People and Computers Agree on the Complexity of Small Art

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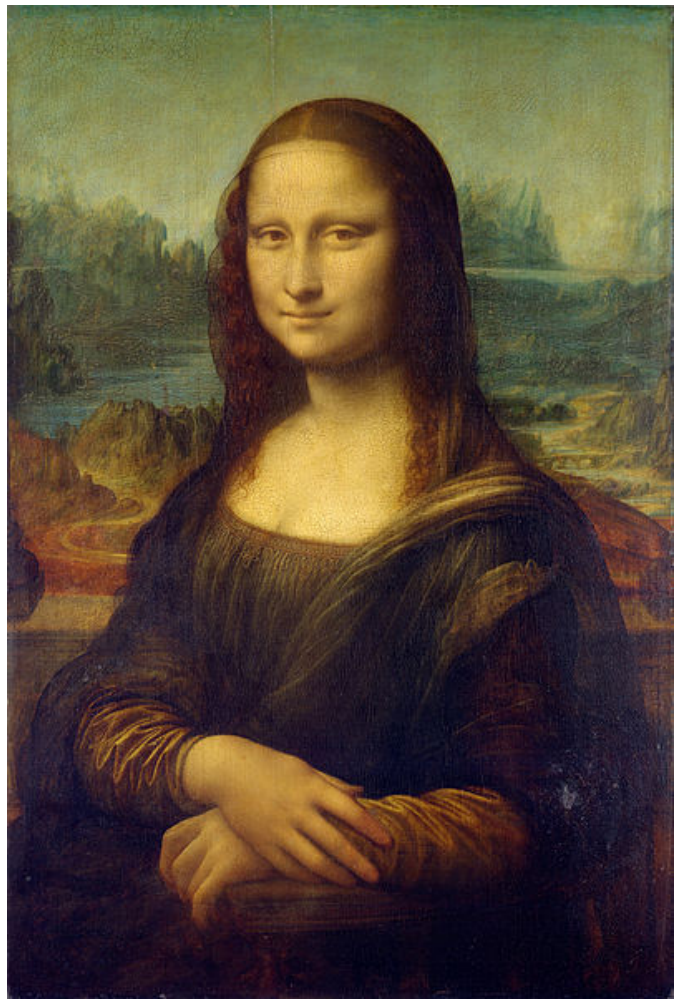
This is art



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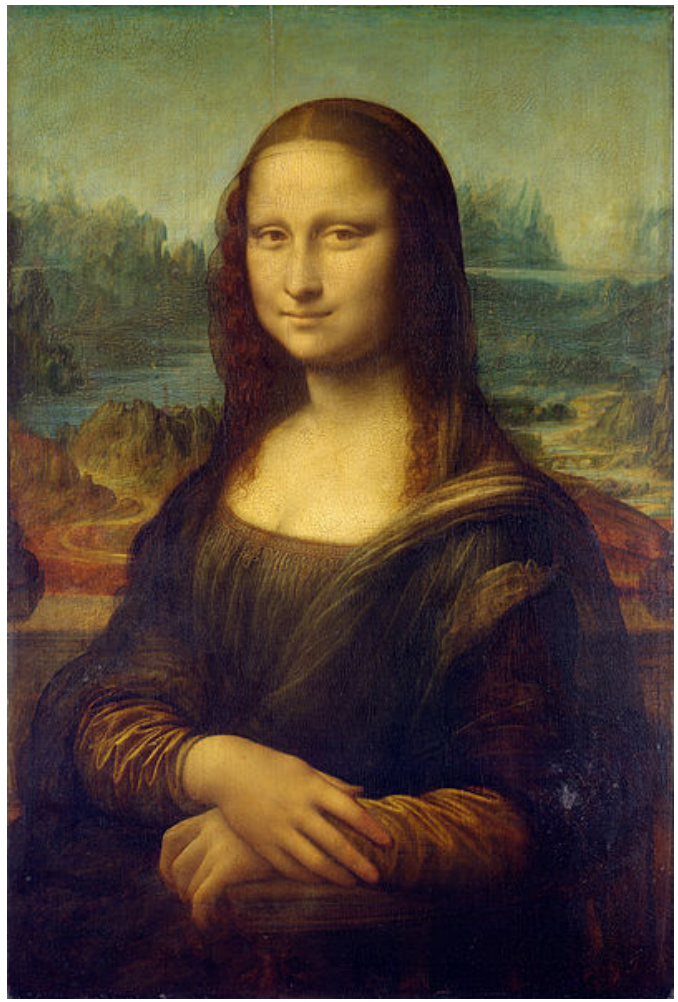
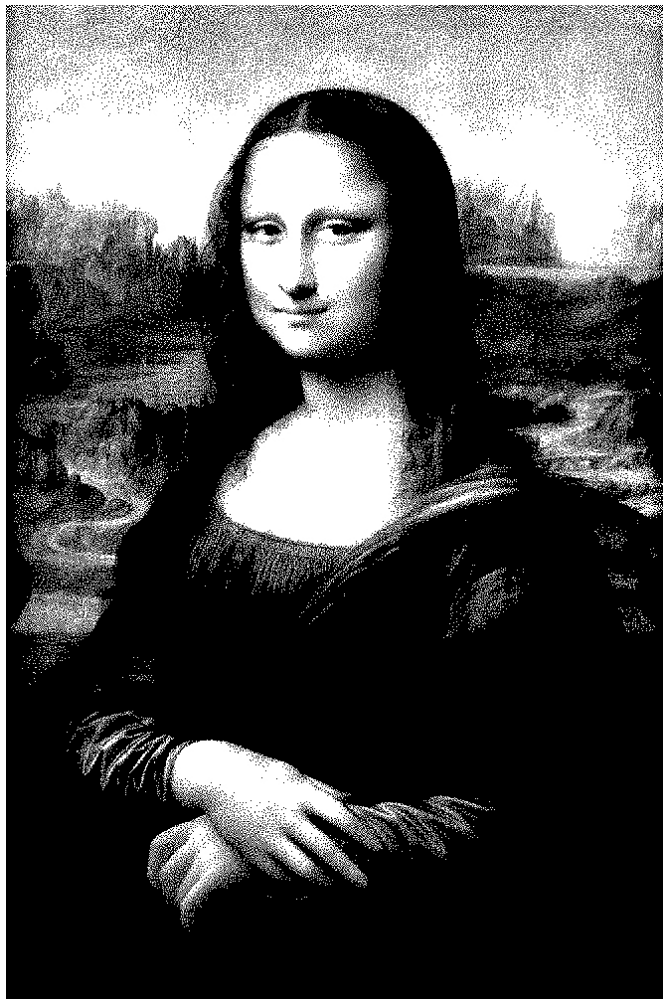


