Question 1

a) $s_{AM}(t) = A_c[1+k_a m(t)]\cos(2\pi f_c t+\theta)$

$s_{AM}(t) = Re[\tilde{s}_{AM}(t)\cdot e^{j2\pi f_c t}]$

i) $s_{AM}(t) = Re\{A_c[1+k_a m(t)]e^{j2\pi f_c t+\theta}\}$

$\Rightarrow \tilde{s}_{AM}(t) = A_c[1+k_a m(t)]e^{j\theta}$

ii) $s^{I}_{AM}(t) = A_c[1+k_a m(t)]\cos\theta$

$s^{Q}_{AM}(t) = A_c[1+k_a m(t)]\sin\theta$

iii) $|s_{AM}(t)| = A_c[1+k_a m(t)]$

$\angle s_{AM}(t) = \theta$

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b) $s_{QAM}(t) = m_1(t)A_c\cos(2\pi f_c t) + m_2(t)A_c\sin(2\pi f_c t)$

$= m_1(t)A_c\cos(2\pi f_c t) + m_2(t)A_c\cos(2\pi f_c t - \pi/2)$

i) $\tilde{s}_{QAM}(t) = m_1(t)A_c + m_2(t)A_c e^{-j\pi/2}$

$= A_c[m_1(t) - j m_2(t)]$

ii) $s^{I}_{QAM}(t) = m_1(t)A_c$

$s^{Q}_{QAM}(t) = -m_2(t)A_c$

iii) $|s_{QAM}(t)| = \sqrt{(m_1(t)A_c)^2 + (-m_2(t)A_c)^2}$

$|s_{QAM}(t)| = \left(\sqrt{m_1(t)^2 + m_2^2(t)}\right)\cdot A_c$

$\angle s_{QAM}(t) = \tan^{-1}\left(\frac{-m_2(t)A_c}{m_1(t)A_c}\right)$

$\angle s_{QAM}(t) = -\tan^{-1}\left(\frac{m_2(t)}{m_1(t)}\right)$

②

c) $s_{USB}(t) = \frac{1}{2} m(t) \cos(2\pi f_c t) - \frac{1}{2} m_h(t) \sin(2\pi f_c t)$

$= \frac{1}{2} m(t) \cos(2\pi f_c t) - \frac{1}{2} m_h(t) \cos(2\pi f_c t - \frac{\pi}{2})$

i) $\tilde{s}_{USB}(t) = \frac{1}{2} m(t) - \frac{1}{2} m_h(t) e^{-j\pi/2}$

$\tilde{s}_{USB}(t) = \frac{1}{2} m(t) + \frac{1}{2} m_h(t) e^{j\pi/2}$

$= \frac{1}{2}[m(t) + j m_h(t)]$

ii) $s^{I}_{USB}(t) = \frac{1}{2} m(t)$

$s^{Q}_{USB}(t) = \frac{1}{2} m_h(t)$

iii) $|s_{USB}(t)| = \sqrt{\left(\frac{1}{2} m(t)\right)^2 + \left(\frac{1}{2} m_h(t)\right)^2}$

$|s_{USB}(t)| = \frac{1}{2}\sqrt{m^2(t) + m_h^2(t)}$

$\angle s_{USB}(t) = \tan^{-1}\left(\frac{\frac{1}{2} m_h(t)}{\frac{1}{2} m(t)}\right) = \tan^{-1}\left(\frac{m_h(t)}{m(t)}\right)$

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d) $s_{FM}(t) = A_c \cos\left(2\pi f_c t + k_f \int_0^t m(\tau) d\tau\right)$

i) $\tilde{s}_{FM}(t) = A_c e^{j k_f \int_0^t m(\tau) d\tau}$

ii) $s^{I}_{FM}(t) = A_c \cos\left(k_f \int_0^t m(\tau) d\tau\right)$

$s^{Q}_{FM}(t) = A_c \sin\left(k_f \int_0^t m(\tau) d\tau\right)$

iii) $|s_{FM}(t)| = A_c$

$\angle s_{FM}(t) = k_f \int_0^t m(\tau) d\tau$

Question 2 :

