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Abstract

Simple experiments involving musical instruments of the woodwind family can be used to demonstrate the basic physics of vibrating air columns in resonance tubes using nothing more than straightforward measurements and data collection hardware and software. More involved experimentation with the same equipment can provide insight into the effects of holes in the tubing and other factors that make simple tubes useful as musical instruments.

Introduction

Exploration of the physics of music and specifically behind the operation of musical instruments is often one of the best ways to make physics seem interesting and relevant to students. What follows are some investigations involving woodwind instruments, specifically the flute and clarinet. The simpler experiments were developed and have proven useful as laboratories in a science of sound and light course for fine-arts students. The more in-depth investigations may be more appropriate for physics students with an interest in music and or musical instruments, perhaps as more advanced laboratory projects or independent or directed studies.

Resonance tubes

Musical instruments of the woodwind family are basically resonance tubes in which a standing wave is generated and maintained in the air column. In the case of the simplest woodwind, the flute, the player generates vibrations in the air column by blowing air against the edge of an 'embouchure hole'. The air column extends approximately from the embouchure hole centre to just beyond the first open 'tone hole' or just beyond the end of the tube if all the tone holes are closed. See figure 1[1]. The air column in a clarinet is excited when the player blows over and vibrates a reed which excites an air column that extends also approximately to the first open tone hole's centre or to the beginning of a flaring 'bell' section that aids in projection of the sound [1] (see figure 2)[2].

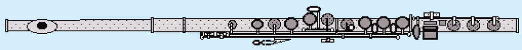

Figure 1. The air column in a flute extends approximately from the embouchure hole (left) to the first open tone hole, or just beyond the end of the tube if all the tone holes are closed.

The basic physics of vibrating air columns is well understood. If a tube is open at both ends, which is the case with the flute, there are pressure nodes or displacement antinodes [2, 3] at both ends of the air column. It is true that the end of a flute's 'head joint', the piece of tubing that contains the embouchure hole, is closed, but the actual vibrating air column that produces the sound

[1] From http://www.phys.unsw.edu.au/music/flute/, used with permission.
[2] From http://www.phys.unsw.edu.au/music/clarinet/, used with permission.

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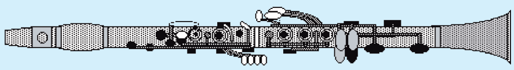

Figure 2. The air column in a clarinet extends approximately from the reed (left) to the first open tone hole, or to the beginning of the flaring bell section if all the tone holes are closed.

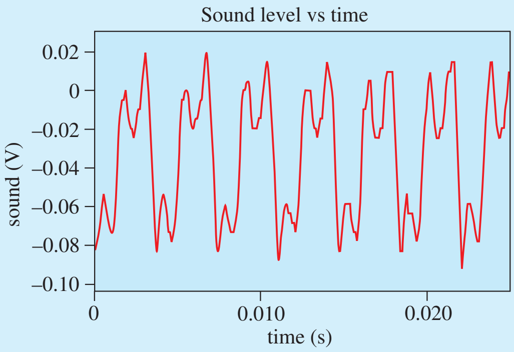

Figure 3. The waveform produced by a flute.

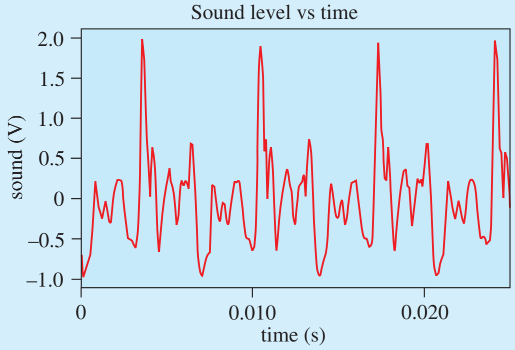

Figure 4. The waveform produced by a clarinet.

begins where the vibrations are initially generated, at the open embouchure hole, not at the closed end of the head joint. The resonant frequencies for an air column open at both ends are given by the expression $f_n = nc/2L$ where L is the length of the tube, c is the speed of sound, approximately 344 m s^{-1} at room temperature, and n is the mode of vibration or harmonic number. The musical 'pitch' that is heard when a flute is played comes from the frequency of the first, $n = 1$, harmonic or the fundamental. The higher harmonics add colour or 'timbre' to the instrument's tone by making the waveform more complex than that of a simple sine wave which is what produces the rather dull sound heard when blowing into a simple tube (see figure 3).