$$\texttt{int} \times \texttt{int} \rightarrow \texttt{color}$$

Grayscale

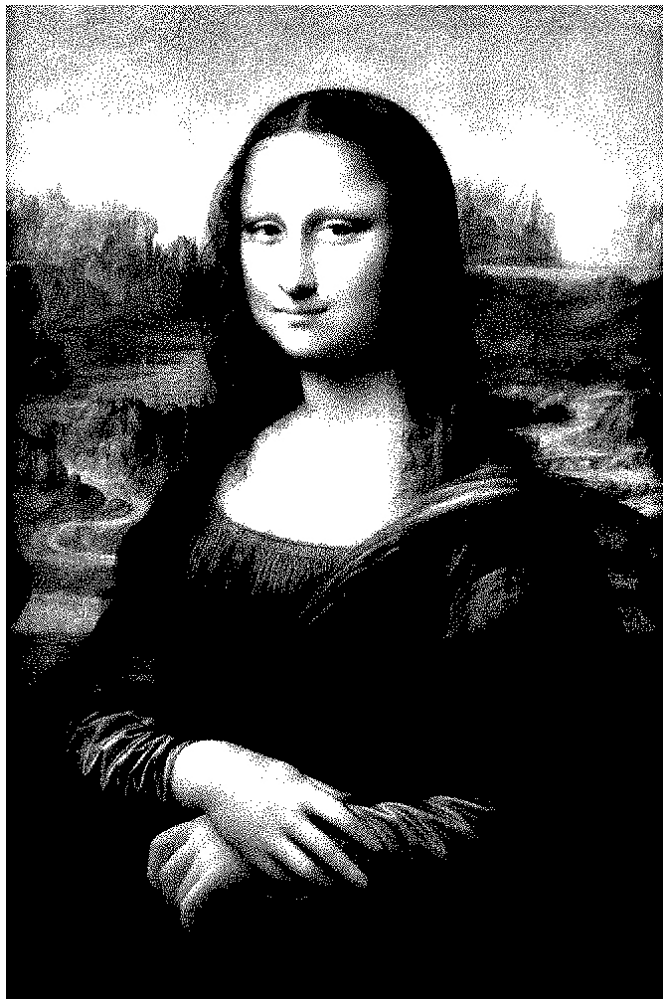
$$\texttt{int} \times \texttt{int} \to \texttt{number}$$

Black and White



$$\texttt{int} \times \texttt{int} \to \{0, 1\}$$

Black and White



$$\texttt{int} \times \texttt{int} \to \texttt{bool}$$

Black and White

$\texttt{int} \times \texttt{int} \to \texttt{bool}$

Generalized

Digital pictures are the output of evaluating a function at every pixel in the picture.