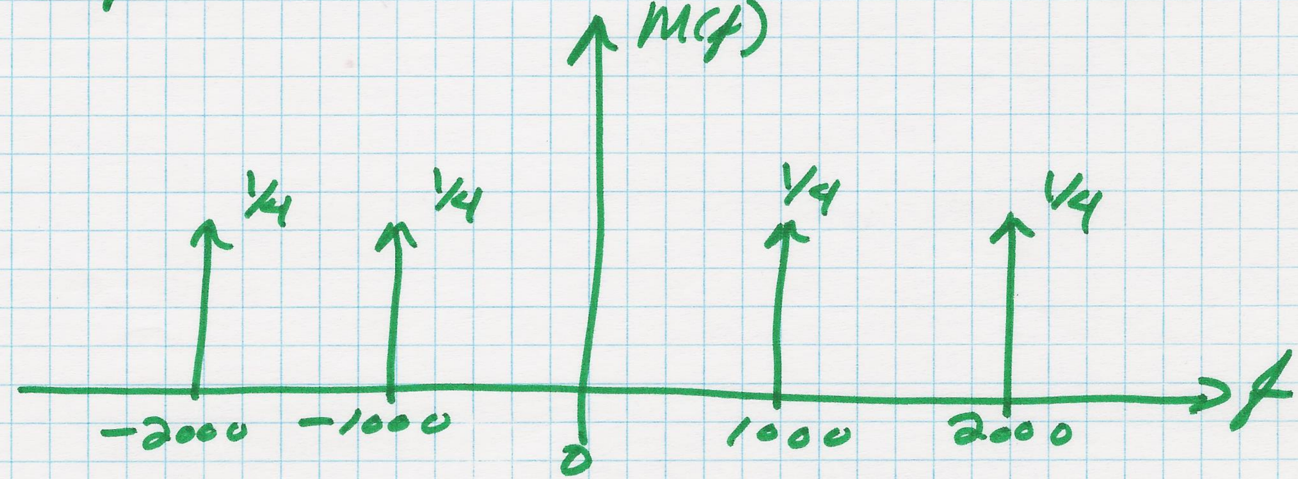
$$m(t) = \cos(1000\pi t) \cdot \cos(3000\pi t)$$

a) Spectre $M(f)$:

$$M(f) = \mathcal{F}[m(t)]$$
$$= \mathcal{F}[\cos(1000\pi t) \cdot \cos(3000\pi t)]$$
$$= \mathcal{F}[\cos(1000\pi t)] * \mathcal{F}[\cos(3000\pi t)]$$

$$M(f) = \frac{1}{2}[\delta(f-500) + \delta(f+500)]$$
$$* \frac{1}{2}[\delta(f-1500) + \delta(f+1500)]$$

$$M(f) = \frac{1}{4}[\delta(f-2000) + \delta(f-1000) + \delta(f+1000) + \delta(f+2000)]$$

