People and Computers Agree on the Complexity of Small Art

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Abstract

Restricting our purview to black and white digital artworks on a grid, we developed a lower-power version of Kolmogorov complexity, and then we found the complexity of every piece of 3x3 art. We also asked people to compare two artworks and decide which one was more visually complex as they understood the term. We used these comparisons to assign every artwork a strength rating (similar to a chess rating), and we found that the human-generated ratings were well correlated with the formula complexity of the artworks. Therefore, computers and humans largely agree on the complexity of small artworks!

Introduction

We compare the complexity of small artworks from two perspectives: that of a computer, and that of a human. To do this in a principled manner we define our artworks, define our notion of complexity for computers, and describe how we measured visual complexity for people. We procede through each task in turn, one per section, before bringing our computer and human results together and describing future work in the last sections.

Our Definition of Art

In order to make this question tractable, and in order to ensure that our artworks are "native" to both the human and computer domains, we restrict our purview to black and white pixel-based digital artworks. This is clearly a subset of all digital artworks, but it is still an expressive subset, as shown in Figure 1.

We can generate a mapping of positions on the grid to black and white pixels in multiple ways. The simplest way to generate the mapping is to simply write down the sequence verbatim. However, considered more generally, each artwork can also be thought of as the output of a function that takes two numbers as input (representing the grid position of each pixel) and returns either true or false (representing whether the pixel at that position is black or white). In computer science terms, every black and white picture is the output of a function of type $\texttt{int}\times\texttt{int}\rightarrow\texttt{bool}$.

This isomorphism between functions of type $\texttt{int}\times\texttt{int}\rightarrow\texttt{bool}$ and black and white artworks is the basis of our work, as it allows us to ask people about the perceived visual complexity of an artwork, and to "ask" computers about the complexity of the corresponding functions. In order to make our problems computationally tractable, we restrict our purview to artworks that are nine pixels laid out in a three by three grid. This allows us to generate all artworks (of which there are $2^9 = 512$), and also, less trivially, to enumerate all possible formulae in an effort to calculate the formula complexity of each artwork.