For black and white pictures, this function is of type:

$$\texttt{int} \times \texttt{int} \rightarrow \texttt{bool}$$

Human complexity has to do with the picture
Computer complexity has to do with the function

Kolmogorov Complexity

The Kolmogorov complexity of an object is equal to the size of the smallest program that outputs that object.

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All Turing-complete programming languages are equivalent
Provably impossible to calculate
Provably impossible to approximate

Formula complexity

For us a "formula" is a well-typed fully-parenthesized expression built out of the atoms:

```
{ 0, 1, x, y, +, *, <, not, and, or, true, false }
```

`x` and `y` are integer variables which hold the coordinates of the grid point.

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The Formula complexity of an object is the size of the smallest formula whose output is that object.

Not Turing complete!
A matter of "mere" computation to calculate and enumerate!

Computing formula complexity

The algorithm to calculate the formula complexity of a picture is simple to describe:

> *Enumerate all well-typed formulae in order of size. Stop when you have enumerated a formula whose output is the picture. The size of that formula is the formula complexity of the picture.*

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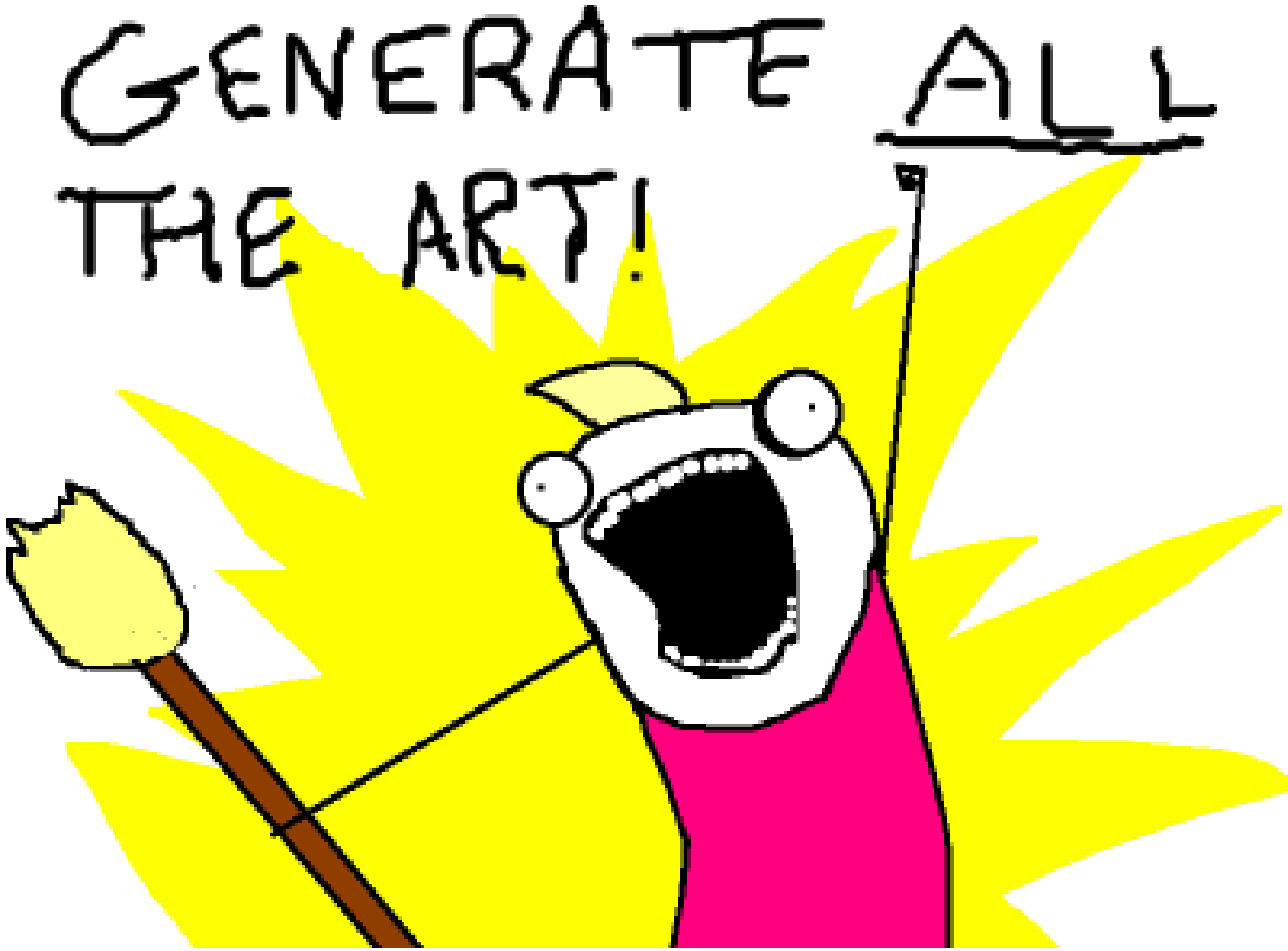
Horrendous runtime. Highly exponential. Do it anyway.

