DeepQ Decoding for Fault Tolerant Quantum Computation

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Topological error correcting codes, and particularly the surface code, currently provide the most feasible roadmap towards large-scale fault tolerant quantum computation. As such, obtaining fast and flexible decoding algorithms for these codes, within the experimentally relevant context of faulty syndrome measurements, is of critical importance. In this work we show that the problem of decoding such codes, in the full fault tolerant setting, can be naturally reformulated as a process of repeated interactions between a decoding agent and a code environment, to which the machinery of reinforcement learning can be applied to obtain decoding agents. As a demonstration, by using deepQ learning, we obtain fast decoding agents for the surface code, for a variety of noise-models, within the fully fault tolerant setting.

I. INTRODUCTION

In order to implement large scale quantum algorithms it is necessary to be able to store and manipulate quantum information in a manner that is robust with respect to the unavoidable errors introduced through the interaction of physical qubits with a noisy environment. A typical strategy for achieving such robustness is to encode a single logical qubit into the state of many physical qubits, via a quantum error correcting code, from which it may be possible to actively diagnose and correct errors that might occur [1, 2]. While many quantum error correcting codes exist, topological quantum codes [1–8] in which only local operations are required to diagnose and correct errors, are of particular interest as a result of their experimental feasibility [9–15]. Recently the surface code has emerged as an espescially promising code for large scale fault tolerant quantum computation, due to the combination of its comparitively low overhead and locality requirements, coupled with the availability of convenient strategies for the implementation of all required logical gates.

Within any such code based strategy for fault tolerant quantum compuation, decoding algorithms play a critical role. At a high level, throughout the course of a computation these algorithms take as input the outcome of diagnostic syndrome measurements and should provide as output suggested corrections for any errors which might have occured, which can then be tracked through the computation and later used to apply corrections to any obtained results. It is particularly important to note that in any physically realistic setting the required syndrome measurements are themselves obtained via small quantum circuits, and are therefore also generically faulty. As such, while the setting of perfect syndrome measurements provides a good test-bed for the development of decoding algorithms, any decoding algorithm which aims to be experimentally feasible should also be capable of dealing with such faulty syndrome measurements. Additionally, such algorithms should also be capable of dealing with experimentally relevant noise models, as well as be fast enought to not present a bottleneck to the execution of computations, even as the size of the code scales to larger code distances.

Due to the importance of decoding algorithms for fault tolerant quantum computation, many different approaches have been developed, each of which tries to satisfy as many of the experimentally required criterion as possible. Perhaps most prominent are algorithms based on minimum-weight perfect matching subroutines [16], however alternative approaches based on techniques such as the renormalization group [17] and locally operating cellular automata [18] have also been put forward. Furthermore, recently techniques from classical machine learning have begun to find application in diverse areas of quantum physics - such as in the efficient representation of many-body quantum states, the identification of phase transitions, and the autonomous design of novel experimental setups MK: TODO: add citations - and various neural-network based decoders have also been proposed [19–25]. However, despite the diversity of decoding algorithms now available, there is not as of yet an algorithm which clearly satisifes all the required criteria, or a clear consensus as to which technique would be the most experimentally feasible in any given experimental context. In particular, while the so-far proposed neural network decoders promise extremely fast decoding times, flexibility with respect to the underlying noise model and the potential to scale to large code distances, all such decoders are so far restricted either to the setting of perfect syndrome measurements, or to the setting in which one is trying to store a logical qubit as long as possible, without the requirement of performing a subsequent logical gate. As such, while this approach seems promising, there remains room for improvement and generalization.

Simultaneously, the last few years have also seen extremely impressive advances in the development of deep reinforcement learning algorithms, which have allowed for the training of neural network based agents capable of obtaining super-human performance in domains such as Atari [26], Chess [27] and Go [28]. These techniques are

particularly powerful in situations where it is necessary to learn strategies for complex sequential decision making, involving consideration of the future effects of ones actions. At a surface level, the problem of decoding within the context of fault-tolerant quantum computation seems like exactly such a setting and as such it natural to ask to what extent reinforcement learning techniques could be used to obtain decoding agents, and what advantages such agents might have over existing approaches. In this work we set out to provide answers to these questions.