When a clarinet is played, the reed end of the air column is closed by the player's lips, creating a pressure antinode or displacement node ([2], p 265, [3]) at the mouthpiece end and thus an air column open only at one end, giving the frequencies $f_n = (2n - 1)c/4L$, only odd harmonics. As with the flute, the perceived pitch comes from the $n = 1$, the fundamental, and the higher, in this case odd only, harmonics add form to the wave, resulting in the characteristic 'darker' tone of the clarinet compared to the brighter tone of a flute. See figure 4 (see footnote 3).

In practice, the displacement antinode of a standing wave is actually found just beyond the open end of a tube because the air continues travelling as if in the tube for a short distance before spreading out. This results in the air column being longer than the actual resonance tube or in a slightly longer 'equivalent length' of the vibrating air column. For a simple tube, the equivalent length is longer than the actual length by an 'end correction' equal to about a third of the diameter of the tube ([2], p 234, [4]). In the more complicated tubes of real musical instruments, corrections for both open and closed tone holes, embouchure holes and mouthpiece cavities are necessary to determine equivalent lengths [5].

Observations

Different musical notes are played on both flutes and clarinets by varying the length of the vibrating air column. When all the tone holes are closed, the air column is at its maximum length so, since the length is in the denominator of the above expressions for the frequencies produced by vibrating air columns, the lowest frequency and therefore lowest musical pitches will sound. Higher pitches are played by opening tone holes to shorten the length of the air column, thus raising the frequency. The instruments are designed so that each successive tone hole makes the air column about 6% shorter, raising the frequency by 6%. In the system of 'equal-tempered' tuning currently used in western music, adjacent musical pitches are separated by a frequency ratio of $\sqrt[12]{2} \cong 1.059$, about 6% ([1] p 147).

Table 1 shows the frequencies sounded by a flute when different musical notes are being

Table 1. The measured frequencies of the musical notes played on a flute, the musical intervals between the measured frequencies and the measured lengths compared to the equivalent lengths calculated from the measured frequencies.

Musical note	Frequency (Hz)	Interval with previous pitch	Interval with first pitch	Expected interval	Length (measured) L (m)	Length (equivalent) L_e (m)	$L_e - L$ (m)
C-4	260	1.00	1.00	1.00	0.600	0.662	0.062
C#-4	275	1.06	1.06	1.06	0.550	0.625	0.075
D4	291	1.06	1.12	1.12	0.524	0.591	0.067
D#-4	310	1.07	1.19	1.19	0.490	0.555	0.065
E4	328	1.06	1.26	1.26	0.457	0.524	0.067
F4	348	1.06	1.34	1.33	0.428	0.494	0.066
F#-4	369	1.06	1.42	1.41	0.401	0.466	0.065
G-4	392	1.06	1.51	1.50	0.376	0.439	0.063
G#-4	415	1.06	1.60	1.59	0.350	0.414	0.064
A-4	441	1.06	1.70	1.68	0.327	0.390	0.063
A#-4	472	1.07	1.82	1.78	0.304	0.364	0.060
B-4	493	1.04	1.90	1.89	0.284	0.349	0.065
C-5	529	1.07	2.03	2.00	0.264	0.325	0.061
C#-5	573	1.08	2.20	2.12	0.233	0.300	0.067

played, first with all the tone holes closed, then with each tone hole in order of increasing distance from the end of the tube open and the corresponding lengths measured from the centre of the embouchure hole to the centre of the first open hole or, when all the tone holes are closed, to the end of the tube. The table also shows the frequency ratios between adjacent pitches and the actual and expected frequency ratios of each pitch with the lowest pitch. Frequencies were determined by using the time axis of the displays shown in figures 3 and 4 to measure the periods of the waveforms then reciprocating them.

Table 2 is the same as table 1 but for the clarinet. Air column lengths were measured from the tip of the reed to the centre of the first open tone hole. The maximum length was measured to the beginning of the flaring bell section. The standing waves for lower harmonics tend to terminate near the beginning of a flaring bell section while higher harmonics penetrate further into the bell [6]. Since the frequency of the fundamental is what determines the pitch of a clarinet, for purposes of comparing frequency and length, the air column can be approximated as ending at the beginning of the flaring section [5].

Initial inspection of the data in tables 1 and 2 shows that adjacent pitches for both the flute and clarinet are indeed close to 6% apart and that the musical intervals, the frequency ratios that each successively higher pitch makes with the lowest pitch are also close to what is expected ([1] p 146).