This terrible algorithm restricts the size of artworks we can consider. In particular, we can only work with 9 pixel artworks — 3 pixels by 3 pixels. There are $2^9 = 512$ such artworks, but we will need to generate hundreds of billions of formulae.

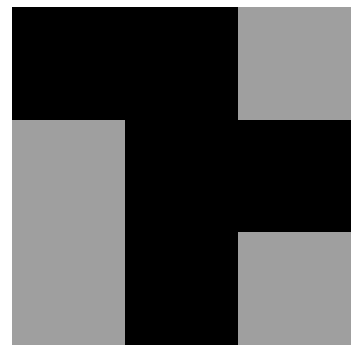


http://hyperboleandahalf.blogspot.com/2010/06/this-is-why-ill-never-be-adult.html

All the 2x2 Art

Formula	FC	Picture
none	0	empty
(true), (false)	1	
none	2	empty
(x < 1), (y < 1), (x < y), (0 < x), (0 < y), (y < x)	3	
(not (x < y)), (not (y < x))	4	
((y + x) < 1), ((y * x) < 1), (0 < (y + x)), (1 < (y + x))	5	
none	6	empty
((y < x) or (x < y))	7	
(not ((y < x) or (x < y)))	8	

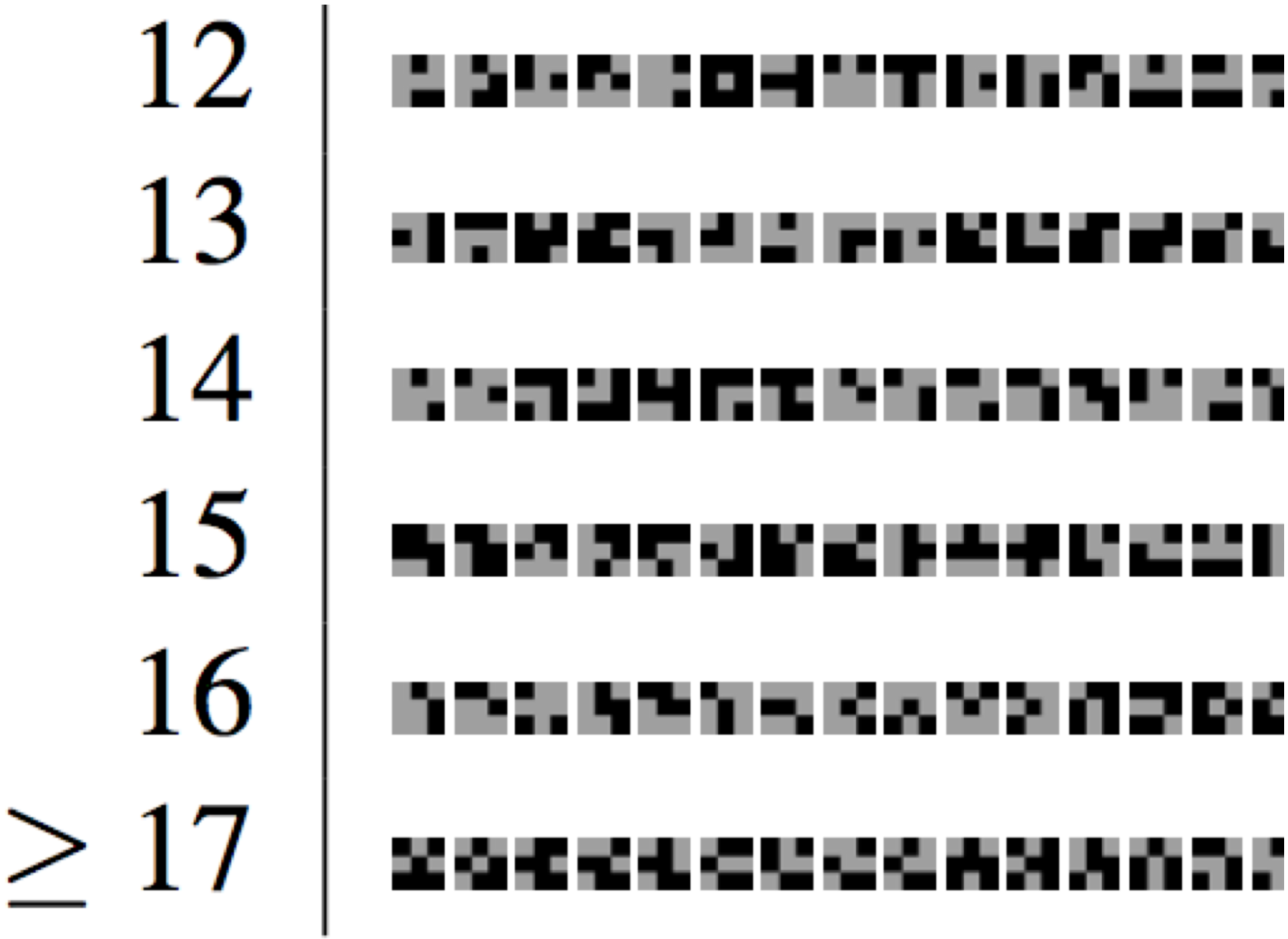
Selected 3x3 art

Formula	FC	Picture
(x < y)	3	
((not (y < (x * y))) and ((y * y) < (x + (y + y))))	16	

All the 3x3 art

FC	Artworks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
≥ 17	


1
2
3
4
5


12
13
14
15
16
≥ 17

Visual complexity is hard to measure

- Intuitive.
- Complex.
- “I know it when I see it.”
- Hard to explain.

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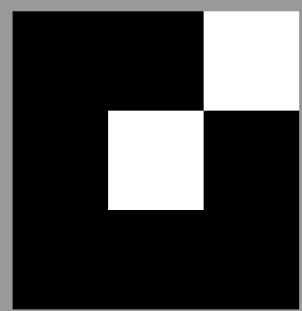
- “I know it when I see it.”
- Hard to explain.

- Can’t be measured absolutely.
- Can be measured relatively!

Our survey

Which picture looks more complex?



20 Tie? 304

We have calculated the Kolmogorov Complexity of every 3x3 artwork. Our hypothesis is that Kolmogorov Complexity is related to visual complexity — which is the fuzzy notion we have of one image being more “complex looking” than another. For each of the images presented, please click on the one that is, in your opinion, more visually complex.

Measure like chess

- Perform repeated pairwise comparisons, asking test subjects "Which of these is more visually complex?"
- Treat each comparison as a match with a winner and a loser.
- Use the TrueSkill algorithm to assign a strength rating to each artwork based on match results.

