Contents

1 Introduction **4**

2 Data and Empirical Analysis **6**

2.1 Data . . . 6

2.1.1 Well Data . . . 6

2.1.2 Geological Survey Data . . . 7

2.1.3 Oil Price Data . . . 7

2.2 Empirical Analysis . . . 8

2.2.1 Correlation between Oil Prices and Horizontal Drilling in North Dakota . . . 8

2.2.2 The Role of Geological Quality in Horizontal Drilling . . . 9

3 A DCDP Model in Continuous Time for Drilling Decisions in Oil and Gas Extraction **14**

3.1 A Limitation of AKS-style Model . . . 14

3.2 Setup . . . 16

3.3 Social Planner's Problem and Necessary Conditions . . . 20

3.3.1 Implications of Necessary Conditions . . . 24

3.4 Firm's Problem . . . 25

4 Equilibrium Dynamics with Oil Prices **27**

4.1 Impacts of Unexpected Demand Shocks . . . 27

4.2 Endogenous vs. Exogenous Oil Prices . . . 30

4.3 Heterogeneous Impacts of Unanticipated Demand Shocks on Drilling Well Sites of Different Quality 30

5 Conclusion **31**

A Appendixes **33**

A.1 Details in Derivations . . . 33

A.1.1 Linearization near the Steady State of the Social Planner's Problem . . . 33

A.1.2 The Value Function for a Well Site i in state k in Continuous Time . . . 34

A.1.3 The Euler Equation for the Firm's Problem . . . 35

A.2 Additional Figure(s) and Table(s) . . . 37

List of Tables

1 Summary Statistics for Wells 7

List of Figures

1 Time Series of the Number of Well Completions in North Dakota 8
2 Simultaneous Drilling of Horizontal Wells with Heterogeneous Geological Quality 11
3 High Sensitivity of Firm-Level Low-Quality Well Drilling to the Negative Price Shocks in 2014-15 12
4 Held-by-Production vs. Non-Held-by-Production Horizontal Well Drilling 13
5 Phase Diagrams for the Social Planner's Problem 23
6 Equilibrium Paths under Unexpected Demand Shocks 28
7 Heterogeneous Impacts of Unexpected Demand Shocks on Drilling and Production 29
8 Time Paths for Drilling and Reserves under Endogenous and Exogenous Oil Prices 31
9 Spatial Distribution of the Estimated Geological Characteristic by Year 37
10 Spatial Distributions of Geological Characteristics 38

1 Introduction

Hotelling's model of exhaustible resource extraction provides simple but useful economic intuitions about the trade-off between extraction today and extraction in the future in the context of the forward-looking resource owners. The framework is flexible in applying to real-world resource extraction problems, such as exploration (Pindyck, 1978; Arrow and Chang, 1982; Swierzbinski and Mendelsohn, 1989; Quyen, 1991), uncertainty over reserves/demand/price (Gilbert, 1979; Pindyck, 1980, 1981; Farrow and Krautkraemer, 1989), taxation effects (Sweeney, 1977; Heaps, 1985), and technological improvement (Stiglitz, 1974; Slade, 1982). Accordingly, economists have utilized this canonical theory of the optimal depletion of nonrenewable resources for many decades to understand how exhaustible resource markets function. Hotelling's model, however, shows a different story in terms of empirical work. The main focus of the empirical literature on the Hotelling framework has been to test the well-known r-percent rule that a resource's shadow price has to rise at the rate of interest r. Unfortunately, such attempts have not been very fruitful due to various econometric issues and the fact that resource rents are generally unobservable.[1] Furthermore, recent empirical work on oil and gas extraction tends not to use Hotelling's theoretical model.[2]

Anderson, Kellogg and Salant (2018) (AKS) extends Hotelling's model by adding a new layer to oil producers' decision-making. AKS allows extractors to manipulate the rate of extraction from each well (the intensive margin) as well as the rate of drilling new wells (the extensive margin). The authors document several stylized facts about oil production: 1) oil production from existing wells is unresponsive to oil prices, which is inconsistent with Hotelling's classic model; however, 2) the rate of drilling is responsive to oil prices. The authors can reconcile these stylized facts with their reformulation of the Hotelling model.

