How does the PSD for this random data case compare with the magnitude spectrum for the deterministic case of Prob. 3–24? What is the spectral efficiency?

3–27 Rework Prob. 3–26 for the case of unipolar RZ signaling where the pulse width is $\tau = \frac{3}{4} T_b$. Compare this magnitude spectrum with PSD of Prob. 3–25. What is the spectral efficiency?

3–28 Consider a deterministic data pattern consisting of alternating binary 1's and 0's. Determine the magnitude spectra (not the PSD) for the following types of signaling formats as a function of T_b, the time needed to send one bit of data:

(a) Polar NRZ signaling.

(b) Manchester NRZ signaling.

How would each of these magnitude spectra change if the test pattern was changed to an alternating sequence of four binary l's followed by two binary 0's?

★ **3–29** Consider a random data pattern consisting of binary 1's and 0's, where the probability of obtaining either a binary 1 or a binary 0 is $\frac{1}{2}$. Calculate the PSD for the following types of signaling formats as a function of T_b, the time needed to send 1 bit of data:

(a) Polar RZ signaling where the pulse width is $\tau = \frac{1}{2} T_b$.

(b) Manchester RZ signaling where the pulse width is $\tau = \frac{1}{4} T_b$. What is the first null bandwidth of these signals? What is the spectral efficiency for each of these signaling cases?

3–30 Obtain the equations for the PSD of the bipolar NRZ and bipolar RZ (pulse width $\frac{1}{2}$ Tb) line codes assuming peak values of ± 3 V. Plot these PSD results for the case of R = 1.544 Mbits/s.

★ **3–31** In Fig. 3–16, the PSDs for several line codes are shown. These PSDs were derived assuming unity power for each signal so that the PSDs could be compared on an equal transmission power basis. Rederive the PSDs for these line codes, assuming that the peak level is unity (i.e., $A = 1$). Plot the PSDs so that the spectra can be compared on an equal peak-signal-level basis.

3–32 Using Eq. (3–36), determine the conditions required so that there are delta functions in the PSD for line codes. Discuss how this affects the design of bit synchronizers for these line codes. [*Hint:* Examine Eq. (3–43) and (6–70d).]

3–33 Consider a random data pattern consisting of binary 1's and 0's, where the probability of obtaining either a binary 1 or a binary 0 is $\frac{1}{2}$. Assume that these data are encoded into a polar-type waveform such that the pulse shape of each bit is given by

$$f(t) = \begin{cases} \cos\left(\dfrac{\pi t}{T_b}\right), & |t| < T_b/2 \\ 0, & |t| \text{ elsewhere} \end{cases}$$

where T_b is the time needed to send one bit.

(a) Sketch a typical example of this waveform.

(b) Find the expression for the PSD of this waveform and sketch it.

(c) What is the spectral efficiency of this type of binary signal?

3–34 The data stream 01101000101 appears at the input of a differential encoder. Depending on the initial start-up condition of the encoder, find two possible differentially encoded data streams that can appear at the output.

3–35 Create a practical block diagram for a differential encoding and decoding system. Explain how the system works by showing the encoding and decoding for the sequence 001111010001. Assume that the reference digit is a binary 1. Show that error propagation cannot occur.

3–36 Design a regenerative repeater with its associated bit synchronizer for a polar RZ line code. Explain how your design works. (*Hint:* See Fig. 3–19 and the discussion of bit synchronizers.)

3–37 Design a bit synchronizer for a Manchester NRZ line code by completing the following steps:

(a) Give a simplified block diagram.

(b) Explain how the synchronizer works.

(c) Specify the synchronizer's filter requirements.

(d) Explain the advantages and disadvantages of using this design for the Manchester NRZ line code compared with using a polar NRZ line code and its associated bit synchronizer.

3–38 Figure 3–22c illustrates an eight-level multilevel signal. Assume that this line code is passed through a channel that filters the signal and adds some noise.

(a) Draw a picture of the eye pattern for the received waveform.

(b) Design a possible receiver with its associated symbol synchronizer for this line code.

(c) Explain how your receiver works.

★ **3–39** The information in an analog waveform is first encoded into binary PCM and then converted to a multilevel signal for transmission over the channel. The number of multilevels is eight. Assume that the analog signal has a bandwidth of 2,700 Hz and is to be reproduced at the receiver output with an accuracy of $\pm 1\%$ (full scale).

(a) Determine the minimum bit rate of the PCM signal.

(b) Determine the minimum baud rate of the multilevel signal.

(c) Determine the minimum absolute channel bandwidth required for transmission of this PCM signal.

★ **3–40** A binary waveform of 9,600 bits/s is converted into an octal (multilevel) waveform that is passed through a channel with a raised cosine-rolloff Nyquist filter characteristic. The channel has a conditioned (equalized) phase response out to 2.4 kHz.

(a) What is the baud rate of the multilevel signal?

(b) What is the rolloff factor of the filter characteristic?

3–41 Assume that the spectral properties of an $L = 64$-level waveform with rectangular RZ-type pulse shapes are to be examined. The pulse shape is given by

$$f(t) = \Pi\left(\frac{2t}{T_s}\right)$$

where T_s is the time needed to send one of the multilevel symbols.

(a) Determine the expression for the PSD for the case of equally likely levels where the peak signal levels for this multilevel waveform are +10 V.

(b) What is the null bandwidth?

(c) What is the spectral efficiency?

3–42 A binary communication system uses polar signaling. The overall impulse response is designed to be of the $(\sin x)/x$ type, as given by Eq. (3–67), so that there will be no ISI. The bit rate is $R = f_s = 300$ bits/s.

(a) What is the bandwidth of the polar signal?

(b) Plot the waveform of the polar signal at the system output when the input binary data is 01100101. Can you discern the data by looking at this polar waveform?

★ **3–43** Equation (3–67) gives one possible noncausal impulse response for a communication system that will have no ISI. For a causal approximation, select

$$h_e(t) = \frac{\sin \pi f_s(t - 4 \times 10^{-3})}{\pi f_s(t - 4 \times 10^{-3})}\,\Pi\left(\frac{t - 4 \times 10^{-3}}{8 \times 10^{-3}}\right)$$

where $f_s = 1{,}000$.

(a) Using a PC, calculate $H_e(f)$ by the use of the Fourier transform integral, and plot $|H_e(f)|$.
(b) What is the bandwidth of this causal approximation, and how does it compare with the bandwidth of the noncausal filter described by Eqs. (3–67) and (3–68)?

3–44 Starting with Eq. (3–69), prove that the impulse response of the raised cosine-rolloff filter is given by Eq. (3–73).

3–45 Consider the raised cosine-rolloff Nyquist filter given by Eqs. (3–69) and (3–73).
(a) Plot $|H_e(f)|$ for the case of $r = 0.75$, indicating f_1, f_0, and B on your sketch in a manner similar to Fig. 3–25.
(b) Plot $h_e(t)$ for the case of $r = 0.75$ in terms of $1/f_0$. Your plot should be similar to Fig. 3–26.

3–46 Find the PSD of the waveform out of an $r = 0.5$ raised cosine-rolloff channel when the input is a polar NRZ signal. Assume that equally likely binary signaling is used and the channel bandwidth is just large enough to prevent ISI.

★ **3–47** Equation (3–66) gives the condition for the absence of ISI (Nyquist's first method). Using that equation with $C = 1$ and $\tau = 0$, show that Nyquist's first method for eliminating ISI is also satisfied if

$$\sum_{k=-\infty}^{\infty} H_e\left(f + \frac{k}{T_s}\right) = T_s \quad \text{for} \quad |f| \leq \frac{1}{2T_s}$$

3–48 Using the results of Prob. 3–47, demonstrate that the following filter characteristics do or do not satisfy Nyquist's criterion for eliminating ISI ($f_s = 2f_0 = 2/T_0$).

(a) $H_e(f) = \dfrac{T_0}{2}\Pi\left(\dfrac{1}{2}fT_0\right)$.

(b) $H_e(f) = \dfrac{T_0}{2}\Pi\left(\dfrac{2}{3}fT_0\right)$.

3–49 Assume that a pulse transmission system has the overall raised cosine-rolloff Nyquist filter characteristic described by Eq. (3–69).
(a) Find the $Y(f)$ Nyquist function of Eq. (3–75) corresponding to the raised cosine-rolloff Nyquist filter characteristic.
(b) Sketch $Y(f)$ for the case of $r = 0.75$.
(c) Sketch another $Y(f)$ that is not of the raised cosine-rolloff type, and determine the absolute bandwidth of the resulting Nyquist filter characteristic.

★ **3–50** An analog signal is to be converted into a PCM signal that is a binary polar NRZ line code. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine-rolloff type with $r = 0.5$.
(a) Find the maximum PCM bit rate that can be supported by this system without introducing ISI.
(b) Find the maximum bandwidth that can be permitted for the analog signal.

3–51 Rework Prob. 3–50 for the case of a multilevel polar NRZ line code when the number of levels is four.

★ **3–52** Multilevel data with an equivalent bit rate of 2,400 bits/s is sent over a channel using a four-level line code that has a rectangular pulse shape at the output of the transmitter. The overall transmission system (i.e., the transmitter, channel, and receiver) has an $r = 0.5$ raised cosine-rolloff Nyquist filter characteristic.
(a) Find the baud rate of the received signal.
(b) Find the 6-dB bandwidth for this transmission system.
(c) Find the absolute bandwidth for the system.

3–53 Assume that a PCM-type system is to be designed such that an audio signal can be delivered at the receiver output. This audio signal is to have a bandwidth of 3,400 Hz and an SNR of at least 40 dB. Determine the bit rate requirements for a design that uses $\mu = 255$ companded PCM signaling.

3–54 Rework Prob. 3–53 for the case of DPCM signaling.

Discuss which of the preceding systems of Prob. 3–53 and Prob. 3–54 would be used in your design and why.

3–55 Refer to Fig. 3–32, which shows typical DM waveforms. Draw an analog waveform that is different from the one shown in the figure. Draw the corresponding DM and integrator output waveforms. Denote the regions where slope overload noise dominates and where granular noise dominates.

3–56 A DM system is tested with a 10-kHz sinusoidal signal, 1 V peak to peak, at the input. The signal is sampled at 10 times the Nyquist rate.

(a) What is the step size required to prevent slope overload and to minimize granular noise?
(b) What is the PSD for the granular noise?
(c) If the receiver input is bandlimited to 200 kHz, what is the average signal–quantizing noise power ratio?

3–57 Assume that the input to a DM is $0.1t^8 - 5t + 2$. The step size of the DM is 1 V, and the sampler operates at 10 samples/s. Sketch the input waveform, the delta modulator output, and the integrator output over a time interval of 0 to 2 s. Denote the granular noise and slope overload regions.

3–58 Rework Prob. 3–57 for the case of an adaptive delta modulator where the step size is selected according to the number of successive binary 1's or 0's on the DM output. Assume that the step size is 1.5 V when there are four or more binary digits of the same sign, 1 V for the case of three successive digits, and 0.5 V for the case of two or fewer successive digits.

★ **3–59** A delta modulator is to be designed to transmit the information of an analog waveform that has a peak-to-peak level of 1 V and a bandwidth of 3.4 kHz. Assume that the waveform is to be transmitted over a channel whose frequency response is extremely poor above 1 MHz.

(a) Select the appropriate step size and sampling rate for a sine-wave test signal, and discuss the performance of the system, using the parameter values you have selected.
(b) If the DM system is to be used to transmit the information of a voice (analog) signal, select the appropriate step size when the sampling rate is 25 kHz. Discuss the performance of the system under these conditions.

3–60 One analog waveform $w_1(t)$ is bandlimited to 3 kHz, and another, $w_2(t)$, is bandlimited to 9 kHz. These two signals are to be sent by TDM over a PAM-type system.

(a) Determine the minimum sampling frequency for each signal, and design a TDM commutator and decommutator to accommodate the signals.
(b) Draw some typical waveforms for $w_1(t)$ and $w_2(t)$, and sketch the corresponding TDM PAM waveform.

★ **3–61** Three waveforms are time-division multiplexed over a channel using instantaneously sampled PAM. Assume that the PAM pulse width is very narrow and that each of the analog waveforms is sampled every 0.15 s. Plot the (composite) TDM waveform when the input analog waveforms are

$$w_1(t) = 3 \sin(2\pi t)$$

$$w_2(t) = \Pi\left(\frac{t-1}{2}\right)$$

and

$$w_3(t) = -\Lambda(t-1)$$

3–62 Twenty-three analog signals, each with a bandwidth of 3.4 kHz, are sampled at an 8-kHz rate and multiplexed together with a synchronization channel (8 kHz) into a TDM PAM signal. This TDM signal is passed through a channel with an overall raised cosine-rolloff Nyquist filter characteristic of $r = 0.75$.

(a) Draw a block diagram for the system, indicating the f_s of the commutator and the overall pulse rate of the TDM PAM signal.

(b) Evaluate the absolute bandwidth required for the channel.

★ **3–63** Two flat-topped PAM signals are time-division multiplexed together to produce a composite TDM PAM signal that is transmitted over a channel. The first PAM signal is obtained from an analog signal that has a rectangular spectrum, $W_1(f) = \Pi(f/2B)$. The second PAM signal is obtained from an analog signal that has a triangular spectrum $W_2(f) = \Lambda\ (f/B)$, where $B = 3$ kHz.

(a) Determine the minimum sampling frequency for each signal, and design a TDM commutator and decommutator to accommodate these signals.

(b) Calculate and sketch the magnitude spectrum for the composite TDM PAM signal.

3–64 Rework Prob. 3–62 for a TDM pulse code modulation system in which an 8-bit quantizer is used to generate the PCM words for each of the analog inputs and an 8-bit synchronization word is used in the synchronization channel.

3–65 Design a TDM PCM system that will accommodate four 300-bit/s (synchronous) digital inputs and one analog input that has a bandwidth of 500 Hz. Assume that the analog samples will be encoded into 4-bit PCM words. Draw a block diagram for your design, analogous to Fig. 3–39, indicating the data rates at the various points on the diagram. Explain how your design works.

★ **3–66** Design a TDM system that will accommodate two 2,400-bit/s synchronous digital inputs and an analog input that has a bandwidth of 2,700 Hz. Assume that the analog input is sampled at 1.11111 times the Nyquist rate and converted into 4-bit PCM words. Draw a block diagram for your design, and indicate the data rate at various points on your diagram. Explain how your TDM scheme works.

3–67 Find the number of the following devices that could be accommodated by a T1-type TDM line if 1% of the line capacity were reserved for synchronization purposes.

(a) 110-bit/s teleprinter terminals.

(b) 8,000-bit/s speech codecs.

(c) 9,600-bit/s computer output ports.

(d) 64-kbit/s PCM VF lines.

(e) 144-kbit/s ISDN terminals.

How would these numbers change if each of the sources was operational an average of 10% of the time?

3–68 Assume that a sine wave is sampled at four times the Nyquist rate using instantaneous sampling.

(a) Sketch the corresponding PWM signal.

(b) Sketch the corresponding PPM signal.

3–69 Discuss why a PPM system requires a synchronizing signal, whereas PAM and PWM can be detected without the need for a synchronizing signal.

★ **3–70** Compare the bandwidth required to send a message by using PPM and PCM signaling. Assume that the digital source sends out 8 bits/character, so that it can send 256 different messages (characters). Assume that the source rate is 10 characters/s. Use the dimensionality theorem, $N/T_0 = 2B$, to determine the minimum bandwidth B required.

(a) Determine the minimum bandwidth required for a PCM signal that will encode the source information.

(b) Determine the minimum bandwidth required for a PPM signal that will encode the source information.

PROBLEMS

4–1 Show that if $v(t) = \text{Re}\{g(t)e^{j\omega_c t}\}$, Eqs. (4–1b) and (4–1c) are correct, where $g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$.

4–2 An AM signal is modulated by a waveform such that the complex envelope is

$$g(t) = A_c\{1 + a[0.2\cos(\pi 250t) + 0.5\sin(\pi 2500t)]\}$$

where $A_c = 10$. Find the value of a such that the AM signal has a positive modulation percentage of 90%. *Hint:* Look at Ex. 4–3 and Eq. (5–5a).

★ **4–3** A double-sideband suppressed carrier (DSB-SC) signal $s(t)$ with a carrier frequency of 3.8 MHz has a complex envelope $g(t) = A_c m(t)$. $A_c = 50$ V, and the modulation is a 1-kHz sinusoidal test tone described by $m(t) = 2\sin(2\pi\ 1{,}000t)$. Evaluate the voltage spectrum for this DSB-SC signal.

4–4 A DSB-SC signal has a carrier frequency of 900 kHz and $A_c = 10$. If this signal is modulated by a waveform that has a spectrum given by Fig. P3–3. Find the magnitude spectrum for this DSB-SC signal.

4–5 Assume that the DSB-SC voltage signal $s(t)$, as described in Prob. 4–3 appears across a 50-Ω resistive load.

(a) Compute the actual average power dissipated in the load.

(b) Compute the actual PEP.

4–6 For the AM signal described in Prob. 4–2 with $a = 0.5$, calculate the total average normalized power.

4–7 For the AM signal described in Prob. 4–2 with $a = 0.5$, calculate the normalized PEP.

4–8 A bandpass filter is shown in Fig. P4–8.

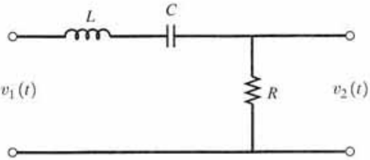

Figure P4–8

(a) Find the mathematical expression for the transfer function of this filter, $H(f) = V_2(f)/V_1(f)$, as a function of R, L, and C. Sketch the magnitude transfer function $|H(f)|$.

(b) Find the expression for the equivalent low-pass filter transfer function, and sketch the corresponding low-pass magnitude transfer function.

★ **4–9** Let the transfer function of an ideal bandpass filter be given by

$$H(f) = \begin{cases} 1, & |f + f_c| < B_T/2 \\ 1, & |f - f_c| < B_T/2 \\ 0, & f \text{ elsewhere} \end{cases}$$

where B_T is the absolute bandwidth of the filter.

(a) Sketch the magnitude transfer function $|H(f)|$.
(b) Find an expression for the waveform at the output, $v_2(t)$, if the input consists of the pulsed carrier

$$v_1(t) = A\Pi(t/T)\cos(\omega_c t)$$

(c) Sketch the output waveform $v_2(t)$ for the case when $B_T = 4/T$ and $f_c \gg B_T$.

(*Hint:* Use the complex-envelope technique, and express the answer as a function of the sine integral, defined by

$$\text{Si}(u) = \int_0^u \frac{\sin\lambda}{\lambda}\,d\lambda$$

The sketch can be obtained by looking up values for the sine integral from published tables [Abramowitz and Stegun, 1964] or by numerically evaluating Si (u).

4–10 Examine the distortion properties of an RC low-pass filter (shown in Fig. 2–15). Assume that the filter input consists of a bandpass signal that has a bandwidth of 1 kHz and a carrier frequency of 15 kHz. Let the time constant of the filter be $\tau_0 = RC = 10^{-5}$ s.

(a) Find the phase delay for the output carrier.
(b) Determine the group delay at the carrier frequency.
(c) Evaluate the group delay for frequencies around and within the frequency band of the signal. Plot this delay as a function of frequency.
(d) Using the results of (a) through (c), explain why the filter does or does not distort the bandpass signal.

★ **4–11** A bandpass filter as shown in Fig. P4–11 has the transfer function

$$H(s) = \frac{Ks}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

where $Q = R\sqrt{C/L}$, the resonant frequency is $f_0 = 1/(2\pi\sqrt{LC})$, $\omega_0 = 2\pi f_0$, K is a constant, and values for R, L, and C are given in the figure. Assume that a bandpass signal with $f_c = 4$ kHz and a bandwidth of 200 Hz passes through the filter, where $f_0 = f_c$.

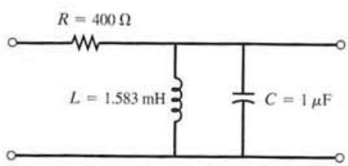

Figure P4–11

(a) Using Eq. (4–39), find the bandwidth of the filter.
(b) Plot the carrier delay as a function of f about f_0.
(c) Plot the group delay as a function of f about f_0.
(d) Explain why the filter does or does not distort the signal.

4–12 An FM signal is of the form

$$s(t) = A_c \cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\sigma)\,d\sigma\right]$$

where $m(t)$ is the modulating signal and $\omega_c = 2\pi f_c$, in which f_c is the carrier frequency. Show that the functions $g(t)$, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$, as given for FM in Table 4–1, are correct.

4–13 The output of a FM transmitter at 96.9 MHz delivers 25kw average power into an antenna system which presents a 50-Ω resistive load. Find the value for the peak voltage at the input to the antenna system.

★ **4–14** Let a modulated signal,

$$s(t) = 100\sin(\omega_c + \omega_a)t + 500\cos\omega_c t - 100\sin(\omega_c - \omega_a)t$$

where the unmodulated carrier is $500\cos\omega_c t$.

(a) Find the complex envelope for the modulated signal. What type of modulation is involved? What is the modulating signal?

(b) Find the quadrature modulation components $x(t)$ and $y(t)$ for this modulated signal.