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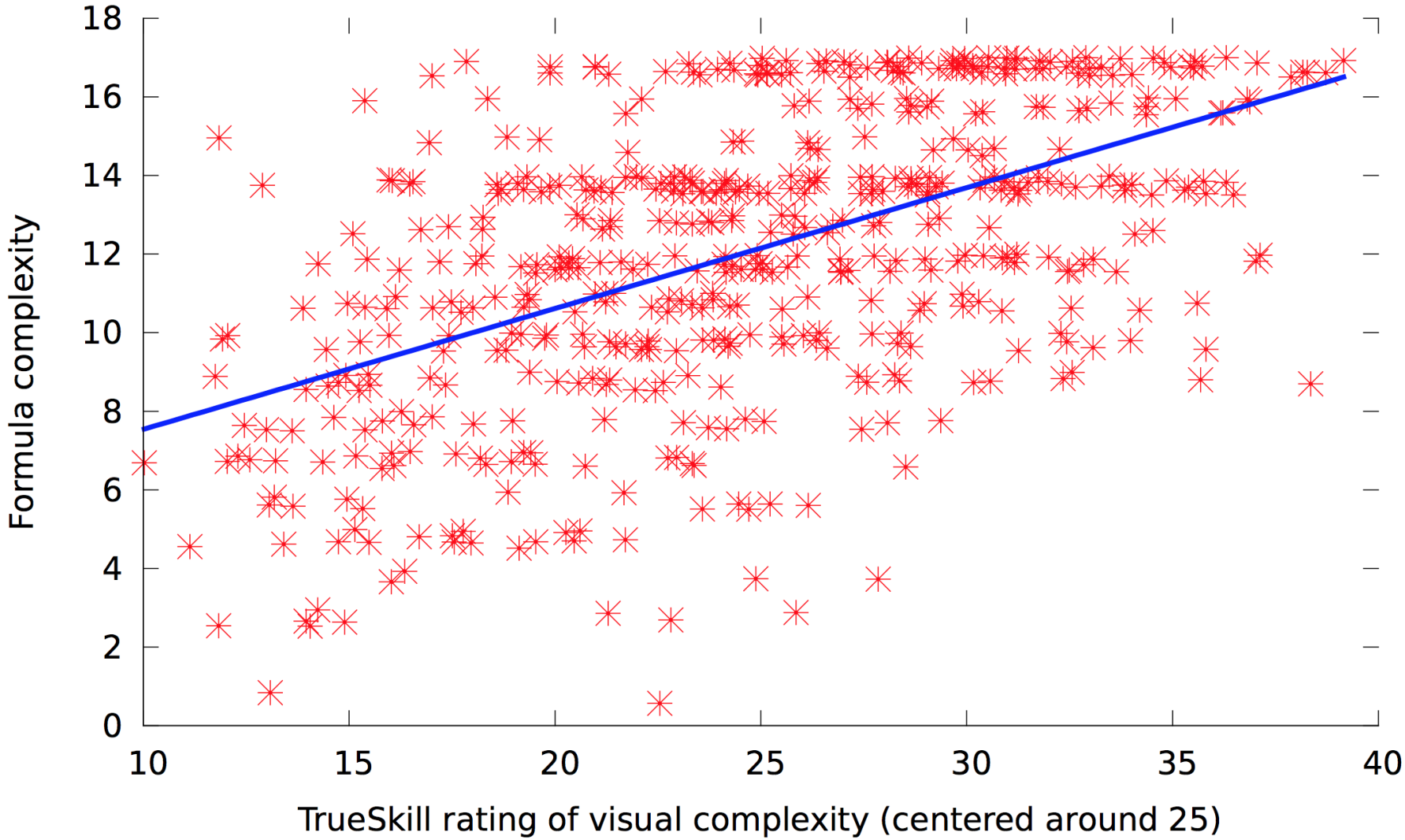
