Consideration of internal torque balance in *Drift of elastic hinges in quasi-two-dimensional oscillating shear flows*[*]

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Equation 15

For Equation (15),

$$T_1 - T_2 + 2\hat{\kappa}(\alpha - \alpha_0) = 0, \tag{15}$$

the appearance of a negative sign in front of the T_2 term (as opposed to that in Equation 12 where the T_2 term is positive) is because of the additional consideration of the internal torque balance at the hinge point O arising from the elasticity of the hinge. The fluid exerts torques T_1 and T_2 on the upper and lower arms respectively, about O. However, elastic hinges also have a restoring torque $\tau = \hat{\kappa}(\alpha - \alpha_0)$, where α is the hinge angle and α_0 is the equilibrium angle. The two arms interact through the hinge, producing internal torques:

- The upper arm exerts a torque $-\tau$ (in the clockwise direction) on the lower arm.
- The lower arm exerts an equal and opposite (counterclockwise) torque τ on the upper arm.

The torques on each individual arm must also balance so the torque balance equations for the upper arm and lower arm are:

- $T_1 + \tau = 0$
- $T_2 - \tau = 0$.

We then get:

$$T_1 - T_2 = -2\tau \Rightarrow T_1 - T_2 + 2\hat{\kappa}(\alpha - \alpha_0) = 0. \tag{15}$$

[*]Numbering of equations and sections is consistent with that in the paper