(c) Find the magnitude and PM components $R(t)$ and $\theta(t)$ for this modulated signal.

(d) Find the total average power, where $s(t)$ is a voltage waveform that is applied across a 50-Ω load.

★ **4–15** Find the spectrum of the modulated signal given in Prob. 4–14 by two methods:

(a) By direct evaluation using the Fourier transform of $s(t)$.

(b) By the use of Eq. (4–12).

4–16 Given a pulse-modulated signal of the form

$$s(t) = e^{-at}\cos[(\omega_c + \Delta\omega)t]u(t)$$

where a, ω_c, and $\Delta\omega$ are positive constants and the carrier frequency, $w_c \gg \Delta\omega$,

(a) Find the complex envelope.

(b) Find the spectrum $S(f)$.

(c) Sketch the magnitude and phase spectra $|S(f)|$ and $\theta(f) = \underline{/S(f)}$.

4–17 In a digital computer simulation of a bandpass filter, the complex envelope of the impulse response is used, where $h(t) = \text{Re}[k(t)e^{j\omega_c t}]$, as shown in Fig. 4–3. The complex impulse response can be expressed in terms of quadrature components as $k(t) = 2h_x(t) + j2h_y(t)$, where $h_x(t) = \frac{1}{2}\text{Re}[k(t)]$ and $h_y(t) = \frac{1}{2}\text{Im}[k(t)]$. The complex envelopes of the input and output are denoted, respectively, by $g_1(t) = x_1(t) + jy_1(t)$ and $g_2(t) = x_2(t) + jy_2(t)$. The bandpass filter simulation can be carried out by using four *real baseband* filters (i.e., filters having real impulse responses), as shown in Fig. P4–17. Note that although there are four filters, there are only two different impulse responses: $h_x(t)$ and $h_y(t)$.

(a) Using Eq. (4–22), show that Fig. P4–17 is correct.

(b) Show that $h_y(t) = 0$ (i.e., no filter is needed) if the bandpass filter has a transfer function with Hermitian symmetry about f_c—that is, if $H(-\Delta f + f_c) = H^*(\Delta f + f_c)$, where $|\Delta f| < B_T/2$ and B_T is the bounded spectral bandwidth of the bandpass filter. This Hermitian symmetry implies that the magnitude frequency response of the bandpass filter is even about f_c and the phase response is odd about f_c.

4–18 Evaluate and sketch the magnitude transfer function for **(a)** Butterworth, **(b)** Chebyshev, and **(c)** Bessel low-pass filters. Assume that $f_b = 10$ Hz and $\epsilon = 1$.

★ **4–19** Plot the amplitude response, the phase response, and the phase delay as a function of frequency for the following low-pass filters, where $B = 100$ Hz:

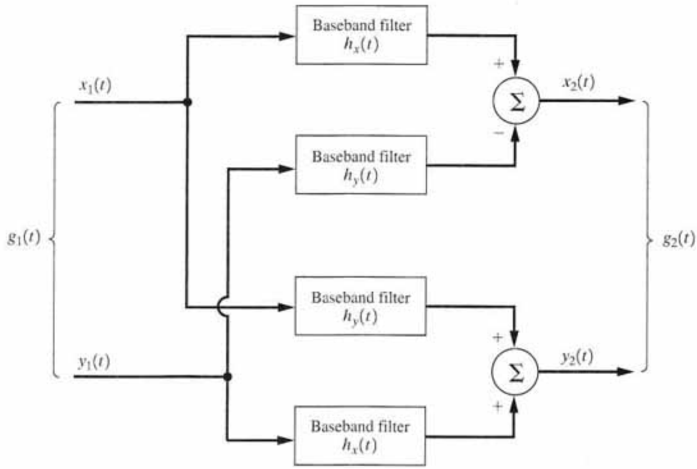

Figure P4–17

(a) Butterworth filter, second order:

$$H(f) = \frac{1}{1 + \sqrt{2}(jf/B) + (jf/B)^2}$$

(b) Butterworth filter, fourth order:

$$H(f) = \frac{1}{[1 + 0.765(jf/B) + (jf/B)^2][1 + 1.848(jf/B) + (jf/B)^2]}$$

Compare your results for the two filters.

4–20 Assume that the output-to-input characteristic of a bandpass amplifier is described by Eq. (4–42) and that the linearity of the amplifier is being evaluated by using a two-tone test.

(a) Find the frequencies of the fifth-order intermodulation products that fall within the amplifier bandpass.

(b) Evaluate the levels for the fifth-order intermodulation products in terms of A_1, A_2, and the K's.

4–21 An amplifier is tested for total harmonic distortion (THD) by using a single-tone test. The output is observed on a spectrum analyzer. It is found that the peak values of the three measured harmonics decrease according to an exponential recursion relation $V_{n+1} = V_n e^{-n}$, where $n = 1, 2, 3$. What is the THD?

★ **4–22** The nonlinear output–input characteristic of an amplifier is

$$v_{\text{out}}(t) = 5v_{\text{in}}(t) + 1.5v_{\text{in}}^2(t) + 1.5v_{\text{in}}^3(t)$$

Assume that the input signal consists of seven components:

$$v_{in}(t) = \frac{1}{2} + \frac{4}{\pi^2}\sum_{k=1}^{6}\frac{1}{(2k-1)^2}\cos[(2k-1)\pi t]$$

(a) Plot the output signal and compare it with the linear output component $5v_{in}(t)$.

(b) Take the FFT of the output $v_{out}(t)$, and compare it with the spectrum for the linear output component.

4–23 For a bandpass limiter circuit, show that the bandpass output is given by Eq. (4–55), where $K = (4/\pi)A_0$. A_0 denotes the voltage gain of the bandpass filter, and it is assumed that the gain is constant over the frequency range of the bandpass signal.

4–24 Discuss whether the Taylor series nonlinear model is applicable to the analysis of **(a)** soft limiters and **(b)** hard limiters.

★ **4–25** Assume that an audio sine-wave testing signal is passed through an audio hard limiter circuit. Evaluate the total harmonic distortion (THD) on the signal at the limiter output.

4–26 Using the mathematical definition of linearity given in Chapter 2, show that the analog switch multiplier of Fig. 4–10 is a linear device.

4–27 An audio signal with a bandwidth of 10 kHz is transmitted over an AM transmitter with a carrier frequency of 1.0 MHz. The AM signal is received on a superheterodyne receiver with an envelope detector. What is the constraint on the *RC* time constant for the envelope detector?

★ **4–28** Assume that an AM receiver with an envelope detector is tuned to an SSB-AM signal that has a modulation waveform given by $m(t)$. Find the mathematical expression for the audio signal that appears at the receiver output in terms of $m(t)$. Is the audio output distorted?

4–29 Referring to Table 4–1, find the equation that describes the output of an envelope detector as a function of $m(t)$, if the input is a

(a) DSB-SC signal.

(b) FM signal.

4–30 Evaluate the sensitivity of the zero-crossing FM detector shown in Fig. 4–18. Assume that the differential amplifier is described by $v_{out}(t) = A\,[v_2(t) - v_3(t)]$, where A is the voltage gain of the amplifier. In particular, show that $v_{out} = Kf_d$, where $f_d = f_i - f_c$, and find the value of the sensitivity constant K in terms of A, R, and C. Assume that the peak levels of the monostable outputs Q and $\overline{Q}$ are 4 V (TTL circuit levels).

4–31 Using Eq. (4–100), show that the linearized block diagram model for a PLL is given by Fig. 4–22.

4–32 Show that Eq. (4–101) describes the linear PLL model as given in Fig. 4–22.

★ **4–33** Using the Laplace transform and the final value theorem, find an expression for the steady-state phase error, $\lim_{t\to\infty}\theta_e(t)$, for a PLL as described by Eq. (4–100). [*Hint:* The final value theorem is $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$.]

4–34 Assume that the loop filter of a PLL is a low-pass filter, as shown in Fig. P4–34.

(a) Evaluate the closed-loop transfer function $H(f) = \dfrac{\Theta_0(f)}{\Theta_i(f)}$ for a linearized PLL.

(b) Sketch the Bode plot $[|H(f)|]_{dB} \triangleq 20\log|H(f)|$ for this PLL.

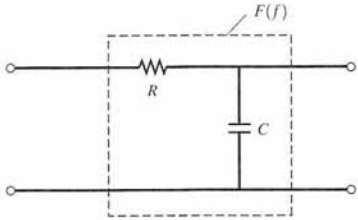

Figure P4–34

★ **4–35** Assume that the phase noise characteristic of a PLL is being examined. The internal phase noise of the VCO is modeled by the input $\theta_n(t)$, as shown in Fig. P4–35.

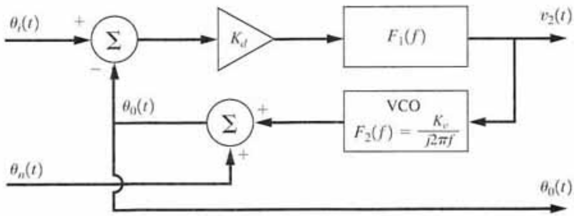

Figure P4–35

(a) Find an expression for the closed-loop transfer function $\Theta_0(f)/\Theta_n(f)$, where $\theta_i(t) = 0$.

(b) If $F_1(f)$ is the low-pass filter given in Fig. P4–34, sketch the Bode plot $[|\Theta_0(f)/\Theta_n(f)|]_{dB}$ for the phase noise transfer function.

4–36 The input to a PLL is $v_{in}(t) = A\sin(\omega_0 t + \theta_i)$. The LPF has a transfer function $F(s) = (s + a)/s$.

(a) What is the steady-state phase error?

(b) What is the maximum hold-in range for the noiseless case?

★ **4–37** **(a)** Refer to Fig. 4–25 for a PLL frequency synthesizer. Design a synthesizer that will cover a range of 144 to 148 MHz in 5-kHz steps, starting at 144.000 MHz. Assume that the frequency standard operates at 5 MHz, that the M divider is fixed at some value, and that the N divider is programmable so that the synthesizer will cover the desired range. Sketch a diagram of your design, indicating the frequencies present at various points of the diagram.

(b) Modify your design so that the output signal can be frequency modulated with an audio-frequency input such that the peak deviation of the RF output is 5 kHz.

4–38 Assume that an SSB-AM transmitter is to be realized using the AM–PM generation technique, as shown in Fig. 4–27.

(a) Sketch a block diagram for the baseband signal-processing circuit.

(b) Find expressions for $R(t)$, $\theta(t)$, and $v(t)$ when the modulation is $m(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$.

4–39 Rework Prob. 4–38 for the case of generating an FM signal.

4–40 Assume that an SSB-AM transmitter is to be realized using the quadrature generation technique, as shown in Fig. 4–28.

(a) Sketch a block diagram for the baseband signal-processing circuit.

(b) Find expressions for $x(t)$, $y(t)$, and $v(t)$ when the modulation is $m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$.

4–41 Rework Prob. 4–40 for the case of generating an FM signal.

★ **4–42** An FM radio is tuned to receive an FM broadcasting station of frequency 96.9 MHz. The radio is of the superheterodyne type with the LO operating on the high side of the 96.9-MHz input and using a 10.7-MHz IF amplifier.

(a) Determine the LO frequency.

(b) If the FM signal has a bandwidth of 180 kHz, give the requirements for the RF and IF filters.

(c) Calculate the frequency of the image response.

4–43 A dual-mode cellular phone is designed to operate with either cellular phone service in the 900-MHz band or PCS in the 1,900-MHz band. The phone uses a superheterodyne receiver with a 500-MHz IF for both modes.

(a) Calculate the LO frequency and the image frequency for high-side injection when the phone receives an 880-MHz signal.

(b) Calculate the LO frequency and the image frequency for low-side injection when the phone receives a 1,960-MHz PCS signal.

(c) Discuss the advantage of using a 500-MHz IF for this dual-mode phone.

(*Note*: Cellular and PCS systems are described in Chapter 8.)

4–44 A superheterodyne receiver is tuned to a station at 20 MHz. The local oscillator frequency is 80 MHz and the IF is 100 MHz.

(a) What is the image frequency?

(b) If the LO has appreciable second-harmonic content, what two additional frequencies are received?

(c) If the RF amplifier contains a single-tuned parallel resonant circuit with $Q = 50$ tuned to 20 MHz, what will be the image attenuation in dB?

4–45 An SSB-AM receiver is tuned to receive a 7.225-MHz lower SSB (LSSB) signal. The LSSB signal is modulated by an audio signal that has a 3-kHz bandwidth. Assume that the receiver uses a superheterodyne circuit with an SSB IF filter. The IF amplifier is centered on 3.395 MHz. The LO frequency is on the high (frequency) side of the input LSSB signal.

(a) Draw a block diagram of the single-conversion superheterodyne receiver, indicating frequencies present and typical spectra of the signals at various points within the receiver.

(b) Determine the required RF and IF filter specifications, assuming that the image frequency is to be attenuated by 40 dB.

4–46 **(a)** Draw a block diagram of a superheterodyne FM receiver that is designed to receive FM signals over a band from 144 to 148 MHz. Assume that the receiver is of the dual-conversion type (i.e., a mixer and an IF amplifier, followed by another mixer and a second IF amplifier), where the first IF is 10.7 MHz and the second is 455 kHz. Indicate the frequencies of the signals at different points on the diagram, and, in particular, show the frequencies involved when a signal at 146.82 MHz is being received.

(b) Replace the first oscillator by a frequency synthesizer such that the receiver can be tuned in 5-kHz steps from 144.000 to 148.000 MHz. Show the diagram of your synthesizer design and the frequencies involved.

★ **4–47** An AM broadcast-band radio is tuned to receive a 1,080-kHz AM signal and uses high-side LO injection. The IF is 455 kHz.

(a) Sketch the frequency response for the RF and IF filters.

(b) What is the image frequency?

4–48 Commercial AM broadcast stations operate in the 540- to 1,700-kHz band, with a transmission bandwidth limited to 10 kHz.

(a) What are the maximum number of stations that can be accommodated?

(b) If stations are not assigned to adjacent channels (in order to reduce interference on receivers that have poor IF characteristics), how many stations can be accommodated?

(c) For 455-kHz IF receivers, what is the band of image frequencies for the AM receiver that uses a down-converter with high-side injection?

PROBLEMS

★ **5–1** An AM broadcast transmitter is tested by feeding the RF output into a 50-Ω (dummy) load. Tone modulation is applied. The carrier frequency is 850 kHz and the FCC licensed power output is 5,000 W. The sinusoidal tone of 1,000 Hz is set for 90% modulation.

(a) Evaluate the FCC power in dBk (dB above 1 kW) units.

(b) Write an equation for the voltage that appears across the 50-Ω load, giving numerical values for all constants.

(c) Sketch the spectrum of this voltage as it would appear on a calibrated spectrum analyzer.

(d) What is the average power that is being dissipated in the dummy load?

(e) What is the peak envelope power?

5–2 An AM transmitter is modulated with an audio testing signal given by $m(t) = 0.2 \sin \omega_1 t + 0.5 \cos \omega_2 t$, where $f_1 = 500$ Hz, $f_2 = 500\sqrt{2}$ Hz, and $A_c = 100$. Assume that the AM signal is fed into a 50-Ω load.

(a) Sketch the AM waveform.

(b) What is the modulation percentage?

(c) Evaluate and sketch the spectrum of the AM waveform.

5–3 For the AM signal given in Prob. 5–2,

(a) Evaluate the average power of the AM signal.

(b) Evaluate the PEP of the AM signal.

★ **5–4** Assume that an AM transmitter is modulated with a video testing signal given by $m(t) = -0.2 + 0.6 \sin \omega_1 t$, where $f_1 = 3.57$ MHz. Let $A_c = 100$.

(a) Sketch the AM waveform.

(b) What are the percentages of positive and negative modulation?

(c) Evaluate and sketch the spectrum of the AM waveform about f_c.

5–5 A 50,000-W AM broadcast transmitter is being evaluated by means of a two-tone test. The transmitter is connected to a 50-Ω load, and $m(t) = A_1 \cos \omega_1 t + A_1 \cos 2\omega_1 t$, where $f_1 = 500$ Hz. Assume that a perfect AM signal is generated.

(a) Evaluate the complex envelope for the AM signal in terms of A_1 and ω_1.

(b) Determine the value of A_1 for 90% modulation.

(c) Find the values for the peak current and average current into the 50-Ω load for the 90% modulation case.

★ **5–6** An AM transmitter uses a two-quadrant multiplier so that the transmitted signal is described by Eq. (5–7). Assume that the transmitter is modulated by $m(t) = A_m \cos \omega_m t$, where A_m is adjusted so that 120% positive modulation is obtained. Evaluate the spectrum of this AM signal in terms of A_c, f_c, and f_m. Sketch your result.

5–7 Repeat Prob. 5–6 using a four-quadrant multiplier.

5–8 Assume that Prob. 5–6 describes the operation of an AM transmitter and its modulating waveform. Let $A_c = 500$ V, $f_c = 60$ kHz, and $f_a = 6$kHz. Using MATLAB, plot the voltage waveform that appears at the output of this transmitter.

5–9 Assume that Prob. 5–6 describes the operation of an AM transmitter and its modulating waveform, except that a four-quadrant multiplier is used. Let $A_c = 500$ V, $f_c = 60$ kHz, and $f_a = 6$kHz. Using MATLAB, plot the voltage waveform that appears at the output of this transmitter.

★ **5–10** A DSB-SC signal is modulated by $m(t) = \cos \omega_1 t + 2 \cos 2\omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 500$ Hz, and $A_c = 1$.

(a) Write an expression for the DSB-SC signal and sketch a picture of this waveform.

(b) Evaluate and sketch the spectrum for this DSB-SC signal.
(c) Find the value of the average (normalized) power.
(d) Find the value of the PEP (normalized).

5–11 Assume that transmitting circuitry restricts the modulated output signal to a certain peak value, say, A_p, because of power-supply voltages that are used and because of the peak voltage and current ratings of the components. If a DSB-SC signal with a peak value of A_p is generated by this circuit, show that the sideband power of this DSB-SC signal is four times the sideband power of a comparable AM signal having the same peak value A_p that could also be generated by this circuit.

5–12 A DSB-SC signal can be generated from two AM signals as shown in Fig. P5–12. Using mathematics to describe signals at each point on the figure, prove that the output is a DSB-SC signal.

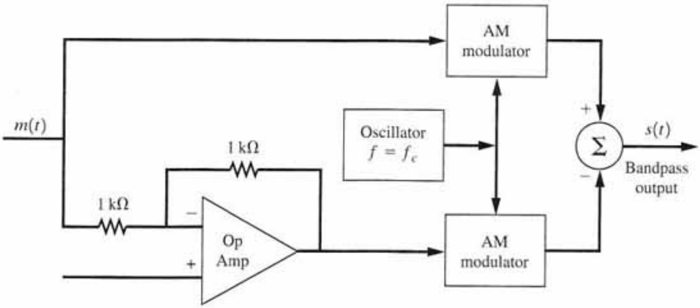

Figure P5–12

5–13 Show that the complex envelope $g(t) = m(t) - j\hat{m}(t)$ produces a lower SSB signal, provided that $m(t)$ is a real signal.

5–14 Show that the impulse response of a $-90°$ phase shift network (i.e., a Hilbert transformer) is $1/\pi t$. *Hint:*

$$H(f) = \lim_{\substack{\alpha \to 0 \\ \alpha > 0}} \begin{cases} -je^{-\alpha f}, & f > 0 \\ je^{\alpha f}, & f < 0 \end{cases}$$

★ **5–15** SSB signals can be generated by the phasing method shown in Fig. 5–5a, by the filter method, of Fig. 5–5b, or by the use of Weaver's method [Weaver, 1956], as shown in Fig. P5–15. For Weaver's method (Fig. P5–15), where B is the bandwidth of $m(t)$,
(a) Find a mathematical expression that describes the waveform out of each block on the block diagram.
(b) Show that $s(t)$ is an SSB signal.

5–16 An SSB-AM transmitter is modulated with a sinusoid $m(t) = 5\cos\omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 500$ Hz, and $A_c = 1$.
(a) Evaluate $\hat{m}(t)$.
(b) Find the expression for a lower SSB signal.

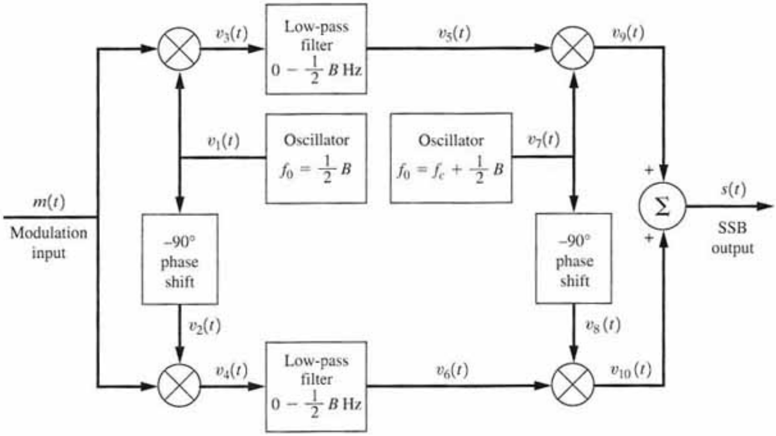

Figure P5–15 Weaver's method for generating SSB.