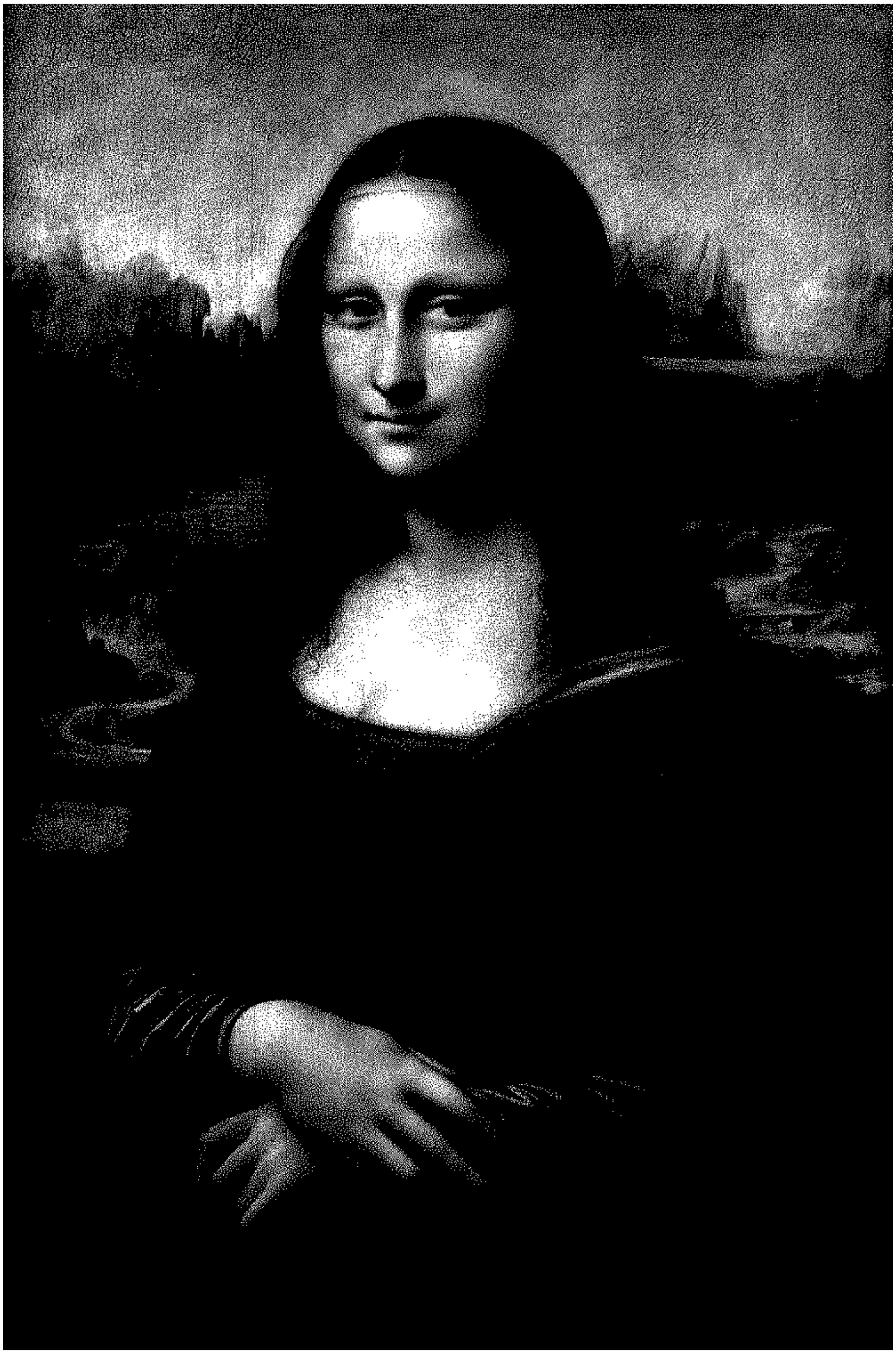


Figure 1 : *A example artwork consisting of black and white (and no gray) pixels on a grid.*

Formula Complexity

Formula complexity is intended to be a "low power" version on Kolmogorov complexity[1]. In Kolmogorov complexity, the complexity of an object is defined to be the size of the smallest program that outputs that object — interestingly, the programming language used does not usually matter, as all Turing-complete programming languages are equivalent up to an additive constant.

Formula complexity is also a property of an object, in this case a 2-dimensional artwork, but the programming language used to write the function of type $\texttt{int}\times\texttt{int}\rightarrow\texttt{bool}$ is not Turing-complete. In particular, in formula complexity it is impossible to define and call functions, which prohibits looping and recursion. Instead of the entirety of mathematical symbols, in formula complexity we restrict ourselves to just the symbols in the set

$$\{\texttt{+, *, 0, 1, x, y, true, false, not, and, or, <}\}.$$

We also require that every expression be fully parenthesized, to eliminate any potential ambiguity of interpretation. In each formula `x` and `y` represent the coordinates of the grid point being interpreted.

Note that some formulae that our language can produce might make no sense. For example, what is the value of `(1 + false)`? Furthermore, other formulae produced may make sense, but not be useful in our context. For example, if our formula is `(x + y)`, then what color should we make the pixel at $(1,0)$? Therefore, we place further restrictions on our formulae:

1. All formulae must be well-typed: numerical operations are only performed on numbers (or subexpressions which evaluate to a number) and logical operations are only performed on logical values (or subexpressions which evaluate to a logical value).
2. All formulae must evaluate to `true` or `false` after substituting the coordinate values in for `x` and `y`.

The first requirement ensures that a formula makes sense, and the second ensures that the formula can be used to define an artwork. Now that these preliminaries are decided, we can define formula complexity.

Definition (Formula Complexity) The *formula complexity* of an artwork is the number of symbols used in the smallest formula which produces that artwork when evaluated at each point on the grid.

Some example artworks, along with their formula complexity and the corresponding formula of minimum size may be seen in Figure 2.

We wrote a program to generate all well-typed formulae in order of size, and then tested each formula as it was generated in order to determine if it was the first formula to produce its corresponding picture. We ran our program for months, but were only able to generate the formulae up to size 17 due to the extremely large number of formulae and the super-exponential explosion in formula count as formula size grows. A complete diagram of all artworks with their corresponding formula complexity may be seen in Figure 3.

Visual Complexity

When compared with the mathematical formalism of formula complexity, visual complexity is a frustratingly slippery concept. We declined to define it at all, instead allowing every survey participant to decide for themselves what it meant. We surveyed people online (through Twitter and through our own social networks) and presented them with a page that showed two artworks and asked them to click on the artwork that was

[1] Also called Chaitin-Kolmogorov complexity. The standard text is Li and Vitányi[3], although Chaitin's article in Scientific American[1] provides a very approachable introduction.

