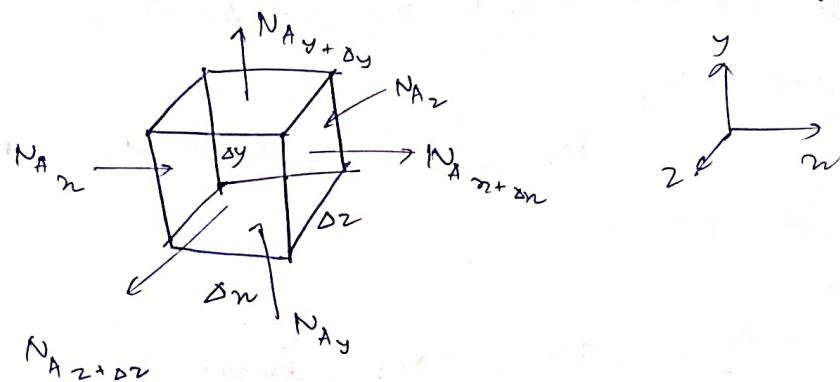


## Section-2

(Based on Lecture Notes)

Q.1) Derive the equation of continuity in terms of molar flux.

Ans: Let us assume a cuboid of volume sides  $\Delta x$ ,  $\Delta y$  and  $\Delta z$ , in the Cartesian coordinate system.



Molar fluxes having components (such as  $N_{Ax}$  for  $x$  direction) are depicted as shown in figure.

Suppose  $R_A$  moles of A are generated per unit volume per unit time in the differential cuboidal element.

Then,

~~Input + Production~~

~~Input + Generation = Output + Accumulation~~

(the accumulation/depletion is taken care of in  $R_A$  itself through the 'tive' sign)

$$\therefore (N_{Ax} \Delta y \Delta z + N_{Ay} \Delta z \Delta x + N_{Az} \Delta x \Delta y + R_A (\Delta x \Delta y \Delta z)) \underset{\substack{\text{CROSS} \\ \text{SECTIONAL} \\ \text{area}}}{\uparrow} \underset{\substack{\text{infinite} \\ \text{time} \\ \text{change}}}{\downarrow} \underset{\substack{\text{volume}}}{\rightarrow} \text{at}$$

(input = flux  $\times$  area)

$$= \cancel{R_A \Delta x \Delta y \Delta z} + (N_{Ax+\Delta x} \Delta y \Delta z + N_{Ay+\Delta y} \Delta z \Delta x + N_{Az+\Delta z} \Delta x \Delta y) \Delta t$$

$$+ \Delta x \Delta y \Delta z \frac{\Delta e}{M_A} \underset{\substack{\text{change in no. of} \\ \text{moles per unit volume}}}{\hookrightarrow}$$

$$\Rightarrow \left( (N_{n+\Delta n} - N_n) \Delta y \Delta z \right. \\ + (N_{y+\Delta y} - N_y) \Delta z \Delta n \\ \left. + (N_{z+\Delta z} - N_z) \Delta n \Delta y \right) + \frac{\partial e}{M_A} \Delta n \Delta y \Delta z \\ - (R_A \Delta n \Delta y \Delta z) dt = 0$$

In the limit  $\Delta n, \Delta y, \Delta z \rightarrow 0$ ,

and using

$$N_{n+\Delta n} = N_n + \frac{\partial N_n}{\partial n} \Delta n : \Delta n \rightarrow 0,$$

we get

$$\left( \frac{\partial N_n}{\partial n} + \frac{\partial N_y}{\partial y} + \frac{\partial N_z}{\partial z} \right) \Delta n \Delta y \Delta z dt \\ + \frac{\partial e}{M_A} \Delta n \Delta y \Delta z - R_A \Delta n \Delta y \Delta z dt = 0$$

$$\left( \frac{\partial N_{An}}{\partial n} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z} \right) \Delta n \Delta y \Delta z$$

$$+ \left( \frac{\partial e}{M_A dt} - R_A \right) \Delta n \Delta y \Delta z = 0$$

In steady state (no accumulation/generation),

$$\boxed{\frac{\partial N_{An}}{\partial n} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z} = 0}$$

This is the continuity equation in terms of molar flux.

1.2. Considering the steady state diffusion in binary gas mixture derive the equation for the flux for the following cases

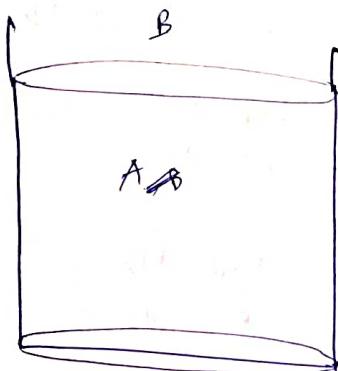
a. Diffusion of A through Non-diffusing B

b. Equimolar counter diffusion

c. Non-Equimolar counter diffusion



Ans: (a)



(Assuming  
constant geometry)

a) A diffusing, B non-diffusing

By Fick's Law in terms of Molar Flux,

$$N_A = -\frac{D_{AB}}{RT} \frac{dp_A}{dz} + N_A \frac{p_A}{P}$$

$$\Rightarrow N_A \left(1 - \frac{p_A}{P}\right) = -\frac{D_{AB}}{RT} \frac{dp_A}{dz}$$

$$\therefore z \frac{N_A p}{D_{AB}} = \int \frac{dp_A}{1 - \frac{p_A}{P}} = -R \ln \frac{P - P_{A2}}{P - P_{A1}}$$

$$\therefore N_A = -\frac{D_{AB} P}{z RT} \ln \left( \frac{P - P_{A2}}{P - P_{A1}} \right)$$

b)

Equimolar counter diffusion

$$N_A = -\frac{D_{AB} P}{RT} - \frac{D_{AB}}{RT} \frac{dp_A}{dz} + 0$$

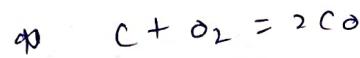
(as  $N_A + N_B = 0$ )

~~Ans~~ ~~Ans~~

$N_A$  is constant with  $z$  in  
steady state

$$\therefore N_A = -\frac{D_{AB} (P_2 - P_A)}{RT \cdot L}$$

(C) Non equimolar



$$N_{CO} = 2N_A - \frac{1}{2} N_C$$

$$N_B \qquad \qquad \qquad N_A$$

$$N_A = -\frac{D_{AB}}{RT} \frac{dP_A}{dz} + \left( N_A - \frac{1}{2} N_A \right) \frac{P_A}{P}$$

$$\therefore \left( N_A - \frac{N_A P_A}{2P} \right) = -\frac{D_{AB}}{RT} \frac{dP_A}{dz}$$

$$\therefore N_A \frac{RT}{-D_{AB}} z = \int \frac{dP_A}{z - \frac{P_A}{2P}}$$

$$= -2P \left( \ln \frac{2P - P_{A2}}{2P - P_{A1}} \right)$$

$$\therefore N_A = \frac{DFT}{RTz} \frac{2P D_{AB}}{2P - P_{A1}} \ln \left( \frac{2P - P_{A2}}{2P - P_{A1}} \right)$$

2.3. Considering the steady state diffusion in binary liquid mixture derive the equation for the flux for the following cases.

a. Diffusion of A through non-diffusing B

b. Equimolar counter diffusion

c. Non-Equimolar counter diffusion,

When  $N_A = N_B/2$ .

Ans: (a) Corresponding results for gas are valid by replacing  $\frac{P}{RT}$  with  $c$  — (i)

$$\therefore N_A = -\frac{D_{AB}}{R} \frac{c}{z} - D_{AB} \frac{c}{e} \ln \left( \frac{c - c_{A_2}}{c - c_{A_1}} \right)_{RT}$$

(from (i))

$$(b) N_A = -\frac{D_{AB}}{R} \frac{c}{z} - D_{AB} \left( \frac{c_{A_2} - c_{A_1}}{e} \right)$$

(from (i))

$$(c) N_A = -\frac{N_B}{2} \Rightarrow N_B = -2N_A$$

$$N_A = -\frac{D_{AB}}{z} \frac{dc_A}{dz} + \frac{(N_A - 2N_A)}{c} c_A$$

$$\therefore N_A \left( 1 + \frac{c_A}{c} \right) = -D_{AB} \frac{dc_A}{dz}$$

$$\therefore \frac{N_A}{D_{AB}} = \int \frac{dc_A}{1 + \frac{c_A}{c}} = c \ln \frac{c + c_{A_2}}{c + c_{A_1}}$$

$$\therefore N_A = \frac{D_{AB}}{z} c \ln \frac{(c + c_{A_2})}{(c + c_{A_1})}$$

(b.i) Show the analogies among mass, heat and momentum transfer.

