Circuit analysis methods

Circuit Theory and Electronics Fundamentals

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing two voltage and current sources connected to various resistors. The two current sources (one dependent, I_b, and one independent, I_d), and two voltage sources (one dependent, V_c, and one independent, V_a) connect to 7 different resistors (R_{1-7}). The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 5.

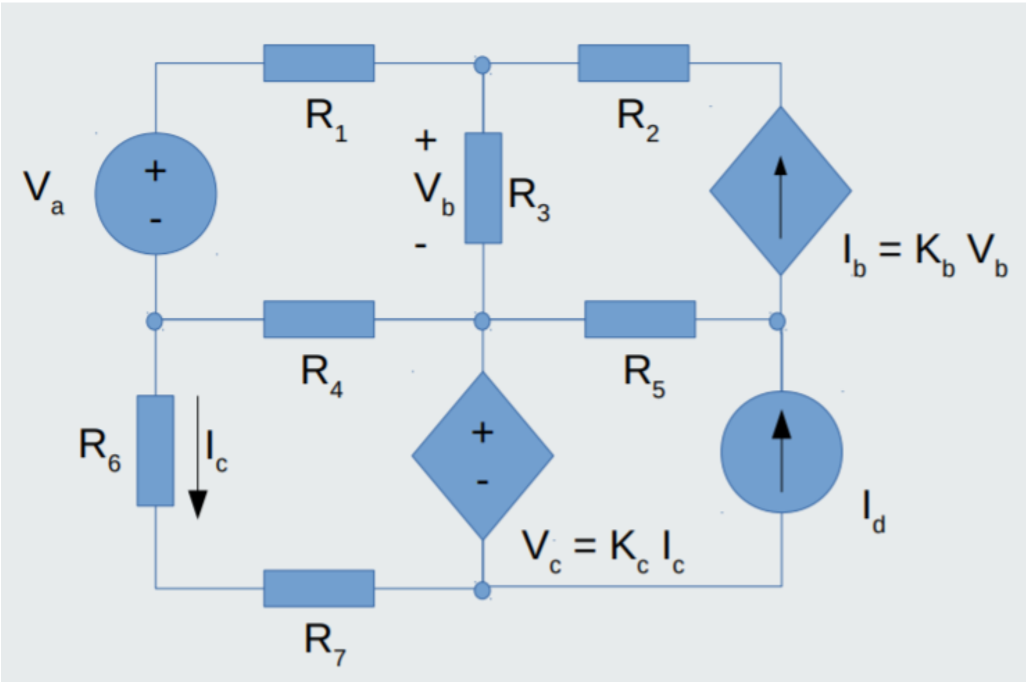

Figure 1: Voltage and Current driven circuit with 7 resistors.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using the mesh and the node methods.

2.1 Mesh analysis

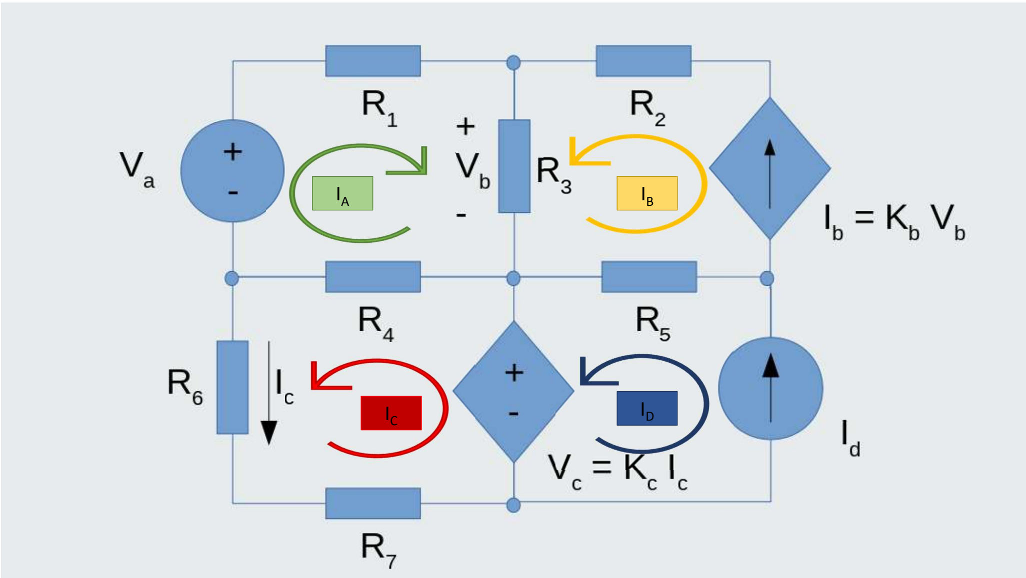

Figure 2: Current flow by mesh.