The end corrections for each individual playing frequency and corresponding tube length shown in tables 1 and 2 were calculated by taking the difference between the actual length, L, of a tube and the equivalent length of the air column given, for a flute, by $L_e = c/2f$ or $L_e = c/4f$ for a clarinet. Both the equivalent air column lengths and end corrections, $\Delta L = L_e - L$ for a flute and clarinet are in tables 1 and 2.

Another observation that can be made from inspection of the data in tables 1 and 2 is when playing the same musical notes, C-4 to G-4, the measured flute tube lengths are about twice the clarinet tube lengths. This is because the fundamental frequencies produced by tubes open at both ends, $f = c/2L$, flutes, are twice, $f = c/4L$, those produced by tubes closed at one end, clarinets.

The software[3] used to capture the sound waves and display the waveforms has a sampling rate that can be varied to a minimum of $\Delta T = 0.02$ ms between data points. The expressions used to calculate the equivalent lengths in tables 1

[3] The waveforms were captured with a microphone and a Lab-Pro interface and displayed with Logger-Pro; all products of Vernier Software and Technologies, www.vernier.com. Any similar hardware and software could be used such as the Science Workshop Interface and Data Studio of Pasco, www.pasco.com.

Table 2. The measured frequencies of the musical notes played on a clarinet, the musical intervals between the measured frequencies and the measured lengths compared to the equivalent lengths calculated from the measured frequencies.

Musical note	Frequency (Hz)	Interval with previous pitch	Interval with first pitch	Expected interval	Length (measured) L (m)	Length (equivalent) L_e (m)	$L_e - L$ (m)
D-3	146	1.00	1.00	1.00	0.582	0.589	0.007
D#-3	155	1.06	1.06	1.06	0.540	0.555	0.015
E-3	165	1.06	1.13	1.12	0.508	0.521	0.013
F-3	175	1.06	1.20	1.19	0.473	0.491	0.018
F#-3	187	1.07	1.28	1.26	0.449	0.460	0.011
G-3	198	1.06	1.36	1.33	0.416	0.434	0.018
G#-3	211	1.07	1.45	1.41	0.393	0.408	0.015
A-3	226	1.07	1.55	1.50	0.367	0.381	0.014
A#-3	237	1.05	1.62	1.59	0.352	0.363	0.011
B-3	250	1.05	1.71	1.68	0.322	0.344	0.022
C-4	263	1.05	1.80	1.78	0.311	0.327	0.016
C#-4	286	1.09	1.96	1.89	0.289	0.301	0.012
D-4	299	1.05	2.05	2.00	0.274	0.288	0.014
D#-4	317	1.06	2.17	2.12	0.254	0.271	0.017
E-4	340	1.07	2.33	2.24	0.241	0.253	0.012
F-4	355	1.04	2.43	2.38	0.232	0.242	0.010
F#-4	377	1.06	2.58	2.52	0.216	0.228	0.012
G-4	400	1.06	2.74	2.67	0.204	0.215	0.011

and 2 could be written for a flute and clarinet respectively as $L = cT/2$ and $L = cT/4$ which would give uncertainties in the calculated equivalent lengths of $\Delta L = (c/2)\Delta T$ and $\Delta L = (c/4)\Delta T$, 3 mm for the flute and 1.5 mm for the clarinet. These values are only about 5% of the calculated end corrections for the flute in table 1 and about 10% of those for the clarinet in table 2. Measuring the actual lengths of the air columns was straightforward with an error of at most $\Delta L =$ 1 mm.

Graphical investigations

Figure 5 is a plot of the measured playing frequencies and corresponding measured tube lengths for a flute (data from table 1)[4]. An equation of the form $f = A/(L + C)^B$ should fit the plot. A should equal $c/2$, c being the speed of sound; B should be close to 1, since a flute's tube is considered open at both ends; and C is the necessary end correction added to the tube length L to equal the equivalent length of the air column.

A curve fit to the plot shown in figure 5, taking $A = 344/2 = 172$ as a known, gave $B = 1.06$. Taking $B = 1$ as the known gave $A = 171$ or $c = 342$ m s^{-1} for the speed of sound. The best agreement with the average value of end correction from the flute data in table 1, 6.5 $\pm$ 0.4 cm, was 6.43 cm, obtained from the fit shown in figure 5 where $A = 172$ and $B = 1$ were both considered knowns.

