Herleitung des ATE + Bias

Notation

$$p = \text{the proportion treated (i.e., the proportion of cases } D = 1\text{)},$$
$$q = \text{the proportion untreated (i.e., the proportion of cases } D = 0\text{)},$$

$$E(Y^1_{D=1}) = E(Y^1 \mid D = 1),$$
$$E(Y^0_{D=1}) = E(Y^0 \mid D = 1),$$
$$E(Y^1_{D=0}) = E(Y^1 \mid D = 0),$$
$$E(Y^0_{D=0}) = E(Y^0 \mid D = 0).$$

Law of iterative Expectation

- Erwartungswert als gewichtete Summe von Erwartungswerten

$$
\begin{aligned}
ATE &= E(Y^1 - Y^0) \\
&= E(Y^1_{D=1})p + E(Y^1_{D=0})q - E(Y^0_{D=1})p - E(Y^0_{D=0})q
\end{aligned}
$$

Weitere Umformung

$$E(Y^1_{D=1})p = E(Y^1_{D=1}) - E(Y^1_{D=1})q$$

- Führt zu:
- eine Variable weniger

$$=E\left(Y^1_{D=1}\right) - E\left(Y^1_{D=1}\right)q + E\left(Y^1_{D=0}\right)q - E\left(Y^0_{D=1}\right) + E\left(Y^0_{D=1}\right)q - E\left(Y^0_{D=0}\right)q$$

HTE-Bias

- Betrachtung der E(...) mit dem Faktor q

$$
\begin{aligned}
HTE &= -E(Y^1_{D=1})q + E(Y^0_{D=1})q + E(Y^1_{D=0})q - E(Y^0_{D=0})q \\
&= -q[+E(Y^1_{D=1}) - E(Y^0_{D=1}) - E(Y^1_{D=0}) + E(Y^0_{D=0})] \\
&= -q[(E(Y^1_{D=1}) - E(Y^0_{D=1})) - (E(Y^1_{D=0}) - E(Y^0_{D=0}))]
\end{aligned}
$$

SDO und Selection Bias

$$= E(Y^1_{D=1}) - E(Y^0_{D=1})$$

- Wir erweitern die Formel durch:

$$E(Y^0_{D=1}) = E(Y^0_{D=0}) + (E(Y^0_{D=1}) - E(Y^0_{D=0}))$$

- Wir Erhalten SDO – Selection Bias:

$$= E(Y^1_{D=1}) - E(Y^0_{D=0}) - (E(Y^0_{D=1}) - E(Y^0_{D=0}))$$

Zusammenfügen

- ATE = SDO – Selection Bias – HTE

$$=E\left(Y^1_{D=1}\right)-E\left(Y^0_{D=0}\right)-\left[E\left(Y^0_{D=1}\right)-E\left(Y^0_{D=0}\right)\right]-(TT-TUT)q,$$

Literatur

Vorlesung Causal Inference and Digital Causality Lab, Prof. Dr. Spindler.

Xie, Y. (2011). Causal inference and heterogeneity bias in social science. Information, knowledge, systems management, 10(1–4), 279–289.s https://doi.org/10.3233/iks-2012-0197