Homework 3

Q1

1. $p \to q$
2. $q \to p$
3. $\neg p \to q$
4. $q \to p$

Q2

2.1

$\text{rk}(p \lor q \to \neg r) = 2$

$\text{rk}(p_1 \land p_2 \land \cdots \land p_n) = n - 1$

$\text{rk}(A \to (B \to A)) \geq 5$

2.2

rank直观上展示了需要进行逻辑运算的次数，表示了公式的结构复杂程度和计算深度；秩越大，则公式更加复杂，需要进行计算的次数越多。

2.3

1. $\text{len}(A) = 1$
2. $\text{len}(\neg B) = \text{len}(B) + 1$
3. $\text{len}(A \land B) = \text{len}(A) + \text{len}(B) + 1$
4. $\text{len}(A \lor B) = \text{len}(A) + \text{len}(B) + 1$
5. $\text{len}(A \to B) = \text{len}(A) + \text{len}(B) + 1$

Q3

3.1

真值表法:

A	B	C	¬A	¬A → (A → (B → C))	A → (B → C)	B → C
1	1	1	0	1	1	1
1	1	0	0	1	0	0
1	0	1	0	1	1	1
1	0	0	0	1	1	1
0	1	1	1	1	1	1
0	1	0	1	1	1	0
0	0	1	1	1	1	1
0	0	0	1	1	1	1

归谬法:

$\neg A \rightarrow (A \rightarrow (B \rightarrow C))$

1   0   0

0   1   0   (矛盾)

树形图法:

3.2

真值表法:

A	B	C	B ∧ C	A ↔ (B ∧ C)	(A ↔ (B ∧ C)) → (A ↔ B) ∨ (A ↔ C)	(A ↔ B) ∨ (A ↔ C)	A ↔ B	A ↔ C
1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	0
1	0	1	0	0	1	1	0	1
1	0	0	0	0	1	0	0	0
0	1	1	1	0	1	0	0	0
0	1	0	0	1	1	1	0	1
0	0	1	0	1	1	1	1	0
0	0	0	0	1	1	1	1	1

归谬法:

$(A \leftrightarrow (B \land C)) \rightarrow (A \leftrightarrow B) \lor (A \leftrightarrow C)$

    1   0   0

                0   0

1   1   1   1   0   1   0   (情况1, 矛盾)

0   0   0   1   0   1   (情况2, 矛盾)

树形图法:

$$\neg\Big(\big(A \leftrightarrow (B \wedge C)\big) \to (A \leftrightarrow B) \vee (A \leftrightarrow C)\Big)$$

$$|$$

$$A \leftrightarrow (B \wedge C) \quad \checkmark$$

$$\neg\big((A \leftrightarrow B) \vee (A \leftrightarrow C)\big) \quad \checkmark$$

$$|$$

$$\neg (A \leftrightarrow B)$$
$$\neg (A \leftrightarrow C)$$

$$\neg A \qquad\qquad A$$
$$\neg (B \wedge C) \qquad B \wedge C$$
$$\qquad\qquad\qquad |$$
$$\qquad\qquad\qquad B$$
$$\neg B \qquad \neg C \qquad C$$

$$\neg A \quad A \quad \neg A \quad \neg C \quad \neg A \qquad A$$
$$B \quad \neg B \quad C \quad A \quad B \quad \neg B$$
$$X \qquad X \quad X \quad X \quad X \qquad X$$