A plot of the clarinet data from table 1 was similar to figure 5 and curve fits, this time with

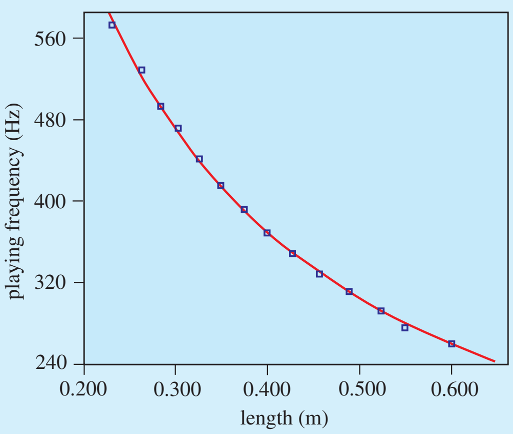

Figure 5. Plot of playing frequency versus air column length for a flute.

[4] Plot and fits done with Graphical Analysis from Vernier Software and Technologies, www.vernier.com.

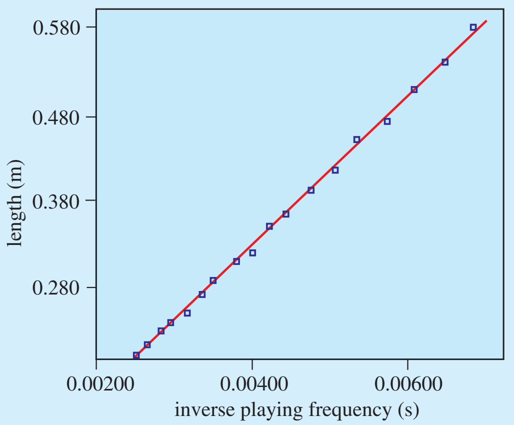

Figure 6. Plot of air column length versus inverse of playing frequency for a clarinet with a linear fit.

$A = c/4$ because the clarinet is closed at the mouthpiece end, gave comparable values for the speed of sound and the exponent, B.

The average end correction for the clarinet data in table 2 was 1.37 ± 0.35 cm. The best agreement with this value came from combining the previous two expressions, which gives $\Delta L = c/4f - L$ or $L = (c/4)*(1/f) - \Delta L$, the equation of a line with slope $c/4$ and a y intercept equal to the negative of the end correction. A linear fit to a plot of L versus $1/f$ for the clarinet data from table 2 is shown in figure 6 (see footnote 4). $c/4 = 86$ gives a speed of sound of 344 m s^{-1} and the y intercept gives an end correction of 1.36 cm.

It can be instructive to plot and analyse the frequency versus length data for both instruments in the manners of both figures 5 and 6. Figure 5 directly shows the inverse proportion between frequency and length, which could be the main objective of the investigation in a less mathematical, more descriptive course, like many physics of music or science of sound courses. The linear plot in figure 6 is a more sophisticated analysis and gives not only the end correction, but also the speed of sound. This analysis would be appropriate in a course of higher mathematical level, such as introductory physics.

More on flutes

As mentioned above, the equivalent length of the vibrating air columns in woodwind instruments can be calculated by considering the effects of both open and closed tone holes as well as the mouthpiece. Just as the antinode of a standing wave is found just beyond the end of a tube it is also found just beyond the end of an open tone hole by an amount that depends on the dimensions of the hole and the tube it is in.

An expression for the distance beyond the centre of a tone hole that the vibrating air column actually ends is

$$\Delta L_{\mathrm{H}} = s\left(\sqrt{1 + 2(t_{\mathrm{e}}/s)(a/b)^2} - 1\right) \qquad (1)$$

where $2s$ is the distance between adjacent tone holes or 'tone hole spacing', a is the inside radius of the tube, b is the inside radius of the tone hole and t is the thickness or height of the tone hole; $t_{\mathrm{e}} = t + 1.5b$ is a corrected tone hole height [7].

The equivalent length, L_{e}, of an actual tube of measured length L in a flute vibrating at a frequency $f = c/2L_{\mathrm{e}}$ can be calculated if the embouchure hole is also accounted for. Empirical measurements over a wide range of frequencies have shown the embouchure hole of a flute adds about $\Delta L_{\mathrm{E}} = 5$ cm of equivalent length to a tube [8, 9].

