I start by splitting the data into a training set and a testing set. The training set consists of 70% of randomly sampled data and the testing set consists of the remaining 30%.

I start the linear regression by removing the variables "density" from the list of variables for the linear regression model, because of its high absolute correlation with residual sugar and alcohol. Since I am dealing with a regression problem, I am going to use the mean squared error (MSE) as a measure of how much our predictions are far away from the real data. My out-of-sample MSE is 0.544 and in-sample MSE is 0.567 with this model.

```
call:
lm(formula = quality ~ . - density, data = train)
Residuals:
             1Q
                 Median
    Min
                              3Q
                                     Max
-3.8156 - 0.4935 - 0.0341
                          0.4643
                                  2.7363
Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
                                  0.013055
                                           450.677
(Intercept)
                       5.883414
                                                       2e-16
                                  0.015319
                                             -2.777
                                                    0.005517 **
fixed.acidity
                      -0.042541
volatile.acidity
                      -0.192080
                                  0.013794
                                            -13.925
                                                     < 2e-16
                      -0.009656
                                  0.014186
                                             -0.681 0.496115
citric.acid
residual.sugar
                       0.142881
                                              9.152
                                  0.015613
                                                     < 2e-16
                      -0.007904
                                  0.013977
                                             -0.566 0.571739
chlorides
                                  0.017067
free.sulfur.dioxide
                                                              ***
                       0.070860
                                              4.152
                                                    3.38e-05
                                             -1.489 0.136486
total.sulfur.dioxide -0.028744
                                  0.019300
                                  0.015084
                       0.026640
                                              1.766 0.077466
рН
                                                             ***
sulphates
                       0.045114
                                  0.013350
                                              3.379 0.000735
                                                     < 2e-16 ***
alcohol
                       0.460309
                                  0.016569
                                             27.781
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.7642 on 3418 degrees of freedom
                               Adjusted R-squared:
Multiple R-squared: 0.2687,
                                                     0.2666
F-statistic: 125.6 on 10 and 3418 DF, p-value: < 2.2e-16
```

Confidence intervals:

```
2.5 % 97.5 %
(Intercept)
fixed.acidity
volatile.acidity
                          5.858
                                  5.909
                         -0.073
                                 -0.013
                         -0.219
                                 -0.165
citric.acid
                         -0.037
                                   0.018
residual.sugar
                          0.112
                                   0.173
chlorides
                         -0.035
                                   0.019
free.sulfur.dioxide
                          0.037
                                  0.104
total.sulfur.dioxide
                         -0.067
                                   0.009
рН
                         -0.003
                                   0.056
sulphates
                          0.019
                                   0.071
alcohol
                          0.428
                                   0.493
```

With the KNN regression, the k that minimizes the out-of-sample error rate is 8, with an error of 0.506. My in-sample MSE is 0.393.