(c) Find the RMS value of the SSB signal.
(d) Find the peak value of the SSB signal.
(e) Find the normalized average power of the SSB signal.
(f) Find the normalized PEP of the SSB signal.

★ **5–17** An SSB-AM transmitter is modulated by a rectangular pulse such that $m(t) = \Pi(t/T)$ and $A_c = 1$.
(a) Prove that

$$\hat{m}(t) = \frac{1}{\pi}\ln\left|\frac{2t+T}{2t-T}\right|$$

as given in Table A–7.
(b) Find an expression for the SSB-AM signal $s(t)$, and sketch $s(t)$.
(c) Find the peak value of $s(t)$.

5–18 For Prob. 5–17,
(a) Find the expression for the spectrum of a USSB-AM signal.
(b) Sketch the magnitude spectrum, $|S(f)|$.

★ **5–19** A USSB transmitter is modulated with the pulse

$$m(t) = \frac{\sin \pi a t}{\pi a t}$$

(a) Prove that

$$\hat{m}(t) = \frac{\sin^2[(\pi a/2)t]}{(\pi a/2)t}$$

(b) Plot the corresponding USSB signal waveform for the case of $A_c = 1$, $a = 2$, and $f_c = 20$ Hz.

5–20 A USSB-AM signal is modulated by a rectangular pulse train:

$$m(t) = \sum_{n=-\infty}^{\infty} \prod[(t - nT_0)/T]$$

Here, $T_0 = 2T$.

(a) Find the expression for the spectrum of the SSB-AM signal.

(b) Sketch the magnitude spectrum, $|S(f)|$.

5–21 A phasing-type SSB-AM detector is shown in Fig. P5–21. This circuit is attached to the IF output of a conventional superheterodyne receiver to provide SSB reception.

(a) Determine whether the detector is sensitive to LSSB or USSB signals. How would the detector be changed to receive SSB signals with the opposite type of sidebands?

(b) Assume that the signal at point A is a USSB signal with $f_c = 455$ kHz. Find the mathematical expressions for the signals at points B through I.

(c) Repeat part (b) for the case of an LSSB-AM signal at point A.

(d) Discuss the IF and LP filter requirements if the SSB signal at point A has a 3-kHz bandwidth.

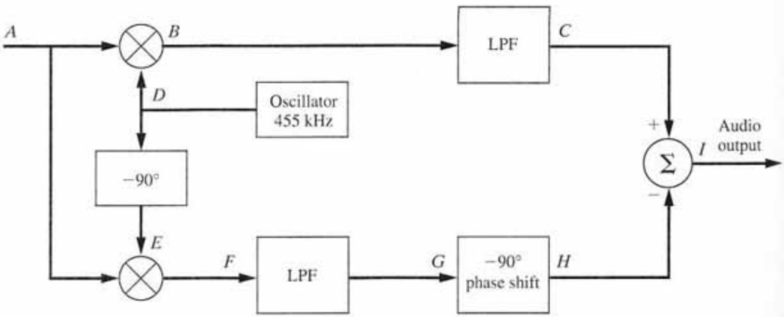

Figure P5–21

5–22 Can a Costas loop, shown in Fig. 5–3, be used to demodulate an SSB-AM signal? Use mathematics to demonstrate that your answer is correct.

5–23 A modulated signal is described by the equation

$$s(t) = 10 \cos [(2\pi \times 10^8)t + 10 \cos (2\pi \times 10^3 t)]$$

Find each of the following:

(a) Percentage of AM.

(b) Normalized power of the modulated signal.

5–24 For the modulated signal described in Prob. 5–23, find the

(a) Maximum phase deviation.

(b) Maximum frequency deviation.

5–25 A transmitter produces a quadrature modulated (QM) signal. The complex envelope is

$$g(t) = A_c[m_1(t) + j\,m_2(t)]$$

Let the carrier frequency be 50 kHz and $A_c = 100$. The transmitter is tested using two sinusoidally modulated waveforms that are $m_1(t) = 3\cos(\omega_1 t)$ and $m_2(t) = 4\cos(\omega_2 t)$ where $f_1 = 2$ kHz and $f_2 = 5$ kHz. Using MATLAB, plot the output voltage waveform for this transmitter.

5–26 Using MATLAB, calculate the actual PEP for the transmitter described in Prob. 5–25 if the resisitive load on the transmitter is 50 Ω.

★ **5–27** A sinusoidal signal $m(t) = \cos 2\pi f_m t$ is the input to an angle-modulated transmitter, where the carrier frequency is $f_c = 1$ Hz and $f_m = f_c/4$.
(a) Plot $m(t)$ and the corresponding PM signal, where $D_p = \pi$.
(b) Plot $m(t)$ and the corresponding FM signal, where $D_f = \pi$.

5–28 A sinusoidal modulating waveform of amplitude 4 V and frequency 1 kHz is applied to an FM exciter that has a modulator gain of 50 Hz/V.
(a) What is the peak frequency deviation?
(b) What is the modulation index?

5–29 An FM signal has sinusoidal modulation with a frequency of $f_m = 15$ kHz and modulation index of $\beta = 2.0$.
(a) Find the transmission bandwidth by using Carson's rule.
(b) What percentage of the total FM signal power lies within the Carson rule bandwidth?

★ **5–30** An FM transmitter has the block diagram shown in Fig. Fig. P5–30. The audio frequency response is flat over the 20-Hz-to-15-kHz audio band. The FM output signal is to have a carrier frequency of 103.7 MHz and a peak deviation of 75 kHz.
(a) Find the bandwidth and center frequency required for the bandpass filter.
(b) Calculate the frequency f_0 of the oscillator.
(c) What is the required peak deviation capability of the FM exciter?

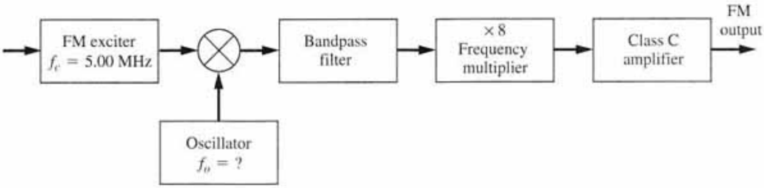

Figure P5–30

5–31 Analyze the performance of the FM circuit of Fig. 5–8b. Assume that the voltage appearing across the reverse-biased diodes, which provide the voltage variable capacitance, is $v(t) = 5 + 0.05\,m(t)$, where the modulating signal is a test tone, $m(t) = \cos\omega_1 t$, $\omega_1 = 2\pi f_1$, and $f_1 = 1$ kHz. The capacitance of each of the biased diodes is $C_d = 100\sqrt{1 + 2v(t)}$ pF. Assume that $C_0 = 180$ pF and that L is chosen to resonate at 5 MHz.
(a) Find the value of L.
(b) Show that the resulting oscillator signal is an FM signal. For convenience, assume that the peak level of the oscillator signal is 10 V. Find the parameter D_f.

★ **5–32** A modulated RF waveform is given by $500 \cos [\omega_c t + 20 \cos \omega_1 t]$, where $\omega_1 = 2\pi f_1$, $f_1 = 1$ kHz, and $f_c = 100$ MHz.

(a) If the phase deviation constant is 100 rad/V, find the mathematical expression for the corresponding phase modulation voltage $m(t)$. What is its peak value and its frequency?

(b) If the frequency deviation constant is 1×10^6 rad/V-s, find the mathematical expression for the corresponding FM voltage $m(t)$. What is its peak value and its frequency?

(c) If the RF waveform appears across a 50-Ω load, determine the average power and the PEP.

5–33 Consider the FM signal $s(t) = 10 \cos [\omega_c t + 100 \int_{-\infty}^{t} m(\sigma)\, d\sigma]$, where $m(t)$ is a polar square-wave signal with a duty cycle of 50%, a period of 1 s, and a peak value of 5 V.

(a) Sketch the instantaneous frequency waveform and the waveform of the corresponding FM signal. (See Fig. 5–9.)

(b) Plot the phase deviation $\theta(t)$ as a function of time.

(c) Evaluate the peak frequency deviation.

5–34 A carrier $s(t) = 100 \cos (2\pi \times 10^9 t)$ of an FM transmitter is modulated with a tone signal. For this transmitter, a 1-V (RMS) tone produces a deviation of 30 kHz. Determine the amplitude and frequency of all FM signal components (spectral lines) that are greater than 1% of the unmodulated carrier amplitude if the modulating signal is $m(t) = 2.5 \cos (3\pi \times 10^4 t)$.

5–35 Rework Prob. 5–34 if the modulating signal is $m(t) = 1 \cos (6\pi \times 10^4 t)$.

5–36 Referring to Eq. (5–58), show that

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$

★ **5–37** Consider an FM exciter with the output $s(t) = 100 \cos [2\pi 1{,}000 t + \theta(t)]$. The modulation is $m(t) = 5 \cos (2\pi 8 t)$, and the modulation gain of the exciter is 8 Hz/V. The FM output signal is passed through an ideal (brick-wall) bandpass filter that has a center frequency of 1,000 Hz, a bandwidth of 56 Hz, and a gain of unity. Determine the normalized average power

(a) At the bandpass filter input.

(b) At the bandpass filter output.

★ **5–38** A 1-kHz sinusoidal signal phase modulates a carrier at 146.52 MHz with a peak phase deviation of 45°. Evaluate the exact magnitude spectra of the PM signal if $A_c = 1$. Sketch your result. Using Carson's rule, evaluate the approximate bandwidth of the PM signal, and see if it is a reasonable number when compared with your spectral plot.

5–39 A 1-kHz sinusoidal signal frequency modulates a carrier at 146.52 MHz with a peak deviation of 5 kHz. Evaluate the exact magnitude spectra of the FM signal if $A_c = 1$. Sketch your result. Using Carson's rule, evaluate the approximate bandwidth of the FM signal, and see if it is a reasonable number when compared with your spectral plot.

5–40 The calibration of a frequency deviation monitor is to be verified by using a Bessel function test. An FM test signal with a calculated frequency deviation is generated by frequency modulating a sine wave onto a carrier. Assume that the sine wave has a frequency of 2 kHz and that the amplitude of the sine wave is slowly increased from zero until the discrete carrier term (at f_c) of the FM signal vanishes, as observed on a spectrum analyzer. What is the peak frequency deviation of the FM test signal when the discrete carrier term is zero? Suppose that the amplitude of the sine wave is increased further until this discrete carrier term appears, reaches a maximum, and then disappears again. What is the peak frequency deviation of the FM test signal now?

5–41 A frequency modulator has a modulator gain of 10 Hz/V, and the modulating waveform is

$$m(t) = \begin{cases} 0, & t < 0 \\ 5, & 0 < t < 1 \\ 15, & 1 < t < 3 \\ 7, & 3 < t < 4 \\ 0, & 4 < t \end{cases}$$

(a) Plot the frequency deviation in hertz over the time interval $0 < t < 5$.
(b) Plot the phase deviation in radians over the time interval $0 < t < 5$.

★ **5–42** A square-wave (digital) test signal of 50% duty cycle phase modulates a transmitter where $s(t) = 10 \cos [\omega_c t + \theta(t)]$. The carrier frequency is 60 MHz and the peak phase deviation is 45°. Assume that the test signal is of the unipolar NRZ type with a period of 1 ms and that it is symmetrical about $t = 0$. Find the exact spectrum of $s(t)$.

5–43 Two sinusoids, $m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$, phase modulate a transmitter. Derive a formula that gives the exact spectrum for the resulting PM signal in terms of the signal parameters A_c, ω_c, D_p, A_1, A_2, ω_1, and ω_2. [*Hint:* Use $e^{ja(t)} = (e^{ja_1(t)})(e^{ja_2(t)})$, where $a(t) = a_1(t) + a_2(t)$.]

5–44 Plot the magnitude spectrum centered on $f = f_c$ for an FM signal where the modulating signal is

$$m(t) = A_1 \cos 2\pi f_1 t + A_2 \cos 2\pi f_2 t$$

Assume that $f_1 = 10$ Hz and $f_2 = 17$ Hz, and that A_1 and A_2 are adjusted so that each tone contributes a peak deviation of 20 Hz.

5–45 For small values of β, $J_n(\beta)$ can be approximated by $J_n(\beta) = \beta^n/(2^n n!)$. Show that, for the case of FM with sinusoidal modulation, $\beta = 0.2$ is sufficiently small to give NBFM.

★ **5–46** A polar square wave with a 50% duty cycle frequency modulates an NBFM transmitter such that the peak phase deviation is 10°. Assume that the square wave has a peak value of 5 V, a period of 10 ms, and a zero-crossing at $t = 0$ with a positive-going slope.
(a) Determine the peak frequency deviation of this NBFM signal.
(b) Evaluate and sketch the spectrum of the signal, using the narrowband analysis technique. Assume that the carrier frequency is 30 MHz.

5–47 Design a wideband FM transmitter that uses the indirect method for generating a WBFM signal. Assume that the carrier frequency of the WBFM signal is 96.9 MHz and that the transmitter is capable of producing a high-quality FM signal with a peak deviation of 75 kHz when modulated by a 1-V (rms) sinusoid of frequency 20 Hz. Show a complete block diagram of your design, indicating the frequencies and peak deviations of the signals at various points.

5–48 An FM signal, $[w_c t + D_f \int_{-\infty}^{t} m(\sigma) d\sigma]$ is modulated by the waveform shown in Fig. P5–48. Let $f_c = 420$ MHz.
(a) Determine the value of D_f so that the peak-to-peak frequency deviation is 25 kHz.
(b) Evaluate and sketch the approximate PSD.
(c) Determine the bandwidth of this FM signal such that spectral components are down at least 40 dB from the unmodulated carrier level for frequencies outside that bandwidth.

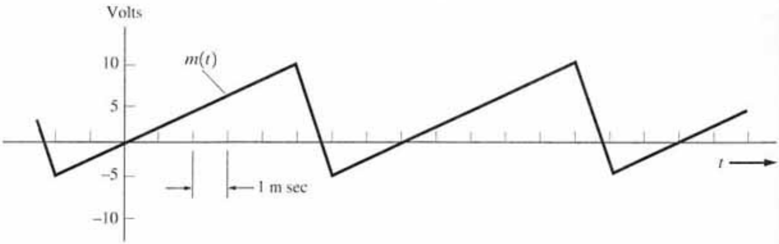

Figure P5–48

★ **5-49** A periodic multilevel digital test signal as shown in Fig. P5–49 modulates a WBFM transmitter. Evaluate and sketch the approximate power spectrum of this WBFM signal if $A_c = 5$, $f_c = 2$ GHz, and $D_f = 10^5$.

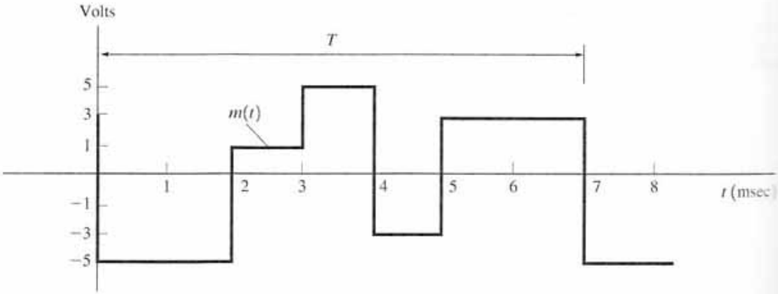

Figure P5–49

5–50 Refer to Fig. 5–16b, which displays the circuit diagram for a commonly used preemphasis filter network.

(a) Show that $f_1 = 1/(2\pi R_1 C)$.

(b) Show that $f_2 = (R_1 + R_2)/(2\pi R_1 R_2 C) \approx 1/(2\pi R_2 C)$.

(c) Evaluate K in terms of R_1, R_2, and C.

5–51 The composite baseband signal for FM stereo transmission is given by

$$m_b(t) = [m_L(t) + m_R(t)] + [m_L(t) - m_R(t)] \cos(\omega_{sc} t) + K \cos(\tfrac{1}{2}\omega_{sc} t)$$

A stereo FM receiver that uses a switching type of demultiplexer to recover $m_L(t)$ and $m_R(t)$ is shown in Fig. P5–51.

(a) Determine the switching waveforms at points C and D that activate the analog switches. Be sure to specify the correct phasing for each one. Sketch these waveforms.

(b) Draw a more detailed block diagram showing the blocks inside the PLL.

(c) Write equations for the waveforms at points A through F, and explain how this circuit works by sketching typical waveforms at each of these points.

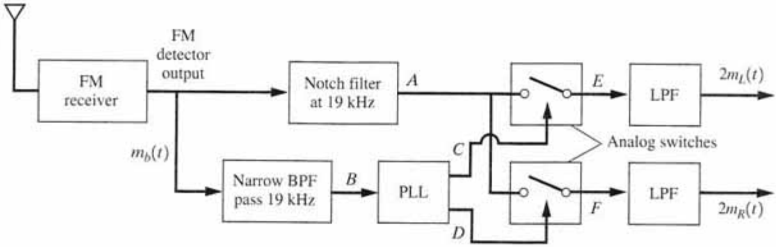

Figure P5–51

★ **5–52** In a communication system, two baseband signals (they may be analog or digital) are transmitted simultaneously by generating the RF signal

$$s(t) = m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t$$

The carrier frequency is 7.250 MHz. The bandwidth of $m_1(t)$ is 5 kHz and the bandwidth of $m_2(t)$ is 10 kHz.

(a) Evaluate the bandwidth of $s(t)$.

(b) Derive an equation for the spectrum of $s(t)$ in terms of $M_1(f)$ and $M_2(f)$.

(c) $m_1(t)$ and $m_2(t)$ can be recovered (i.e., detected) from $s(t)$ by using a superheterodyne receiver with two switching detectors. Draw a block diagram for the receiver and sketch the digital waveforms that are required to operate the samplers. Describe how they can be obtained. Show that the Nyquist sampling criterion is satisfied. (*Hint:* See Fig. P5–51.)

★ **5–53** A digital baseband signal consisting of rectangular binary pulses occurring at a rate of 24 kbits/s is to be transmitted over a bandpass channel.

(a) Evaluate the magnitude spectrum for OOK signaling that is keyed by a baseband digital test pattern consisting of alternating 1's and 0's.

(b) Sketch the magnitude spectrum and indicate the value of the first null-to-null bandwidth. Assume a carrier frequency of 150 MHz.

5–54 Referring to Prob. 5–53 for the case of a random data pattern and OOK signaling, find the PSD and plot the result. Compare this result with that for the magnitude spectrum obtained in Prob. 5–53.

5–55 Repeat Prob. 5–53 for the case of BPSK signaling.

5–56 Referring to Prob. 5–53 for the case of a random data pattern with BPSK signaling, find the PSD and plot the result. Compare this result with that for the magnitude spectrum obtained in Prob. 5–55.

★ **5–57** A carrier is angle modulated with a polar baseband data signal to produce a BPSK signal $s(t) = 10 \cos [\omega_c t + D_p m(t)]$, where $m(t) = \pm 1$ corresponds to the binary data 10010110. $T_b = 0.0025$ sec and $\omega_c = 1{,}000\pi$. Using MATLAB, plot the BPSK signal waveform and its corresponding FFT spectrum for the following digital modulation indices:

(a) $h = 0.2$.

(b) $h = 0.5$.

(c) $h = 1.0$.

5–58 Evaluate the magnitude spectrum for an FSK signal with alternating 1 and 0 data. Assume that the mark frequency is 50 kHz, the space frequency is 55 kHz, and the bit rate is 2,400 bits/s. Find the first null-to-null bandwidth.

5–59 Assume that 4,800-bit/s random data are sent over a bandpass channel by BPSK signaling. Find the transmission bandwidth B_T such that the spectral envelope is down at least 35 dB outside the band.

5–60 As indicated in Fig. 5–22a, a BPSK signal can be demodulated by using a coherent detector wherein the carrier reference is provided by a Costas loop for the case of $h = 1.0$. Alternatively, the carrier reference can be provided by a squaring loop that uses a $\times 2$ frequency multiplier. A block diagram for a squaring loop is shown in Fig. P5–60.

(a) Using the squaring loop, draw an overall block diagram for a BPSK receiver.

(b) By using mathematics to represent the waveforms, show how the squaring loop recovers the carrier reference.

(c) Demonstrate that the squaring loop does or does not have a 180° phase ambiguity problem.

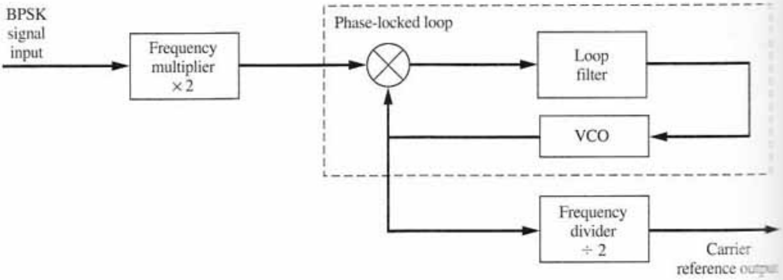

Figure P5–60

5–61 A binary data signal is differentially encoded and modulates a PM transmitter to produce a differentially encoded phase-shift-keyed signal (DPSK). The peak-to-peak phase deviation is 180° and f_c is harmonically related to the bit rate R.