In all of the three transport phenomena, the flux is given as

$$J = \eta \frac{d(\text{driving force})}{dn}$$

$$\text{Heat} \rightarrow \vec{q} = -k \frac{dT}{dn} = \frac{k}{\rho c_p} \frac{de_p}{dn}$$

$\hookrightarrow$  Fourier's Law thermal diffusivity

$$\text{Mass} \rightarrow J = D_{AB} \frac{dc_A}{dn}$$

$\hookrightarrow$  mass diffusivity

$$\text{Momentum} \rightarrow \text{Stress} = \eta \frac{d\vec{v}_y}{dn} = \frac{\eta}{e} \frac{d\vec{v}_y}{dn}$$

$\hookrightarrow$  momentum diffusivity / kinematic viscosity

Thus, all the three are analogous to each other.

## Section 2

(Based on the worked out problems  
in the text book)

from book: Principle of Mass Transfer  
by B. K. Datta

Ex 2.2 Pg. No. 26

Example 2.2 (Diffusion of A through non diffusing B)

There is a 2 mm thick layer of water on the floor of a room. The water vaporizes and diffuses through a stagnant film of air of estimated thickness of 2.5 mm on the water surface. Under the condition of evaporation, the water temperature is essentially equal to its wet-bulb temperature. If the ambient temperature is  $28^{\circ}\text{C}$ , calculate the time required for the water layer to disappear completely for the following cases:

(a) the ambient air has a relative humidity of 60%; and (b) the floor has micropores and water penetrates the floor at a constant rate of  $0.1 \text{ kg/m}^2\text{ s}$  for ambient air having a humidity as in part (a).

Read the wet-bulb temperature from the humidity chart and calculate the vapour pressure of water using the Antoine equation given below. The diffusivity of water vapour in air is  $0.853 \text{ ft}^2/\text{h}$  at 1 atm and  $0^{\circ}\text{C}$ .

Vapour pressure  $p_v$  (in bar) of water is given by:

$$\ln p_v = 13.8573 - 5180.2/T_1 \quad \text{where } T_1 \text{ is the temperature in K.}$$

Ans = (a)

Putting  $T = 28^\circ\text{C} = 301\text{K}$  in the given Ans,

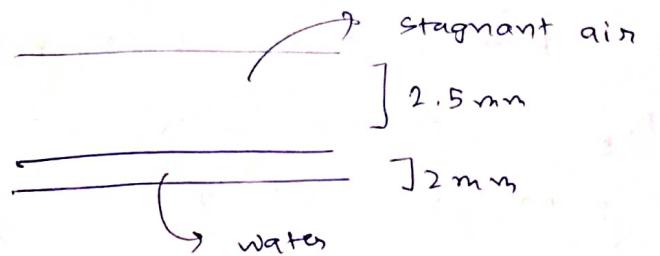
equation, we get

$$\ln \frac{P_v}{P_r} = 23.8573 - \frac{5260.2}{301}$$

$$\therefore P_v = e^{23.8573 - \frac{5260.2}{301}}$$

$$= 0.0374 \text{ bar}$$

From psychometric chart, at 60% RH,  
wet bulb temperature is  $22.5^\circ\text{C}$ .



This corresponds to water vapour (A) diffusing through a 2.5 mm thick stagnant layer of film of air (B) which is non-diffusing. The temperature of this film varies from  $22.5^\circ\text{C}$  to  $28^\circ\text{C}$ ; thus we take mean temperature

$$\text{i.e. } \frac{28^\circ\text{C} + 22.5^\circ\text{C}}{2} = 25.25^\circ\text{C} \sim 25.2^\circ\text{C}$$

$$\text{i.e. } 298.2\text{ K}$$

By Fuller Correlation,

$$D_{AB} \propto T^{1.75}$$

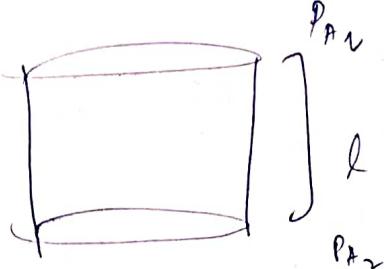
$$\therefore \frac{D_{AB}|_{25.2^\circ\text{C}}}{D_{AB}|_{0^\circ\text{C}}} = \left( \frac{(273 + 25.2)\text{K}}{273\text{K}} \right)^{1.75} =$$

$$\therefore D_{AB}|_{25.2^\circ\text{C}} = 0.853 \text{ ft}^2/\text{h} \times \left( \frac{298.2}{273} \right)^{1.75}$$

$$= 0.853 \times \left( \frac{298.2}{273} \right)^{1.75} \times \frac{(30.48)^2 \text{ cm}^2/\text{s}}{3600}$$

$$= 0.2567 \text{ cm}^2/\text{s}$$

$$= 2.567 \times 10^{-5} \text{ m/s}$$



We know that, in steady state,

$$N_A = \frac{D_{AB} C}{L} \ln \frac{C - C_1}{C - C_2}$$

$$= \frac{D_{AB} \rho}{RTL} \ln \left( \frac{\rho - P_{A1}}{\rho - P_{A2}} \right)$$

Putting  $P_{A1} = 0$ ,  $P_{A2} = P_{\text{vap}} = 0.02718 \text{ bar}$ ,

$\rho = 1 \text{ atm} = 1.013 \text{ bar}$ ,

$D_{AB} = 2.567 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $L = 0.0025 \text{ m}$ ,

$T = 298.2 \text{ K}$ , we get

$$N_A = 2.01 \times 10^{-6} \frac{\text{kmol}}{\text{m}^2 \text{s}}$$

$$\therefore N_A M = 2.01 \times 10^{-6} \times (18 \times 10^{-3} \times 10^3) \text{ kg/m}^2 \text{s}$$

$$= 3.623 \times 10^{-5} \text{ kg/m}^2 \text{s}$$

Also,

$$N_A a = \frac{d \frac{\text{kg}}{\text{m}^2}}{dt} = \frac{d \left( \frac{\text{kg}}{\text{m}^2} \right)}{dt} \xrightarrow{\text{long m of width column}}$$

$$= \frac{e a \frac{d L}{dt}}{M}$$

$$\therefore t = \frac{e a L}{N_A M a} = \frac{e L}{N_A M} = \frac{10^3 \times 10^{-3} \times 2}{3.623 \times 10^{-5}} \text{ s}$$

$$= 5.52 \times 10^4 \text{ s}$$

$$= 15.3 \text{ h}$$

$$\begin{aligned}
 \text{Total rate of loss} &= \text{Rate penetration} \\
 &\quad + \text{Rate vaporisation} \\
 &= 0.1 + 3.623 \times 10^{-5} \times 3600 \\
 &= 0.2304 \text{ kg/m}^2 \text{ h}
 \end{aligned}$$

$$\therefore t = \frac{2 \times 10^{-3} \times 10^3}{0.2304} \text{ h} = 8.68 \text{ h}$$

Example 2.3 (Calculation of flux and velocity) Ammonia (A), diffuses through a stagnant layer of air (B), 1cm thick, at 25°C and 1 atm total pressure. The partial pressures of  $\text{NH}_3$  on the two sides of the air layer are  $p_{A_0} = 0.9 \text{ atm}$  and  $p_{A_1} = 0.1 \text{ atm}$  respectively. Air is non-diffusing.

Calculate (a) the molar flux of  $\text{NH}_3$ , (b) the velocities of the individual components with respect to a stationary observer, (c) the molar and the mass average velocities of the components, and (d) the molar flux of  $\text{NH}_3$  with respect to an observer moving with the mass average velocity.

Also prepare the plots of partial pressure distribution of ammonia and air along the diffusion path.

Given :  $D_{AB} = 0.249 \text{ cm}^2/\text{s}$ .