Adding the heterogeneous geological features of different well sites is a natural augmentation to the AKS theoretical model. As discussed in Agerton (2020), variation in geological characteristics, which we denote *resource quality*, is a key driver of well-level productivity and firms' extraction decisions. We extend the AKS framework to incorporate heterogeneity in resource quality and well-level cost shocks. This extension has several benefits. First, we can accommodate what we see empirically in U.S. production—that firms develop high- and low-quality resources at the same time. Second, our specification is both analytically tractable, allowing for analysis with standard optimal control methods, as well as empirically tractable, allowing for econometric estimation.

To examine the validity of our extended model, we empirically analyze the resource quality of horizontal wells in North Dakota. As is well known, the geological quality of a given well site is usually observable only by

[1] See Gaudet (2007) and Thille and Slade (2009).

[2] Kellogg (2014) examines the relationship between drilling investments in Texas and oil price volatility. Fitzgerald (2015) studies experiential gains in hydraulic fracturing. Muehlenbachs, Spiller and Timmins (2015) investigates the impacts of shale gas development on the housing market. Boomhower (2019) examines the effects of bankruptcy protection on industry structure and environmental outcomes. Lewis (2019) studies the effects of a complex patchwork of mineral ownership on the oil and gas extraction outcomes. Using a government oil lease lottery, Brehm and Lewis (2021) shows that initial assignment results in different trade, drilling, and production outcomes.

extraction firms, though there are publicly available data that we can exploit to gauge it, such as the geological survey data published by the North Dakota Geological Survey (NDGS). Inspired by Herrnstadt, Kellogg and Lewis (2020), we use Robinson's partial linear model with detailed well-level data on horizontal wells drilled in North Dakota to estimate resource quality in each location. This estimation process provides us with two interesting stylized facts. One interesting fact is that fracking firms in North Dakota drilled well sites with different levels of resource quality simultaneously. The other empirical fact we discovered is that drilling of low-quality well locations decreased more than that of high-quality ones in response to the oil price drops during the second half of 2014.

The stylized facts from our analysis of the quality of drilled well sites in North Dakota raise two issues in modeling fracking firms' drilling activity based on the extended AKS framework. First, we find that the AKS-style model incorporating well sites' heterogeneous resource quality cannot rationalize the simultaneous drilling of well sites with different quality levels. Simply put, the model is not able to explain the empirical finding. Second, the simultaneous drilling of well locations with heterogeneous resource quality is inconsistent with the well-known least-cost-first extraction rule in exhaustible resource extraction. Holland (2003), which shows that limited extraction capacity causes the rule not to hold, seems to indicate that we need to include additional extraction-capacity-associated constraints that are highly sophisticated to model to make our extended model describe the empirical facts well. In this context, we suggest adopting a different approach to developing an economic model that enables us to explain fracking firms' drilling activity observed in North Dakota convincingly.

In this paper, following Arcidiacono et al. (2016), we develop a Discrete Choice Dynamic Programming (DCDP) framework in continuous time to model firms' drilling decisions. In this theoretical model, we formulate the decision to drill as an optimal stopping problem that trades off drilling a given well site today against drilling it at some time in the future. This trade-off is also the central idea of Hotelling's classic model. In our formulation, we introduce choice-specific cost shocks ϵ's that allow us to address the two problems we faced with respect to the extended AKS-style model. Because the cost shocks reflect a range of constraints that affect oil production companies' drilling decisions but are difficult to quantify by econometricians, they allow us to avoid adding various constraints to our model. Our model incorporating the cost shocks can also rationalize the empirical finding that fracking firms in North Dakota drilled horizontal wells with heterogenous quality simultaneously. Furthermore, under the assumption of a continuum of infinitesimally small well sites, our framework enables us to compute market-level drilling and production by aggregating the drilling decision for the marginal well location. In addition to its analytical tractability, one of the main advantages of the DCDP framework is that the model is estimable empirically using microeconomic data, which are available from both commercial and government databases.