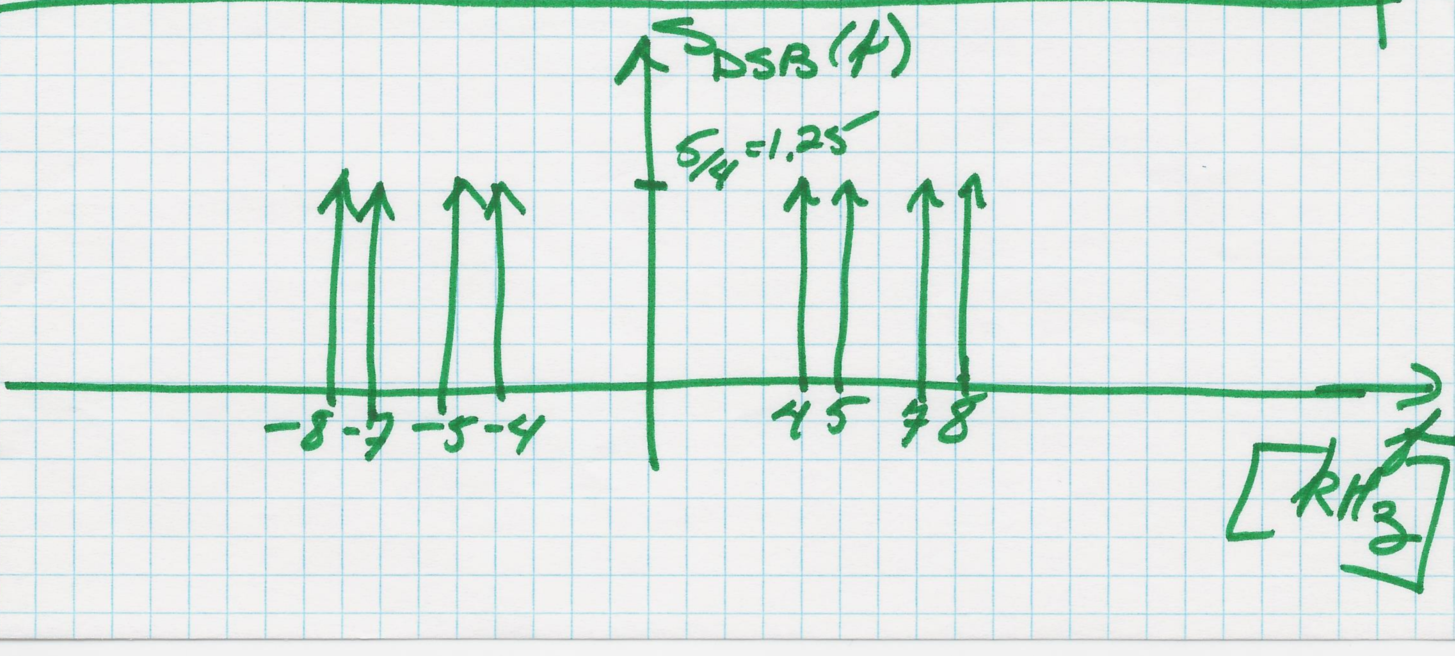


b) $s_{DSB}(t) = 10\, m(t) \cos(12000\pi t)$

$$S_{DSB}(f) = \mathcal{F}[s_{DSB}(t)]$$
$$= \mathcal{F}[10\, m(t) \cos(12000\pi t)]$$
$$= 10 \cdot \mathcal{F}[m(t)] * \mathcal{F}[\cos(12000\pi t)]$$
$$= 10 \cdot M(f) * \frac{1}{2}[\delta(f-6000) + \delta(f+6000)]$$

$$S_{DSB}(f) = 5[M(f-6000) + M(f+6000)]$$

$$\boxed{\begin{aligned} S_{DSB}(f) = \frac{5}{4}\Big[&\delta(f-8000) + \delta(f-7000) \\ &+ \delta(f-5000) + \delta(f-4000)\Big] \\ + \frac{5}{4}\Big[&\delta(f+4000) + \delta(f+5000) \\ &+ \delta(f+7000) + \delta(f+8000)\Big] \end{aligned}}$$



c) <u>Bandes latérales hautes :</u>

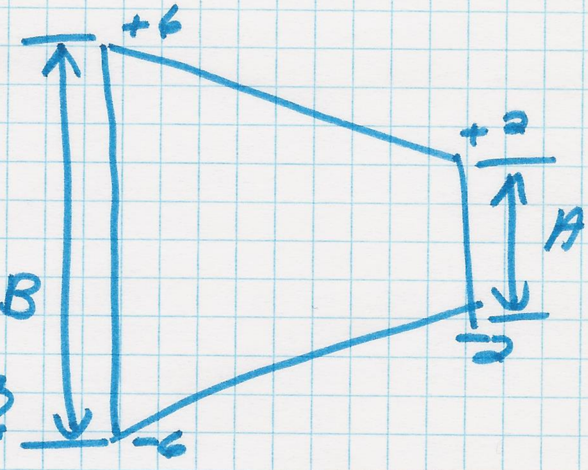
$-8000\ Hz,\ -7000\ Hz,\ +7000\ Hz,\ +8000\ Hz$