Ans = (a) This again is a case of A diffusing through non-diffusing B.

$$P_{A0} = 0.9 \text{ atm}, P_{Ae} = 0.1 \text{ atm}, D_{AB} = 0.214 \text{ cm}^2/\text{s}$$

$$T = 298 \text{ K}, \rho = 1.0 \text{ atm}, l = 1 \text{ cm}.$$

$$\therefore N_A = \frac{D_{AB} l}{RTl} \frac{\ln \frac{l - P_{Ae}}{l - P_{A0}}}{\frac{P_{Ae}}{P_{A0}}}$$

$$= \frac{0.214(1)}{82.1(298)(1)} \ln \frac{l - 0.1}{l - 0.9}$$

$$= 2.922 \times 10^{-5} \text{ mol/cm}^2 \text{ s}$$

(b), (c) (same as  $N_A$  has 0 relative velocity component)

$$\text{Molar avg velocity } v = \frac{N_A}{c} = \frac{1.922 \times 10^{-5}}{\frac{1}{82.1(298)}}$$

$$c = \frac{P}{RT}$$

$$= 0.17 \text{ cm/s}$$

Mass

$$\text{avg velocity, } v = \frac{e_A u_A}{e_A + e_B} = \left( \frac{e_A}{e_A + e_B} \right) u_A$$

$$= \left( \frac{m_A}{m_A + m_B} \right) u_A = \frac{m_A u_A}{m_A + m_B}$$

$$= \frac{m_A u_A}{m_A + m_B} \cdot \frac{m_A}{m_A + m_B}$$

$$= \left( \frac{m_A u_A}{m_A + m_B} \right) \frac{m_A}{m_A + m_B}$$

$$= \frac{u_A}{y_A m_A + y_B m_B}$$

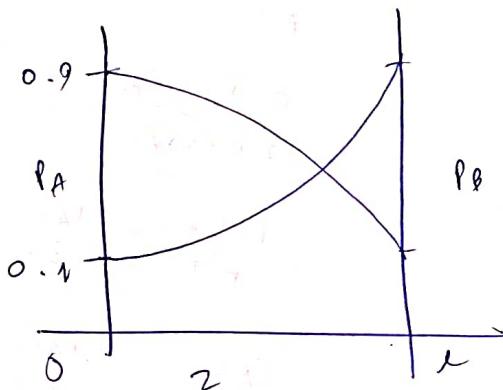
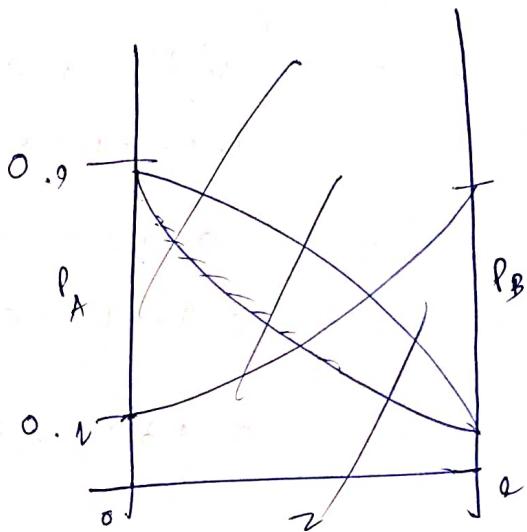
$$\begin{aligned}
 \text{From } v_i &= \frac{P_i}{P} \\
 \therefore v &= \left( \frac{V M_A}{P_A M_A + P_B M_B} \right) P \\
 &= \frac{0.47 \times 27 \times 1}{0.9 \times 27 + 0.1 \times 29} \\
 &= 0.439 \text{ cm/s}
 \end{aligned}$$

(d) We need to find  $Z_A$ .

$$\begin{aligned}
 Z_A &= C_A (v_A - v) \\
 &= \frac{P_A}{P_T} (v_A - v) \\
 &= \frac{0.9}{82.1 (298)} (0 \\
 &= \frac{P_A v_A}{P_T} - \frac{P_A v}{P_T} \\
 &= C_A v_A - \frac{P_A v}{P_T} \\
 &= N_A - \frac{P_A v}{P_T} \\
 &= 1.922 \times 10^{-5} \text{ g mol/cm}^2 \\
 &\quad - \frac{0.9 (0.439)}{82.1 \times 298} \text{ g mol/cm}^2 \\
 &= 3.05 \times 10^{-6} \text{ g mol/cm}^2
 \end{aligned}$$

By steady state  $N_A$  equation, we have

$$\begin{aligned}
 N_A &= 1.922 \times 10^{-5} = \frac{0.214 (0.9)}{82.1 (298)^2} \ln \frac{1 - P_A}{1 - 0.9} \\
 \therefore P_A &= (1 - 0.1 e^{2.1972}) \text{ atm} \\
 P_B &= 1 - P_A = 0.1 e^{2.1972} \text{ atm}
 \end{aligned}$$



Example 2.1. (Flux, velocity and pressure gradient)

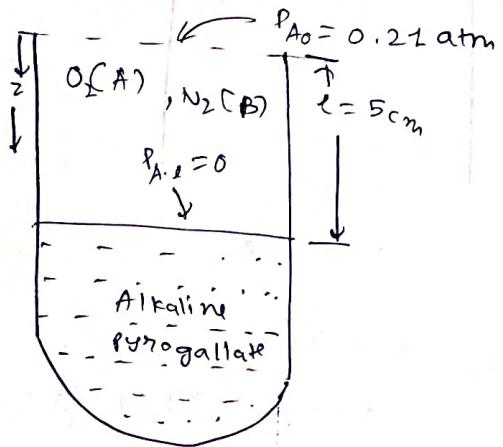
A test tube 1.5 cm in diameter and 12 cm tall is partly filled with a solution of alkaline pyrogallate. The depth of the empty space above the solution is 5 cm. The temperature is  $25^\circ\text{C}$  and total pressure is 1 atm. Air may be assumed to contain 21%  $\text{O}_2$  and 79%  $\text{N}_2$ . The diffusivity of  $\text{O}_2$  in  $\text{N}_2$  at the given condition is  $0.21 \text{ cm}^2/\text{s}$ .

- Calculate the rate of absorption of oxygen from air in the solution at steady state if air flows gently over the open end of the test tube. Make plots of the distribution of partial pressures of the gas along the diffusion path.
- Calculate the partial pressure gradient of oxygen midway in the diffusion path.
- Calculate the molar average velocity of the mixture and the 'diffusion velocity' of the two components ( $\text{O}_2$  and  $\text{N}_2$ ) at

the top one, at the middle and at the liquid surface.

(d) Calculate the flux of the components midway of the diffusion path with respect to an observer moving with twice the molar average velocity at the location in the direction away from the liquid surface.

Solution: The system is shown as



(a)

$$N_A = \frac{D_{AB} \Delta P}{P} \frac{RT}{l} \ln \left( \frac{P - P_e}{P - P_0} \right)$$
$$= \frac{2.1 \times 10^{-5}}{0.08317 \text{ } 298} \frac{(1.013)}{0.05} \ln \left( \frac{1.013}{1.013 - 0.21} \right) \text{ } \frac{\text{m}^3 \text{ bar / kmol s}}{\text{m}^2}$$
$$= 1.05 \times 10^{-6} \text{ kmol / m}^2 \text{ s}$$

Area,

$$a = \pi d^2 = \pi \frac{\pi}{4} \times 0.025^2 = 1.767 \times 10^{-4} \text{ m}^2$$

$\therefore N_A =$

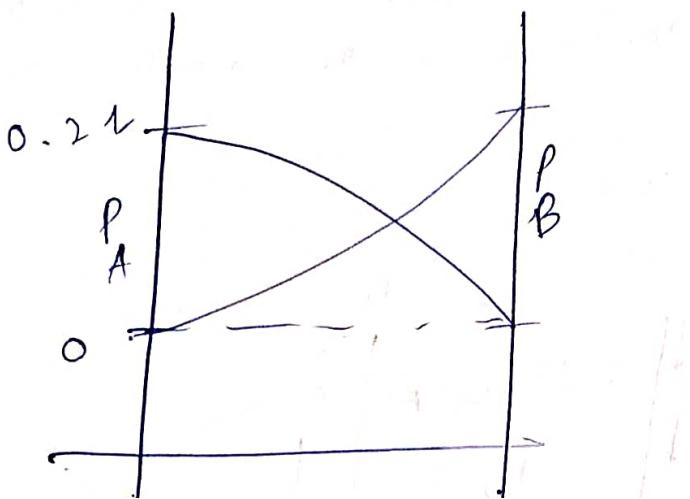
$$\frac{d n_0}{dt}_{\text{oxygen}} = N_A a = 1.767 \times 10^{-4} \times 1.05 \times 10^{-6} \text{ kmol/s}$$
$$= 7.15 \times 10^{-20} \text{ kmol/s}$$