(a) Draw a block diagram for the transmitter, including the differential encoder.

(b) Show typical waveforms at various points on the block diagram if the input data sequence is 01011000101.

(c) Assume that the receiver consists of a superheterodyne circuit. The detector that is used is shown in Fig. P5-61, where $T = 1/R$. If the DPSK IF signal $v_1(t)$ has a peak value of A_c volts, determine the appropriate value for the threshold voltage setting V_T.

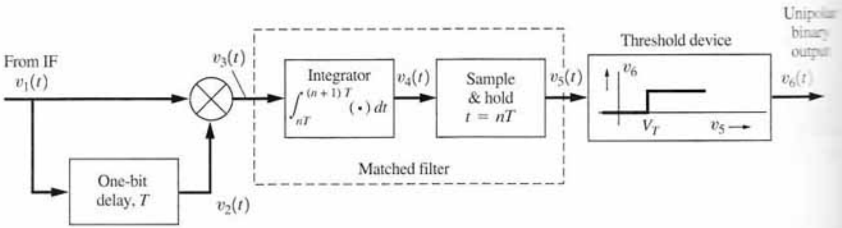

Figure P5–61

(d) Sketch the waveforms that appear at various points of this detector circuit for the data sequence in part (b).

★ **5–62** A binary baseband signal is passed through a raised cosine-rolloff filter with a 50% rolloff factor and is then modulated onto a carrier. The data rate is 64 kbits/s. Evaluate

(a) The absolute bandwidth of a resulting OOK signal.

(b) The approximate bandwidth of a resulting FSK signal when the mark frequency is 150 kHz and the space frequency is 155 kHz.

(*Note:* It is interesting to compare these bandwidths with those obtained in Probs. 5–53 and 5–58.)

5–63 Evaluate the exact magnitude spectrum of the FSK signal that is emitted by a Bell-type 103 modem operating in the answer mode at 300 bits/s. Assume that the data are alternating 1's and 0's.

5–64 Starting with Eq. (5–84), work out all of the mathematical steps to derive the result given in Eq. (5–85).

5–65 When FSK signals are detected using coherent detection as shown in Fig. 5–28b, it is assumed that $\cos \omega_1 t$ and $\cos \omega_2 t$ are orthogonal functions. This is approximately true if $f_1 - f_2 = 2\,\Delta F$ is sufficiently large. Find the *exact* condition required for the mark and space FSK signals to be orthogonal. (*Hint:* The answer relates f_1, f_2, and R.)

5–66 Show that the approximate transmission bandwidth for FSK is given by $B_T = 2R\,(1 + h/2)$, where h is the digital modulation index and R is the bit rate.

★ **5–67** Assume that a QPSK signal is used to send data at a rate of 30 Mbits/s over a satellite transponder. The transponder has a bandwidth of 24 MHz.

(a) If the satellite signal is equalized to have an equivalent raised cosine filter characteristic, what is the rolloff factor r required?

(b) Could a rolloff factor r be found so that a 50-Mbit/s data rate could be supported?

★ **5–68** A QPSK signal is generated with nonrectangular data pulses on the I and Q channels. The data pulses have spectra corresponding to the transfer function of a square-root raised cosine-rolloff filter.

(a) Find the formula for the PSD of the QPSK signal.

(b) Plot the PSD for the complex envelope of the QPSK signal, where the rolloff factor is $r = 0.35$ and the data rate is normalized to $R = 1$ bit/s. Plot your result in dB versus frequency normalized to the bit rate, similar to that shown in Fig. 5–33.

5–69 Show that two BPSK systems can operate simultaneously over the same channel by using quadrature (sine and cosine) carriers. Give a block diagram for the transmitters and receivers. What is the overall aggregate data rate R for this quadrature carrier multiplex system as a function of the channel null bandwidth B_T? How does the aggregate data rate of the system compare to the data rate for a system that time-division multiplexes the two sources and then transmits the TDM data via a QPSK carrier?

5–70 An 18,000-ft twisted-pair telephone line has a usable bandwidth of 750 kHz. Find the maximum data rate tha can be supported on this line to produce a null-to-null bandwidth of 750 kHz for the case of QPSK signaling (rectangular pulses) with a single carrier.

5–71 Rework Prob. 5–70 for the case of OFDM signaling with QPSK carriers.

5–72 Assume that a telephone line channel is equalized to allow bandpass data transmission over a frequency range of 400 to 3,100 Hz so that the available channel bandwidth is 2,700 Hz and the midchannel frequency is 1,750 Hz. Design a 16-symbol QAM signaling scheme that will allow a data rate of 9,600 bits/s to be transferred over the channel. In your design, choose an appropriate rolloff factor r and indicate the absolute and 6-dB QAM signal bandwidth. Discuss why you selected the particular value of r that you used.

5–73 Assume that $R = 9{,}600$ bits/s. For rectangular data pulses, calculate the *second* null-to-null bandwidth of BPSK, QPSK, MSK, 64PSK, and 64QAM. Discuss the advantages and disadvantages of using each of these signaling methods.

5–74 Referring to Fig. 5–31b, sketch the waveforms that appear at the output of each block, assuming that the input is a TTL-level signal with data 110100101 and $\ell = 4$. Explain how this QAM transmitter works.

5–75 Design a receiver (i.e., determine the block diagram) that will detect the data on a QAM waveform having an $M = 16$-point signal constellation as shown in Fig. 5–32. Explain how your receiver works. (*Hint:* Study Fig. 5–31b.)

5–76 Using MATLAB, plot the QPSK and OQPSK in-phase and quadrature modulation waveforms for the data stream

$$\{-1, -1, -1, +1, +1, +1, -1, -1, -1, +1, -1, -1, +1, -1, -1\}$$

Use a rectangular pulse shape. For convenience, let $T_b = 1$.

★ **5–77** For $\pi/4$ QPSK signaling, calculate the carrier phase shifts when the input data stream is 10110100101010, where the leftmost bits are first applied to the transmitter.

5–78 For $\pi/4$ QPSK signaling, find the absolute bandwidth of the signal if $r = 0.5$ raised cosine-rolloff filtering is used and the data rate is 1.5 Mbits/s.

5–79 **(a)** Fig. 5–34 shows the $x(t)$ and $y(t)$ waveforms for Type II MSK. Redraw these waveforms for the case of Type I MSK.
(b) Show that Eq. (5–114b) is the Fourier transform of Eq. (5–114a).

5–80 Using MATLAB, plot the MSK Type I modulation waveforms $x(t)$ and $y(t)$ and the MSK signal $s(t)$. Assume that the input data stream is

$$\{+1, -1, -1, +1, -1, -1, +1, -1, -1, +1, +1, -1, +1, +1, -1\}$$

Assume also that $T_b = 1$ and that f_c has a value such that a good plot of $s(t)$ is obtained in a reasonable amount of computer time.

5–81 Repeat Prob. 5-80, but use the data stream

$$\{-1, -1, -1, +1, +1, +1, -1, -1, -1, +1, -1, -1, +1, -1, -1\}$$

This data stream is the differentially encoded version of the data stream in Prob. 5-80. The generation of FFSK is equivalent to Type I MSK with differential encoding of the data input. FFSK has a one-to-one relationship between the input data and the mark and space frequencies.

★ **5–82** Using MATLAB, plot the MSK Type II modulation waveforms $x(t)$ and $y(t)$ and the MSK signal $s(t)$. Assume that the input data stream is

$$\{-1, -1, -1, +1, +1, +1, -1, -1, -1, +1, -1, -1, +1, -1, -1\}$$

Assume also that $T_b = 1$ and f_c has a value such that a good plot of $s(t)$ is obtained in a reasonable amount of computer time.

5–83 Show that MSK can be generated by the serial method of Fig. 5–36c. That is, show that the PSD for the signal at the output of the MSK bandpass filter is the MSK spectrum as described by Eqs. (5–115) and (5–2b).

★ **5–84** GMSK is generated by filtering rectangular-shaped data pulses with a Gaussian filter and applying the filtered signal to an MSK transmitter.

(a) Show that the Gaussian filtered data pulse is

$$p(t) = \left(\sqrt{\frac{2\pi}{\ln 2}}\right)(BT_b)\int_{\frac{t}{TB_b}-\frac{1}{2}}^{\frac{t}{TB_b}+\frac{1}{2}} e^{-\left[\frac{2\pi^2}{\ln 2}(BT_b)^2 x^2\right]}\,dx$$

[*Hint:* Evaluate $p(t) = h(t) * \Pi(t/T_b)$, where $h(t) = \mathscr{F}^{-1}[H(f)]$ and $H(f)$ is described by Eq. (5–116).]

(b) Plot $p(t)$ for $BT_b = 0.3$ and T_b normalized to $T_b = 1$.

5–85 Recompute the spectral efficiencies for all of the signals shown in Table 5–7 by using a 40-dB bandwidth criterion.

5–86 Evaluate and plot the PSD for OFDM signaling with $N = 64$. Find the bandwidth of this OFDM signal if the input data rate is 10 Mbits/s and each carrier uses 16PSK modulation.

5–87 Prove that Eq. (5–123)—the autocorrelation for an m-sequence PN code—is correct. *Hint:* Use the definition of the autocorrelation function, $R_c(\tau) = \langle c(t)\, c(t+\tau)\rangle$ and Eq. (5–122), where

$$c(t) = \sum_{-\infty}^{\infty} c_n p(t - nT_c)$$

and

$$p(t) = \begin{cases} 1, & 0 < t < T_c \\ 0, & t \text{ elsewhere} \end{cases}$$

5–88 Find an expression for the PSD of an m-sequence PN code when the chip rate is 10 MHz and there are eight stages in the shift register. Sketch your result.

5–89 Referring to Fig. 5–40a, show that the complex Fourier series coefficients for the autocorrelation of an m-sequence PN waveform are given by Eq. (5–128).

★ **5–90** Assume that the modulator and demodulator for the FH-SS system of Fig. 5–42 are of the FSK type.

(a) Find a mathematical expression for the FSK-FH-SS signal $s(t)$ at the transmitter output.

(b) Using your result for $s(t)$ from part (a) as the receiver input in Fig. 5–42b [i.e., $r(t) = s(t)$], show that the output of the receiver bandpass filter is an FSK signal.

SA6–4 PSD for a Bandpass Process A bandpass process is described by

$$v(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

where $y(t) = x(t)$ is a WSS process with a PSD as shown in Fig. 6–22a. θ_c is an independent random variable uniformly distributed over $(0, 2\pi)$. Find the PSD for $v(t)$.

Solution From Eq. (6–130), we know that $v(t)$ is a WSS bandpass process, with $\mathcal{P}_v(f)$ given by Eq. (6–133d). Thus, $\mathcal{P}_g(f)$ needs to be evaluated. Also,

$$v(t) = \mathrm{Re}\{g(t)e^{j(\omega_c t+\theta_c)}\}$$

where $g(t) = x(t) + jy(t) = x(t) + jx(t) = (1 + j)x(t)$. We have, then,

$$R_g(\tau) = \overline{g^*(t)g(t+\tau)} = \overline{(1-j)(1+j)x(t)x(t+\tau)}$$
$$= (1+1)R_x(\tau) = 2R_x(\tau)$$

Thus,

$$\mathcal{P}_g(f) = 2\mathcal{P}_x(f) \qquad (6\text{–}199)$$

Substituting Eq. (6–199) into Eq. (6–133d), we get

$$\mathcal{P}_v(f) = \frac{1}{2}\left[\mathcal{P}_x\left(f - f_c\right) + \mathcal{P}_x\left(-f - f_c\right)\right] \qquad (6\text{–}200)$$

$\mathcal{P}_v(f)$ is plotted in Fig. 6–22b for the $\mathcal{P}_x(f)$ of Fig. 6–22a.

PROBLEMS

6–1 Let a random process $x(t)$ be defined by

$$x(t) = At + B$$

(a) If B is a constant and A is uniformly distributed between -1 and $+1$, sketch a few sample functions.
(b) If A is a constant and B is uniformly distributed between 0 and 2, sketch a few sample functions.

★ **6–2** Let a random process be given by

$$x(t) = A\cos(\omega_0 t + \theta)$$

where A and ω_0 are constants and θ is a random variable. Let

$$f(\theta) = \begin{cases} \dfrac{2}{\pi}, & 0 \le \theta \le \dfrac{\pi}{2} \\ 0, & \text{elsewhere} \end{cases}$$

(a) Evaluate $\overline{x(t)}$.
(b) From the result of part (a), what can be said about the stationarity of the process?

6–3 Using the random process described in Prob. 6–2,

(a) Evaluate $\langle x^2(t) \rangle$.

(b) Evaluate $\overline{x^2(t)}$.

(c) Using the results of parts (a) and (b), determine whether the process is ergodic for these averages.

6–4 Let $x(t)$ be a sinusoidal random process that has a uniformly distributed phase angle as described in Case 1 of Example 6–2. Using MATLAB, plot the PDF for the random process where $A = 5$ volts.

★ **6–5** A conventional average-reading AC voltmeter (volt-ohm multimeter) has a schematic diagram as shown in Fig. P6–5. The needle of the meter movement deflects proportionally to the average current flowing through the meter. The meter scale is marked to give the RMS value of sine-wave voltages. Suppose that this meter is used to determine the RMS value of a noise voltage. The noise voltage is known to be an ergodic Gaussian process having a zero mean value. What is the value of the constant that is multiplied by the meter reading to give the true RMS value of the Gaussian noise? (*Hint*: The diode is a short circuit when the input voltage is positive and an open circuit when the input voltage is negative.)

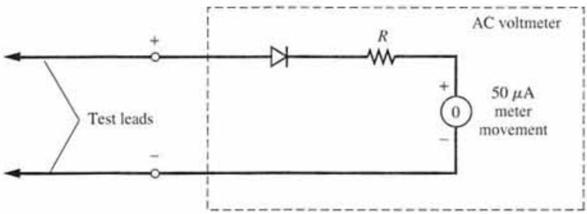

Figure P6–5

6–6 Let $x(t) = A_0 \sin(\omega_0 t + \theta)$ be a random process, where θ is a random variable that is uniformly distributed between 0 and 2π and A_0 and ω_0 are constants.

(a) Find $R_x(\tau)$.

(b) Show that $x(t)$ is wide-sense stationary.

(c) Verify that $R_x(\tau)$ satisfies the appropriate properties.

6–7 Let $r(t) = A_0 \cos \omega_0 t + n(t)$, where A_0 and ω_0 are constants. Assume that $n(t)$ is a wide-sense stationary random noise process with a zero mean value and an autocorrelation of $R_n(\tau)$.

(a) Find $\overline{r(t)}$ and determine whether $r(t)$ is wide-sense stationary.

(b) Find $R_r(t_1, t_2)$.

(c) Evaluate $\langle R_r(t, t+\tau) \rangle$, where $t_1 = t$ and $t_2 = t + \tau$.

6–8 Let an additive signal-plus-noise process be described by the equation $r(t) = s(t) + n(t)$.

(a) Show that $R_r(\tau) = R_s(\tau) + R_n(\tau) + R_{sn}(\tau) + R_{ns}(\tau)$.

(b) Simplify the result for part (a) for the case when $s(t)$ and $n(t)$ are independent and the noise has a zero mean value.

★ **6–9** Consider the sum of two ergodic noise voltages:

$$n(t) = n_1(t) + n_2(t)$$

The power of $n_1(t)$ is 5 W, the power of $n_2(t)$ is 10 W, the DC value of $n_1(t)$ is -2 V, and the DC value n_2 is $+1$ V. Find the power of $n(t)$ if

(a) $n_1(t)$ and $n_2(t)$ are orthogonal.

(b) $n_1(t)$ and $n_2(t)$ are uncorrelated.

(c) The cross-correlation of $n_1(t)$ and $n_2(t)$ is 2 for $\tau = 0$.

6–10 Assume that $x(t)$ is ergodic, and let $x(t) = m_x + y(t)$, where $m_x = \overline{x(t)}$ is the DC value of $x(t)$ and $y(t)$ is the AC component of $x(t)$. Show that

(a) $R_x(\tau) = m_x^2 + R_y(\tau)$.

(b) $\lim_{\tau \to \infty} R_x(\tau) = m_x^2$.

(c) Can the DC value of $x(t)$ be determined from $R_x(\tau)$?

★ **6–11** Determine whether the following functions satisfy the properties of autocorrelation functions:

(a) $\sin \omega_0 \tau$.

(b) $(\sin \omega_0 \tau)/(\omega_0 \tau)$.

(c) $\cos \omega_0 \tau + \delta(\tau)$.

(d) $e^{-a|\tau|}$, where $a < 0$.

(e) (*Note*: $\mathcal{F}[R(\tau)]$ must also be a nonnegative function.)

6–12 A random process $x(t)$ has an autocorrelation function given by $R_x(\tau) = 5 + 8e^{-3|\tau|}$. Find

(a) The RMS value for $x(t)$.

(b) The PSD for $x(t)$.

★ **6–13** The autocorrelation of a random process is $R_x(\tau) = 4e^{-\tau^2} + 3$. Plot the PSD for $x(t)$ and evaluate the RMS bandwidth for $x(t)$.

6–14 Show that two random processes $x(t)$ and $y(t)$ are uncorrelated (i.e., $R_{xy}(\tau) = m_x m_y$) if the processes are independent.

6–15 If $x(t)$ contains periodic components, show that

(a) $R_x(\tau)$ contains periodic components.

(b) $\mathcal{P}_x(f)$ contains delta functions.

6–16 Find the PSD for the random process described in Prob. 6–2.

6–17 Determine whether the following functions can be valid PSD functions for a real process:

(a) $2e^{-2\pi|f-45|}$.

(b) $4e^{-2\pi|f^2-16|}$.

(c) $25 + \delta(f - 16)$.

(d) $10 + \delta(f)$.

★ **6–18** The PSD of an ergodic random process $x(t)$ is

$$\mathcal{P}_x(f) = \begin{cases} \frac{1}{B}(B - |f|), & |f| \le B \\ 0, & f \text{ elsewhere} \end{cases}$$

where $B > 0$. Find

(a) The RMS value of $x(t)$.

(b) $R_x(\tau)$.

6–19 Referring to the techniques described in Example 6–4, evaluate the PSD for a PCM signal that uses Manchester NRZ encoding. (See Fig. 3–15.) Assume that the data have values of $a_n = \pm 1$, which are equally likely, and that the data are independent from bit to bit.

6–20 Using MATLAB, plot the PSD for a Manchester NRZ line code that has values of ± 1 that are equally likely. Assume that the data are independent from bit to bit and that the bit rate is 9600 b/s. *Hint:* Look at your solution for Prob 6–19 or look at Eq. (3–46c).

6–21 The *magnitude* frequency response of a linear time-invariant network is to be determined from a laboratory setup as shown in Fig. P6–21. Discuss how $|H(f)|$ is evaluated from the measurements.

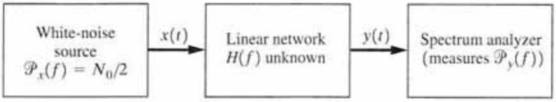

Figure P6–21

★ **6–22** A linear time-invariant network with an unknown $H(f)$ is shown in Fig. P6–22.

(a) Find a formula for evaluating $h(t)$ in terms of $R_{xy}(\tau)$ and N_0.

(b) Find a formula for evaluating $H(f)$ in terms of $\mathcal{P}_{xy}(f)$ and N_0.

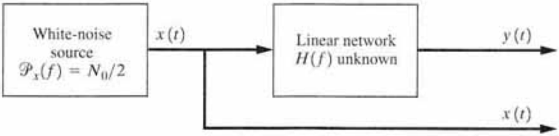

Figure P6–22

6–23 The output of a linear system is related to the input by $y(t) = h(t) * x(t)$, where $x(t)$ and $y(t)$ are jointly wide-sense stationary. Show that

(a) $R_{xy}(\tau) = h(\tau) * R_x(\tau)$.

(b) $\mathcal{P}_{xy}(f) = H(f)\,\mathcal{P}_x(f)$.

(c) $R_{yx}(\tau) = h(-\tau) * R_x(\tau)$.

(d) $\mathcal{P}_{yx}(f) = H^*(f)\,\mathcal{P}_x(f)$.

[*Hint:* Use Eqs. (6–86) and (6–87).]

6–24 Using MATLAB, plot the PSD for the noise out of a RC LPF for the case of white noise at the filter input. Let $N_0 = 2$ and $B_{3dB} = 3$ kHz.

★ **6–25** Ergodic white noise with a PSD of $\mathcal{P}_n(f) = N_0/2$ is applied to the input of an ideal integrator with a gain of K (a real number) such that $H(f) = K/(j2\pi f)$.

(a) Find the PSD for the output.

(b) Find the RMS value of the output noise.