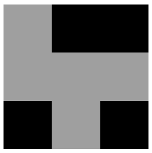
Artwork	Formula Complexity	Formula of minimum size
	1	`true`
	5	`(1 < (x + y))`
	16	`((not (x < (x * y))) and ((x * x) < (x + (x + y))))`

Figure 2 : *Some example artworks, listed with their formula complexity and a formula with that complexity. Black corresponds to false and gray to true, and the bottom left corner pixel* $(0,0)$.

more visually complex. After they clicked on one artwork, we presented them with another, and another, until they decided for themselves to stop taking our survey.

Our task was then to distill these ratings into a ranking of visual complexity. We did this by treating each survey response as a "game result" between the two artworks and then turned the win-loss record of each artwork into a strength rating using the TrueSkill algorithm[2], which is a rating system similar to the one used in chess but with provably better convergence times[2].

Comparing Complexity Results

Armed with our computational results and our survey results, the only thing left to do was see whether these two complexity rankings were well-correlated! A complete chart of our results may be seen in Figure 4. From the figure, two things are clear: First, our correlation is definitely not perfect because the dots do not form an increasing line; Second, there is some correlation, because there are very few dots in the lower right or in the upper left.

When we calculate Pearson's correlation coefficient, we get a correlation of .55 and a very low p-value of $4 \cdot 10^{-39}$, which implies that it is safe to reject the null hypothesis that these two distributions have nothing to do with one another. After rejecting the null hypothesis we can safely conclude that, for small digital artworks, visual complexity is positively correlated with formula complexity!

Conclusion and Future Work

We defined a class of art that is explicable by both humans and computers. We also defined a "powered down" version of Kolmogorov complexity which we called formula complexity and calculated the formula complexity of all 3x3 artworks. We then took survey results where we asked people to compare these artworks to each other and choose the one that is most complex, and we assigned a complexity rating to each artwork based on the number of comparisons it won and the strength of the artworks it beat. We then showed

[2]The TrueSkill algorithm was developed at Microsoft and is currently used to rank players in XBox Live.

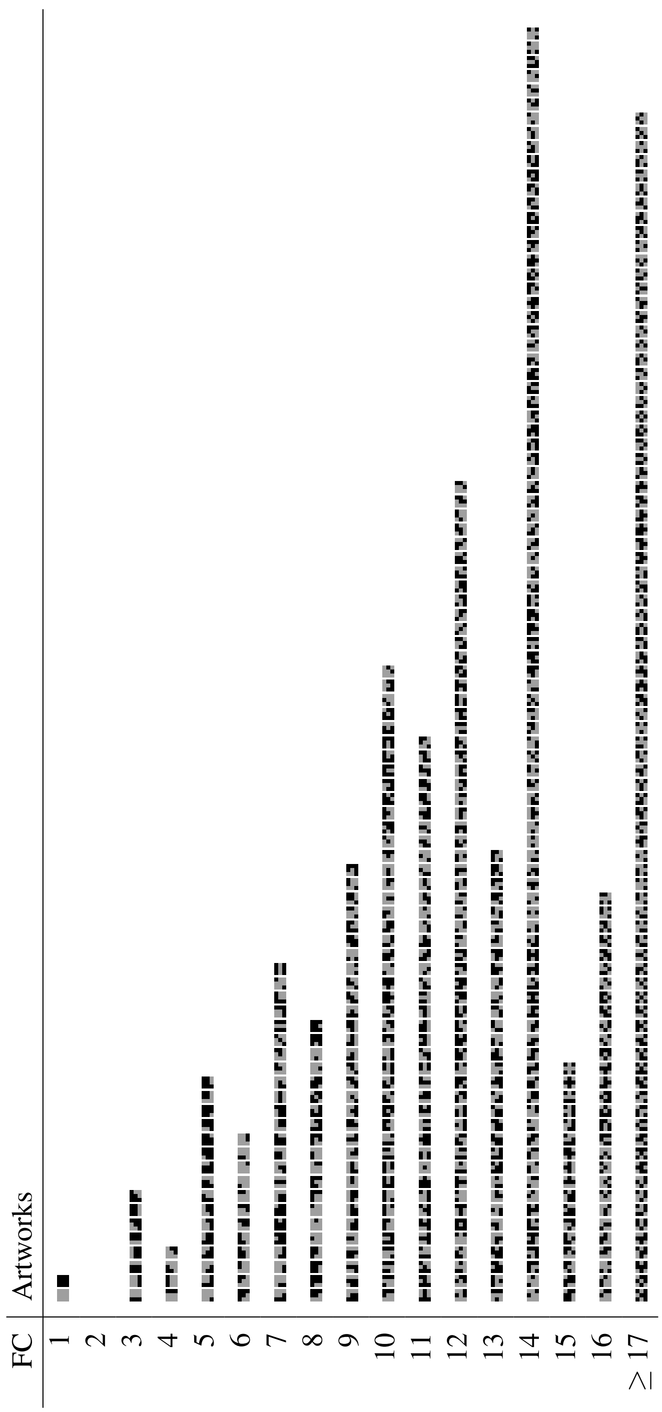


Figure 3 : *All the three by three artworks and the formula complexity of each. The last category is labeled* ≥ 17 *because the formula complexity of those artworks is unknown, except that it is at least 17.*