Also plot

$$N_A = 1.05 \times 10^{-6} = \frac{2.1 \times 10^{-5} (1.013)}{0.08317 (298) R} \ln \left( \frac{1.013}{1.013 - 0.21} \right)$$

$$\therefore \ln \frac{1}{1 - \frac{P}{1.013}} = 4.72 \text{ R}$$

$$\therefore P_A = 1.013 \left(1 - e^{-A \cdot 72 n}\right)$$
$$P_B = P_{\text{atm}} - P = 1.013 e^{-A \cdot 72 n}$$



$$(b) N_A = - \frac{D_{AB}}{RT} \frac{dp}{dz} + \frac{N_A}{P} p_A$$

for  $\ln P - P_A \approx i_3$   
linear in  $i_3$ ,  
 $\therefore P_A = P - \sqrt{P_B P_D}$   
 $= 1 - \sqrt{1 - i_3}$

$$\therefore \frac{dp}{dz} = - N_A \left( \frac{P - P_A}{P} \right) \frac{RT}{D_{AB}}$$

$$= - \frac{0.08317 (298)}{2.2 \times 10^{-5} (1.013)} \times 1.05 \times 10^{-6}$$

$$\therefore P - P_A = P_B = \sqrt{P_B P_D}$$

$$= \sqrt{1 - 0.222}$$

$$= \sqrt{0.778}$$

$$= 0.88$$

$$= -9.25 \text{ cm/m}$$

$$\therefore P_A = 1 - 0.888$$

$$= 0.113$$

$$(c)$$

$$\left\{ \begin{array}{l} v_A = \frac{N_A}{C_A} = N_A \frac{R_T}{P_A} = \frac{1.05 \times 10^{-6}}{0.113} 0.08317 (298) \\ v_B = \frac{N_B}{C_B} = 0 \end{array} \right.$$

$= 8.88 \times 10^{-9} \text{ m/s}$

stationary observer

$$V = \frac{1}{2} (v_A C_A + v_B C_B) = \frac{C_A}{C} v_A = \frac{P_A}{P} v_A$$

$$= 0.113 (8.88 \times 10^{-9})$$

$$= 9.9 \times 10^{-9} \text{ m/s}$$

Diffusion velocity of A

$$v_{A,d} = v_A - V = 8.88 \times 10^{-9} - 9.9 \times 10^{-9}$$

$$= 7.9 \times 10^{-9}$$

Diffusion velocity of B

$$v_{B,d} = v_B - V = 0 - 9.9 \times 10^{-9} = -9.9 \times 10^{-9} \text{ m/s}$$

~~A~~ 2 = 0,

$$v_{A,d} = v_A + 2.72 \times 10^{-9} \text{ m/s}$$

$$v_{B,d} = -9.9 \times 10^{-9} \text{ m/s}$$

2 = 0.5

$$v_A = \frac{n_A}{c_A} = \frac{n_A}{0} \rightarrow \infty$$

$$v_{A,d} \rightarrow \infty$$

$$v_{B,d} = -9.9 \times 10^{-9}$$

(d)

Molar avg. velocity,

$$\bar{v} = 9.9 \times 10^{-9} \text{ m/s}$$

$$V = -20 = -1.98 \times 10^{-9}$$

$$\therefore n_A' = c_A (v_A - V) = n_A - V \frac{P_A}{R_T}$$

$$= 4.05 \times 10^{-6} \frac{(-1.98 \times 10^{-9}) / (0.113)}{0.08314 (298)}$$

$$= 4.95 \times 10^{-6} \text{ kmol/m}^2 \text{ s}$$

$$n_B' = c_B (v_B - V) = 0 - V \frac{P_B}{R_T}$$

$$= -1.013 - 0.113 \frac{(-1.98 \times 10^{-9})}{0.08314 \times 298}$$

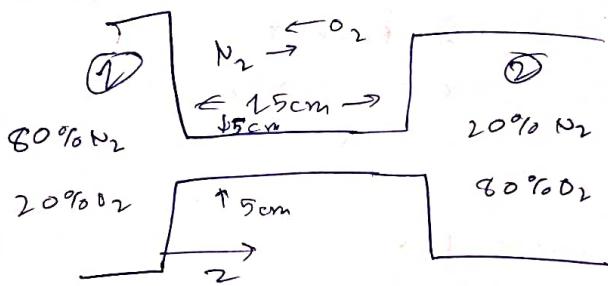
$$= 7.2 \times 10^{-6} \text{ kmol/m}^2 \text{ s}$$

## (Equal molar countercurrent diffusion)

Two large vessels are connected by a tube 5cm in diameter and 15cm in length. Vessel 1 contains 80%  $N_2$  and 20%  $O_2$  (B) ; vessel 2 contains 20%  $N_2$  (A) and 80%  $O_2$ . The temperature is  $20^\circ C$  and the total pressure is 2 atm.

Calculate (a) the steady-state flux and the rate of transport of  $N_2$  from vessel 1 to vessel 2, (b) the same quantity for  $O_2$ , (c) the partial pressure of  $N_2$  and its gradient in the tube 0.05m from vessel 1 and (d), the net mass flux with respect to a stationary observer. Given: the diffusivity of  $N_2-O_2$  pair is  $0.23 \text{ cm}^2/\text{s}$  at  $326\text{K}$  and 1 atm.

Ans = (a)



$$N_A = - \frac{D_{AB}}{RT} \frac{dp}{dz} + 0$$

$\underbrace{\text{const.}}_{\text{wjmz}}$        $\underbrace{\text{const.}}_{\text{gas}}$

$$\Rightarrow N_A = -N_B$$

$$\Rightarrow N_A + N_B = 0$$

$$\therefore N_A = - \frac{D_{AB}}{RT} \left( \frac{p_1 - p_2}{L} \right)$$

$\underbrace{\text{as } \frac{dp}{dz} \text{ is const.}}$

$D_{AB}$  is calculated using the Fick's relation as

$$D_{AB} = \frac{1}{2} \left( \frac{293}{326} \right)^{1.75}$$

$\underbrace{2 \text{ atm}}$

$$\therefore D_{AB} = 0.23 \times \frac{1}{2} \times \left( \frac{293}{326} \right)^{1.75} = 1.01 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Also, } P_{A1} = 2(0.8) = 1.6 \text{ atm},$$

$$P_{A2} = 2(0.2) = 0.4 \text{ atm}$$

(a)

$$N_A = \frac{DAB}{RT^2} (P_{A1} - P_{A2})$$

$$= \frac{1.01 \times 10^{-5}}{0.0821 \times 293 \times 0.15} (1.6 - 0.4)$$

$$= 3.36 \times 10^{-4} \text{ kmol/m}^2\text{s}$$

$$\text{area of cross section} = \pi \frac{d^2}{4} = \pi 0.05^2 = 1.963 \times 10^{-3} \text{ m}^2$$

$$\therefore \text{rate of } N_2 \text{ transfer from 1 to 2} = a N_A = 1.963 \times 10^{-3} \times 3.36 \times 10^{-4}$$

$$= 6.6 \times 10^{-9} \text{ kmol/s}$$

$$(b) N_B = -N_A = -3.36 \times 10^{-6} \text{ kmol/m}^2\text{s}$$

$\leftarrow$  equimolar counter diffusion

(c)

$$\frac{dp}{dz} = \text{const.} = \frac{P_{A1} - P_{A2}}{L} = \frac{0.4 - 0.6}{0.15} = -8 \text{ atm/m}$$

$\hookrightarrow$  as  $N_A$ ,  $R$  and  $T$   
are constant in  
steady state

(d) Not on total mass flux

$$n_T = M_A N_A + M_B N_B = (M_A - M_B) N_A$$

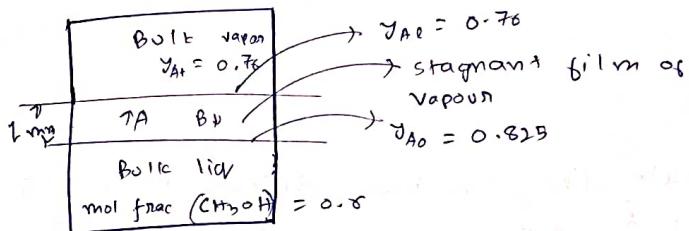
(as  $N_A = -N_B$ )

$$= (28 - 32) \times 3.36 \times 10^{-6}$$

$$= -1.344 \times 10^{-5} \text{ kmol/m}^2\text{s}$$

Example 2.7. (Non-equimolar countercurrent diffusion in distillation of a binary mixture) An aqueous solution of methanol is being separated by distillation in a column. Methanol (A), which is the more volatile component, moves from the liquid phase to the vapour phase while water (B), the less volatile component, gets transported in the opposite direction. At a section of the column, the vapour phase contains 0.76 mole fraction methanol and the liquid has 0.6 mole fraction of H. The temperature is  $71.2^{\circ}\text{C}$  and total pressure is essentially atmospheric. The diffusional resistance offered is equivalent to that of a 'stagnant' vapour film of 1 mm thickness. If the latent heat of vaporization of methanol is 274.6 kcal/kg and that of water is 557.7 kcal/kg at the given temperature, calculate the flux of methanol and that of water vapour. Given; if the mole fraction of methanol in the solution is 0.6, its mole fraction in the equilibrium vapour would be 0.825. The vapour phase mutual diffusivity,  $D_{AB} = 1.82 \times 10^{-5} \text{ m}^2/\text{s}$ .

$$hs =$$



Let us represent the counter diffusion as

$$N_A = -n N_B$$

We need  $n$ .