Table 3 compares the equivalent lengths of the flute tubes of actual lengths, L, vibrating at frequencies f, calculated by $L_{\mathrm{e}} = c/2f$ to effective lengths calculated from $L_{\mathrm{e}} = L + \Delta L_{\mathrm{H}} + \Delta L_{\mathrm{E}}$. For each tube of successively shorter length, ΔL_H is calculated from equation (1) for the first open hole, the one furthest 'up' the tube or closest to the embouchure hole. The dimensions of the flute and tone holes are $2a = 1.87$ cm, $2b = 1.61$ cm, $t = 0.43$ cm and $2s$, which can vary somewhat from hole to hole, taken for each hole as the length of a tube segment that extends the sum of the distances from the centre of a tone hole halfway to the next tone hole in either direction[5]; see table 3 for results.

Note the large error in the shortest tube length. The tone hole for this length is smaller than the others. In general, the smaller the tone hole, the further down the tube the antinode is found ([5] p 170, [10]). This assertion is verified by use of equation (1) as the value of ΔL_{H} for this hole is larger than for the others, but this resulted in the largest discrepancy in calculated equivalent lengths.

[5] Using the average tone hole spacing is considered permissible ([3] p 1598).

Table 3. Comparison between equivalent lengths for a flute's air column calculated from the playing frequencies to those calculated with corrections for the embouchure hole and tone holes.

Musical note	f (Hz)	$L_e - L$ (table 2) (m)	ΔL^a (m)	$L + \Delta L$ (m)	L_e (from f) (m)	Difference (%)
C-4	260	0.062	0.076	0.676	0.662	2.25
C#-4	275	0.075	0.064	0.614	0.625	−1.79
D4	291	0.067	0.064	0.588	0.591	−0.47
D#-4	310	0.065	0.065	0.555	0.555	−0.005
E4	328	0.067	0.065	0.522	0.524	−0.41
F4	348	0.066	0.065	0.493	0.494	−0.27
F#-4	369	0.065	0.065	0.466	0.466	−0.12
G-4	392	0.063	0.064	0.440	0.439	0.34
G#-4	415	0.064	0.064	0.414	0.414	−0.06
A-4	441	0.063	0.064	0.391	0.39	0.27
A#-4	472	0.060	0.064	0.368	0.364	0.94
B-4	493	0.065	0.064	0.348	0.349	−0.38
C-5	529	0.061	0.063	0.327	0.325	0.66
C#-5	573	0.067	0.087	0.320	0.300	6.67

[a] For all air column lengths other than that of C-4, ΔL is the sum of the embouchure and tone hole correction. For the length at note C-4, the entire length of the instrument, ΔL is the sum of the embouchure hole correction, the sum of all closed tone hole corrections and the standard $0.6a$ end correction.

When all the tone holes are closed, ΔL_H is replaced with the standard end correction of $0.6a$ and a correction for all the closed tone holes in the tube. Since they add volume to a tube ([7] p 448), closed tone holes tend to increase the effective length of a tube by an amount based on their dimensions. Closed holes increase the effective length of their segment of a tube by an amount $\Delta L_C = 2s(E - 1)$, where $(E - 1)$ is a percentage increase, usually between 2 and 5% ([7] p 449) in the segment of the tube $2s$, which is equivalent to the tone hole spacing. E is a numerical factor;

$$E = \left[1 + \frac{1}{2}\left(\frac{b}{a}\right)\left(\frac{t}{2s}\right)^2\right]. \qquad (2)$$

ΔL_C was calculated for each tone hole in the 'lattice' using the lengths of each segment and equation (2). The sum of all the closed tone hole corrections ([3] p 1597) and $0.6a$ for the correction at the end of the tube was used to determine the equivalent length of the flute with all the tone holes closed, the longest length, playing musical note C-4. Note that equation (2) was only used when all the tone holes were closed because equation (1) calculates the position of the antinode, how far beyond the centre of the tone hole that it is located, not just an amount to add along with the correction for the embouchure hole to the measured length of the tube ([9] chapter 8, section 5). It should also be noted that the above-mentioned smallest tone hole and two others present further up the tube have values from equation (2) very close to $E = 1$ so therefore do not contribute significantly to the equivalent length of a flute when all the tone holes are closed.

More on clarinets

Comparisons like those shown in table 3 are more difficult for a clarinet than a flute because the dimensions of the clarinet tone holes are more difficult to determine without removing the keys. This is of course possible, but once removed they are difficult to put back on correctly. This should be left to experts in the playing and/or maintenance of clarinets[6].

