

1. Entropy of Y

$$S = [12+, 9-] \rightarrow P(c1) = 12/21 \text{ \& } P(c2) = 9/21$$

$$H(Y) = -(12/21)\log(12/21) - (9/21)\log(9/21) = \mathbf{.985}$$

2. Information Gain

Values for (x1) = T, F

$$S|T \rightarrow [7+, 1-] \rightarrow P(c1) = 7/8 \text{ \& } P(c2) = 1/8$$

$$S|F \rightarrow [5+, 8-] \rightarrow P(c1) = 5/13 \text{ \& } P(c2) = 8/13$$

$$\text{Entropy of T} \rightarrow -(7/8)\log(7/8) - (1/8)\log(1/8) \rightarrow .543$$

$$\text{Entropy of F} \rightarrow -(5/13)\log(5/13) - (8/13)\log(8/13) \rightarrow .961$$

$$IG(x1) = H(Y) - H(Y|x1) = .98522 - (8/21)(.543) - (13/21)(.961) = \mathbf{.183}$$

Values for (x2) = T, F

$$S|T \rightarrow [7+, 3-] \rightarrow P(c1) = 7/10 \text{ \& } P(c2) = 3/10$$

$$S|F \rightarrow [5+, 6-] \rightarrow P(c1) = 5/11 \text{ \& } P(c2) = 6/11$$

$$\text{Entropy of T} \rightarrow -(7/10)\log(7/10) - (3/10)\log(3/10) \rightarrow .881$$

$$\text{Entropy of F} \rightarrow -(5/11)\log(5/11) - (6/11)\log(6/11) \rightarrow .994$$

$$IG(x2) = H(Y) - H(Y|x2) = .98522 - (10/21)(.881) - (11/21)(.994) = \mathbf{.045}$$

Variable x1 contains the larger information gain

3. ID3 tree

The information gained decides which feature comes first.

The final nodes are determined by ratio.

TT is 100% +

TF is 4:1 +

FT is 4:3 +

FF is 5:1 -