By assuming all the energy exchanges occur due to latent heats (i.e. system is isolated), we have, by energy balance,

$$N_A \Delta H_A = -N_B \Delta H_B \Rightarrow n = \frac{\Delta H_B}{\Delta H_A} = \frac{557.7 \times 18}{274.6 \times 32} = \frac{1}{0.8753}$$

$$\therefore N_B = -0.8753 N_A$$

Using Fick's Law, we have

$$N_A = -\frac{D_{AB}}{RT} \frac{dp}{dz} + \frac{(N_A - n N_A) p_A}{P}$$

$$\therefore N_A \left(1 - \frac{0.8753}{P} p_A\right) = -\frac{D_{AB}}{RT} \frac{dp}{dz}$$

$$\therefore N_A \frac{RT}{P P_{AB}} \int_{z=0}^{y_A} dz = \int_{y_A}^1 \left( \frac{dp_A}{1 - 0.1247 \frac{p_A}{P}} \right) \frac{1}{\bar{P}} \frac{dp_A}{P} = \frac{dy_A}{\bar{P}} = dy_A$$

$$\frac{p_A}{P} = y_A$$

$$\therefore N_A = \frac{D_{AB} P}{0.1247 P_T} \ln \frac{1 - 0.1247 y_A}{1 - 0.1247 y_0}$$

$$y_{A0} = 0.6, \quad y_{A \text{ liquid}} = 0.6, \quad y_{A0} = 0.825$$

$$P = 1 \text{ atm}, \quad l = 0.001 \text{ m}, \quad T = 344.2 \text{ K}$$

$$\therefore N_A = \frac{1.816 \times 10^{-5}}{0.1247 \times 0.0821 \times 344.2 \times 10^{-3}} \ln \frac{1 - 0.1247(0.76)}{1 - 0.1247(0.825)}$$

$$= 1.65 \times 10^{-5} \text{ kmol/m}^2 \text{s}$$

$$\text{Rate of diffusion of } B = -N_A \times 0.8753$$

$$= -1.65 \times 10^{-5} \times 0.8753$$

$$= -4.07 \times 10^{-6} \text{ kmol/m}^2 \text{s}$$

2.19. Estimate the Knudsen diffusivity of ethylene within a  $100 \text{ \AA}$  pore of a catalyst at 600 K.

Ans:

$$D_K = \frac{d_{\text{pore}}}{3} \sqrt{\frac{8kT}{\pi M_A}} = 4850 d_{\text{pore}} \sqrt{\frac{T}{M_A}}$$

$$T = 600 \text{ K}, \quad d_{\text{pore}} = 100 \text{ \AA},$$

$$M_A = 2 \times 12 + 4 = 28$$

$$\therefore D_K = 4850 (100) \sqrt{\frac{600}{28}} \times (10^{-8}) \text{ cm}^2/\text{s}$$

$\hookrightarrow \text{ \AA to cm}$

$$= 0.0225 \text{ cm}^2/\text{s}$$

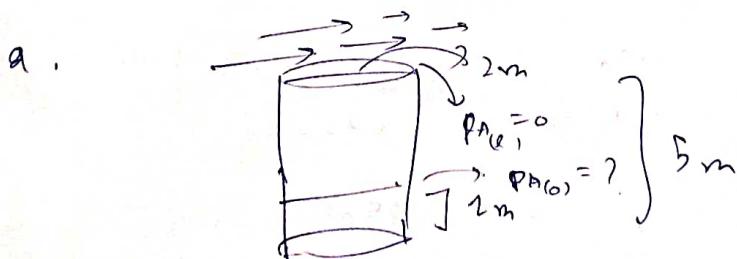
### Section 3

(Assume atmospheric temperature and pressure, and required constant is not mentioned)

- 3.1. A cylindrical tank with a diameter of 2m and total height of 5m contains liquid methanol ( $\text{MeOH}$ , molecular weight 32 g/mol), which is present at the bottom of a tank at a level of 1m, open to the atmosphere. The  $\text{MeOH}$  vapours are quickly dispersed after they leave the tank. The gas space inside is stagnant. At  $30^\circ\text{C}$ , liquid  $\text{MeOH}$  exerts a vapour pressure of 163 mm Hg, and at  $40^\circ\text{C}$ , that pressure rises to 265 mm Hg. The diffusion coefficient of  $\text{MeOH}$  in air is  $1.62 \times 10^{-5} \text{ m}^2/\text{s}$  at  $25^\circ\text{C}$ .

- a. What is the rate of  $\text{MeOH}$  vapour emission from the tank, expressed in kg  $\text{MeOH}/\text{day}$  when the tank is at a temperature of  $30^\circ\text{C}$ ? State all assumptions and boundary conditions.
- b. If the temperature of the tank is raised to  $40^\circ\text{C}$ , what is the new methanol emission rate?

S:



By Fuller's correlation,

$$\frac{D_{AB}|_{30}}{1.62 \times 10^{-5}} = \left( \frac{273 + 30}{273 + 29} \right)^{1.75} = 1.0295$$

$$\therefore D_{AB}|_{30} = 1.62 \times 10^{-5} \times 1.0295 = 1.668 \times 10^{-5} \text{ m}^2/\text{s}$$

We use the steady state equation for  $N_A$  molar

$$N_A = + \frac{D_{AB} P}{\bar{e}} \ln \left( \frac{P - P_{A2}}{P - P_{A0}} \right)$$

$$= 1.668 \times 10^{-5} \times 1.01 \times 10^5 \ln \frac{1.01 - 0}{1.01 - \frac{26.9 \times 10^2 \times 23.6 \times 0.81}{10^5 \times 1}} \xrightarrow{\text{Hg to atm}}$$
$$\times 10^{3/2}$$

$$= 1.668 \times 1.01 \times \ln \frac{1.01}{1.01 - \frac{26.9 \times 10^2 \times 23.6 \times 0.81}{10^5}}$$

$$= 0.7259 \text{ mol} / 0.4085 \text{ mol/m}^2 \text{s}$$

$$a = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 2^2 = \pi = 3.14 \text{ m}^2$$

$$\therefore \frac{dm_A}{dt} = N_A \times a = 0.9085 \times 3.14 \text{ mol/s} = 2.88 \text{ mol/s}$$

$$\therefore \frac{dm}{dt} = \frac{dm_A M}{dt} = 1.28 \times 28 \text{ g/s}$$

$$= \frac{1.28 \times 28}{10^3} \times 24 \times 3600 \text{ kg/day}$$

$$= 3096.58 \text{ kg/day}$$

b. Only factor changing in the above derivation is temperature.

Thus, as

$$D_{AB} \propto T^{2.75}$$

$$\therefore \alpha D_{AB} \frac{P}{\bar{e}} \ln \left( \frac{P - P_{A2}}{P - P_{A0}} \right) \propto T^{2.75}$$

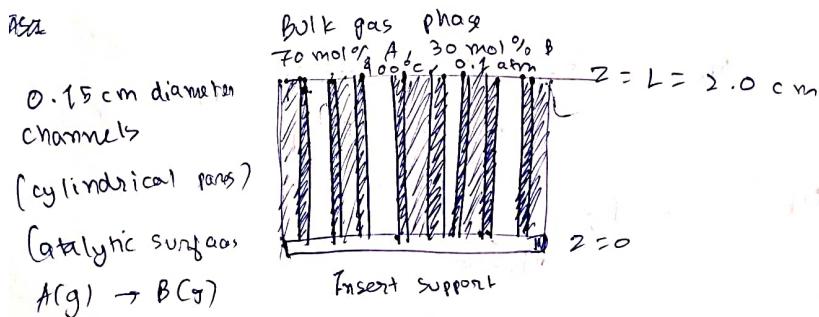
$$\therefore \frac{dm}{dt} \propto T^{2.75}$$

$$\therefore \frac{m_{A0}}{3096.58} = \left( \frac{273 + A_0}{273 + 30} \right)^{2.75}$$

$$= 1.0585$$

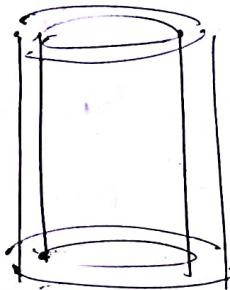
$$\therefore m_{A0} = 3277.63 \times 3096.58 \times 1.0585$$
$$= 3277.63 \text{ kg/day}$$

Q.2. A reactor consists of cylindrical channels which has catalyst coated at its inner walls and a sealed base as shown in the figure. This catalyst promotes isomerisation of species A to species B. The gas phase above the channels contains species A at a constant composition of 70% and rest is species B. Gas phase species A diffuses down a straight channel of diameter = 0.15 cm and length = 2 cm. The production rate of B is diffusion limited. The quiescent gas space in the channel consists of only species A and B.