This circuit is composed by four primary meshes in which we assume the current flows counter-clockwise in every mesh but one, as seen in Figure 2.

Note that this is merely a convention used in our theoretical computations. The actual physical direction of the current can be obtained by analysing the algebraic sign of the current in each branch.

To find out the current that flows in each mesh we use the Kirchhoff Voltage Law (KVL) followed by Ohm's Law.

$$\sum_{k=1}^{n} V_k = 0. \quad , \quad V = RI. \tag{1}$$

In some cases, two currents flow on the same branch. To solve this we use Kirchhoff Current Law (KCL), to find the current on that branch.

$$\sum_{k=1}^{n} I_k = 0. \tag{2}$$

In mesh a (mesh where the current I_a flows) we assume that the voltage source is providing energy to the circuit and therefore the current has to flow clock-wise. This means that the sum of the voltages in each resistor (R_1, R_3 and R_4) has to equal the voltage V_a.

In meshes b, c and d we followed the same process as in mesh a, defining the way in which the current flows through current and voltage sources. To exemplify this process, the equation for mesh a is the following:

$$V_a = R_1 I_a + R_3(I_a + I_b) + R_4(I_a + I_c). \tag{3}$$

With the aid of octave, we can solve the four equations to obtain the following table (table 1).

Name	**Value [mA]**
Ia	0.194523
Ib	-0.204136
Ic	0.961761
Id	1.034000

Table 1: Results from the mesh analysis, from *Octave*.

Since we know the current on each mesh we can calculate the current on every branch. The results are present on table 2.

Name	**Value [mA]**
Ib	-0.204136
Id	1.034000
R1	0.194523
R2	-0.204136
R3	-0.009614
R4	1.156284
R5	1.238136
R6	0.961761
R7	0.961761

Table 2: Current on each branch.

2.2 Node analysis

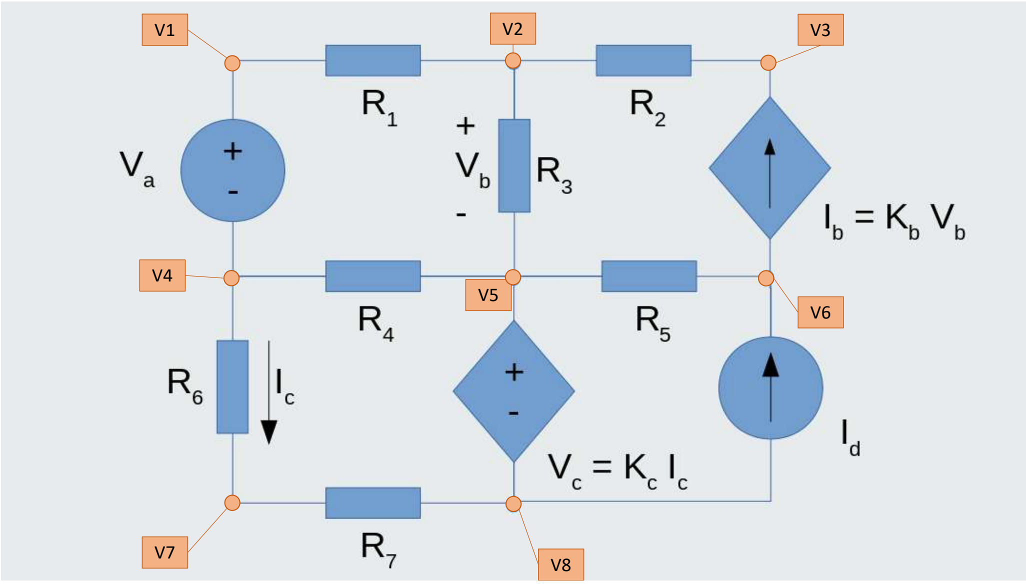

Figure 3: Nodes.

There are 8 nodes in total which means that we need 8 equations to find out the voltages in each node and solve the circuit. Therefore, we use the Kirchhoff Current Law (KCL, equation 2) in every node that is not connected to a voltage source (nodes 2, 3, 6 and 7, which are identified in Figure 3).

We assume node 4 (V_4) as the reference node, which means that its value is 0. This node was chosen because usually, the negative terminal of a voltage source is connected to the ground.

It is also known that the value of a voltage source is equivalent to the difference of the voltages in each node to which the source is connected. That allows us to create two more equations, since there are two voltage sources in the circuit.

For the 8^{th} equation we can create a supernode with nodes 5 and 8 since the dependent voltage source is not connected to the reference node. This way, nodes 5 and 8 are considered as one by ignoring the voltage source between them.

