Problem Set 7 Part I

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1 Redefine the problem

This submission is the revised version of the Part 1 of the problem set 7.

With the slowdown of the growth on domestic e-commerce platforms, many foreign SME companies are seeking opportunities on Amazon in the United States. Many of them adopt a common business model: transporting their products to Amazon's distribution centers, entrusting Amazon to fulfill their orders, and paying the service charges, which include fixed monthly fee and variable cost based on the goods sold. How to compete with others on this "new" platform is not an easy question. Based on the 4Ps marketing mix defined by McCarthy [1], price is one of the most critical variables impacting sales. Here I take an attempt to study whether the Dynamic Programming technique can be used to find an optimal pricing strategy for those new players on Amazon U.S.

2 The model

In this model, the agent is the SME companies who adopt the above business model. The purpose is to maximize profit for a given initial quantity of the products. Therefore, the preference:

$$\sum_{t=1}^{T} \beta_t \Pi(s_t) - C_l * Q, \forall Q \tag{2.1}$$

Where:

$\Pi(\cdot)$: profit function; here I assume: $\Pi' > 0, \Pi'' < 0$

s_t : the sales in time t

C_l : the unit total landing cost of the product

Q : the initial quantity of the product

β_t : the discount rate, a function of opportunity cost of capital

$$\beta_t = \frac{1}{(1+r)^t} \tag{2.2}$$

Q, s_t are state variable of the model.

The profit function Π is defined:

$$\Pi(s_t) = p_t s_t - C_v s_t - C_f \tag{2.3}$$

Where:

p_t : the price of time period t, which is the control variable of this model

C_v : the unit variable cost of goods sold in the time t, which is a constant

C_f : the fixed cost of service charge in the time t

It is impossible to directly control how many products can be sold in each period, i.e.,s_t. However, it is possible to control sales indirectly by adjusting the prices based on the idea of the demand curve. By adjusting the prices to an optimal point, the companies can realize the expected sales, which generate the maximized profit. This is the productive technology of the model. The p_t is the control variable.

The demand function:

$$s_t = \varepsilon_t a p_t + b \tag{2.4}$$

Where:

ε_t : the market shock in the time t;

a : the constant part of the slope of the demand curve

b : the intercept of the demand curve, which is a constant

The slope of the demand curve is not permanent. It changes because of various market shocks. To make the model not so complicated, here I assume:

1. The shock only impacts the slope of the demand curve but not the intercept.
2. There is only two possible value of the shock: $\varepsilon_H, \varepsilon_L$.
3. ε follows a First-Order Markov Process.
4. The transition probabilities of the shock decided by a 2x2 matrix with four constants.
5. The companies know ε and the transition probabilities matrix at the beginning of the operations.

The above is the information technology of this model.
Again, to make the model less complicated, I assume the initial Q is given,

and there is no endowment in other periods. And there are no enforcement technology, and matching technology should be defined in this model.

The Bellman equation:

$$V_T(Q) = \max_{p_t} \sum_{t=1}^{T} \beta_t \Pi(s_t) - C_l * Q, \forall Q \tag{2.5}$$

Where:

$$Q = \sum_{t=1}^{T} s_t, \forall Q \tag{2.6}$$

The Lagrangian:

$$L = \max_{p_t} \sum_{t=1}^{T} \beta_t \Pi(s_t) - C_l * Q + \lambda(Q - \sum_{t=1}^{T} s_t), \forall Q \tag{2.7}$$

The FOCs:

$$\beta_t \Pi'(s_t) = \lambda, \forall t \tag{2.8}$$

Therefore:

$$\beta_t \Pi'(s_t) = \beta_{t+1} \Pi'(s_{t+1}) \tag{2.9}$$

After transformation:

$$\Pi'(s_t) = \beta_1 \Pi'(s_{t+1}) \tag{2.10}$$

Add the market shock into the Bellman equation:

$$V_T(Q, \varepsilon) = \max_{p_t} \beta_1(\Pi(Q, \varepsilon) + \beta_1 E_{\varepsilon'|\varepsilon} V_{T-1}(Q', \varepsilon')), \forall Q \tag{2.11}$$

$$V_{T-1}(Q', \varepsilon') = \max_{p_t} \beta_1 E_{\varepsilon'|\varepsilon}(\beta_1 \Pi(Q', \varepsilon') + \beta_1 E_{\varepsilon''|\varepsilon'} V_{T-2}(Q'', \varepsilon'')), \forall Q \tag{2.12}$$

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Here $Q = Q' + s_1, Q' = Q'' + s_2 ...$

3 Results

I had been struggling with how to add market shock to the model. According to the Markov Chain, when I know the constant 2x2 transition matrix and the value of high and low shocks on period one, I should be able to estimate the shock for each following period. The sequence of the time is important here. However, the order of time is neglected in the VFI or PFI because the state is relevant to the time. Therefore, I assumed the shock was gone in my solution.

If neglecting the impact of shock, I can use almost the same method as Dr.Debacker's notebook. Although I want to control the price to impact the sales for each period to maximize the profit, however, the solution per se can be reversed: calculating the optimal sales for each time period then know the optimal price.

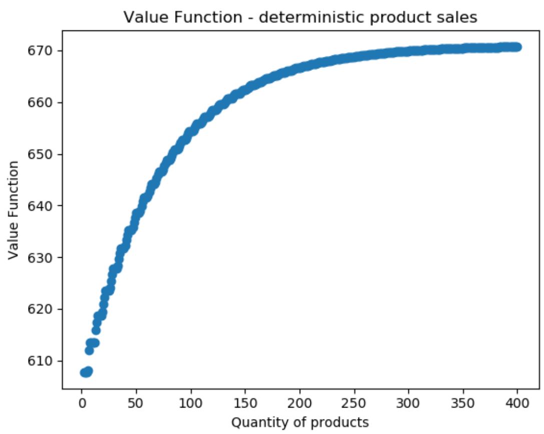

Figure 1: Value changes with quantity

Pic.1 is the value function plot. Here I didn't consider the initial landing cost because it is a constant when Q is given and will not influence the price decisions. The value increases along with the quantity of products increase but the rate of increase decreases when there are more products are ready to sell.

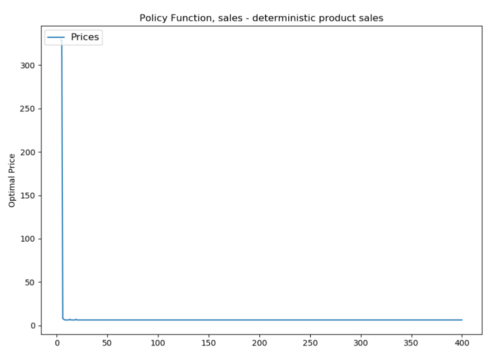

Figure 2: Price changes with quantity

Pic.2 is the policy function plot, which shows the optimal prices along with the different quantity for sell. The optimal prices are stable at around 8. But when the quantity is very low, the optimal price will be very high.

References

[1] Eugene J McCarthy. Basic marketing: A managerial approach, rev. ed. *Homewood, IL: Richard D. Irwin*, 1964.