- State three relevant assumptions for the mass transfer process.
- Based on your assumptions, simplify the general differential equation for the mass transfer of A, leaving the equation in terms of the flux  $J_A$ .
- Using equations for the flux of A in your determined equation, express the general differential equation in terms of the concentration  $c_A$ .
- Specify relevant boundary conditions for the gas phase concentration  $c_A$ .

Ans =



(a) Assumptions:

- (i) Inside cylinder, radial direction,  $\dot{F}_B = 0$ , diffusion only in
- (ii) Steady state,  $B$  non diffusing,
- (iii) Radially uniform diffusion (also axially uniform,  $N_A = f(z)$ )  
 $\dot{F}_B \rightarrow$  direction uniform

~~✓~~

(b) In

the cylindrical coordinates,

$$N_A = -D_{AB} \frac{dP_A}{RTdn} + \left( \frac{N_A + N_B}{P_A + P_B} \right) P_A$$

$$N_B = 0, \quad P_B \mid_{\text{inside}} = 0.$$

$$N_A = -\frac{D_{AB}}{RT} \frac{dP_A}{dn} + \frac{N_A P_A}{P}$$

→ 3D version of Fick's Law

$$\therefore N_A = -D_{AB} \frac{dP_A}{RTdn} + N_A \frac{P_A}{P} \rightarrow \text{by radial symmetry, axial symmetry assumption (iii)}$$

$$\boxed{N_A = -\frac{D_{AB}}{1 - \frac{P_A}{P}} \frac{dP_A}{RTdn}}$$

Diff. eqn. in terms of  $N_A$

(c)

$$P_A = RT C_A$$

$$P = RT C$$

$$\boxed{N_A = -\frac{D_{AB}}{1 - \frac{C_A}{C}} \frac{dC_A}{RTdn}}$$

General  
diffusion law (radial)  
in terms of  $C_A$

(d) Boundary conditions

$$C_A \mid \text{other side of } = C$$

3.3. A vapor mixture of A and B is getting distilled when in contact with the liquid solution of A and B. A being more volatile is getting transferred from liquid to vapor phase whereas B is getting transferred in opposite direction. The molecular weight ratio of A/B is around 2.5. The energy required for condensation of B is giving the required energy for vaporization of A. Both are diffusing through a gas film of 0.15 mm thickness. The temperature of is 372 K and pressure is  $1.01 \times 10^5$  Pa. At this condition, the enthalpy of vaporization of A and B are 900 and 2950 kJ/kg respectively. Develop the flux equation for A vapour.

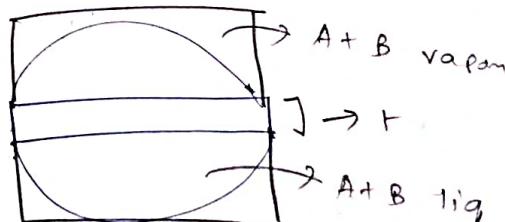
Then develop the flux equation assuming that the components have equimolar heats of vaporization.

State the relevant assumptions along with a diagram showing the system,

Ans=

Assumptions:

- (i) No heat loss
- (ii) ~~perfect exchange~~  
Steady state



Ques

$$N_A = -dC_A$$

$$N_A \Delta H_A M_A = N_B \Delta H_B M_B$$

$\Rightarrow$

$$N_B = N_A \frac{\Delta H_A}{\Delta H_B} \frac{M_A}{M_B}$$

$$= N_A \frac{2950}{900} N_A \frac{900}{2950} (2.5)$$

$$= \frac{45}{49} N_A$$

$$N_A = -n A - (say)$$

Then,

$$N_A = -\frac{D_{AB}}{RT} \frac{dP_A}{dz} + (N_A - n A) \frac{P_A}{P}$$

$$\Rightarrow N_A \left( 1 - \frac{(1-n)P_A}{P} \right) = -\frac{D_{AB}}{RT} \frac{dP_A}{dz}$$

$$\therefore \int_{1 - \frac{(1-n)P_A}{P}} dP_A = \int_{-\frac{RT}{D_{AB}}} N_A dz$$

$$P \ln \left( \frac{P - (1-n)P_{A2}}{P - (1-n)P_{A1}} \right) = -\frac{RT}{D_{AB}} N_A z$$

$$P = 1 \text{ atm}, z = 0.00025$$

$$\therefore P \ln \left( \frac{1 - (1-n)P_{A2}}{1 - (1-n)P_{A1}} \right) = -\frac{RT}{D_{AB}} N_A z$$

$$\downarrow 1.02 \times 10^5 = -\frac{8.314 \times 372 (0.00025)}{D_{AB}}$$

$$N_A$$

$$\therefore N_A = -\frac{D_{AB}}{RT} \frac{P}{z} \ln \left( \frac{1 - (1-n)P_{A2}}{1 - (1-n)P_{A1}} \right)$$

$$= -\frac{D_{AB}}{2.18 \times 10^5} \ln \left( \frac{1 - \frac{9}{19} P_{A2}}{1 - \frac{9}{19} P_{A1}} \right) \text{ in atm}$$

$$(i) \quad N_B \Delta H_B M_B = -N_A \Delta H_A M_A$$

$$\therefore N_B = -N_A \frac{\Delta H_A}{\Delta H_B} \frac{M_A}{M_B} = N_A \frac{900 \times 2.5}{2450} \frac{1}{2}$$

$$= -\frac{25}{19} N_A$$

$$\therefore n = \frac{25}{19}$$

$$\therefore N_A = -\frac{D_{AB}}{2.18 \times 10^5} \ln \left( \frac{1 - \frac{9}{19} P_{A2}}{1 - \frac{9}{19} P_{A1}} \right)$$

$$(ii) \quad \Delta H_B M_B = \Delta H_A M_A$$

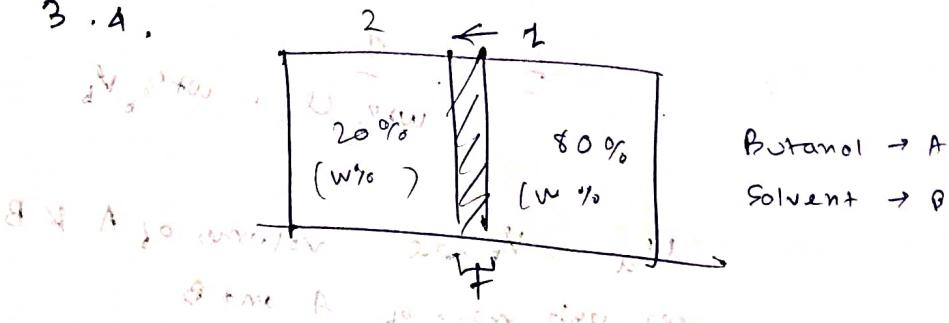
$\therefore N_B = -N_A \rightarrow$  equilibrium case in diffusion

$$\therefore N_A = -\frac{D_{AB}}{RT} \frac{dP_A}{dz} + 0 \Rightarrow N_A = -\frac{D_{AB}}{RT} (P_{A2} - P_{A1})$$

$$\text{Thus, flow equation is } N_A = -\frac{D_{AB}}{2.18 \times 10^5} (P_{A2} - P_{A1}) \rightarrow \text{in atm}$$

3.A. Determine the rate of diffusion of butanol at  $20^{\circ}\text{C}$ ,  
conditions of undisturbed steady state when the  
concentrations of butanol on opposite sides of the film  
are, respectively, 20% and 80% butanol by weight.  
The thickness of the film in water is 0.5 cm. Butanol's  
diffusivity in aqueous solution is  $5.9 \times 10^{-6} \text{ cm}^2/\text{s}$ .  
At  $20^{\circ}\text{C}$ , a solution with 20% butanol has a density  
of 0.971 g/cc, while a solution with 1% butanol has  
a density of 0.992 g/cc.