6–26 A linear system has a power transfer function $|H(f)|^2$ as shown in Fig. P6–26. The input $x(t)$ is a Gaussian random process with a PSD given by

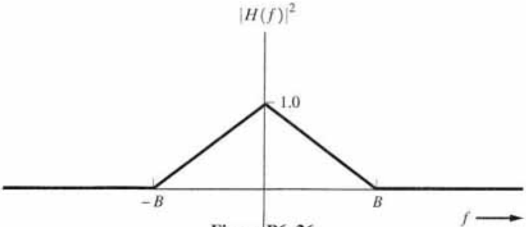

Figure P6–26

$$\mathcal{P}_x(f) = \begin{cases} \frac{1}{2}N_0, & |f| \le 2B \\ 0, & f \text{ elsewhere} \end{cases}$$

(a) Find the autocorrelation function for the output $y(t)$.
(b) Find the PDF for $y(t)$.
(c) When are the two random variables $y_1 = y(t_1)$ and $y_2 = y(t_2)$ independent?

6–27 A linear filter evaluates the T-second moving average of an input waveform, where the filter output is

$$y(t) = \frac{1}{T}\int_{t-(T/2)}^{t+(T/2)} x(u)\,du$$

and $x(t)$ is the input. Show that the impulse response is $h(t) = (1/T)\,\Pi\,(t/T)$.

6–28 For Problem 6–27 show that

$$R_y(\tau) = \frac{1}{T}\int_{-T}^{T}\left(1 - \frac{|u|}{T}\right)R_x(\tau - u)\,du$$

If $R_x(\tau) = e^{-|\tau|}$ and $T = 1$ sec, plot $R_y(\tau)$, and compare it with $R_x(\tau)$.

★ **6–29** As shown in Example 6–8, the output signal-to-noise ratio of an *RC* LPF is given by Eq. (6–95) when the input is a sinusoidal signal plus white noise. Derive the value of the RC product such that the output signal-to-noise ratio will be a maximum.

6–30 Assume that a sine wave of peak amplitude A_0 and frequency f_0, plus white noise with $\mathcal{P}_n(f) = N_0/2$, is applied to a linear filter. The transfer function of the filter is

$$H(f) = \begin{cases} \frac{1}{B}\,(B - |f|), & |f| < B \\ 0, & f \text{ elsewhere} \end{cases}$$

where B is the absolute bandwidth of the filter. Find the signal-to-noise power ratio for the filter output.

6–31 For the random process $x(t)$ with the PSD shown in Fig. P6–31, determine
(a) The equivalent bandwidth.
(b) The RMS bandwidth.

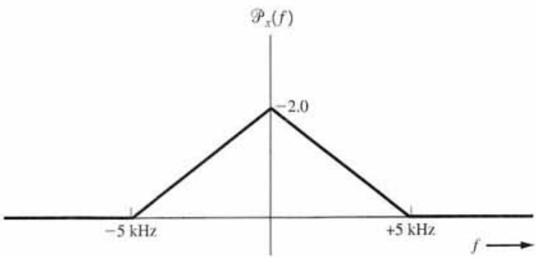

Figure P6–31

6–32 If $x(t)$ is a real bandpass random process that is wide-sense stationary, show that the definition of the RMS bandwidth, Eq. (6–100), is equivalent to

$$B_{\text{rms}} = 2\sqrt{\overline{f^2} - (f_0)^2}$$

where $\overline{f^2}$ is given by Eq. (6–98) or Eq. (6–99) and f_0 is given by Eq. (6–102).

6–33 In the definition for the RMS bandwidth of a *bandpass* random process, f_0 is used. Show that

$$f_0 = \frac{1}{2\pi R_x(0)}\left(\frac{d\hat{R}_x(\tau)}{d\tau}\right)\bigg|_{\tau=0}$$

where $\hat{R}_x(\tau)$ is the Hilbert transform of $R_x(\tau)$.

★ **6–34** Two identical *RC* LPFs are coupled in cascade by an isolation amplifier that has a voltage gain of 10.

(a) Find the overall transfer function of the network as a function of *R* and *C*.

(b) Find the 3-dB bandwidth in terms of *R* and *C*.

6–35 Let $x(t)$ be a Gaussian process in which two random variables are $x_1 = x(t_1)$ and $x_2 = x(t_2)$. The random variables have variances of σ_1^2 and σ_2^2 and means of m_1 and m_2. The correlation coefficient is

$$\rho = \overline{(x_1 - m_1)(x_2 - m_2)}/(\sigma_1\sigma_2)$$

Using matrix notation for the $N = 2$-dimensional PDF, show that the equation for the PDF of $\mathbf{x}$ reduces to the bivariate Gaussian PDF as given by Eq. (B–97).

6–36 A bandlimited white Gaussian random process has an autocorrelation function that is specified by Eq. (6–125). Show that as $B \to \infty$, the autocorrelation function becomes $R_n(\tau) = \frac{1}{2}N_0\delta(\tau)$.

★ **6–37** Let two random processes $x(t)$ and $y(t)$ be jointly Gaussian with zero-mean values. That is, $(x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_M)$ is described by an $(N + M)$-dimensional Gaussian PDF. The cross-correlation is

$$R_{xy}(\tau) = \overline{x(t_1)y(t_2)} = 10\sin(2\pi\tau)$$

(a) When are the random variables $x_1 = x(t_1)$ and $y_2 = y(t_2)$ independent?

(b) Show that $x(t)$ and $y(t)$ are or are not independent random processes.

6–38 Starting with Eq. (6–121), show that

$$\mathbf{C}_y = \mathbf{H}\mathbf{C}_x\mathbf{H}^T$$

(*Hint:* Use the identity matrix property, $\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.)

★ **6–39** Consider the random process

$$x(t)A_0\cos(\omega_c t + \theta)$$

where A_0 and ω_0 are constants and θ is a random variable that is uniformly distributed over the interval $(0, \pi/2)$. Determine whether $x(t)$ is wide-sense stationary.

★ **6–40** Referring to Prob. 6–39, find the PSD for $x(t)$.

★ **6–41** Referring to Prob. 6–39, if θ is uniformly distributed over $(0, 2\pi)$, is $x(t)$ wide-sense stationary?

6–42 A bandpass WSS random process $v(t)$ is represented by Eq. (6–129a), where the conditions of Eq. (6–129) are satisfied. The PSD of $v(t)$ is shown in Fig. P6–42, where $f_c = 1$ MHz. Using MATLAB

(a) Plot $\mathcal{P}_x(f)$.

(b) Plot $\mathcal{P}_{xy}(f)$.

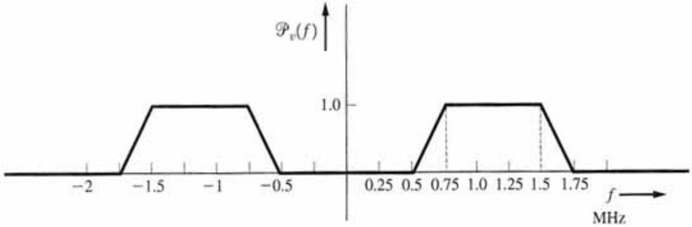

Figure P6–42

★**6–43** The PSD of a bandpass WSS process $v(t)$ is shown in Fig. P6–43. $v(t)$ is the input to a product detector, and the oscillator signal (i.e., the second input to the multiplier) is $5 \cos (\omega_c t + \theta_0)$, where $f_c = 1$ MHz and θ_0 is an independent random variable with a uniform distribution over $(0, 2\pi)$. Using MATLAB, plot the PSD for the output of the product detector.

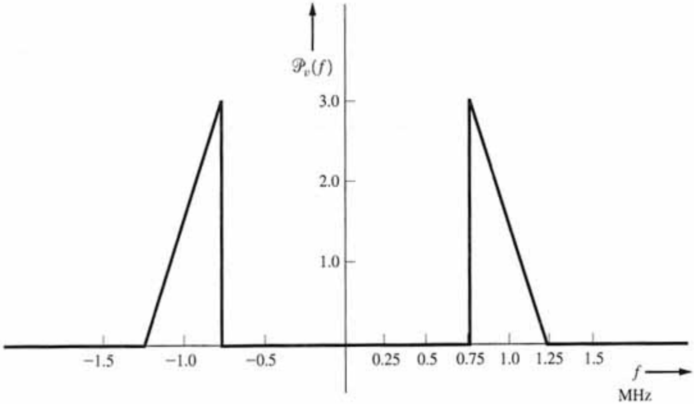

Figure P6–43

6–44 A WSS bandpass process $v(t)$ is applied to a product detector as shown in Fig. 6–11, where $\theta_c = 0$.

(a) Derive an expression for the autocorrelation of $w_1(t)$ in terms of $R_v(\tau)$. Is $w_1(t)$ WSS?

(b) Use $R_{w_1}(\tau)$ obtained in part (a) to find an expression for $\mathcal{P}_{w_1}(f)$. (*Hint:* Use the Wiener–Khintchine theorem.)

★ **6–45** A USSB signal is

$$v(t) = 10\,\mathrm{Re}\{[x(t) + j\hat{x}(t)]e^{j(\omega_c t+\theta_c)}\}$$

where θ_c is a random variable that is uniformly distributed over $(0, 2\pi)$. The PSD for $x(t)$ is given in Fig. P6–31. Find

(a) The PSD for $v(t)$.

(b) The total power of $v(t)$.

6–46 Show that $R_{\hat{x}}(\tau) = R_x(\tau)$.

6–47 Show that $R_{x\hat{x}}(\tau) = \hat{R}_x(\tau)$,

where the caret symbol denotes the Hilbert transform.

6–48 A bandpass random signal can be represented by

$$s(t) = x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

where the PSD of $s(t)$ is shown in Fig. P6–48. θ_c is an independent random variable that is uniformly distributed over $(0, 2\pi)$. Assume that $f_3 - f_2 = f_2 - f_1$. Find the PSD for $x(t)$ and $y(t)$ when

(a) $f_c = f_1$. This is USSB signaling, where $y(t) = \hat{x}(t)$.

(b) $f_c = f_2$. This represents independent USSB and LSSB signaling with two different modulations.

(c) $f_1 < f_c < f_2$. This is vestigial sideband signaling.

(d) For which, if any, of these cases are $x(t)$ and $y(t)$ orthogonal?

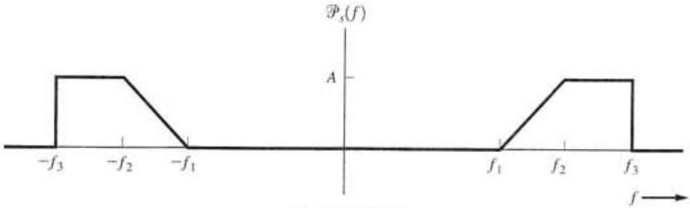

Figure P6–48

★ **6–49** Referring to Prob. 6–48(b), how are the two modulations $m_1(t)$ and $m_2(t)$ for the independent sidebands related to $x(t)$ and $y(t)$? Give the PSD for $m_1(t)$ and $m_2(t)$, where $m_1(t)$ is the modulation on the USSB portion of the signal and $m_2(t)$ is the modulation on the LSSB portion of $s(t)$.

6–50 For the bandpass random process, show that Equation (6–133m) is valid (property 13).

6–51 For the bandpass random process, show that Equation (6–133i) is valid (property 9).

★ **6–52** Referring to Example 6–12, find the PSD for a BPSK signal with Manchester-encoded data. (See Fig. 3–15.) Assume that the data have values of $a_n = \pm 1$ which are equally likely and that the data are independent from bit to bit.

6–53 The input to an envelope detector is an ergodic bandpass Gaussian noise process. The RMS value of the input is 2 V and the mean value is 0 V. The envelope detector has a voltage gain of 10. Find

(a) The DC value of the output voltage.

(b) The RMS value of the output voltage.

6–54 A narrowband-signal-plus-Gaussian noise process is represented by the equation

$$r(t) = A\cos(\omega_c t + \theta_c) + x(t)\cos(\omega_c t + \theta_c) - y(t)\sin(\omega_c t + \theta_c)$$

where $A\cos(\omega_c t + \theta_c)$ is a sinusoidal carrier and the remaining terms are the bandpass noise with independent Gaussian I and Q components. Let the noise have an RMS value of σ and a mean of 0. The signal-plus-noise process appears at the input to an envelope detector. Show that the PDF for the output of the envelope detector is

$$f(R) = \begin{cases} \dfrac{R}{\sigma^2} e^{-[(R^2+A^2)/(2\sigma^2)]} I_0\left(\dfrac{RA}{\sigma^2}\right), & R \geq 0 \\ 0, & R < 0 \end{cases}$$

where

$$I_0(z) \triangleq \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta}\, d\theta$$

is the modified Bessel function of the first kind of zero order. $f(R)$ is known as a *Rician* PDF in honor of the late S. O. Rice, who was an outstanding engineer at Bell Telephone Laboratories.

6–55 Assume that

$$s(t) = \begin{cases} \dfrac{A}{T} t \cos\omega_c t, & 0 \leq t \leq T \\ 0, & t \text{ otherwise} \end{cases}$$

is a known signal. This signal plus white noise is present at the input to a matched filter.

(a) Design a matched filter for $s(t)$. Sketch waveforms analogous to those shown in Fig. 6–16.

(b) Sketch the waveforms for the correlation processor shown in Fig. 6–18.

★ **6–56** A baseband digital communication system uses polar signaling with a bit rate of $R = 2{,}000$ bits/s. The transmitted pulses are rectangular and the frequency response of channel filter is

$$H_c(f) = \frac{B}{B + jf}$$

where $B = 6{,}000$ Hz. The filtered pulses are the input to a receiver that uses integrate-and-dump processing, as illustrated in Fig. 6–17. Examine the integrator output for ISI. In particular,

(a) Plot the integrator output when a binary "1" is sent.

(b) Plot the integrator output for an all-pass channel, and compare this result with that obtained in part (a).

6–57 Refer to Fig. 6–19 for the detection of a BPSK signal. Suppose that the BPSK signal at the input is

$$r(t) = s(t) = \sum_{n=0}^{7} d_n p(t - nT)$$

where

$$p(t) = \begin{cases} e^{-t}\cos\omega_c t, & 0 < t < T \\ 0, & t \text{ otherwise} \end{cases}$$

and the binary data d_n is the 8-bit string $\{+1, -1, -1, +1, +1, -1, +1, -1\}$. Use MATLAB to

(a) Plot the input waveform $r(t)$.

(b) Plot the integrator output waveform $r_0(t)$.

★ **6–58** A matched filter is described in Fig. P6–58.

(a) Find the impulse response of the matched filter.

(b) Find the pulse shape to which this filter is matched (the white-noise case).

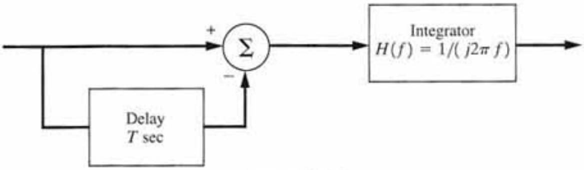

Figure P6–58

★ **6–59** An FSK signal $s(t)$ is applied to a correlation receiver circuit that is shown in Fig. P6–59. The FSK signal is

$$s(t) = \begin{cases} A\cos(\omega_1 t), & \text{when a binary 1 is sent} \\ A\cos(\omega_2 t), & \text{when a binary 1 is sent} \end{cases}$$

where $f_1 = f_c + \Delta F$ and $f_2 = f_c - \Delta F$. Let ΔF be chosen to satisfy the MSK condition, which is $\Delta F = 1/(4T)$. T is the time that it takes to send one bit, and the integrator is reset every T seconds. Assume that $A = 1$, $f_c = 1{,}000$ Hz, and $\Delta F = 50$ Hz.

(a) If a binary 1 is sent, plot $v_1(t)$, $v_2(t)$, and $r_0(t)$ over a T-second interval.

(b) Refer to Fig. 6–16, and find an expression that describes the output of a filter that is matched to the FSK signal when a binary 1 is sent. Plot the filter output.

(c) Discuss how the plots obtained for parts (a) and (b) agree or disagree.

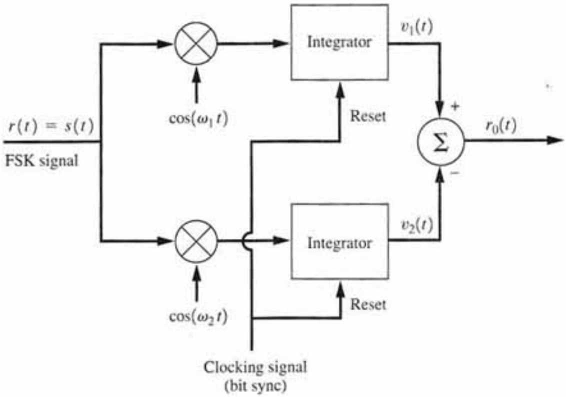

Figure P6–59

6–60 Let

$$s(t) = \text{Re}\{g(t)e^{j(\omega_c t+\theta_c)}\}$$

be a wideband FM signal, where

$$g(t) = A_c e^{jD_f \int_{-\infty}^{t} m(\lambda)\, d\lambda}$$

and $m(t)$ is a random modulation process.

(a) Show that

$$R_g(\tau) = A_c^2[\overline{e^{jD_f \tau m(t)}}]$$

when the integral $\int_t^{t+\tau} m(\lambda)\, d\lambda$ is approximated by $\tau m(t)$.

(b) Using the results of part (a), prove that the PSD of the wideband FM signal is given by Eq. (5–66). That is, show that

$$\mathcal{P}_s(f) = \frac{\pi A_c^2}{2D_f}\left[f_m\left(\frac{2\pi}{D_f}(f - f_c)\right) + f_m\left(\frac{2\pi}{D_f}(-f - f_c)\right)\right]$$

or

$$I = \frac{B^3}{3f_1^3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]} \tag{7–154}$$

Because $I_{dB} = 10 \log (I)$, we get

$$I_{dB} = 30 \log\left(\frac{B}{f_1}\right) - 10 \log\left\{3\left[\frac{B}{f_1} - \tan^{-1}\left(\frac{B}{f_1}\right)\right]\right\} \tag{7–155}$$

For $f_1 = 2.1$ kHz and $B = 15$ kHz, Eq. (7–155) gives

$$I_{dB} = 13.2\,\text{dB}$$

This checks with the value for I_{dB} that is obtained from Fig. 7–26.

PROBLEMS

★ **7–1** In a binary communication system the receiver test statistic, $r_0(t_0) = r_0$, consists of a polar signal plus noise. The polar signal has values $s_{01} = +A$ and $s_{02} = -A$. Assume that the noise has a *Laplacian* distribution, which is

$$f(n_0) = \frac{1}{\sqrt{2}\sigma_0} e^{-\sqrt{2}|n_0|/\sigma_0}$$

where σ_0 is the RMS value of the noise.

(a) Find the probability of error P_e as a function of A/σ_0 for the case of equally likely signaling and V_T having the optimum value.

(b) Plot P_e as a function of A/σ_0 decibels. Compare this result with that obtained for Gaussian noise as given by Eq. (7–26a).

7–2 Using MATLAB, plot the PDF for a Laplacian distribution (see Prob. 7–1) where $\sigma_0 = 1$. Also, plot the PDF for a Gaussian distribution where the standard deviation is 1 and the mean is 0. Compare the two PDFs.

7–3 Using Eq. (7–8), show that the optimum threshold level for the case of antipodal signaling with additive white Gaussian noise is

$$V_T = \frac{\sigma_0^2}{2s_{01}} \ln\left[\frac{P(s_2\,\text{sent})}{P(s_1\,\text{sent})}\right]$$

Here the receiver filter has an output with a variance of σ_0^2. s_{01} is the value of the sampled binary 1 signal at the filter output. $P(s_1$ sent) and $P(s_2$ sent) are the probabilities of transmitting a binary 1 and a binary 0, respectively.

7–4 A baseband digital communication system uses polar signaling with matched filter in the receiver. The probability of sending a binary 1 is p, and the probability of sending a binary zero is $1 - p$.

(a) For $E_b/N_0 = 10$ dB, plot P_e as a function of p using a log scale.

(b) Referring to Eq. (1–8), plot the entropy, H, as a function of p. Compare the shapes of these two curves.

7–5 A *whole* binary communication system can be modeled as an *information channel*, as shown in Fig. P7–5. Find equations for the four transition probabilities $P(\widetilde{m}|m)$, where both $\widetilde{m}$ and m can be binary 1's or binary 0's. Assume that the test statistic is a linear function of the receiver input and that additive white Gaussian noise appears at the receiver input. [*Hint:* Look at Eq. (7–15).]

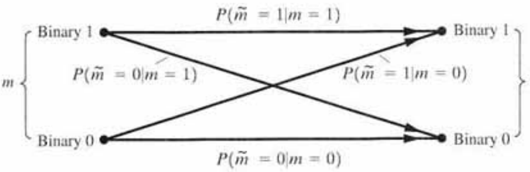

Figure P7–5

★ **7–6** A baseband digital communication system uses unipolar signaling (rectangular pulse shape) with matched-filter detection. The data rate is $R = 9{,}600$ bits/sec.

(a) Find an expression for bit error rate (BER), P_e, as a function of $(S/N)_{\text{in}}$. $(S/N)_{\text{in}}$ is the signal-to-noise power ratio at the receiver input where the noise is measured in a bandwidth corresponding to the equivalent bandwidth of the matched filter. [*Hint:* First find an expression of E_b/N_0 in terms of $(S/N)_{\text{in}}$.]