Note in table 2 that the end corrections for a clarinet are smaller than those for the flute in table 1. This is because the correction for the clarinet mouthpiece, where the reed is attached to the tube, at the closed end, is much less than the correction for the flute embouchure hole. The mouthpiece cavity adds an equivalent length to the cylinder to which it is attached equal to that of a cylinder of the same diameter, having the same volume as the mouthpiece ([5] p 174, [11]) and,

[6] The author speaks on this point from unfortunate experience.

Table 4. Comparison between equivalent lengths for a clarinet's air column calculated from the playing frequencies to those calculated with corrections for the mouthpiece and tone holes.

Musical note	f (Hz)	L_m (m)	ΔL_H (m)	L_e (m)	L_e (from f) (m)	Difference (%)
G-3	198	0.416	0.022	0.432	0.434	−0.607
G#-3	211	0.393	0.0283	0.415	0.408	1.83
A-3	226	0.367	0.0283	0.389	0.381	2.24
C-4	263	0.311	0.0246	0.329	0.327	0.700
C#-4	286	0.289	0.0287	0.311	0.301	3.55
D#-4	317	0.254	0.0409	0.289	0.271	6.40

furthermore, the volume is not the actual volume of the mouthpiece cavity, it is an equivalent volume that is very difficult to determine; however experimentation has shown that for frequencies below 700 Hz the equivalent volume of a clarinet mouthpiece and reed combination is about $V' = 13.25$ cm^2 ([7] p 472), a small amount more than a measured value of about $V = 11$ cm^2 for the clarinet used. The volume of the mouthpiece can be measured by taping the hole that is usually covered by the reed shut and filling the mouthpiece with water. The difference in the mass in grams between the mouthpiece filled with water and empty will be numerically equal to the mouthpiece volume, V, in cubic centimetres. Since the diameter of the mouthpiece decreases in the direction of the actual end of the tube, the equivalent length will actually be *less* than the measured length of the mouthpiece by a small amount. Subtracting the equivalent length from that provided by tone holes results in overall smaller end corrections.

Several of the tone holes on a clarinet are not covered with keys; the effective length of air columns ending with these holes can be determined by adding a correction from equation (1) and a correction for the mouthpiece to the measured length of the air column. The effective volume of the mouthpiece along with the radius of the clarinet tubing to which it is attached, b, can be used to calculate the mouthpiece effective length, $\Delta L_M = V'/\pi b^2$. This length can be added to the length of the tube after the actual length of the mouthpiece is subtracted.

For example, the first uncovered clarinet tone hole (G-3; see table 4) was a measured distance of 41.6 cm from the closed end of the tube. The actual length of the mouthpiece portion of the tube, 9 cm, was subtracted and $\Delta L_M =$ 8.36 cm, calculated from the equivalent volume $V' = 13.25$ cm^3 and tube radius $b = 0.71$ cm was added along with a tone hole correction of $\Delta L_H = 2.2$ cm calculated with equation (1). Note that the equivalent length for the mouthpiece is actually negative. This gave an equivalent cylinder length of 43.2 cm compared to 43.4 cm calculated from the playing frequency of 198 Hz, a difference of about 0.6%. See table 4 for results for other uncovered clarinet tone holes. The differences between effective lengths are similar in all cases resulting in higher percentage errors for the shorter air column lengths.

Conclusion

These experiments or a combination thereof could be useful for students of various physics courses. Graphically verifying that the air columns in flutes and clarinets behave similarly to simple open and closed resonance tubes can be completed in a laboratory period and could be useful even in a general introductory physics course showing real-world examples of standing waves in air columns. The former experiments as well as those comparing the musical pitches and intervals produced by the instruments to those that are expected and those comparing actual and equivalent lengths may be more appropriate for a musical acoustics course, but can also be easily completed in a single laboratory period.

If the computer hardware and software necessary to measure the playing frequencies produced by the instruments are not available, or time does not allow it, an alternative is to use an electronic tuner to tune the instruments. If not available in a student physics laboratory, one could be borrowed from the music department. The experiments can then be carried out with the assumption that the playing frequencies of the instruments are those of the musical pitches when the instruments are played precisely in tune[7]. The more in-depth investigations involving tone holes may only be appropriate for more advanced laboratory students or perhaps as individual or independent study projects.

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[7] [1] p 153, has a table of the frequencies of the notes in the tempered scale.