3.4.



Butanol  $\rightarrow$  A  
Solvent  $\rightarrow$  B

Given,

$$D_{AB} = 1.5 \times 10^{-10} \text{ m}^2/\text{s}$$

$$\text{Total vapor pressure} = 1.5 \times 10^{-3} \text{ m}$$

$$C_A = 20 \% (\text{wt})$$

$$C_B = 80 \% (\text{wt})$$

We need  $C_A$  and  $C_B$  in terms of mol/L.

Given, at  $20^\circ\text{C}$ ,  $\rho_A = 0.992 \text{ g/cc}$

$$A \rightarrow 10 \% (\text{wt}) + 20 \% = 30 \% \text{ (approx)}$$

$$\rho_{10\%} = 0.992 \text{ g/cc}$$

$$A \rightarrow 10 \% (\text{wt}) = A - 20\%$$

$$\rho_{10\%} = 0.972 \text{ g/cc}$$

Now, we can calculate  $m_a$  and  $m_b$ .

$$\frac{\rho_{10\%}}{\rho_{30\%}} = \frac{m_a}{m_a + m_b} = \frac{0.972}{0.992} = \frac{m_a}{m_a + m_b}$$

$$= \frac{m_a}{m_a + m_b} = \frac{1}{1 + \frac{m_b}{m_a}}$$

$$\frac{m_b}{m_a} = \frac{m_a}{m_a + m_b} - 1 = \frac{m_a}{m_a + m_b} - 1 = \frac{m_b}{m_a + m_b}$$

$$\text{Let } \frac{m_a + m_b}{m_a} = t, \text{ then, } m_a = \frac{w\%}{100\%} \cdot m_a$$

$$m_a = w\% \cdot m_a$$

$$= \frac{w\% \cdot m_a}{m_a} = \frac{1}{1 + \frac{m_b}{m_a}}$$

$$\text{Total vapor pressure} = \frac{m_a}{m_a + m_b(m_a)} \cdot \frac{m_b v_b}{m_a + m_b(m_b)}$$

$$\rho = \frac{1}{w\%_a c_a + w\%_b c_b}$$

where,  
c<sub>a</sub> & c<sub>b</sub> are  
specific  
volumes  
of A & B

∴ c<sub>a</sub>, c<sub>b</sub> are volumes of A & B  
per unit mass of A and B

~~∴ for 1.2. specific volumes~~

(constant for a species at same temp).

Thus, we have

$$w\%_a c_a + w\%_b c_b = \frac{1}{\rho}$$

Using this, we have

At A → 1%, B → 99% ~~is mixed~~

$$0.01 c_a + 0.99 c_b = \frac{1}{0.992}$$

At A → 10%, B → 90%

$$0.1 c_a + 0.9 c_b = \frac{1}{0.971}$$

Solving these two, we have

$$c_a = 1.357 \text{ cc/gm}$$

$$c_b = 0.994 \text{ cc/gm}$$

∴ A.

$$\text{Now, } \rho = \frac{m_A}{V} = \frac{m_A}{m_A c_a + m_B c_b} = \frac{m_A}{m_A + m_B}$$

$$= \frac{m_A}{m_A c_a + m_B c_b} \quad \text{(for } m_A \text{ unit total mass)}$$

~~unit~~

$$\rho = \frac{m_A (w\%_a c_a + w\%_b c_b)}{m_A + m_B} \rightarrow \text{for unit total mass}$$

(i)  $A \rightarrow 20\%$ ,  $B \rightarrow 80\%$

$$M_A = M_{\text{butanol}} = 74 \text{ g/mol}, M_B = M_{\text{water}} = 18 \text{ g}$$

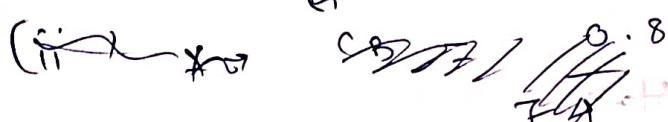
$$C_A = 0.2$$

$$74(0.2 \times 1.357 + 0.8 \times 0.991)$$

$$= 2.53 \times 10^{-3} \text{ mol/cc}$$

$$= 2.53 \text{ mol/L}$$

concentration of  $\times 0.2$  is



$$C_B = \frac{0.8}{18(0.2 \times 1.357 + 0.8 \times 0.991)}$$

$$= 41.67 \text{ mol/L}$$

Ans

(ii)  $A \rightarrow 80\%$ ,  $B \rightarrow 20\%$

$$C_A = \frac{0.8}{74(0.2 \times 1.357 + 0.8 \times 0.991)}$$

$$= 10.19 \text{ mol/L}$$

$$C_B = \frac{0.2}{18(0.2 \times 1.357 + 0.8 \times 0.991)}$$

$$= 10.41 \text{ mol/L}$$

Ans

$$\text{Rate} = -D_{AB} \frac{C_A}{l} \ln \left( \frac{C_A}{C_A + C_B} \right)$$

(A)  $\times 10^{-9} \rightarrow \text{Fick's law} \times 10^{-9}$

$$J_A = \frac{D_{AB} \cdot 10^{-10}}{0.005} \times 32.375 \ln \left( \frac{C_B}{C_A} \right) \times 10^{-3}$$

$\downarrow$  mean taken

$$J_A \text{ from F.D. } =$$

$$= 6 \times 5.3 \times 10^{-9} \text{ mol/m}^2\text{s}$$

$\rightarrow$  Steady state

$$(A) \times 10^{-9} + 6 \times 10^{-9} \times 10^{-3} = 8.9$$

$$J_A = -D_{AB} \frac{dC_A}{dx} + \frac{C_A N_A}{C_A + N_B}$$

$$\therefore N_A = -D_{AB} \frac{dC_A}{dx} - \frac{C_A N_A}{C_A + N_B}$$

$$N_A = -D_{AB} \frac{dC_A}{dx} \ln \left( \frac{C_A}{C_A + C_B} \right)$$

$$J_A = -D_{AB} \frac{dC_A}{dx} \ln \left( \frac{C_A}{C_A + C_B} \right)$$

$$J_A = 5.3 \times 10^{-9} \text{ mol/m}^2\text{s}$$

$$J_A \text{ from F.D. } =$$

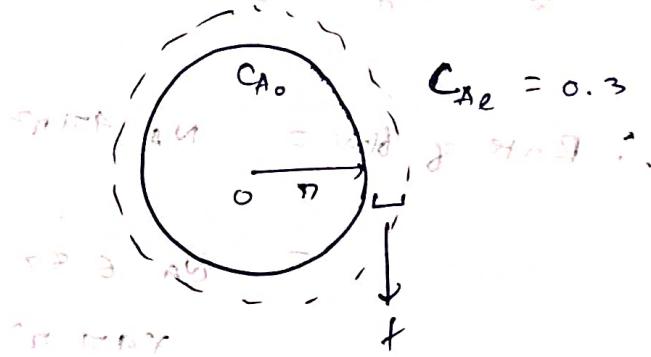
3.5.

An absorbent sphere made up of a basic oxide is suspended in a very large degree volume of a mixture of 70% air and 30% carbon dioxide at a temperature of  $40^{\circ}\text{C}$  and a total pressure of 1 atm. The gas is vigorously agitated, and the diffusion of carbon dioxide to the surface of the absorbent takes place through a stationary gas layer that is 3.5 mm thick and encircles the sphere. The thickness of the gas is far less than the sphere's radius. Because carbon dioxide is quickly absorbed at the surface of the sphere, the amount of gas that is present is almost negligible. Determine the rate of carbon dioxide flow as well as the velocity of its diffusion in the middle of the gas layer. The diffusivity of carbon dioxide in air is calculated to be  $1.62 \times 10^{-5} \text{ m}^2/\text{s}$  at  $25^{\circ}\text{C}$  and 1 atm.

Ans

3.5.