(b) Plot P_e vs. $(S/N)_{\text{in}}$ in dB units on a log scale over a range of $(S/N)_{\text{in}}$ from 0 to 15 dB.

7–7 Rework Prob. 7–6 for the case of polar signaling.

7–8 Examine how the performance of a baseband digital communication system is affected by the receiver filter. Equation (7–26a) describes the BER when a low-pass filter is used and the bandwidth of the filter is large enough that the signal level at the filter output is $s_{01} = +A$ or $s_{02} = -A$. Instead, suppose that a RC low-pass filter with a restricted bandwidth is used where $T = 1/f_0 = 2\pi\, RC$. T is the duration (pulse width) of one bit, and f_0 is the 3-dB bandwidth of the RC low-pass filter as described by Eq. (2–147). Assume that the initial conditions of the filter are reset to zero at the beginning of each bit interval.

(a) Derive an expression for P_e as a function of E_b/N_0.

(b) On a log scale, plot the BER obtained in part (a) for E_b/N_0 over a range of 0 to 15 dB.

(c) Compare this result with that for a matched-filter receiver (as shown in Fig. 7–5).

★ **7–9** Consider a baseband digital communication system that uses polar signaling (rectangular pulse shape) where the receiver is shown in Fig. 7–4a. Assume that the receiver uses a second-order Butterworth filter with a 3-dB bandwidth of f_0. The filter impulse response and transfer function are

$$h(t) = \left[\sqrt{2}\,\omega_0 e^{-(\omega_0/\sqrt{2})t} \sin\left(\frac{\omega_0}{\sqrt{2}}t\right)\right]u(t)$$

$$H(f) = \frac{1}{(jf/f_0)^2 + \sqrt{2}(jf/f_0) + 1}$$

where $\omega_0 = 2\pi f_0$. Let $f_0 = 1/T$, where T is the bit interval (i.e., pulse width), and assume that the initial conditions of the filter are reset to zero at the beginning of each bit interval.

(a) Derive an expression for P_e as a function of E_b/N_0.

(b) On a log scale, plot the BER obtained in part (a) for E_b/N_0 over a range of 0 to 15 dB.

(c) Compare this result with that for a matched-filter receiver (as shown in Fig. 7–5).

7–10 Consider a baseband unipolar communication system with equally likely signaling. Assume that the receiver uses a simple RC LPF with a time constant of $RC = \tau$ where $\tau = T$ and $1/T$ is the bit rate. (By "simple," it is meant that the initial conditions of the LPF are *not* reset to zero at the beginning of each bit interval.)

(a) For signal alone at the receiver input, evaluate the approximate worst-case signal to ISI ratio (in decibels) out of the LPF at the sampling time $t = t_0 = nT$, where n is an integer.

(b) Evaluate the signal to ISI ratio (in decibels) as a function of the parameter K, where $t = t_0 = (n + K)T$ and $0 < K \leq 1$.

(c) What is the optimum sampling time to use to maximize the signal-to-ISI power ratio out of the LPF?

(d) Repeat part (a) for the case when the equivalent bandwidth of the RC LPF is $2/T$.

7–11 Examine a baseband polar communication system with equally likely signaling and no channel noise. Assume that the receiver uses a simple RC LPF with a time constant of $\tau = RC$. (By "simple," it is meant that the initial conditions of the LPF are *not* reset to zero at the beginning of each bit interval.) Evaluate the worst-case approximate signal to ISI ratio (in decibels) out of the LPF at the sampling time $t = t_0 = nT$, where n is an integer. This approximate result will be valid for $T/\tau > \frac{1}{2}$. Plot this result as a function of T/τ for $\frac{1}{2} \leq T/\tau \leq 5$.

★ **7–12** Consider a baseband unipolar system described in Prob. 7–10d. Assume that white Gaussian noise is present at the receiver input.

(a) Derive an expression for P_e as a function of E_b/N_0 for the case of sampling at the times $t = t_0 = nT$.

(b) Compare the BER obtained in part (a) with the BER characteristic that is obtained when a matched-filter receiver is used. Plot both of these BER characteristics as a function of $(E_b/N_0)_{\text{dB}}$ over the range 0 to 15 dB.

7–13 Rework Prob. 7–12 for the case of *polar* baseband signaling.

7–14 For bipolar signaling, the discussion leading up to Eq. (7–28) indicates that the optimum threshold at the receiver is $V_T = \dfrac{A}{2} + \dfrac{\sigma_0^2}{A}\ln 2$.

(a) Prove that this is the optimum threshold value.

(b) Show that $A/2$ approximates the optimum threshold if $P_e < 10^{-3}$.

★ **7–15** For unipolar baseband signaling as described by Eq. (7–23),

(a) Find the matched-filter frequency response and show how the filtering operation can be implemented by using an integrate-and-dump filter.

(b) Show that the equivalent bandwidth of the matched filter is $B_{\text{eq}} = 1/(2T) = R/2$.

7–16 Equally likely polar signaling is used in a baseband communication system. Gaussian noise having a PSD of $N_0/2$ W/Hz plus a polar signal with a peak level of A volts is present at the receiver input. The receiver uses a matched-filter circuit having a voltage gain of 1,000.

(a) Find the expression for P_e as a function of A, N_0, T, and V_T, where $R = 1/T$ is the bit rate and V_T is the threshold level.

(b) Plot P_e as a function of V_T for the case of $A = 8 \times 10^{-3}$ V, $N_0/2 = 4 \times 10^{-9}$ W/Hz, and $R = 1200$ bits/sec.

★ **7–17** Consider a baseband polar communication system with matched-filter detection. Assume that the channel noise is white and Gaussian with a PSD of $N_0/2$. The probability of sending a binary 1 is $P(1)$ and the probability of sending a binary 0 is $P(0)$. Find the expression for P_e as a function of the threshold level V_T when the signal level out of the matched filter is A, and the variance of the noise out of the matched filter is $\sigma^2 = N_0/(2T)$, where $R = 1/T$ is the bit rate.

7–18 Design a receiver for detecting the data on a bipolar RZ signal that has a peak value of $A = 5$ volts. In your design assume that an RC low-pass filter will be used and the data rate is 2,400 (bits/sec).

(a) Draw a block diagram of your design and explain how it works.

(b) Give the values for the design parameters R, C, and V_T.

(c) Calculate the PSD level for the noise N_0 that is allowed if P_e is to be less than 10^{-6}.

7–19 A BER of 10^{-5} or less is desired for an OOK communication system where the bit rate is $R = 10$ Mb/s. The input to the receiver consists of the OOK signal plus white Gaussian noise.

(a) Find the minimum transmission bandwidth required.

(b) Find the minimum E_b/N_0 required at the receiver input for coherent matched-filter detection.

(c) Rework part (b) for the case of noncoherent detection.

7–20 Rework Prob. 7–19 for the case when FSK signaling is used. Let $2\Delta F = f_2 - f_1 = 1.5R$.

★ **7–21** In this chapter, the BER for a BPSK receiver was derived under the assumption that the coherent receiver reference (see Fig. 7–7) was exactly in phase with the received BPSK signal. Suppose that there is a phase error of θ_e between the reference signal and the incoming BPSK signal. Obtain new equations that give the P_e in terms of θ_e, as well as the other parameters. In particular,

(a) Obtain a new equation that replaces Eq. (7–36).

(b) Obtain a new equation that replaces Eq. (7–38).

(c) Plot results from part (b) where the log plot of P_e is given as a function of θ_e over a range $-\pi < \theta_e < \pi$ for the case when $E_b/N_0 = 10$ dB.

★ **7–22** Digital data are transmitted over a communication system that uses nine repeaters plus a receiver, and BPSK signaling is used. The P_e for each of the regenerative repeaters (see Sec. 3–5) is 5×10^{-8}, assuming additive Gaussian noise.

(a) Find the overall P_e for the system.

(b) If each repeater is replaced by an ideal amplifier (no noise or distortion), what is the P_e of the overall system?

★ **7–23** Digital data are to be transmitted over a toll telephone system using BPSK. Regenerative repeaters are spaced 50 miles apart along the system. The total length of the system is 600 miles. The telephone lines between the repeater sites are equalized over a 300- to 2,700-Hz band and provide an E_b/N_0 (Gaussian noise) of 15 dB to the repeater input.

(a) Find the largest bit rate R that can be accommodated with no ISI.

(b) Find the overall P_e for the system. (Be sure to include the receiver at the end of the system.)

7–24 A BPSK signal is given by

$$s(t) = A \sin [\omega_c t + \theta_c + (\pm 1)\beta_p], \quad 0 < t \leq T$$

The binary data are represented by (± 1), where $(+1)$ is used to transmit a binary 1 and (-1) is used to transmit a binary 0. β_p is the phase modulation index as defined by Eq. (5–47).

(a) For $\beta_p = \pi/2$, show that this BPSK signal becomes the BPSK signal as described by Eq. (7–34).

(b) For $0 < \beta_p < \pi/2$, show that a discrete carrier term is present in addition to the BPSK signal as described by Eq. (7–34).

7–25 Referring to the BPSK signal described in Prob. 7–24, find P_e as a function of the modulation index β_p, where $0 < \beta_p \leq \pi/2$. Find P_e as a function of A, β_p, N_0, and B for the receiver that uses a narrowband filter.

7–26 Rework Prob. 7–25 and find P_e as a function of E_b, N_0, and β_p for the receiver that uses a matched filter. (E_b is the average BPSK signal energy that is received during one bit.)

7–27 Referring to the BPSK signal described in Prob. 7–24, let $0 < \beta_p < \pi/2$.

(a) Show a block diagram for the detection of the BPSK signal where a PLL is used to recover the coherent reference signal from the BPSK signal.

(b) Explain why Manchester-encoded data are often used when the receiver uses a PLL for carrier recovery, as in part (a). (*Hint:* Look at the spectrum of the Manchester-encoded PSK signal.)

7–28 In obtaining the P_e for FSK signaling with coherent reception, the energy in the difference signal E_d was needed, as shown in Eq. (7–46). For *orthogonal* FSK signaling the cross-product integral was found to be zero. Suppose that f_1, f_2, and T are chosen so that E_d is maximized.

(a) Find the relationship as a function of f_1, f_2, and T for maximum E_d.

(b) Find the P_e as a function of E_b and N_0 for signaling with this FSK signal.

(c) Sketch the P_e for this type of FSK signal and compare it with a sketch of the P_e for the orthogonal FSK signal that is given by Eq. (7–47).

★ **7–29** An FSK signal with $R = 110$ bits/sec is transmitted over an RF channel that has white Gaussian noise. The receiver uses a noncoherent detector and has a noise figure of 6 dB. The impedance of the antenna input of the receiver is 50 ω. The signal level at the receiver input is 0.05 μV, and the noise level is $N_0 = kT_0$, where $T_0 = 290$ K and k is Boltzmann's constant. (See Sec. 8–6.) Find the P_e for the digital signal at the output of the receiver.

7–30 Rework Prob. 7–29 for the case of DPSK signaling.

7–31 An analysis of the noise associated with the two channels of an FSK receiver precedes Eq. (7–44). In this analysis, it is stated that $n_1(t)$ and $n_2(t)$ are independent when they arise from a common white Gaussian noise process because $n_1(t)$ and $n_2(t)$ have nonoverlapping spectra. Prove that this statement is correct. [*Hint:* $n_1(t)$ and $n_2(t)$ can be modeled as the outputs of two linear filters that have nonoverlapping transfer functions and the same white noise process, $n(t)$, at their inputs.]

7–32 In most applications, communication systems are designed to have a BER of 10^{-5} or less. Find the minimum E_b/N_0 decibels required to achieve an error rate of 10^{-5} for the following types of signaling.

(a) Polar baseband.

(b) OOK.

(c) BPSK.

(d) FSK.

(e) DPSK.

★ **7–33** Digital data are to be transmitted over a telephone line channel. Suppose that the telephone line is equalized over a 300- to 2,700-Hz band and that the signal-to-Gaussian-noise (power) ratio at the output (receive end) is 25 dB.

(a) Of all the digital signaling techniques studied in this chapter, choose the one that will provide the largest bit rate for a P_e of 10^{-5}. What is the bit rate R for this system?

(b) Compare this result with the bit rate R that is possible when an ideal digital signaling scheme is used, as given by the Shannon channel capacity stated by Eq. (1–10).

★ **7–34** An analog baseband signal has a uniform PDF and a bandwidth of 3500 Hz. This signal is sampled at an 8 samples/s rate, uniformly quantized, and encoded into a PCM signal having 8-bit words. This PCM signal is transmitted over a DPSK communication system that contains additive white Gaussian channel noise. The signal-to-noise ratio at the receiver input is 8 dB.

(a) Find the P_e of the recovered PCM signal.

(b) Find the peak signal/average noise ratio (decibels) out of the PCM system.

7–35 A spread spectrum (SS) signal is often used to combat narrowband interference and for communication security. The SS signal with direct sequence spreading is (see Sec. 5–13)

$$s(t) = A_c c(t) m(t) \cos(\omega_c t + \theta_c)$$

where θ_c is the start-up carrier phase, $m(t)$ is the polar binary data baseband modulation, and $c(t)$ is a polar baseband spreading waveform that usually consists of a pseudonoise (PN) code. The PN code is a binary sequence that is N bits long. The "bits" are called *chips*, since they do not contain data and since many chips are transmitted during the time that it takes to transmit 1 bit of the data [in $m(t)$]. The same N-bit code word is repeated over and over, but N is a large number, so the chip sequence in $c(t)$ looks like digital noise. The PN sequence may be generated by using a clocked r-stage shift register having feedback so that $N = 2^r - 1$. The autocorrelation of a long sequence is approximately

$$R_c(\tau) = \Lambda\left(\frac{\tau}{T_c}\right)$$

where T_c is the duration of one chip (the time it takes to send one chip of the PN code). $T_c \ll T_b$, where T_b is the duration of a data bit.

(a) Find the PSD for the SS signal $s(t)$. [*Hint:* Assume that $m(t)$, $c(t)$, and θ_c are independent. In addition, note that the PSD of $m(t)$ can be approximated by a delta function, since the spectral width of $m(t)$ is very small when compared to that for the spreading waveform $c(t)$.]

(b) Draw a block diagram for an optimum coherent receiver. Note that $c(t)\,m(t)$ is first coherently detected and then the data, $m(t)$, are recovered by using a correlation processor.

(c) Find the expression for P_e.

★ **7–36** Examine the performance of an AM communication system where the receiver uses a product detector. For the case of a sine-wave modulating signal, plot the ratio of $[(S/N)_{\text{out}}/(S/N)_{\text{in}}]$ as a function of the percent modulation.

★ **7–37** An AM transmitter is modulated 40% by a sine-wave audio test tone. This AM signal is transmitted over an additive white Gaussian noise channel. Evaluate the noise performance of this system and determine by how many decibels this system is inferior to a DSB-SC system.

7–38 A phasing-type receiver for SSB signals is shown in Fig. P7–38. (This was the topic of Prob. 5–21.)

(a) Show that this receiver is or is not a linear system.

(b) Derive the equation for SNR out of this receiver when the input is an SSB signal plus white noise with a PSD of $N_0/2$.

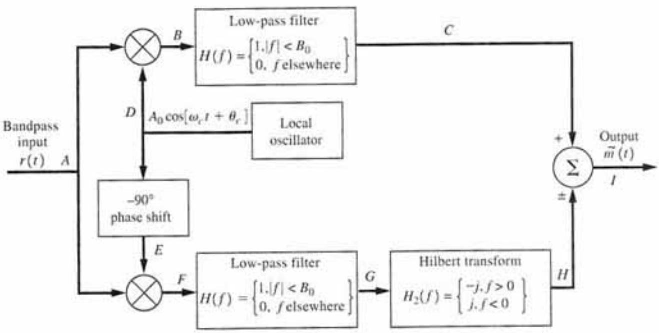

Figure P7–38

7–39 Referring to Fig. P7–38, suppose that the receiver consists only of the upper portion of the figure, so that point C is the output. Let the input be an SSB signal plus white noise with a PSD of $N_0/2$. Find $(S/N)_{\text{out}}$.

7–40 Consider the receiver shown in Fig. P7–40. Let the input be a DSB-SC signal plus white noise with a PSD of $N_0/2$. The mean value of the modulation is zero.

(a) For A_0 large, show that this receiver acts like a product detector.

(b) Find the equation for $(S/N)_{\text{out}}$ as a function of A_c, $\overline{m^2}$, N_0, A_0, and B_T when A_0 is large.

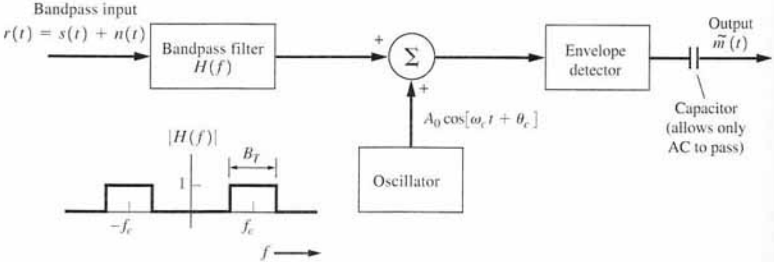

Figure P7–40

7–41 Compare the performance of AM, DSB-SC, and SSB systems when the modulating signal $m(t)$ is a Gaussian random process. Assume that the Gaussian modulation has a zero mean value and a peak value of $V_p = 1$, where $V_p \approx 4\sigma_m$. Compare the noise performance of these three systems by plotting $(S/N)_{\text{out}}/(S/N)_{\text{baseband}}$ for

(a) The AM system.

(b) The DSB-SC system.

(c) The SSB system.

7–42 If linear modulation systems are to be compared on an equal *peak* power basis (i.e., all have equal peak signal values), show that

(a) SSB has a $(S/N)_{\text{out}}$ that is 3 dB better than DSB.

(b) SSB has a $(S/N)_{\text{out}}$ that is 9 dB better than AM.

(*Hint:* See Prob. 5–11.)

7–43 Using Eq. (7–132), plot the output SNR threshold characteristic of a discriminator for the parameters of a conventional broadcast FM system. ($\Delta F = 75$ kHz and $B = 15$ kHz.) Compare plots of the $(S/N)_{\text{out}}$ vs. $(S/N)_{\text{baseband}}$ for this system with those shown in Fig. 7–24.

★ **7–44** An FM receiver has an IF bandwidth of 25 kHz and a baseband bandwidth of 5 kHz. The noise figure of the receiver is 12 dB, and it uses a 75-μsec deemphasis network. An FM signal plus white noise is present at the receiver input, where the PSD of the noise is $N_0/2 = kT/2$. $T = 290$ K. (See Sec. 8–6.) Find the minimum input signal level (in dBm) that will give a SNR of 35 dB at the output when sine-wave test modulation is used.

7–45 Referring to Table 5–4, note that a two-way FM mobile radio system uses the parameters $\beta_f = 1$ and $B = 5$ kHz.

(a) Find $(S/N)_{\text{out}}$ for the case of no deemphasis.

(b) Find $(S/N)_{\text{out}}$ if deemphasis is used with $f_1 = 2.1$ kHz. It is realized that B is not much larger than f_1 in this application.

(c) Plot $(S/N)_{\text{out}}$ vs. $(S/N)_{\text{baseband}}$ for this system, and compare the plot with the result for FM broadcasting as shown in Fig. 7–26.

7–46 Compare the performance of two FM systems that use different deemphasis characteristics. Assume that $\beta_f = 5$, $\overline{(m/V_p)^2} = \frac{1}{2}$, $B = 15\,\text{kHz}$, and that an additive white Gaussian noise channel is used. Find $(S/N)_{\text{out}} / (S/N)_{\text{baseband}}$ for:

(a) 25-μsec deemphasis.

(b) 75-μsec deemphasis.

7–47 A baseband signal $m(t)$ that has a Gaussian (amplitude) distribution frequency modulates a transmitter. Assume that the modulation has a zero-mean value and a peak value of $V_p = 4\sigma_m$. The FM signal is transmitted over an additive white Gaussian noise channel. Let $\beta_f = 3$ and $B = 15$ kHz. Find $(S/N)_{\text{out}} / (S/N)_{\text{baseband}}$ when

(a) No deemphasis is used.

(b) 75-μsec deemphasis is used.

7–48 In FM broadcasting, a preemphasis filter is used at the audio input of the transmitter, and a deemphasis filter is used at the receiver output to improve the output SNR. For 75-μsec emphasis, the 3-dB bandwidth of the receiver deemphasis LPF is $f_1 = 2.1$ kHz. The audio bandwidth is $B = 15$ kHz. Define the improvement factor I as a function of B/f_1 as

$$I = \frac{(S/N)_{\text{out}} \text{ for system with preemphasis–deemphasis}}{(S/N)_{\text{out}} \text{ for system without preemphasis–deemphasis}}$$

For $B = 15$ kHz, plot the decibel improvement that is realized as a function of the design parameter f_1 where 50 Hz $< f_1 <$ 15 kHz.

★ **7–49** In FM broadcasting, preemphasis is used, and yet $\Delta F = 75$ kHz is defined as 100% modulation. Examine the incompatibility of these two standards. For example, assume that the amplitude of a 1-kHz audio test tone is adjusted to produce 100% modulation (i.e., $\Delta F = 75$ kHz).