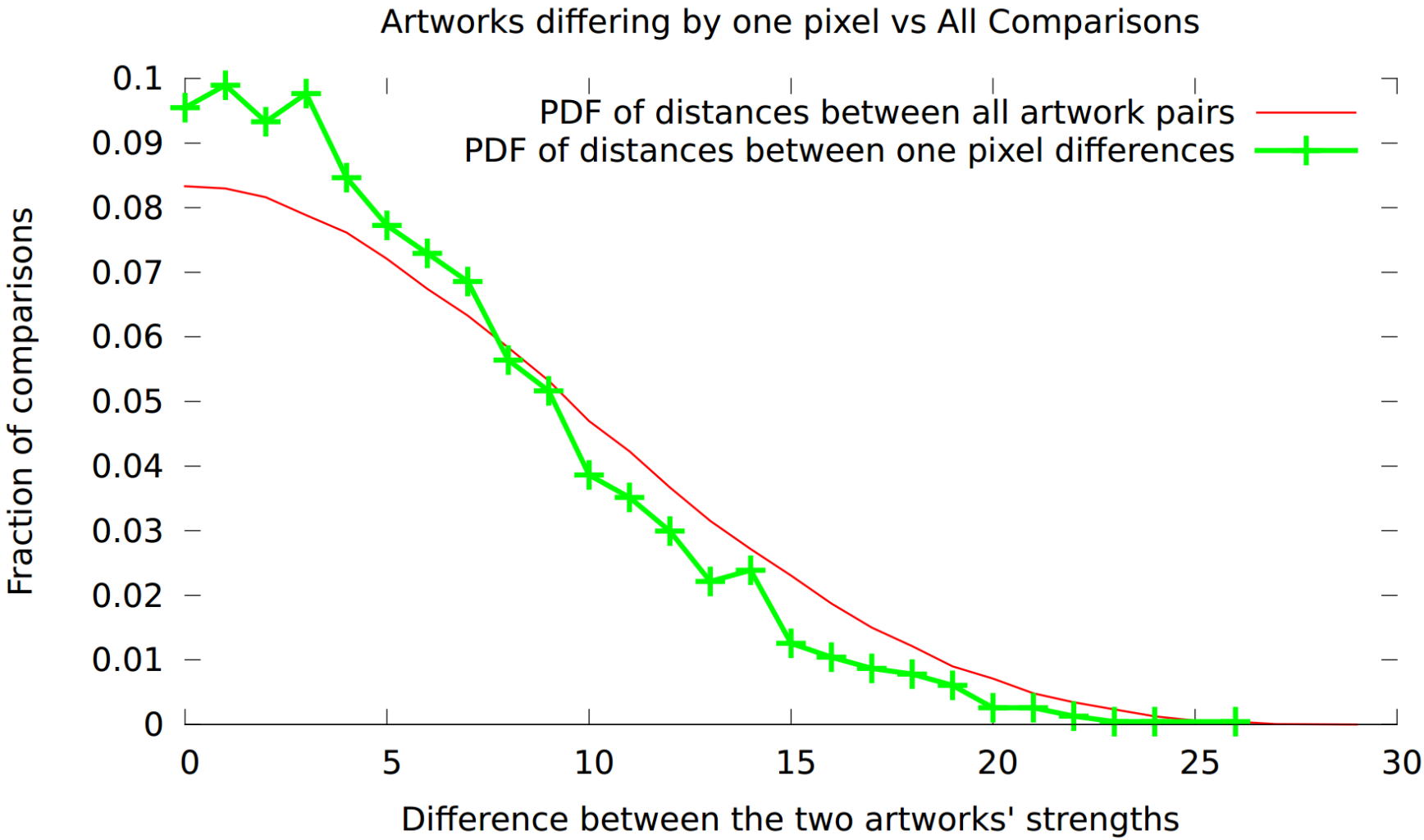
- Passed it around among undergrads and Twitter, got many thousands of comparisons.

