# SECOND EDITION

# FUNDAMENTALS OF GLAGIER DYNAMICS

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C.J. VAN DER VEEN



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CRC Press Taylor & Francis Group 6000 Broken Sound Parkway NW, Suite 300 Boca Raton, FL 33487-2742

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International Standard Book Number-13: 978-1-4398-3567-8 (eBook - PDF)

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# Contents

Preface			1X
Preface to the	he Firs	t Edition	xi
About the A	uthor		xiii
Chapter 1	Mathematical Tools		
	1.1	Vectors and Tensors	
	1.2	Stress and Strain	
	1.3	Error Analysis	
	1.4	Parametric Uncertainty Analysis	
	1.5	Calculating Strain Rates	20
CI 4 2			25
Chapter 2	Ice L	Deformation	25
	2.1	Creep of Glacier Ice	25
	2.2	Constitutive Relation	
	2.3	More about the Constitutive Relation	
	2.4	Fabric Effects in Glacier Ice	
	2.5	Creep in Axially Symmetric Ice	41
Chapter 3	Mechanics of Glacier Flow		
	3.1	Force Balance	45
	3.2	Interpreting Force Balance	51
	3.3	The Force-Budget Technique	55
	3.4	Bridging Effects	57
	3.5	Stokes Equation Applied to Glacier Flow	59
	3.6	Creep Closure of Englacial Tunnels	
Chapter 4	Modeling Glacier Flow		
	4.1	Introduction	
	4.2	Lamellar Flow	
	4.3	Including Lateral Drag	
	4.4	Glacier Flow Controlled by Lateral Drag	
	4.5	Ice-Shelf Spreading	89
	4.6	Along-Flow Variations in Glacier Flow	95
	47	Flow Near an Ice Divide	107

vi Contents

Chapter 5	Equilibrium Profiles of Glaciers		
	5.1	Perfect Plasticity	113
	5.2	Continuity Equation	
	5.3	Steady-State Profiles along a Flowline	
	5.4	Steady-State Profile of an Axisymmetric Ice Sheet	
	5.5	Steady-State Profile of a Free-Floating Ice Shelf	
	5.6	Flow Controlled by Lateral Drag	
Chapter 6	Glacier Thermodynamics		
	6.1 Conservation of Energy		
	6.2	Steady-State Temperature Profiles	
	6.3	Effect of Horizontal Heat Advection	
	6.4	Thermal Response of a Glacier to Changes in Climate	
	6.5	Radiation Balance at the Surface of a Glacier	
	6.6	Turbulent Heat Fluxes	
	6.7	Physical Properties of Firn	
	6.8	Calculated Near-Surface Snow Temperatures at South	107
	0.0	Pole Station	173
Chapter 7	Subglacial Processes		
	7.1	Introductory Concepts	181
	7.2	Sliding with Cavitation	
	7.3	Glacier Flow over a Soft Bed	
	7.4	Subglacial Hydraulics	
	7.5	Tunnel Drainage	
Chapter 8	Fractures		215
	8.1	Surface Crevasses	215
	8.2	Fracture Mechanics	
	8.3	Two-Dimensional Crevasse Propagation	
	8.4	Basal Crevasses	
	8.5	Iceberg Calving	
Chapter 9	Numerical Ice-Sheet Models		
	9.1	Introductory Remarks	257
	9.2	Numerical Methods	
	9.3	Model Driven by Shear Stress Only	
	9.4	Flowband Model	
	9.5	Calculating the Temperature Field	
	9.6	Geodynamics	
	9.7	Ice-Shelf Models	

Contents

11.4

Response to Changes in Surface Mass Balance......301 10.1 10.2 10.3 10.4 10.5 Introductory Remarks......341 11.1 11.2 Estimating the Role of Gradients in Longitudinal Stress..... 352 11.3

References 363
Index 383

Estimating Resistance from Lateral Drag......358

vii

## **Preface**

In the 14 years since publication of *Fundamentals of Glacier Dynamics*, the field of glaciology has changed significantly. Satellite measurements and other remotesensing techniques afford monitoring of glaciers and ice sheets on unprecedented spatial and temporal scales and have revealed large and rapid changes occurring in both Greenland and Antarctica. Identifying the processes responsible for these changes is paramount to understanding the interactions between ice sheets and the climate system and their future contribution to global sea level rise. The primary goal of this revised edition is to provide a theoretical framework for quantitatively interpreting observed glacier changes and for developing models of glacier flow. This framework is based on the partitioning of full stresses into a lithostatic component associated with gravitational action and resistive stresses that oppose this action. In my experience, this provides a more physically intuitive discussion of force budget than the more traditional approach involving deviatoric stresses.

In an effort to streamline the presentation and focus in this edition of *Fundamentals* of Glacier Dynamics, the more descriptive sections, including much of the last four chapters that appeared in the first edition, have been eliminated. This is not to imply that observations are of secondary importance—quite the contrary. However, many sources are available that provide excellent reviews of glacier observations, and a repeat would be redundant. The introductory Chapter 1 reviews the most important mathematical tools used throughout the remainder of the book. The new Chapter 8 discusses fracture mechanics and iceberg calving. Applications of the force-budget technique using measurements of surface velocity to locate mechanical controls on glacier flow are now consolidated into Chapter 11.