(a) If the frequency is changed to 15 kHz, find the ΔF that would be obtained ($f_1 = 2.1$ kHz). What is the percent modulation?

(b) Explain why this phenomenon does not cause too much difficulty when typical audio program material (modulation) is broadcast.

7–50 Stereo FM transmission was studied in Sec. 5–7. At the transmitter, the left-channel audio, $m_L(t)$, and the right-channel audio, $m_R(t)$, are each preemphasized by an $f_1 = 2.1$-kHz network. These preemphasized audio signals are then converted into the composite baseband modulating signal $m_b(t)$, as shown in Fig. 5–17. At the receiver, the FM detector outputs the composite baseband signal that has been corrupted by noise. (Assume that the noise comes from a white Gaussian noise channel.) This corrupted composite baseband signal is demultiplexed into corrupted left- and right-channel audio signals, $\widetilde{m}_L(t)$ and $\widetilde{m}_R(t)$, each having been deemphasized by a 2.1-kHz filter. The noise on these outputs arises from the noise at the output of the FM detector that occurs in the 0- to 15-kHz and 23- to 53-kHz bands. The subcarrier frequency is 38 kHz. Assuming that the input SNR of the FM receiver is large, show that the stereo FM system is 22.2 dB more noisy than the corresponding monaural FM system.

★ **7–51** An FDM signal, $m_b(t)$, consists of five 4-kHz-wide channels denoted by C1, C2, …, C5, as shown in Fig. P7–51. The FDM signal was obtained by modulating five audio signals (each with 4-kHz bandwidth) onto USSB (upper single-sideband) subcarriers. This FDM signal, $m_b(t)$,

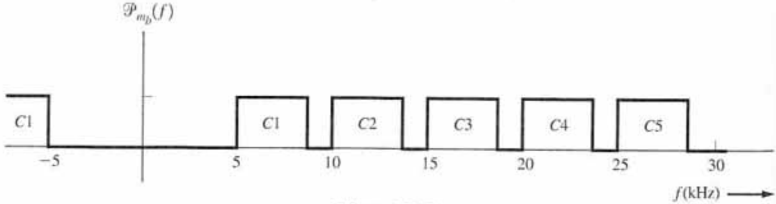

Figure P7–51

modulates a DSB-SC transmitter. The DSB-SC signal is transmitted over an additive white Gaussian noise channel. At the receiver the average power of the DSB-SC signal is P_s and the noise has a PSD of $N_0/2$.

(a) Draw a block diagram for a receiving system with five outputs, one for each audio channel.

(b) Calculate the output SNR for each of the five audio channels.

7–52 Rework Prob. 7–51 for the case when the FDM signal is frequency modulated onto the main carrier. Assume that the RMS carrier deviation is denoted by ΔF_{rms} and that the five audio channels are independent. No deemphasis is used.

7–53 Refer to Fig. 7–27 and

(a) Verify that the PCM curve is correct.

(b) Find the equation for the PCM curve for the case of QPSK signaling over the channel.

(c) Compare the performance of the PCM/QPSK system with that for PCM/BPSK and for the ideal case by sketching the $(S/N)_{\text{out}}$ characteristic.

7–54 In Sec. 7–9, the performance of an ideal analog system is discussed. Using MATLAB, plot the $(S/N)_{\text{out}}$ as a function of $(S/N)_{\text{baseband}}$ where both are expressed in dB units. Let $B_T/B = 12$.

The range of this wireless link could be increased by increasing the transmit power, reducing the receiver noise figure, or implementing a spread spectrum (SS) system. If direct sequence SS is used with a $r = 4$ stage shift register, as shown in Fig. 5–37, then the PN code length is $N = 15$. Assuming that code length spans one bit of data, then the chip rate is $R_c = NR$. From Eq. (5–131), this SS system would provide a processing gain of $G_p = R_c/R = N = 15$, or $G_{p\text{dB}} = 11.76$ dB. If despreading at the receiver is implemented after the IF stage (i.e., after the internal noise source of the receiver), this processing gain would increase the CNR at the detector input by 11.76 dB. This would increase the useful range to around 500 feet. That is, referring to Fig. 8–40 at $d = 500$ feet, we know that a processing gain of 11.76 dB would result in a CNR of 13.0 dB, which corresponds to a BER of 2.1×10^{-5}.

PROBLEMS

★ **8–1** A remote terminal for a telephone company services 300 VF subscribers and 150 G.Lite DSL subscribers (1.5 Mb/s data plus a VF signal that is converted to a DS-0 signal). Compute the minimum data rate needed for the receive fiber-optic line that terminates at the RT from the CO.

8–2 A 50-pair line provides telephone service to 50 subscribers in a rural subdivision via local loops to the CO. How many subscribers can be served if the 50 pairs are converted to T1 lines and a remote terminal is installed in the subdivision?

8–3 Assume that a telephone company has remote terminals connected to its CO via T1 lines. Draw a block diagram that illustrates how the T1 lines are interfaced with the CO switch if the CO uses:
(a) An analog switch.
(b) An integrated digital switch.

8–4 Indicate whether a conference-call connection is better or may be worse than a single-party call if a digital switch is used (at the CO) instead of an analog switch. Explain your answer.

★ **8–5** Full-duplex data of 24 kb/s in each direction from a personal computer is sent via a twisted-pair telephone line having a bandpass from 300–2700 Hz. Explain why modems are needed at each end of the line.

8–6 A satellite with twelve 36-MHz bandwidth transponders operates in the 6/4-GHz bands with 500 MHz bandwidth and 4-MHz guardbands on the 4-GHz downlink, as shown in Fig. 8–10. Calculate the percentage bandwidth that is used for the guardbands.

8–7 An Earth station uses a 3-m-diameter parabolic receiving antenna to receive a 4-GHz signal from a geosynchronous satellite. If the satellite transmitter delivers 10 W into a 3-m-diameter transmitting antenna and the satellite is located 36,000 km from the receiver, what is the power received?

8–8 Figure 8–12b shows an FDM/FM ground station for a satellite communication system. Find the peak frequency deviation needed to achieve the allocated spectral bandwidth for the 6240–MHz signal.

★ **8–9** A microwave transmitter has an output of 0.1 W at 2 GHz. Assume that this transmitter is used in a microwave communication system where the transmitting and receiving antennas are parabolas, each 4 ft in diameter.
(a) Evaluate the gain of the antennas.
(b) Calculate the EIRP of the transmitted signal.
(c) If the receiving antenna is located 15 miles from the transmitting antenna over a free-space path, find the available signal power out of the receiving antenna in dBm units.

8–10 A digital TV station transmits on Channel 7 with an ERP of 18 kW. The signal from this TV station is received 50 miles away via a LOS path with a receiving antenna that has a gain of 8 dB on Channel 7. Calculate the power at the receiving antenna output-connector in:

(a) dBm units.

(b) in dBmV units across a 75 Ω load.

(c) in μV across a 75 Ω load.

8–11 Rework Prob. 8–10 if the TV station operates on Channel 42.

8–12 A digital TV station operates on Channel 7 and another digital TV station operates on Channel 51. Using MATLAB, plot the free-space loss (in dB) for the Channel 7 station as a function of distance in miles. Repeat the plot for the Channel 51 station. Compare these two results by plotting them on the same figure. Do these losses change if these are analog TV stations instead of digital?

8–13 Using MATLAB, plot the free-space loss in dB as a function of distance in miles for a cellular link at 850 MHz. Also, plot the LOS loss for a cellular link at 1900 MHz. Compare these two results by plotting them on the same figure. Comparing these two plots, which link costs the cellular company less to operate for the same cell-site coverage area?

★ **8–14** Using MATLAB, plot the PSD for a thermal noise source with a resistance of 10 kΩ over a frequency range of 10 through 100,000 GHz where $T = 300$ K.

8–15 Given the *RC* circuit shown in Fig. P8–15, where *R* is a physical resistor at temperature *T*, find the RMS value of the noise voltage that appears at the output in terms of *k*, *T*, *R*, and *C*.

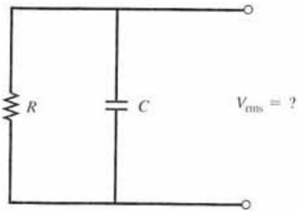

Figure P8–15

8–16 A receiver is connected to an antenna system that has a noise temperature of 100 K. Find the noise power that is available from the source over a 20-MHz band.

★ **8–17** A bipolar transistor amplifier is modeled as shown in Fig. P8–17. Find a formula for the available power gain in terms of the appropriate parameters.

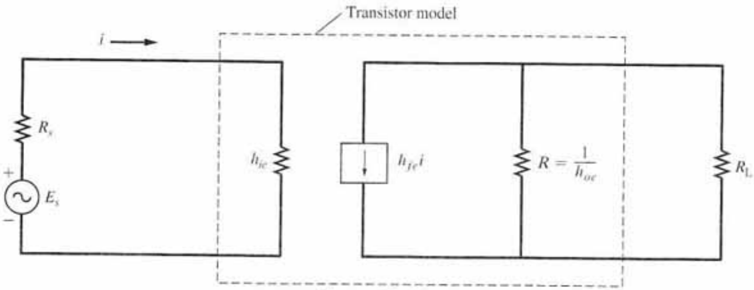

Figure P8–17

8–18 Using the definition for the available power gain, $G_a(f)$, as given by (8–18), show that $G_a(f)$ depends on the driving source impedance as well as the elements of the device and that $G_a(f)$ does not depend on the load impedance. [*Hint*: Calculate $G_a(f)$ for a simple resistive network.]

8–19 Show that the effective input-noise temperature and the noise figure can be evaluated from measurements that use the Y-factor method. With this method the device under test (DUT) is first connected to a noise source that has a relatively large output denoted by its source temperature, T_h, where the subscript h denotes "hot," and then the available noise power at the output of the DUT, P_{aoh} is measured with a power meter. Next, the DUT is connected to a source that has relatively low source temperature, T_c, where the subscript c denotes "cold," and noise power at the output of the DUT is measured, P_{aoc}. Show that

(a) The effective input noise temperature of the DUT is

$$T_e = \frac{T_h - YT_c}{Y - 1}$$

where $Y = P_{aoh}/P_{aoc}$ is obtained from the measurements.

(b) The noise figure of the DUT is

$$F = \frac{[(T_h/T_0) - 1] - Y[(T_c/T_0) - 1]}{Y - 1}$$

where $T_0 = 290$ K.

8–20 If a signal plus noise is fed into a linear device, show that the noise figure of that device is given by $F = (S/N)_{\text{in}}/(S/N)_{\text{out}}$ (*Hint*: Start with the basic definition of noise figure that is given in this chapter.)

★ **8–21** An antenna is pointed in a direction such that it has a noise temperature of 30 K. It is connected to a preamplifier that has a noise figure of 1.6 dB and an available gain of 30 dB over an effective bandwidth of 10 MHz.

(a) Find the effective input noise temperature for the preamplifier.

(b) Find the available noise power out of the preamplifier.

8–22 A 10-MHz SSB-AM signal, which is modulated by an audio signal that is bandlimited to 5 kHz, is being detected by a receiver that has a noise figure of 10 dB. The signal power at the receiver input is 10^{-10} mW, and the PSD of the input noise, $\mathcal{P}(f) = kT/2$, is 2×10^{-21}. Evaluate

(a) The IF bandwidth needed.

(b) The SNR at the receiver input.

(c) The SNR at the receiver output, assuming that a product detector is used.

★ **8–23** An FSK signal with $R = 110$ b/s is transmitted over an RF channel that has white Gaussian noise. The receiver uses a noncoherent detector and has a noise figure of 6 dB. The impedance of the antenna input of the receiver is 50 Ω. The signal level at the receiver input is 0.05 μV, and the noise level is $N_0 = kT_0$, where $T_0 = 290$ K and k is Boltzmann's constant. Find the P_e for the digital signal at the output of the receiver.

8–24 Work Prob. 8–23 for the case of DPSK signaling.

8–25 Prove that the overall effective input-noise temperature for cascaded linear devices is given by Eq. (8–37).

★ **8–26** A TV set is connected to an antenna system as shown in Fig. P8–26. Evaluate

(a) The overall noise figure.

(b) The overall noise figure if a 20-dB RF preamp with a 3-dB noise figure is inserted at point B.

(c) The overall noise figure if the preamp is inserted at point A.

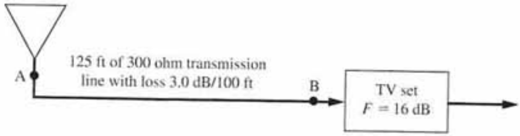

Figure P8–26

8–27 An Earth station receiving system consists of a 20-dB gain antenna with $T_{AR} = 80$ K, an RF amplifier with $G_a = 40$ dB and $T_e = 30$ K, and a down converter with $T_e = 15{,}000$ K. What is the overall effective input-noise temperature of this receiving system?

8–28 A low-noise amplifier (LNA), a down converter, and an IF amplifier are connected in cascade. The LNA has a gain of 40 dB and a T_e of 25 K. The down converter has a noise figure of 8 dB and a conversion gain of 6 dB. The IF amplifier has a gain of 60 dB and a noise figure of 14 dB. Evaluate the overall effective input-noise temperature for this system.

8–29 A geosynchronous satellite transmits 13.5 dBW EIRP on a 4-GHz downlink to an Earth station. The receiving system has a gain of 60 dB, an effective input-noise temperature of 30 K, an antenna source noise temperature of 60 K, and an IF bandwidth of 36 MHz. If the satellite is located 24,500 miles from the receiver, what is the $(C/N)_{dB}$ at the input of the receiver detector circuit?

★ **8–30** An antenna with $T_{AR} = 160$ K is connected to a receiver by means of a waveguide that has a physical temperature of 290 K and a loss of 2 dB. The receiver has a noise bandwidth of 1 MHz, an effective input-noise temperature of 800 K, and a gain of 120 dB from its antenna input to its IF output. Using MATLAB find

(a) The system noise temperature at the input of the receiver.

(b) The overall noise figure.

(c) The available noise power at the IF output.

8–31 The *Intelsat IV* satellite uses 36-MHz transponders with downlinks operating in the 4-GHz band. Each satellite transponder has a power output of 3.5 W and may be used with a 17° global coverage antenna that has a gain of 20 dB.

(a) For users located at the subsatellite point (i.e., directly below the satellite), show that $(C/N)_{dB} = (G_{AR}/T_{syst})_{dB} - 17.1$, where $(G_{AR}/T_{syst})_{dB}$ is the Earth receiving station figure of merit.

(b) Design a ground receiving station (block diagram) showing reasonable specifications for each block so that the IF output CNR will be 12 dB. Discuss the trade-offs that are involved.

★ **8–32** The efficiency of a parabolic antenna is determined by the accuracy of the parabolic reflecting surface and other factors. The gain is $G_A = 4\pi\eta A/\lambda^2$, where η is the antenna efficiency. In Table 8–4 an efficiency of 56% was used to obtain the formula $G_A = 7A/\lambda^2$. In a ground receiving system for the *Intelsat IV* satellite, assume that a (G_{AR}/T_{syst}) of 40 dB is needed. Using a 30-m antenna, give the required antenna efficiency if the system noise temperature is 85 K. How does the required antenna efficiency change if a 25-m antenna is used?

8–33 Evaluate the performance of a TVRO system. Assume that the TVRO terminal is located in Miami, Florida (26.8° N latitude, 80.2° W longitude). A 10-ft-diameter parabolic receiving antenna is used and is pointed toward the satellite with parameters as described in Table 8–5, Fig. 8–25, and Fig. 8–27.
(a) Find the TVRO antenna look angles to the satellite and find the slant range.
(b) Find the overall receiver system temperature.
(c) Find the $(C/N)_{dB}$ into the receiver detector.
(d) Find the $(S/N)_{dB}$ out of the receiver.

8–34 Repeat Prob. 8–33 for the case when the TVRO terminal is located at Anchorage, Alaska (61.2° N latitude, 149.8° W longitude), and a 8-m-diameter parabolic antenna is used.

8–35 A TVRO receive system consists of an 8-ft-diameter parabolic antenna that feeds a 50-dB, 25K LNA. The sky noise temperature (with feed) is 32 K. The system is designed to receive signals from a satellite. This satellite system is described in Table 8-5, Fig. 8-25, and Fig. 8-27. The LNA has a post-mixer circuit that down-converts the satellite signal to 70 MHz. The 70-MHz signal is fed to the TVRO receiver via a 120-ft length of 75-Ω coaxial cable. The cable has a loss of 3 dB/100 ft. The receiver has a bandwidth of 36 MHz and a noise temperature of 3800 K. Assume that the TVRO site is located in Los Angeles, California (34° N latitude, 118.3° W longitude), and vertical polarization is of interest.
(a) Find the TVRO antenna look angles to the satellite and find the slant range.
(b) Find the overall system temperature.
(c) Find the $(C/N)_{dB}$ into the receiver detector.

★ 8–36 An Earth station receiving system operates at 11.95 GHz and consists of a 20-m antenna with a gain of 65.53 dB and $T_{AR} = 60$ K, a waveguide with a loss of 1.28 dB and a physical temperature of 290 K, a LNA with $T_e = 50$ K and 60 dB gain, and a down converter with $T_e = 11{,}000$ K. Using MATLAB, find $(G/T)_{dB}$ for the receiving system evaluated at
(a) The input to the LNA.
(b) The input to the waveguide.
(c) The input to the down converter.

8–37 Rework Ex. 8-4 for the case of reception of TV signals from a direct broadcast satellite (DBS). Assume that the system parameters are similar to those given in Table 8–5, except that the satellite power is 316 kW EIRP radiated in the 12-GHz band. Furthermore, assume that a 0.5-m parabolic receiving antenna is used and that the LNA has a 50-K noise temperature.

★ 8–38 A distant object from our sun is Pluto, which is located at a distance from the Earth of 7.5×10^9 km. Assume that an unmanned spacecraft with a 2-GHz, 10-W transponder is in the vicinity of Pluto. A receiving Earth station with a 64-m antenna is available that has a system noise temperature of 16 K at 2 GHz. Calculate the size of a spacecraft antenna that is required for a 300-b/s BPSK data link to Earth that has a 10^{-3} BER (corresponding to E_b/N_0 of 9.88 dB). Allow 2 dB for incidental losses.

8–39 Using MATLAB, or some spreadsheet program that will run on a personal computer (PC), design a spreadsheet that will solve Prob. 8–35. Run the spreadsheet on your PC, and verify that it gives the correct answers. Print out your results. Also, try other parameters, such as those appropriate for your location, and print out the results.

8–40 Assume that you want to analyze the overall performance of a satellite relay system that uses a "bent pipe" transponder. Let $(C/N)_{up}$ denote the carrier-to-noise ratio for the transponder as evaluated in the IF band of the satellite transponder. Let $(C/N)_{dn}$ denote the IF CNR ratio at the downlink receiving ground station when the satellite is sending a perfect (noise-free) signal. Show that the overall operating CNR ratio at the IF of the receiving ground station, $(C/N)_{ov}$, is given by

$$\frac{1}{(C/N)_{ov}} = \frac{1}{(C/N)_{up}} + \frac{1}{(C/N)_{dn}}$$

8–41 For the wireless device link that is described in Study Aid Example SA8–3, use MATLAB to plot the path loss (in dB) as a function of distance over a range of 50 to 500 feet.

★ **8–42** A PCS cellular site base station operates with 10W into an antenna with 18 dB gain at 1800 MHz. The path loss reference distance is $d_0 = 0.25$ miles, $X_{dB} = 0$, and the PCS phone antenna has 0 dB gain. Find the received power in dBm at the antenna output of the PCS phone when the PCS phone is located at distances of 1 mile, 2 miles, 5 miles, and 10 miles from the BS if the path loss exponent is

(a) $n = 2$ (free space condition).

(b) $n = 3$.

(c) $n = 4$.

8–43 The transmitter of a PCS CDMA cellular phone feeds 200 mW into its 0 dB gain antenna. The PCS signal is spread over a bandwidth of 1.25 MHz in the 1900 MHz band. Calculate the signal power received in dBm at the output of the base station antenna where the PCS phone is located 2 km from the BS. Assume a path loss model given by Eq. (8–47), where $d_0 = 100$ m, $n = 3$ and $X_{dB} = 0$. The BS antenna has a gain of 16 dB.

8–44 Given a CDMA cellular phone link as described in Prob. 8–43 and by the IS-95 standard shown in Table 8–11. The base station receiving system has a noise figure of 8 dB and multiple access interference (MAI) from other phones adds 20 dB to the noise level at the front end of the BS receiver. Compute the CNR, $(C/N)_{dB}$, after despreading (i.e., at the data detector input) in the BS receiver. Also, find $(E_b/N_0)_{dB}$ after despreading. Assume that the bandwidth after despreading is B = 19.2 kHz, since the data rate with coding is 19.2 kb/s (9.6 kb/s payload data rate with rate $\frac{1}{2}$ coding) on each I and Q BPSK carrier of the QPSK signal.