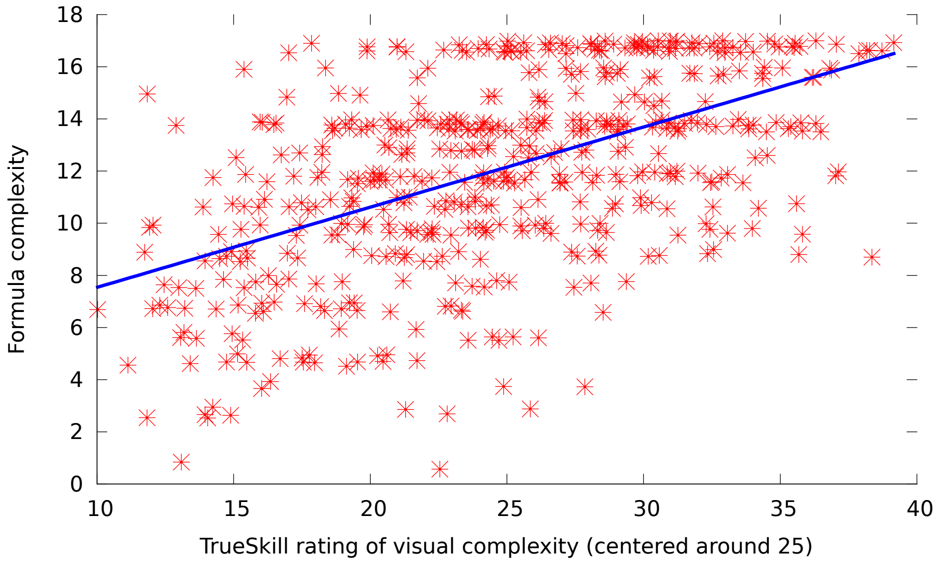


Figure 4: *A scatter plot comparing visual complexity and formula complexity for all 512 artworks, along with the line of best-fit. The formula complexity is artificially clamped at 17. The correlation coefficient between the two measurements is .55 and the p-value is* $4 \cdot 10^{-39}$.

that these two, very different, complexity measures are actually highly correlated. From all this, we conclude that, for small digital artworks, humans and computers agree about what is complex and what is simple!

The future work we lay out is related to the limits of both of our complexity surveys, potential ways of increasing the power of our computer complexity measure, and a search for a computer complexity measure that comes even closer to the human one.

Our computational survey was unfortunately limited by available machine power. It may be fruitful to run this computation again in a few years once transistor density has increased again. Perhaps future programmers could find the formula complexity of all of the small artworks!

Our human survey was limited to college students and Twitter friends. Because we are computer scientists, it is likely that the college students and Twitter users we know are not representative of the population at large. Does this correlation increase as when people take computer science classes? Are we only seeing a correlation because we surveyed people who had taken CS classes?

Our complexity measure of formula complexity was partly driven by expediency and explicability. Other areas of math and CS have developed programming languages and formula schema that allow for richer expressions than we have here. For example, it might be interesting to replicate this result using the simply-typed lambda calculus, which allows function definition and just forbids recursion. For another example, it might be interesting to replicate this result using Levin complexity, which is even closer to Kolmogorov complexity!

Finally, and most interestingly, it could be quite fruitful to search for the set of atoms which provide for a formula complexity that maximally matches the measured visual complexity. The discovery of this set of atoms would potentially have deep implications for understanding how human brains perceive the world.

References

[1] Gregory J. Chaitin. Randomness and mathematical proof. *Scientific American*, 232:47–52, May 1975.

[2] Ralf Herbrich, Tom Minka, and Thore Graepel. TrueSkill(TM): A Bayesian Skill Rating System. In *Advances in Neural Information Processing Systems 20*, January 2007.

[3] Ming Li and Paul M. B. Vitányi. *An introduction to Kolmogorov complexity and its applications (2. ed.)*. Graduate texts in computer science. Springer, 1997.