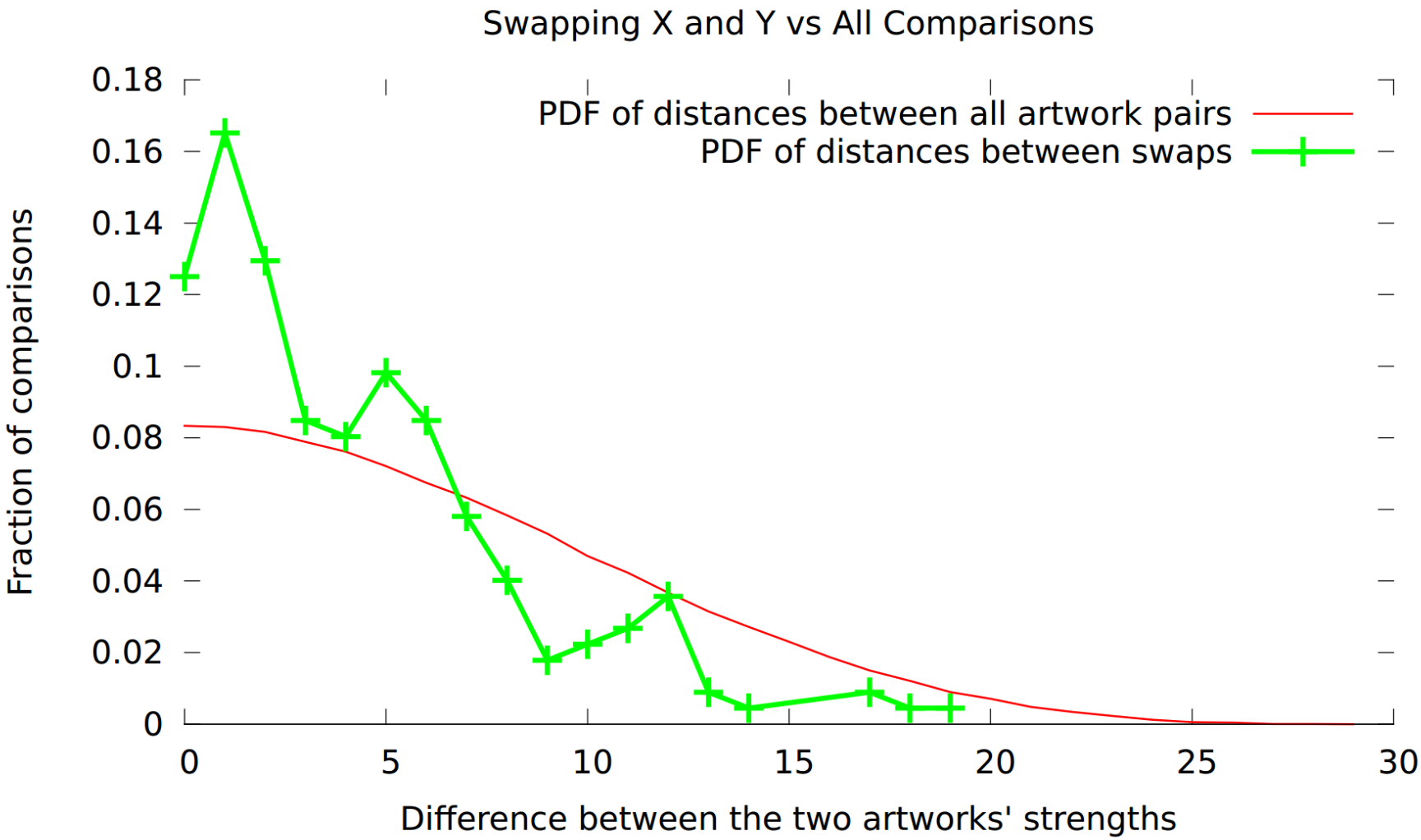
It correlates!