Many colleagues contributed directly and indirectly to the completion of this second edition. Special thanks are due to Leigh Stearns for reading through the entire draft manuscript and providing valuable comments and suggestions, and to Jeremy Bassis for his comments on several chapters. I am indebted to others who commented on individual chapters: Ed Bueler, Richard Hindmarsh, Faezeh Nick, Christian Schoof, and Dirk van As. Of course, I take full responsibility for the final text. Throughout my career as a glaciologist, my research has been funded mostly by the Office of Polar Programs of the National Science Foundation and by the Cryospheric Sciences Program of the National Aeronautics and Space Administration; this support is gratefully acknowledged.

C. J. van der Veen

## Preface to the First Edition

This book is not intended to serve as a ski lift, transporting the reader effortlessly to the peaks of glaciological knowledge. Instead, it is designed to act as a trail guide, helping the reader achieve the basic level of understanding required to describe and model the flow and dynamics of glaciers. The emphasis is more on developing and outlining procedures than on providing a complete overview of all aspects of glacier dynamics. To this end, derivations leading to frequently used equations are presented step by step to allow the reader to grasp the mathematical details as well as physical approximations involved without having to consult the original works. While going through these derivations may be tedious, the reward is that in the end, the reader will have gained the understanding needed to apply similar techniques to somewhat different applications.

The choice of material presented in this book is, not surprisingly, based to a large extent on research that I have conducted over the past 15 years, first under the watchful eye of Hans Oerlemans at the University of Utrecht, and later in collaboration with Ian Whillans and others at the Byrd Polar Research Center. No claim is made here to present a complete and exhaustive review of the glaciological literature. In many instances, topics are discussed based on one or two key papers that I found very helpful and that may provide a good starting point for readers interested in a more in-depth review of the literature. While an effort was made to keep the entire text up to date, the fact that this book was written over a five-year period necessarily implies that some recent work and developments may have been overlooked. My apologies to those authors who feel slighted. The alternative would have been to continue updating the text, but in all likelihood, this would have postponed publication indefinitely.

The discussion of glacier dynamics is based on the force-budget technique. There is nothing magical or special about this technique. It is based on a different approach to solving Newton's second law of motion, which states that forces acting on a section of glacier must sum to zero (accelerations being negligible in glacier flow). Full stresses are partitioned into a lithostatic component associated with the action of gravity and resistive stresses that oppose this action. The main advantage of this approach is the distinction that is made between various physical processes controlling glacier flow. Consequently, the implications of omitting one or more terms in the balance equations becomes more clear than when the usual balance equations involving deviatoric stresses are considered. As should be the case, the final results are the same, whichever approach is used.

One of the most important steps in developing models to describe glacier flow is to determine which of the terms in the force-balance equations are important and should be retained. One procedure is to introduce scaling parameters and derive expressions for first- and higher-order solutions. Another approach, adopted here, is to use measurements of glacier speed and geometry to evaluate the role of various resistances to flow on a particular glacier. Based on these results, the balance

equations can be simplified and a model for that particular glacier formulated. In either case, it is important for the modeler to keep the model limitations in mind when discussing predicted ice-sheet behavior. Equally important is to verify model results against available data. Because of the importance of these issues, evaluating the budget of forces using glacier measurements is discussed extensively in this book.

The organization of this book is fairly traditional, starting with developing the elementary tools, and ending with models of the Greenland and Antarctic ice sheets. Chapter 1 presents the obligatory overview of ice in the climate system. The essential theory needed to develop glacier models is discussed in Chapters 2–4. Simple analytical solutions are derived in Chapters 5 and 6, while Chapter 7 focuses on the thermodynamic aspects of glacier flow and the interactions between temperature and climate forcing. In most instances, numerical methods have to be used to solve the equations governing glacier flow. An overview of commonly used time-evolving models is given in Chapter 8. Some general aspects of glacier dynamics and discussion of important feedback mechanisms can be found in Chapter 9. The final three chapters discuss applications specific to smaller mountain glaciers, the Greenland Ice Sheet, and the Antarctic Ice Sheet, respectively. In the applications in later chapters, the important starting equations and underlying assumptions are summarized so that readers not interested in the details need not go through most of the earlier chapters.

The majority of my research over the past decade or so has been funded by the Office of Polar Programs of the National Science Foundation; this support is gratefully acknowledged. Further, I would like to thank my colleagues, and in particular Ian Whillans, for discussions and mutual criticisms, which have helped me in obtaining a better understanding of the topics discussed here.

C. J. van der Veen *May 13, 1999* 

# About the Author

**C. J. van der Veen** is a glaciologist interested in the dynamics of fast-moving glaciers and ice streams. His research focuses on using measurements of ice velocity and glacier geometry to identify mechanical controls on glaciers and how changes in these controls affect glacier flow and stability. After completing his PhD at the University of Utrecht in the Netherlands, he spent 20 years at The Ohio State University as a research scientist with the Byrd Polar Research Center (1986–2006). He is currently a professor in the Geography Department and research scientist with the Center for Remote Sensing of Ice Sheets at the University of Kansas.