In particular, we reformulate the problem of decoding within the setting of fault-tolerant quantum computation as a process of sequential competitive interaction between a decoding agent and a code environment. This reframing provides a conceptual framework which allows for the application of various deep reinforcement learning algorithms to obtain neural network based decoding agents. As a proof-of-principle, we then utilize to deepQ learning to obtain fast surface code decoders, for a variety of noise models, within the fully fault-tolerant setting. These results then provide a foundation for extension via both more sophisticated reinforcement learning techniques and more sophisticated neural network models.

In this work we begin by providing an introductory overview of the surface code in Section II, before presenting a description of the decoding problem for fault-tolerant quantum computation in Section III. After a brief introduction to the formalism of reinforcement learning and Q-functions in Section IV we are then able to provide the conceptual reframing of decoding as a reinforcement learning problem in Section V, which is one of the primary results of this work. In Section VI we then present deepQ surface code decoders for a variety of noise models, before finally in Section VII discussing both the advantages and disadvantages of the approach presented here, along with various potential strategies for building upon the results presented in this work.

II. THE SURFACE CODE

We begin by providing a brief high-level and hopefully intuitive description of the surface code. The framework and methods presented in this work are not restricted to the surface code, and could be applied to any stabilizer code, but we choose to restrict ourselves to this code here for both simplicity of presentation and experimental relevance. We will focus here on presenting the elements of the surface code necessary for understanding the decoding problem that we aim to address, in the process omitting many details, however there exists a rich underlying theory to both topological error correcting codes, and stabilizer codes more generally, and for a more rigorous or condensed matter perspective we refer to the refs *a,b, and c.*

As illustrated in Fig. 1, we will consider $d \times d$ lattices with a *data qubit* on each vertex, such that states of the lattice are elements of the Hilbert space $\mathcal{H} = \bigotimes_{v \in V} \mathbb{C}^2$, where we have indicated the set of all vertices as V. A surface code on such a lattice is defined by a set S of plaquette stabilizer operators, of which there are four distinct types; Given single-qubit Pauli operators X and Z, bulk (boundary) plaquette stabilizers X_p and Z_p are local four (two) body operators,

$$X_p \equiv \bigotimes_{q \in p} X_q, \quad Z_p \equiv \bigotimes_{q \in p} Z_q, \tag{1}$$

which act non-trivially only on the vertex qubits of plaquette p. With reference to Fig 1, if we denote the set of all blue (orange) plaquettes as B_p (O_p), then the set S of stabilizers defining the surface code is given by,

$$S = \big\{ X_p | p \in B_p \big\} \cup \big\{ Z_p | p \in O_p \big\}, \tag{2}$$

from which we can define the Hamiltonian of the surface code as $H_{\text{sc}} \equiv -\sum_{\hat{O} \in S} \hat{O}$.

At this point it is possible to understand how we can encode one *logical qubit* into the surface code. We begin by defining the *code space* $\mathcal{H}_{\text{sc}}$ as the ground state space of the surface code Hamiltonian H_{sc}. This code space contains all surface code states $|\psi\rangle \in \mathcal{H}$ which are simultaneous $+1$ eigenstates of all stabilizers $\hat{O} \in S$ - i.e. states which commute with all stabilizers in S. A simple counting arguement allows one to verify that this code space is in fact two-dimensional - i.e. $\mathcal{H}_{\text{sc}} \simeq \mathbb{C}^2$ - such that we can effectively identify the code space with the state space of a single qubit, which is the logical qubit associated with the surface code. In addition, as illustrated in Fig. 1, let us define Z_L (X_L) as any continuous string of single vertex Z (X) operators connecting the left and right (top and bottom) boundaries of the code. Despite the fact that these *logical operators* cannot be expressed as the product of stabilizer operators in S, the eigenstates of these operators are elements of the code space, and these operators map states in the code space into other states in the code space. As such, we see that we can use the eigenstates $\{|0_L\rangle, |1_L\rangle\}$ of Z_L to define a basis for the code space, and additionally, given a state within the code space, i.e. a state of the logical qubit, we can manipulate this state by using products of logical operators.