(A)



$$P_{AB} \Big|_{40^\circ\text{C}} = D_{AB} \Big|_{25^\circ\text{C}} \times \left( \frac{273 + 40}{273 + 25} \right)^{1.75}$$

At 1 atm FCF 8.0

(by Fuller correlation)

$$\begin{aligned} & \text{at } 40^\circ\text{C} \text{ and } 1 \text{ atm} \\ & D_{AB} = 2.62 \times 10^{-5} \times \left( \frac{323}{298} \right)^{1.75} \\ & = 1.77 \times 10^{-5} \end{aligned}$$

$$N_A = - \frac{D_{AB}}{r_A} \frac{dP_A}{dr} + N_A \frac{P_A}{P}$$

$$N_A = - \frac{D_{AB}}{r_A} \frac{dP_A}{dr} \quad \text{at } P = 1 \text{ atm}$$

neglecting  $\frac{dP_A}{dr}$

$$N_A = - \frac{D_{AB}}{r_A} \frac{dP_A}{dr} \quad \text{at } P = 1 \text{ atm}$$

$$N_A = - \frac{D_{AB}}{r_A} \ln \left( \frac{P - P_{A_e}}{P - P_{A_0}} \right) \quad \text{at } P = 1 \text{ atm}$$

$$= -1.77 \times 10^{-5} \ln \left( \frac{1 - 0.3}{1 - P_{A_e}} \right) \times \frac{1.01 \times 10^5}{0.0035} \quad \text{in atm}$$

$$= -6.8 \times 10^{-9} \ln \left( \frac{1 - P_{A_e}}{0.7} \right) \times \frac{1.01 \times 10^8}{3.5}$$

Or  $N_A$  rate of flow C. A. =

$$= N_A a =$$

At suppose  $P_{A_0} = 0$  (no  $\text{CO}_2$  in sphere)

$$\therefore N_A = 6.8 \times 10^{-9} \ln\left(\frac{1}{0.7}\right) \times \frac{1.02}{3.5} \times 10^8$$

~~Rate of flow =  $N_A A \pi r^2$~~

$$= N_A 6.8 \times 10^{-9} \ln \frac{1}{0.7} \times \frac{1.02}{3.5} \times 10^8 \times \pi r^2$$

$$= 0.8797 r^2 \text{ mol/s}$$

(continued next page)

$$(b') \text{ Rate of flow} = \frac{\text{Flux at } n + \frac{1}{2}}{\text{Concentration at } n + \frac{1}{2}}$$

$$= 6.8 \times 10^{-9} \ln\left(\frac{1}{0.7}\right) \times \frac{1.02}{3.5} \times 10^8$$

$$= \frac{P_{A,n+\frac{1}{2}}}{P_{T_{total}}} = \frac{P_{A,n+\frac{1}{2}}}{P_{T_{total}}}$$

$$P_{A,n+\frac{1}{2}} = P_{T_{total}} \left( \frac{P - P_r}{P - P_o} \right)^{\frac{1}{2}}$$

$$= \left( 1 - (1-0) \left( \frac{1-0.3}{1-0} \right)^{\frac{1}{2}} \right) \rightarrow \text{Concentration profile in steady state}$$

$$= 0.1633 \times 2.01 \times 10^5 \text{ Pa}$$

$$\therefore r = \frac{6.8 \times 10^{-9} \ln\left(\frac{1}{0.7}\right) \times \frac{1.02}{3.5} \times 10^8}{0.1633 \times 2.01 \times 10^5} \text{ m/s}$$

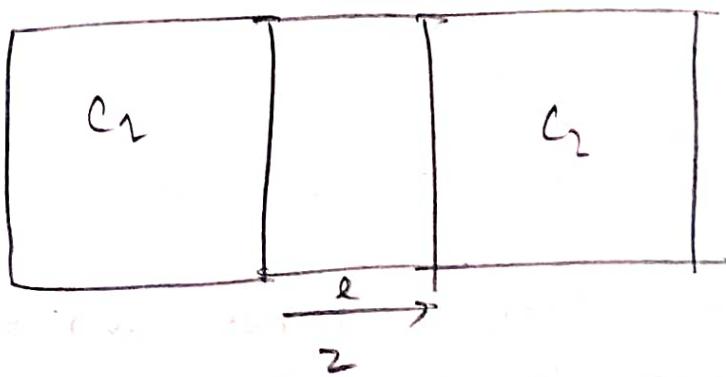
$$= 4.21 \times 10^{-6} \text{ m/s}$$

$$\text{Am} = 0.8797 r^2 \text{ mol/s}$$

$$(b) 4.21 \times 10^{-6} \text{ m/s}$$

3.6 At  $25^\circ\text{C}$ , oxygen diffuses through a 1 mm-thick layer of stagnant water. On two sides of the film, the concentrations of dissolved oxygen are  $0.014 \text{ kg/m}^3$  and  $0.008 \text{ kg/m}^3$ , respectively. Calculate the oxygen flux while ignoring bulk flow and the concentration profile as a function of the position  $z$  within the liquid film.

Ans =



$$N_A = \frac{\partial dA}{dz} - D_{AB} \frac{dC_A}{dz} + \frac{N_A}{C} C_A$$

$$\therefore N_A = -D_{AB} C \ln \frac{C - C_{A2}}{C - C_{A1}}$$

$$= -\frac{D_{AB} C}{z} \ln \frac{C - C_{A2}}{C - C_{A1}}$$

$$\therefore \left( \frac{C - C_{A2}}{C - C_{A1}} \right)^{\frac{1}{z}} = \left( \frac{C - C_{A2}}{C - C_{A1}} \right)^{\frac{1}{L}}$$

$$\therefore C_{A2} = C - (C - C_{A1}) \left( \frac{C - C_{A2}}{C - C_{A1}} \right)^{\frac{z}{L}}$$

3.6 (contd.)

Thus, putting values, and using

$$C_A = \frac{m_A}{M_A V} = \left( \frac{1000}{M_A} \right) \cdot \frac{1}{V}$$

We have

$$C_A|_2 = 0.0146 = 4.375 \times 10^{-3}$$
$$\frac{32 \times 10^3}{40} = 0.4375 \text{ mol/m}^3$$
$$= 0.4375 \times 10^{-3} \text{ mol/l}$$

$$C_{A_{\text{ex}}}|_2 = \frac{0.008}{32 \times 10^{-3}} = 0.25 \text{ mol/m}^3$$
$$= 0.25 \times 10^{-3} \text{ mol/l}$$

Total conc. ( $C$ ) &  $D_{AB}$  are not given,  
So we keep the expression in terms of those.

Thus, we have

(i) Flux of oxygen

$$N_A = -D_{AB} C \frac{\ln \left( \frac{C - 0.4375 \times 10^{-3}}{C - 0.25 \times 10^{-3}} \right)}{0.002}$$

$$= -D_{AB} C \times 10^3 \ln \left( \frac{C - 0.0004375}{C - 0.00025} \right)$$

(ii) Cone profile (Subnormal)  $\beta = \delta$   
Base radius  $b = 0.0004375$

$$C_{A2} = C - \frac{(C - 0.0004375)}{0.0004375} \left( \frac{C - 0.00025}{C - 0.0004375} \right)^{\frac{z}{0.002}}$$

(Calculated from  $z=0$  at  $C=0.0004375$ )

$$\text{Standard F.A.} = \frac{1}{A+0.0004375} = \frac{1}{A}$$

$$\text{Condition of F.A.} = \frac{1}{A+0.00025} = \frac{1}{A}$$

3.7. A test tube, 5cm in diameter and 50cm long, has 5g camphor in it. How long will it take for the camphor to completely sublime into vapour leaving no residue? The pressure is atmospheric, and the temperature is  $20^\circ\text{C}$ . The sublimation pressure of camphor at this temperature is 97.5mm Hg, and the diffusivity of camphor is  $6.5 \times 10^{-6} \text{ m}^2/\text{s}$ .

Ans:



$$N_A = \frac{D_{AB} P}{RT l} \ln \left( \frac{P - P_{A_2}}{P - P_{A_1}} \right)$$

(Assuming  $P_{A_2} = 0$ ,  $P_{A_1} = 97.5 \text{ mm Hg}$

$$l = 760 \text{ mm Hg} = 1.01 \times 10^5 \text{ Pa}$$

$$N_A \alpha M = \frac{dm}{dt}$$

$$\therefore t = \frac{m}{N_A \alpha M}$$

$$M = 152.23 \text{ g/mol} = 1.52 \times 10^{-1} \text{ kg/mol}$$

$$\alpha = \pi \frac{0.05^2}{4} = 1.96 \times 10^{-3}$$

$$m = 5g = 0.005 \text{ kg}$$

$$N_A = \frac{6.5 \times 10^{-6} \times 1.02 \times 10^5 \ln\left(\frac{760}{760 - 97.5}\right)}{8.314 \times 293 \times 0.5}$$

$$= 7.4 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\therefore t = \frac{0.005}{7.4 \times 10^{-5} \times 1.96 \times 10^{-3} \times 0.152}$$

$$= 226797.75$$

$$= 6244.63 \text{ h}$$

Ans = 63 hours.