<u>Bandes latérales basses :</u>

$-4000\ Hz,\ -5000\ Hz,\ +5000\ Hz,\ +4000\ Hz$

Question 3

Méthode des trapèzes



a)

$$m_a = \frac{B - A}{B + A} \times 100\%$$

$$m_a = \left(\frac{12 - 4}{12 + 4}\right) \times 100\% = \frac{8}{16} \times 100\%$$

$$\boxed{m_a = 50\% = 0.5}$$

b) $m(t) = 10 \cos(200\pi t + \pi/4)$

et $s_{AM}(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$

$m_a = \max|k_a m(t)|$

$50\% = k_a \max|m(t)| = 10 k_a$

$$\Rightarrow \boxed{k_a = 0.05}$$

Question 4 $\quad m(t) = \cos(2\pi t) + 2\cos(12\pi t)$

$$s(t) = A_c \cos[2\pi f_c t + \theta(t)]$$

où $A_c = 10[V]$ et $f_c = 200[Hz]$

a) modulation FM avec $k_f = 36\pi \left[\frac{rad/s}{V}\right]$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right]$$

où $\int_0^t m(\tau)d\tau = \int_0^t [\cos(2\pi\tau) + 2\cos(12\pi\tau)]d\tau$

$$= \frac{1}{2\pi}\sin(2\pi t) + \frac{2}{12\pi}\sin(12\pi t)$$

$$\Rightarrow s_{FM}(t) = 10\cos\left[400\pi t + 36\pi \cdot \left[\frac{1}{2\pi}\sin(2\pi t) + \frac{2}{12\pi}\sin(12\pi t)\right]\right]$$

$$\boxed{s_{FM}(t) = 10\cos[400\pi t + 18\sin(2\pi t) + 6\sin(12\pi t)]}$$

b) $f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$

$$= 200 + \frac{36\pi}{2\pi}[\cos(2\pi t) + 2\cos(12\pi t)]$$

$$\boxed{f_i(t) = 200 + 18\cos(2\pi t) + 36\cos(12\pi t)}$$

c) $\Delta f_{max} = \frac{k_f}{2\pi} \max\left[\underbrace{|\cos(2\pi t) + 2\cos(12\pi t)|}_{\text{à } t=0 \Rightarrow \max = 3}\right]$

$$\boxed{\Delta f_{max} = 18 \cdot 3 = 54 \text{ [Hz]}}$$

2

d) $\beta_{FM} = \dfrac{\Delta f_{max}}{W} = \dfrac{54\,[Hz]}{6\,[Hz]} = 9$

car $m(t) = \cos(\underbrace{2\pi t}_{f_1 = 1\,Hz}) + 2\cos(\underbrace{12\pi t}_{f_2 = 6\,Hz})$

$\Rightarrow W = 6\,[Hz]$

$\Rightarrow$ $\boxed{\beta_{FM} = 9 \Rightarrow \text{modulation FM à large bande}}$

e) $B_T \cong 2(\beta_{FM}+1)\,W$

$\cong 2(9+1)\,6\,[Hz]$

$$\boxed{B_T \cong 120\,[Hz]}$$

Question 5

$f_{IF} = 500\,MHz$

a) mode analogique

$f_{osc} = f_{RF} + f_{IF}$

$f_{RF} = 870\,MHz$

(°) $\boxed{f_{osc} = 870 + 500 = 1370\,MHz}$

$\boxed{f_{image} = f_{RF} + 2f_{IF} = 1870\,MHz}$

b) mode numérique

$f_{osc} = f_{RF} - f_{IF}$

$f_{RF} = 1940\,MHz$

(°) $\boxed{f_{osc} = 1940 - 500 = 1440\,MHz}$

$\boxed{f_{image} = f_{RF} - 2f_{IF} = 940\,MHz}$

c) avantages :

⇒ 1 seul circuit (étage) IF pour les 2 modes d'opération