Problem Set 7 Part I

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1 Introduction

Farmer Jet attends a city fair every month. He rents a booth there and sells watermelon from his farm. The fair is two days. After the fair, Jet donates the remaining watermelon to a local primary school because the transportation fee is too expensive to cover the value of those watermelon. He hopes he can maximize his profit during the fair by adjusting the price to sell more watermelon. I will use dynamic programming to help him find a solution.

2 Define the problem

The preference:

$$\Pi(s_1) + \Pi(s_2) - C(q_3)$$

Where:

s_1 : sales at day 1

s_2 : sales at day 2

q_3 : remaining watermelon at day 3

Π : profit function

C : cost function

Other parameters:

p_1 : price at day 1

p_2 : price at day 2

q_1 : the number of watermelon at day 1

q_2 : the number of watermelon at day 2

Transition equation:

$q_2 = q_1 - s_1$

$q_3 = q_2 - s_2$

Price and demand function:

$s_1 = \alpha * p_1 + b$

$s_2 = \alpha * p_2 + b$

Where:

α : slop of the demand curve. Because the higher price is associated with the lower demand, the $\alpha is negative$

b : the intercept of the demand curve

Bellman equation:

$$V \equiv \max_{p_1, p_2} \Pi(s_1) + \Pi(s_2) - C(q_3) \tag{1}$$

It can be easily transform to:

$$V \equiv \max_{p_1, p_2} s_1 * p_1 + s_2 * p_2 - c * (s_1 + s_2 + q_3) \tag{2}$$

Because the cost is sunk, the profit function can be substituted by revenue function.

$$V \equiv \max_{p_1, p_2} s_1 * p_1 + s_2 * p_2 \tag{3}$$

We can assume the p_2 is p_1 with a discount: $p_2 = \beta * p_1$. Therefore, the new Bellman equation is:

$$V \equiv \max_{\beta} R(p_1) + R(\beta * p_1) \tag{4}$$

Where:

$R(p)$: revenue function

β : price discount coefficient

And also:

Control variable: We can control β to change the sales in day 2.

State variable: The q_1 is the state variable of this system.

First order condition:

$$(2 * \alpha + 2 * \alpha * \beta^2) * p_1 + b + b * \beta = 0 \tag{5}$$