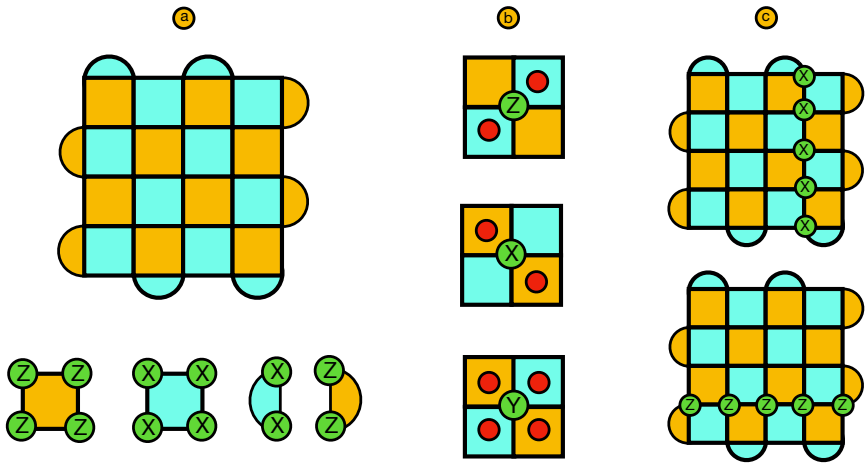

FIG. 1. An overview of the surface code. (a) We consider square $d \times d$ lattices, with a data qubit on each vertex of the lattice. Additionally, there exist four types of local stabilizer operators: bulk X (Z) stabilizers, which are four-body operators acting on the vertex qubits of blue (orange) plaquettes in the bulk of the lattice, and boundary X (Z) stabilizers, which are two-body operators acting on the vertex qubits of blue (orange) plaquettes on the boundary of the lattice. (b) Assuming that each data qubit is initialized in the $+1$ eigenstate of the single-qubit Z operator, single qubit Pauli flips on vertex qubits will cause the global lattice state to anti-commute with certain stabilizers supported on this vertex. Here we have illustrated the stabilizers *violated* (those which anti-commute with the global lattice state) after different Pauli flips are applied to a single data qubit. (c) Logical X_L and Z_L operators for the surface code are provided by continuous strings of single qubit X or Z operators connecting the top and bottom or left and right boundaries of the code respectively.

At this point it is natural to ask for the motivation behind such a redundant encoding. To answer this question, let's assume that the global lattice state $|\psi\rangle$ is an element of the code space - i.e. a simultaneous $+1$ eigenstate of all the stabilizer operators in S. In the process of storing (or manipulating) this state it may happen that some unavoidable error or noise process causes an operation, such as a single qubit Pauli flip on a single data qubit, which is not a logical operation, and therefore causes the global lattice state to move outside of the code space. At this point, the global lattice state *will not* commute with all the stabilizers, and will instead anti-commute with some of the stabilizers supported on the flipped data-qubit (as illustrated in Fig. 1). If we define a *syndrome* as a binary string encoding the outcome of a simultaneous measurement of all the stabilizers in S, then we can see that the syndrome obtained from the global lattice state after the single qubit Pauli flip will contain a -1 entry in each position corresponding to a stabilizer with which the global lattice state anti-commutes, and $+1$ entries everywhere else. We say that stabilizers which anti-commute with the global lattice state are *violated*. As we will discuss in detail in the next section, unlike a truly single qubit encoding of the state $|\psi\rangle$, there is now a chance that from the measured syndrome we will be able to diagnose the error which occured and implement a correction which moves the global lattice state back into the code space.

