

# Homework 1

ANN and Backpropagation

Jiaming Zhou

G28650757

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# Problem 1

(a):

Input:  $x_i \in \{x_1 = 0, x_2 = 1, x_0 = b = 1\}$

Net Input:  $z = b w_{hib} + x_1 w_{hix1} + x_2 w_{hix2}, i = 1, 2$

Active Function:  $g(z) = \frac{1}{1+e^{-z}}$

Node  $h_1$ :

$$z_1 = 1*1.5 + 0*2.5 + 1*1 = 2.5$$

$$g(z_1) = \frac{1}{1+e^{-2.5}} \approx 0.92$$

output value at node  $h_1$  is about 0.92.

Node  $h_2$ :

$$z_2 = 1*2 + 0*(-1.5) + 1*(-3) = -1$$

$$g(z_2) = \frac{1}{1+e^1} \approx 0.27$$

o=output value at node  $h_2$  is about 0.27.

(b):

node  $\hat{y}$ :

$$\text{net input: } z = 1*(-1) + 1*0.92 + 0.5*0.27 = 0.055$$

$$\text{output} = \frac{1}{1+e^{0.055}} \approx 0.49$$

$$y = 1, O = 0.49$$

derivatives of backpropagation algorithm:

$$\text{error part: } \frac{\partial E}{\partial} = (y - O)$$

$$\text{active function part: } \frac{\partial g}{\partial z} = g(z)(1 - g(z))$$

Dot product part:  $\{x_i\}$

For hidden layer weights:

$$\begin{aligned} \text{gradient } k &= (y - O) * g(z)(1 - g(z)) * \{h_i\} \\ &\approx [0.127, 0.117, 0.034]^T \end{aligned}$$

$$\text{new weights: } w_{\hat{y}h_i}' = w_{\hat{y}h_i} - \alpha * \text{gradient} =$$

$$w_{\hat{y}h_i}' = [-1, 1, 0.5]^T - 0.1 * [0.127, 0.117, 0.034]^T$$

$$w_{\hat{y}h_i}' = [-1.0127, 0.9883, 0.4966]^T$$

for initial weights:

$$\text{gradient } k_{h1} = g(0.117)(1-g(0.117)) * \{x_i\} \approx [0.249, 0, 0.249]^T$$

$$\text{gradient } k_{h2} = g(0.034)(1-g(0.034)) * \{x_i\} \approx [0.250, 0, 0.250]^T$$

$$\text{new weights to node } h_1: w_{h_1x_i}' = [1.5, 2.5, 1]^T - 0.1 * [0.249, 0, 0.249]^T$$

$$w_{h_1x_i}' = [1.4751, 2.5, 0.9751]^T$$

$$\text{new weights to node } h_2: w_{h_2x_i}' = [1.975, 1.5, -3.025]^T$$