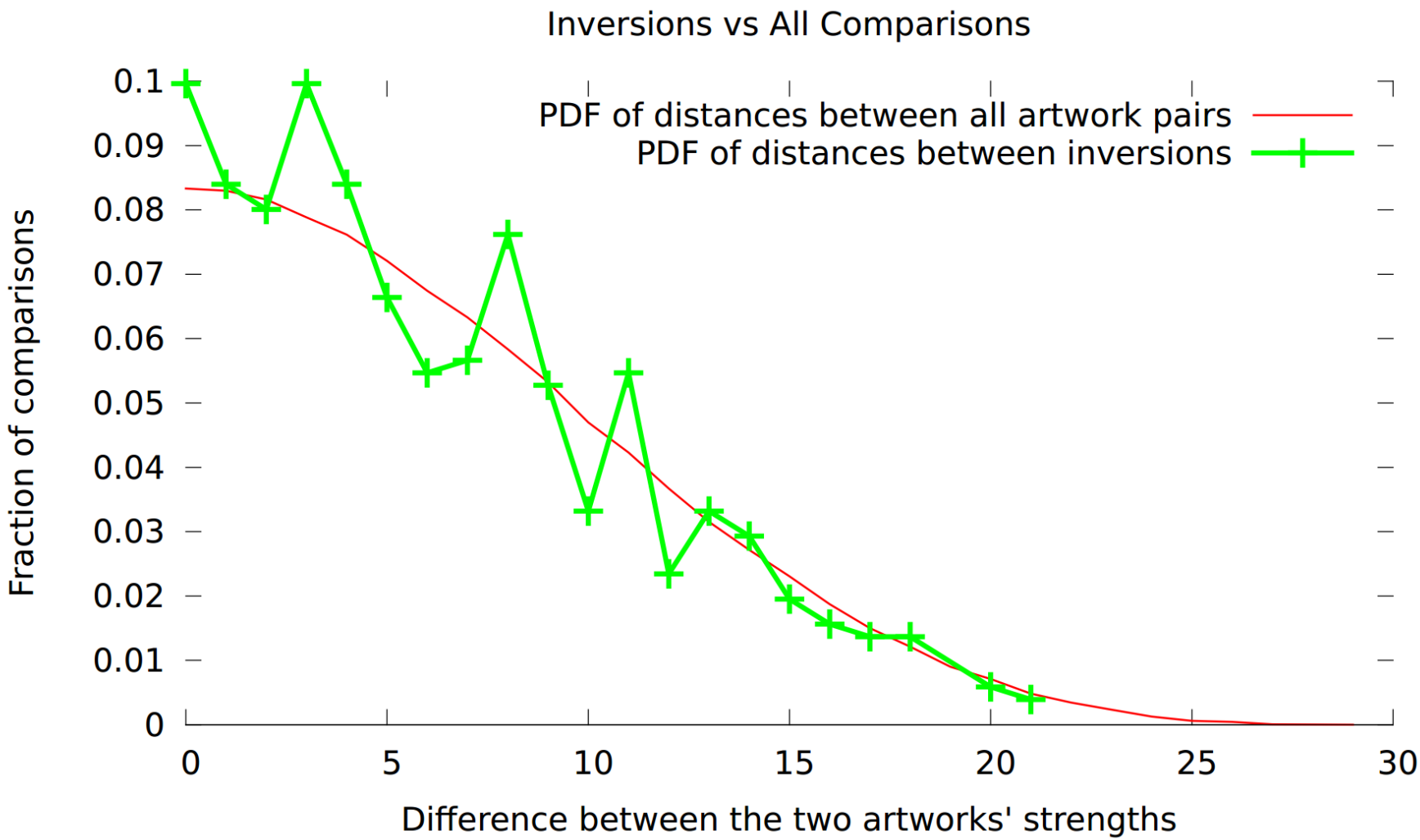
One-pixel differences



Swap x and y


Swapping X and Y vs All Comparisons
PDF of distances between all artwork pairs
PDF of distances between swaps
Fraction of comparisons
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
Difference between the two artworks' strengths

Swap black and white



Threats to validity

- We only surveyed WEIRD[1] CS people.
- The missing data of the ≥ 17 artworks
- The art was tiny

[1] Western, educated, industrialized, rich, democratic

Future work

- Wait for computers to get faster, re-run the generation process to find the missing data.
- Try using Levin Complexity instead of formula complexity or Kolmogorov complexity.
- What atoms make for a formula complexity that best matches people's visual complexity?

Summary

We have:

1. . . . established a mapping between functions and art
2. . . . defined "formula complexity", a low-power version of KC
3. . . . found the formula complexity of almost all 3x3 artworks
4. . . . asked people to compare the visual complexity of these artworks
5. . . . generated a TrueSkill rating for the visual complexity of each artwork
6. . . . found that the two measures correlate!

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Questions?