8–45 Assume that an analog TV station is licensed to operate on TV Channel 35 with an effective radiated visual power of 1000 W. The tower height is 400 ft, and the transmission line is 450 ft long. Assume that a $1\frac{5}{8}$ -in.-diameter semirigid 50-Ω coaxial transmission line will be used that has a loss of 0.6 dB/100 ft (at the operating frequency). The antenna has a gain of 5.6 dB, and the duplexer loss is negligible. Find the PEP at the visual transmitter output.

★ **8–46** An analog TV visual transmitter is tested by modulating it with a periodic test signal. The envelope of the transmitter output is viewed on an oscilloscope as shown in Fig. P8–46, where K is an unknown constant. The output of the transmitter is connected to a 50-Ω dummy load that has

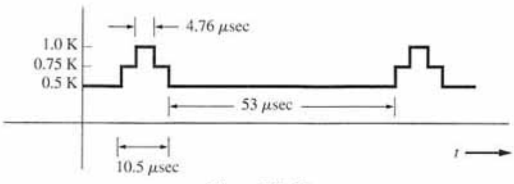

Figure P8–46

a calibrated average reading wattmeter. The wattmeter reads 6.9 kW. Find the PEP of the transmitter output.

8–47 Specifications for the MTS stereo audio system for analog TV are given in Fig. 8–33. Using this figure,

(a) Design a block diagram for TV receiver circuitry that will detect the stereo audio signals and the second audio program (SAP) signal.

(b) Describe how the circuit works. That is, give mathematical expressions for the signal at each point on your block diagram and explain in words. Be sure to specify all filter transfer functions, VCO center frequencies, and so on.

8–48 For the R-Y and B-Y signals in an analog color-TV subcarrier demodulation system, the $G–Y$ signal is obtained from the $R–Y$ and $B–Y$ signals. That is,

$$[m_G(t) - m_Y(t)] = K_1[m_R(t) - m_Y(t)] + K_2[m_B(t) - m_Y(t)]$$

(a) Find the values for K_1 and K_2 that are required.

(b) Draw a possible block diagram for a $R–Y$, $B–Y$ system, and explain how it works.

★ **8–49** Referring to the digital DTV standards in Table 8–14, the payload data rate with sync is 19.39 Mb/s. Show that with the addition of coding bits and training bits, the 8VSB signal has a symbol rate of 10.76 Mbaud.

8–50 8VSB signaling is used in the US for DTV transmission. To prevent ISI, the spectrum of the 8VSB signal is designed to roll off with a square root raised cosine-rolloff filtering characteristic, as shown in Fig. 8–39.

(a) Show that the absolute bandwidth of a VSB signal with raised cosine-rolloff filtering is

$$B_{\text{VSB}} = \frac{1}{2}(1 + 2r)D$$

where D is the symbol rate and r is the rolloff factor. [*Hint*: The derivation of this formula is similar to that for Eq. (3–74).]

(b) Using the parameters for DTV from Table 8–14 in the formula developed in part **(a)**, show that the absolute bandwidth for DTV is 6 MHz.

8–51 A United States digital TV station operates on Channel 9 with an ERP of 4.9kW. The signal from this station is received 25 miles away via a LOS path with a receiving antenna that has a gain of 8 dB. Assume that the receiving system has a noise figure of 3 dB. Calculate the $(E_b/N_o)_{dB}$ at the receiver detector input.

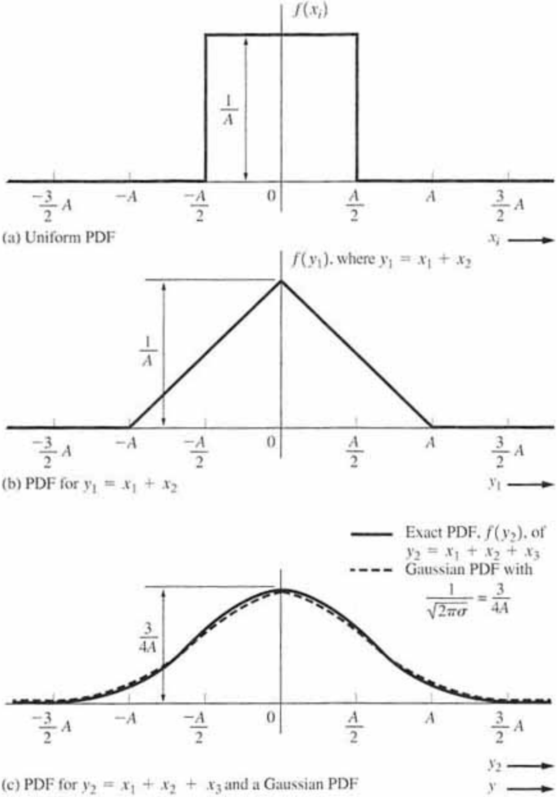

Figure B–14 Demonstration of the central limit theorem (Ex. B–8).

PROBLEMS

★ **B–1** A long binary message contains 1,428 binary 1s and 2,668 binary 0s. What is the probability of obtaining a binary 1 in any received bit?

★ **B–2** **(a)** Find the probability of getting an 8 in the toss of two dice.
(b) Find the probability of getting either a 5, 7, or 8 in the toss of two dice.

B–3 Show that

$$P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$$

★ **B–4** A die is tossed. The probability of getting any face is $P(x) = \frac{1}{6}$, where $x = k = 1, 2, 3, 4, 5$, or 6. Find the probability of getting an odd-numbered face.

★ **B–5** For the die toss of Prob. B–4, find the probability of getting a 4 when an even-numbered face is obtained on a toss.

B–6 Which of the following functions satisfy the properties for a PDF. Why?

(a) $f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$

(b) $f(x) = \begin{cases} |x|, & |x| < 1 \\ 0, & x \text{ otherwise} \end{cases}$

(c) $f(x) = \begin{cases} \frac{1}{6}(8-x), & 4 \le x \le 10 \\ 0, & x \text{ otherwise} \end{cases}$

(d) $f(x) = \sum_{k=0}^{\infty} \frac{3}{4}\left(\frac{1}{4}\right)^k \delta(xk)$

B–7 Show that all cumulative distribution functions must satisfy the properties given in Sec. B–5.

B–8 Let $f(x) = Ke^{-b|x|}$, where K and b are positive constants. Find the mathematical expression for the CDF and sketch the results.

★ **B–9** Find the probability that $-\frac{1}{4}A \le y_1 \le \frac{1}{4}A$ for the triangular distribution shown in Fig. B–14b.

B–10 A triangular PDF is shown in Fig. B–14b.
(a) Find a mathematical expression that describes the CDF.
(b) Sketch the CDF.

B–11 Evaluate the PDFs for the two waveforms shown in Fig. PB–11.

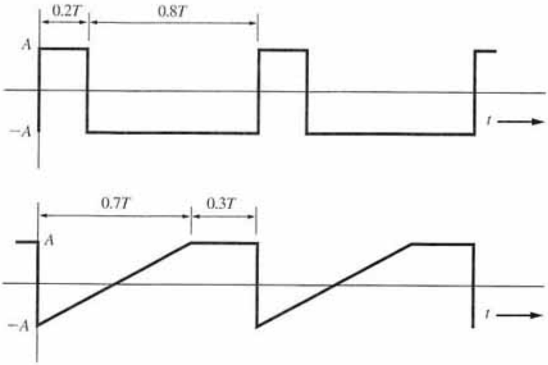

Figure PB–11

★ **B–12** Let a PDF be given by $f(x) = Ke^{-bx}$, for $x \geq 0$, and $f(x) = 0$, for $x < 0$, where K and b are positive constants.

(a) Find the value required for K in terms of b.

(b) Find m in terms of b.

(c) Find σ^2 in terms of b.

B–13 A random variable x has a PDF

$$f(x) = \begin{cases} \frac{3}{32}(-x^2 + 8x - 12), & 2 < x < 6 \\ 0, & x \text{ elsewhere} \end{cases}$$

(a) Demonstrate that $f(x)$ is a valid PDF.

(b) Find the mean.

(c) Find the second moment.

(d) Find the variance.

B–14 Determine the standard deviation for the triangular distribution shown in Fig. B–14b.

B–15 **(a)** Find the terms in a binomial distribution for $n = 7$, where $p = 0.5$.

(b) Sketch the PDF for this binomial distribution.

(c) Find and sketch the CDF for this distribution.

(d) Find $\overline{x^3}$ for this distribution.

B–16 For a binomial distribution, show that $\sigma^2 = np(1 - p)$.

★ **B–17** A binomial random variable x_k has values of k, where

$$k = 0, 1, \ldots, n; \quad P(k) = \binom{n}{k} p^k q^{n-k}; \quad q = 1 - p$$

Assume that $n = 160$ and $p = 0.1$.

(a) Plot $P(k)$.

(b) Compare the plot of part (a) with a plot $P(k)$ using the Gaussian approximation

$$\binom{n}{k} p^k q^{n-k} \approx \frac{1}{\sqrt{2\pi}\sigma} e^{-(k-m)^2/2\sigma^2}$$

which is valid when $npq \gg 1$ and $|k - np|$ is in the neighborhood of $\sqrt{npq}$, where $\sigma = \sqrt{npq}$ and $m = np$.

(c) Also plot the Poisson approximation

$$\binom{n}{k} p^k q^{n-k} \approx \frac{\lambda^k}{k!} e^{-\lambda}$$

where $\lambda = np$, n is large, and p is small.

B–18 An order of $n = 3{,}000$ transistors is received. The probability that each transistor is defective is $p = 0.001$. What is the probability that the number of defective transistors in the batch is 6 or less? (*Note:* The Poisson approximation is valid when n is large and p is small.)

B–19 In a fiber-optic communication system, photons are emitted with a Poisson distribution as described in Table B–1. $m = \lambda$ is the average number of photons emitted in an arbitrary time interval, and $P(k)$ is the probability of k photons being emitted in the same interval.

(a) Plot the PDF for $\lambda = 0.5$.

(b) Plot the CDF for $\lambda = 0.5$.

(c) Show that $m = \lambda$.

(d) Show that $\sigma = \sqrt{\lambda}$.

B–20 Let x be a random variable that has a Laplacian distribution. The Laplacian PDF is $f(x) = (1/2b)$ $e^{-|x-m|/b}$, where b and m are real constants and $b > 0$.

(a) Find the mean of x in terms of b and m.

(b) Find the variance of x in terms of b and m.

B–21 Referring to your solution for Prob. B–20, use MATLAB to plot the Laplacian PDF for $m = 15$ and $\sigma = 5$.

B–22 Given the Gaussian PDF

$$f(x) = \frac{1}{\sqrt{2\pi}\beta} e^{-(x-m)^2/(2\beta^2)}$$

show that the variance of this distribution is β^2.

★ **B–23** In a manufacturing process for resistors, the values obtained for the resistors have a Gaussian distribution where the desired value is the mean value. If we want 95% of the manufactured 1-kΩ resistors to have a tolerance of ±10%, what is the required value for σ?

B–24 Assume that x has a Gaussian distribution. Find the probability that

(a) $|x - m| < \sigma$.

(b) $|x - m| < 2\sigma$.

(c) $|x - m| < 3\sigma$.

Obtain numerical results by using MATLAB, or tables if necessary.

B–25 Show that

(a) $Q(z) = \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$.

(b) $Q(-z) = 1 - Q(z)$.

(c) $Q(z) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right]$.

B–26 Using MATLAB, plot $Q(z)$ as defined by Eq. (B–60) and Prob. B–25.

B–27 For a Gaussian distribution, show that

(a) $F(a) = \frac{1}{2}\operatorname{erfc}\left(\frac{m-a}{\sqrt{2}\sigma}\right)$. **(b)** $F(a) = \frac{1}{2}\left[1 + \operatorname{erfc}\left(\frac{a-m}{\sqrt{2}\sigma}\right)\right]$.

B–28 Using MATLAB, plot the CDF for a Gaussian random variable where $m = 10$ and $\sigma = 2$.

B–29 A noise voltage has a Gaussian distribution. The RMS value is 5 V, and the DC value is 1.0 V. Find the probability of the voltage having values between −5 and +5 V.

★ **B–30** Suppose that x is a Gaussian random variable with $m = 5$ and $\sigma = 0.6$.

(a) Find the probability that $x \le 1$.

(b) Find the probability that $x \le 6$.

B–31 The Gaussian random variable x has a zero mean and a variance of 2. Let A be the event such that $|x| < 3$.

(a) Find an expression for the conditional PDF $f(x|A)$.

(b) Plot $f(x|A)$ over the range $|x| < 5$.

(c) Plot $f(x)$ over the range $|x| < 5$ and compare these two plots.

B–32 Let x have a sinusoidal distribution with a PDF as given by Eq. (B–67). Show that the CDF is

$$F(a) = \begin{cases} 0, & a \le -A \\ \dfrac{1}{\pi}\left[\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{a}{A}\right)\right], & |a| \le A \\ 1, & a \ge A \end{cases}$$

B–33 **(a)** If x has a sinusoidal distribution with the peak value of x being A, show that the RMS value is $\sigma = A/\sqrt{2}$. [*Hint*: Use Eq. (B–67).]

(b) If $x = A\cos\psi$, where ψ is uniformly distributed between $-\pi$ and $+\pi$, show that the RMS value of x is $\sigma = A/\sqrt{2}$.

★ **B–34** Given that $y = x^2$ and x is a Gaussian random variable with mean value m and variance σ^2, find a formula for the PDF of y in terms of m and σ^2.

B–35 x is a uniformly distributed random variable over the range $-1 \le x \le 1$ plus a discrete point at $x = \frac{1}{2}$ with $P\left(x = \frac{1}{2}\right) = \frac{1}{4}$.

(a) Find a mathematical expression for the PDF for x, and plot your result.

(b) Find the PDF for y, where

$$y = \begin{cases} x^2, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

Sketch your result.

B–36 A saturating amplifier is modeled by

$$y = \begin{cases} Ax_0, & x > x_0 \\ Ax, & |x| \le x_0 \\ -Ax_0, & x < -x_0 \end{cases}$$

Assume that x is a Gaussian random variable with mean value m and variance σ^2. Find a formula for the PDF of y in terms of A, x_0, m, and σ^2.

B–37 Using MATLAB and your results for Prob. B–36, plot the PDF for the output of a saturating amplifier with a Gaussian input if $x_0 = 5$, $A = 10$, $m = 2$, and $\sigma = 1.5$.

★ **B–38** A sinusoid with a peak value of 8 V is applied to the input of a quantizer. The quantizer characteristic is shown in Fig. 3–8a. Calculate and plot the PDF for the output.

B–39 A voltage waveform that has a Gaussian distribution is applied to the input of a full-wave rectifier circuit. The full-wave rectifier is described by $y(t) = |x(t)|$, where $x(t)$ is the input and $y(t)$ is the output. The input waveform has a DC value of 1 V and an RMS value of 2 V.

(a) Plot the PDF for the input waveform.

(b) Plot the PDF for the output waveform.

★ **B–40** Refer to Example B–10 and Eq. (B–75), which describe the PDF for the output of an ideal diode (half-wave rectifier) characteristic. Find the mean (DC) value of the output.

B–41 Given the joint density function,

$$f(x_1, x_2) = \begin{cases} e^{-(1/2)(4x_1 + x_2)}, & x_1 \ge 0, x_2 \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Verify that $f(x_1, x_2)$ is a density function.
(b) Show that x_1 and x_2 are either independent or dependent.
(c) Evaluate $P(1 \le x_1 \le 2, x_2 \le 4)$.
(d) Find ρ.

★ **B–42** A joint density function is

$$f(x_1, x_2) = \begin{cases} K(x_1 + x_1 x_2), & 0 \le x_1 \le 1, \quad 0 \le x_2 \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find K.
(b) Determine if x_1 and x_2 are independent.
(c) Find $F_{x_1 x_2}(0.5, 2)$.
(d) Find $F_{x_2|x_1}(x_2|x_1)$.

B–43 Let $y = x_1 + x_2$, where x_1 and x_2 are uncorrelated random variables. Show that
(a) $\bar{y} = m_1 + m_2$, where $m_1 = \overline{x_1}$ and $m_2 = \overline{x_2}$.
(b) $\sigma_y^2 = \sigma_1^2 + \sigma_2^2$, where $\sigma_1^2 = \overline{(x_1 - m_1)^2}$ and $\sigma_2^2 = \overline{(x_2 - m_2)^2}$.
[*Hint*: Use the ensemble operator notation similar to that used in the proof for Eq. (B–29).]

B–44 Let $x_1 = \cos\theta$ and $x_2 = \sin\theta$, where θ is uniformly distributed over $(0, 2\pi)$. Show that
(a) x_1 and x_2 are uncorrelated.
(b) x_1 and x_2 are not independent.

★ **B–45** Two random variables x_1 and x_2 are jointly Gaussian. The joint PDF is described by Eq. (B–97), where $m_1 = m_2 = 0$, $\sigma_{x_1} = \sigma_{x_2} = 1$ and $\rho = 0.5$. Plot $f(x_1, x_2)$ for x_1 over the range $|x_1| < 5$ and $x_2 = 0$. Also give plots for $f(x_1, x_2)$ for $|x_1| < 5$ and $x_2 = 0.4, 0.8, 1.2$, and 1.6.

B–46 Show that the marginal PDF of a bivariate Gaussian PDF is a one-dimensional Gaussian PDF. That is, evaluate

$$f(x_1) = \int_{-\infty}^{\infty} f(x_1, x_2)\, dx_2$$

where $f(x_1, x_2)$ is given by Eq. (B–97). [*Hint*: Factor some terms outside the integral containing x_1 (but not x_2). Complete the square on the exponent of the remaining integrand so that a Gaussian PDF form is obtained. Use the property that the integral of a properly normalized Gaussian PDF is unity.]

B–47 The input to a receiver consists of a binary signal plus some noise. That is, assume that the input y is $y = x + n$, where x is random binary data with values of $\pm A$ Volts that are equally likely to occur. Let n be independent Gaussian random noise with a mean of m and a standard deviation of σ. Find the PDF for y as a function of A, m, and σ.

B–48 Referring to your solution for Prob. B–47 and using MATLAB, plot the PDF for y, where $A = 12$, $m = 2$, and $\sigma = 4$.

B–49 Referring to your solution for Prob. B–47 and using MATLAB, calculate the probability that the voltage at the receiver input is between 10 and 14 Volts, where $A = 12$, $m = 2$, and $\sigma = 4$.

B–50 **(a)** $y = A_1 x_1 + A_2 x_2$, where A_1 and A_2 are constants and the joint PDF of x_1 and x_2 is $f_{\mathbf{x}}(x_1, x_2)$. Find a formula for the PDF of y in terms of the (joint) PDF of $\mathbf{x}$.
(b) If x_1 and x_2 are independent, how can this formula be simplified?

B–51 Two independent random variables—x and y—have the PDFs $f(x) = 5e^{-5x}\,u(x)$ and $f(y) = 2e^{-2y}u(y)$. Plot the PDF for w where $w = x + y$.

★ **B–52** Two Gaussian random variables x_1 and x_2 have a mean vector $\mathbf{m}_x$ and a covariance matrix $\mathbf{C}_x$ as shown. Two new random variables y_1 and y_2 are formed by the linear transformation $\mathbf{y} = \mathbf{T}\mathbf{x}$.

$$\mathbf{m}_x = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \mathbf{C}_x = \begin{bmatrix} 5 & -2/\sqrt{5} \\ -2/\sqrt{5} & 4 \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$$

(a) Find the mean vector for $\mathbf{y}$, which is denoted by $\mathbf{m}_y$.
(b) Find the covariance matrix for $\mathbf{y}$, which is denoted by $\mathbf{C}_y$.
(c) Find the correlation coefficient for y_1 and y_2. (*Hint:* See Sec. 6–6.)

B–53 Three Gaussian random variables x_1, x_2, and x_3 have zero mean values. Three new random variables y_1, y_2, and y_3 are formed by the linear transformation $\mathbf{y} = \mathbf{T}\mathbf{x}$, where

$$C_x = \begin{bmatrix} 6.0 & 2.3 & 1.5 \\ 2.3 & 6.0 & 2.3 \\ 1.5 & 2.3 & 6.0 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 5 & 2 & -1 \\ -1 & 3 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

(a) Find the covariance matrix for $\mathbf{y}$, which is denoted by $\mathbf{C}_y$.
(b) Write an expression for the PDF $f(y_1, y_2, y_3)$. (*Hint*: See Sec. 6–6).

★ **B–54** **(a)** Find a formula for the PDF of $y = Ax_1\,x_2$, where x_1 and x_2 are random variables having the joint PDF $f_{\mathbf{x}}(x_1, x_2)$.
(b) If x_1, and x_2 are independent, reduce the formula obtained in part (a) to a simpler result.

B–55 $y_2 = x_1 + x_2 + x_3$, where x_1, x_2, and x_3 are independent random variables. Each of the x_i has a one-dimensional PDF that is uniformly distributed over $-(A/2) \le x_i \le (A/2)$. Show that the PDF of y_2 is given by Eq. (B–109).

★ **B–56** Use the built-in random number generator of MATLAB to demonstrate the central limit theorem. That is,
(a) Compute samples of the random variable y, where $y = \sum x_i$ and the x_i values are obtained from the random number generator.
(b) Plot the PDF for y by using the histogram function of MATLAB.