III. THE DECODING PROBLEM

Given the foundations from the previous section, we are able to succinctly state a simple preliminary version of the decoding problem:

Assume that at $t = 0$ one is given a global lattice state $|\psi\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$, which is an element of the code space. At some time $t_1 > 0$ a syndrome measurement is performed which indicates that one or more stabilizers are violated - i.e. some errors have occured on data qubits. From the given syndrome, determine a set of corrections which should be applied to the code lattice such that the subsequent global state of the lattice is equal to the state $|\psi\rangle$ at $t = 0$.

Before proceeding to discuss more subtle and technical versions of the decoding problem, let's examine why even the above problem is indeed potentially difficult. The most important observation is that the map from error configurations to syndromes is many-to-one, i.e. many different sets of errors can lead to the same syndrome. As an example, consider the error configurations illustrated in lattices (a) and (b) of Fig. 2, both of which lead to the same syndrome. If the probability of an error on a single data qubit is low, given such a syndrome one might reasonably assume that the error in lattice (b) occured, as one single qubit error is a much more likely event than four single qubit errors. Given

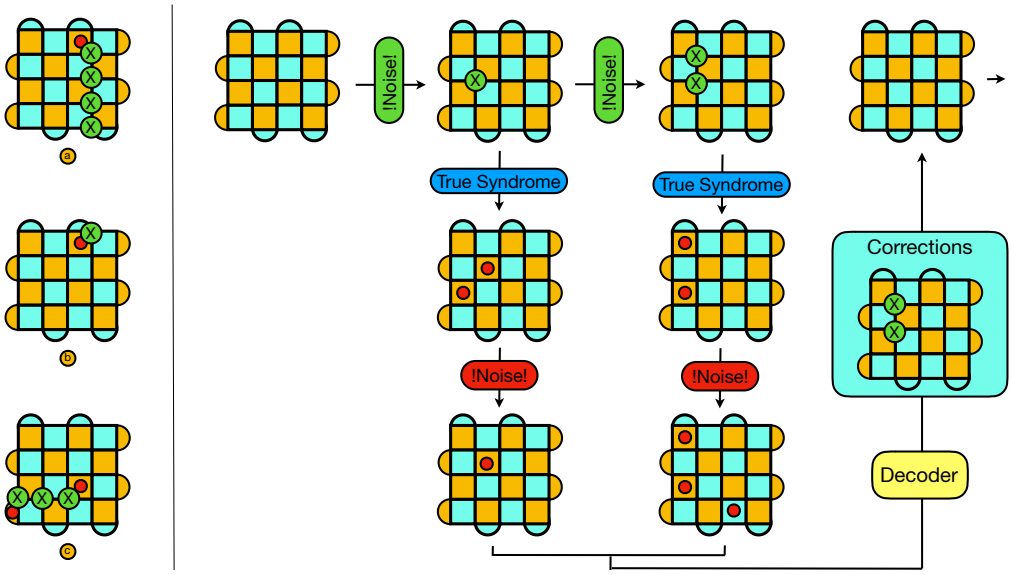

FIG. 2. In the left hand panel, the syndromes associated with various error configurations are illustrated. Note that the error configurations in (a) and (b) lead to the same syndrome. Also, as can be seen in (c), note that continuous strings of errors result in violated stabilizers only at the endpoint of the string. In the right hand panel a typical decoding cycle is illustrated, for the simplified faulty measurements scenario in which one imagines each time step consisting of an initial physical error process generating errors on the data qubits, followed by a second measurement error process which corrupts the true syndrome. The decoding algorithm then has access to a sequence of potentially faulty syndromes.

this reasoning, one might then suggest to correct by applying an X flip on the data qubit in the first row and fourth column. If indeed the error in lattice (b) occured then this would be the correct operation. However, if an unlikely event occured and the error in lattice (a) occured, then the combination of the original errors with the correction would effectively implement a logical X_L operation. As a result, even though the post-correction state is back in the code space, it will be in a different state to the original state and the information we were trying to preserve would have been corrupted. From this simple example one can see that most often solving the decoding problem as stated above involves deciding, given an inherently ambiguous syndrome and (possibly imperfect) knowledge of the underlying error model, which error configuration was most likely to have occured.