To demonstrate the process, the node 2 equation resulting from the KCL application is the following:

$$(V_2 - V_3)G_2 = (V_5 - V_2)G_3 + (V_1 - V_2)G_1. \tag{4}$$

Doing this on all possible nodes/supernode and adding the extra equations, regarding the voltage sources, a system of linear equations can be obtained and solved with the aid of *Octave*. The results are presented on table 3.

Name	**Value [V]**
V1	5.008942
V2	4.808960
V3	4.394159
V4	0.000000
V5	4.837862
V6	8.700771
V7	-2.008723
V8	-2.970917

Table 3: Results from the node analysis, from *Octave*.

3 Simulation Analysis

This section discusses the circuit simulation, performed using *Ngspice*.

This circuit was entered into the *Ngspice* simulation environment. This tool is used to simulate analog electronic circuits and predict circuit behaviour. This *Ngspice* simulation begins by defining the ground node, which is the node with potential 0 (by convention). In *Ngspice*, the ground node is represented by V_0. In our theoretical analysis, the ground node was defined to be node 4. However node 4 is still needed because we must introduce a voltage source with 0V potential, in order to measure the current Ic for the dependent voltage source, since *Ngspice* considers the voltage sources as Ammeters. The new diagram, which represents more accurately what was introduced in *Ngspice*, is shown on Figure 4.

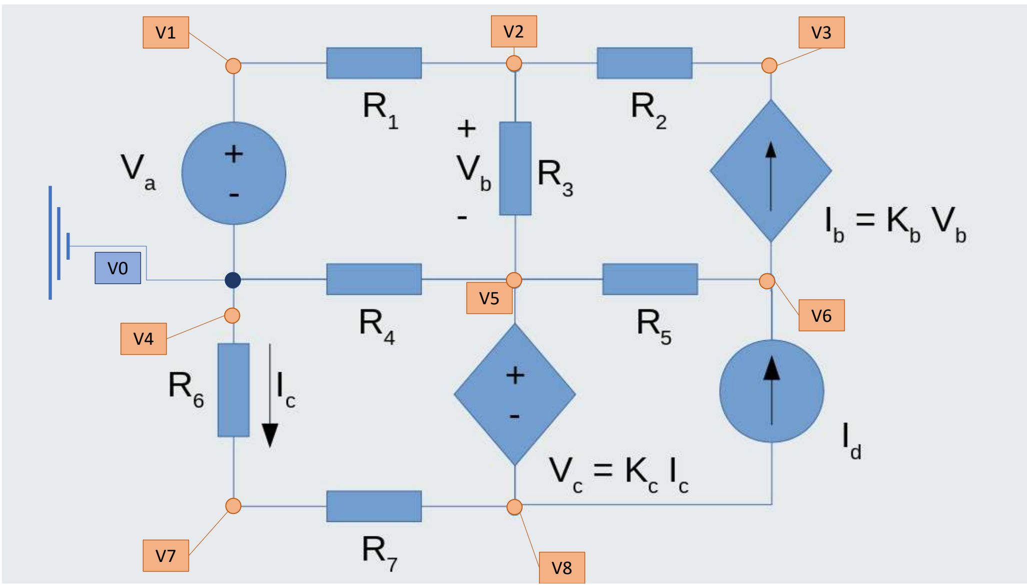

Figure 4: Voltage and Current driven circuit with 7 resistors.

Table 4 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
@gcs[i]	-2.04136e-04
@id[current]	1.033653e-03
@r1[i]	1.945228e-04
@r2[i]	-2.04136e-04
@r3[i]	-9.61339e-06
@r4[i]	1.156284e-03
@r5[i]	1.237790e-03
@r6[i]	9.617611e-04
@r7[i]	9.617611e-04
v(1)	5.008942e+00
v(2)	4.808960e+00
v(3)	4.394160e+00
v(4)	0.000000e+00
v(5)	4.837861e+00
v(6)	8.699687e+00
v(7)	-2.00872e+00
v(8)	-2.97092e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Results Analysis

The simulation results match the predicted ones from the theoretical analysis with precision. This was expected due to the fact that this circuit only contains linear (dependent and independent) components. Nevertheless, there are very small differences. This could probably originate from numerical approximation on either *Octave* or *Ngspice*.

5 Conclusion

In this laboratory assignment, the objective of analysing a circuit with dependent and independent current and voltage sources has been achieved. This circuit has been analysed using the Mesh and the Node methods, which are based on Kirchhoff's Circuit Laws (KVL and KCL, respectively).

The theoretical equations derived from applying the Mesh and Node methods have been computed and solved using the *Octave* Maths tool. These results have then been confronted with the simulation results, which were obtained by using the *Ngspice* tool, resulting in a match.