As previously mentioned, in addition to the inherent difficulty resulting from syndrome ambiguity, in experimental settings the process of extracting the syndrome is also subject to noise, and as such the syndrome one receives may also contain errors. In practice, each stabilizer is associated with an ancilla qubit, and the syndrome value for that particular stabilizer is obtained by first executing a small quantum circuit which entangles the ancilla with each of the data qubits on which the corresponding stabilizer is supported, before extracting the syndrome value via a measurement of the ancilla qubit. In order to fully account for errors during the process of syndrome extraction one should therefore model this entire circuit, in which errors can occur on both the data qubits and ancilla qubits at each time step, and importantly, errors on the ancilla qubits can propogate onto the data qubits via the required entangling gates. However, the essential aspect of the additional difficulty from faulty syndrome measurements can be more simply modelled by imagining each time step as consisting of two distinct error processes, as illustrated in Fig. 2. In the first error process, an error occurs on each data qubit with some probability. One then imagines extracting the perfect syndrome before a second error process occurs, in which with a given probabiliy an error occurs on each stabilizer measurement outcome.

RS: TO DO: syndrome volume instead of single syndrome in faulty measurements case, and then, differentiate single-shot decoding from multi-cycle decoding! in the context of fault tolerant quantum computation we always imagine that there is a decoded gate or measurement process still to come, and the idea is not necessarily to always put the qubit back into the ground state, but rather to avoid the accumulation of errors for as long as possible, such that the decoding process involved in the next circtui element may be succesfull. As a proxy for whether or not this would work we can imagine a decoder which has access to the true syndrome, and is trying to solve the single shot decoding problem - we say that the logical qubit is "alive" if at any moment in time such a static referee decoder could decode it, and we say that the qubit is dead if at any moment errors have accumulated to the point where a referee decoder would not be succesful in single shot decoding with perfect syndrome measurements. voila.

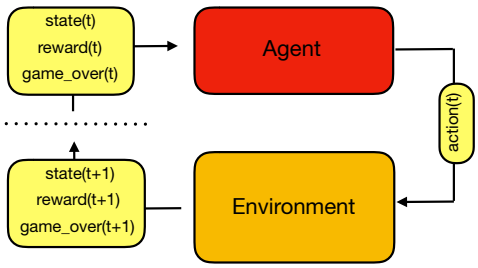

FIG. 3. An illustration of the signals passed between an agent and the environment through the duration of the sequential turn based episodes we consider here.

IV. REINFORCEMENT LEARNING AND Q-FUNCTIONS

In this section we shift focus and introduce some of the fundamental concepts of reinforcement learning and Q-functions, which will be crucial to our conceptual reframing of the decoding problem in Section V. Again, we will omit many interesting details, which can be found in the standard textbook. As illustrated in Fig. 3, we imagine an agent interacting with some environment in discrete time steps. In each time step t the agent is in some state S_t, which is generically a combination of its knowledge of the environment, its memory of its own previous actions, and possibly the state of additional internal variables. From this state the agent must then choose some valid action A_t, which it applies to the environment. In response to this action, via some process which is typically not directly known by the agent, the environment updates and provides the agent with a potentially reduced description of its new state (i.e. a description of the part of the environment which is *visible* to the agent), along with a a scalar reward R_{t+1} and a boolean signal which indicates whether the game is over - i.e. whether the agent has failed, died or lost. In principal it is also possible to imagine settings in which the sequence of interactions can carry on indefinitely, but we will consider here only episodic actions, which end once the environment is placed into a terminal state (which may not be unique) by the actions of the agent. Generically, we consider to goal of the agent to be to avoid dying, while accumulating as much reward as possible.

Typically the way that the agent chooses its action, the way that the environment is effected by the action, and the reward that is generated can all be stochastic, and in the context of finite state and action spaces this process can be formalized within the framework of finite Markov decision processes (FMDP):

$$p(s', r|s, a) \equiv \text{pr}(S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a). \tag{3}$$

To formalize the decision making process of the agent, we define an agent's *policy* π, in essence the agent's strategy, as a mapping from states to probabilities of specific actions - i.e. $\pi(a|s)$ is the probability that $A_t = a$ if $S_t = s$. for FMDP's we then define the *value* of a state s under policy π as,

$$v_\pi(s) = \mathbb{E}_\pi[G_t|S_t = s] = \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\Big|S_t = s\Big] \qquad \forall S_t \in \mathcal{S} \tag{4}$$

where G_t is the discounted return (discounted cumulative reward), with discount factor $0 \leq \gamma \leq 1$, and in practice the infinite sum terminates whenever state S_{t+k+1} is a terminal state. We call v the *state-value function*, which provides the expected discounted cumulative reward the agent would obtain when following policy π from state s. It is an important conceptual point to note that by using the metric of the discounted cumulative reward the value of any given state depends not only the immediate rewards an agent would obtain from following a specific policy, and hence strategies which involve some element of succesful future planning may lead to higher state values. As such, we see that the value of a state reflects accurately the ability of an agent to achieve its long-term goals when following its policy from that state. Similarly, we can define the *action-value function* (referred to as the Q-function) for policy π via:

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t|S_t = s, A_t = a] = \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\Big|S_t = s, A_t = a\Big] \tag{5}$$

Importantly, both state and action value functions satisfy a relation (actually system of equations) known as Bellman's equation, which for action-value functions looks as follows:

$$q_\pi(s,a) = \mathbb{E}_\pi[G_t|S_t = s, A_t = a] \tag{6}$$
$$= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1}|S_t = s, A_t = a] \tag{7}$$
$$= \sum_{s'}\sum_{r} p(s', r|s, a)\big[r + \gamma \mathbb{E}_\pi[G_{t+1}|S_{t+1} = s']\big] \tag{8}$$
$$= \sum_{s'}\sum_{r} p(s', r|s, a)\big[r + \gamma v_\pi(s')\big] \tag{9}$$

In addition, value functions allow us to place an order over policies ($\pi > \pi' \iff v_\pi(s) > v_{\pi'}(s) \quad \forall s \in \mathcal{S}$) such that we can define an optimal policy π^*, with respect to which the action-value function will be the unique solution of the following system of equations:

$$q_*(s,a) = \mathbb{E}\big[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a')\big|S_t = s, A_t = a\big] \tag{10}$$

Now, the idea of iterative Q-learning is to start from an arbitary q function, and then use eqn. (1) as an update function for an agent which uses the q function to make decisions as to how to interact with the environment. The hope (can be proven under certain constraints) is that the q function will eventually converge to q_*, which is a stationary point of eqn. (1).

More specifically, for *deep*-Q learning we parameterize q with a neural network, and use eqn. (1) to construct the cost function on which we train the network. In particular we let the agent interact with the environment, using some ϵ-greedy policy, in the process generating tuples of experience of the following form:

$$[S_t, A_t, R_{t+1}, S_{t+1}]$$

From these tuples we can construct a loss on which to train the Q-network, by using the cost function:

$$C = y_{\text{pred}} - y_{\text{true}} \tag{11}$$
$$= q(S_t, A_t) - \big[R_{t+1} + \gamma \max_{a'} q(S_{t+1}, a')\big] \tag{12}$$

which, by staring at eqn. (1), we can see will be 0 only for the optimal policy.

V. DECODING AS A REINFORCEMENT LEARNING PROBLEM

VI. RESULTS

VII. CONCLUSION

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