We examine the equilibrium dynamics implied by our DCDP model in continuous time, especially focusing on how hydraulic fracking firms adjust their drilling, and thus oil production, in response to changes in oil prices under different conditions. First, we investigate the impact of unexpected demand shocks on the evolution of optimal drilling paths. Our simulation shows that a negative demand shock results in an immediate decrease in drilling, oil production, and the equilibrium oil price and that the equilibrium oil price gradually increases after the discontinuous drop. Second, we simulate how firms' drilling activity on well locations with heterogenous resource quality responds to unexpected price shocks. The time paths from this simulation demonstrate an interesting result that is consistent with our empirical observation: they reduced the drilling of low-quality well sites more than that of high-quality ones. Third, we compute the time paths of optimal drilling of horizontal wells and oil production from them under two distinct types of oil prices—exogenous and endogenous oil prices. The obtained equilibrium paths show that exogenous oil prices cause a higher drilling rate over the early period in our simulation.

The rest of this paper proceeds as follows. Section 2 discusses a set of data utilized for this research and the results from our empirical analysis. In Section 3, we develop a continuous-time DCDP model for drilling decisions in oil and gas extraction. Section 4 presents, under distinct conditions, the time paths of optimal drilling and oil production implied by our model, and Section 5 concludes.

2 Data and Empirical Analysis

2.1 Data

This section summarizes data on wells, geology, and oil prices, which are utilized to conduct empirical analysis and estimate a model of drilling behaviors observed in North Dakota.

2.1.1 Well Data

We scrape data for well in North Dakota's Bakken region from the data portal of the North Dakota Industrial Commission (NDIC), the regulator for the drilling and production of oil and gas in North Dakota.[3] NDIC-providing well data include a complete index of all wells permitted in North Dakota. The data contains basic information for each well, such as the type of well, completion and spud dates, location, the first and the current operator names, and targeting pool. Individual well's production and injection histories, including producing days during each month, are also contained in the data.

Detailed well completion data are also obtained from the NDIC.[4] The data contain how much water and proppant were consumed during well simulations.

The regulatory body also provides well-level survey data. The survey data include detailed information on the path of each wellbore, including the direction and length of each lateral.

[3]NDIC's well data are available at Official Portal for North Dakota State Government.
[4]Using Form 6, Well Completion or Recompletion Report, filed by operators, the NDIC has developed the detailed data.

Table 1: Summary Statistics for Wells

	Mean	(S.D.)	P25	P50	P75
Production					
Cumulative Oil Production (Bbls)	211,491.30	(108,012.00)	136,811.50	193,354.00	267,052.20
Cumulative Producing Days (Days)	1,451.37	(356.58)	1,343.75	1,601.00	1,697.00
Drilling					
Water (Bbls)	101,716.50	(249,157.50)	42,032.75	65,856.00	130,219.20
Sand/Proppants (Lbs)	4,553,380.00	(4,560,985.00)	2,312,951.00	3,441,914.00	5,543,660.00
Horizontal Drilling (Feet)	23,866.77	(15,134.08)	19,276.23	20,025.88	21,227.25
Geological Characteristics					
Thickness (Feet)	44.79	(13.48)	37.50	44.50	44.50
Thermal Maturity	0.64	(0.20)	0.50	0.50	0.75
Total Organic Content	13.62	(1.98)	12.24	13.34	15.13

Note: This table presents summary statistics for 694,294 horizontal wells whose cumulative production month equals or exceeds 24.

Throughout this paper, we use the sample of 24,520 horizontal wells that targeted the Bakken pool.[5] Summary statistics for those wells are presented in Table 1.

2.1.2 Geological Survey Data

We obtain geological survey data from the North Dakota Geological Survey (NDGS).[6] We follow Covert (2015) to match the three geological characteristics with each horizontal well in our sample.[7] Figure 10 demonstrates the spatial distribution of well-level geological features.

2.1.3 Oil Price Data

We collect the monthly per-barrel spot prices for West Texas Intermediate at the Cushing, Oklahoma from the Energy Information Administration.[8] As shown in Figure 1, there was a striking movement in oil prices between 2014 and 2016. Specifically, oil prices, maintained at around \$100 per bbl during the first half of 2014, had continued to plunge, reaching less than \$50 per bbl in January 2015. After recovering to \$60 per bbl during the first half of 2015, oil prices had fallen to \$30 per bbl by the end of the year. Since then, oil prices have gradually risen until they declined again during the final quarter of 2018.

[5] According to Lee (2019), the Bakken pool includes Bakken, Three Forks, and Sanish formations.
[6] To be specific, we exploit NDGS maps GI-59 and GI-63.
[7] Refer to Section 2.4.1 of Covert (2015).
[8] Time series data for Cushing, OK WTI Spot Price FOB is available here.

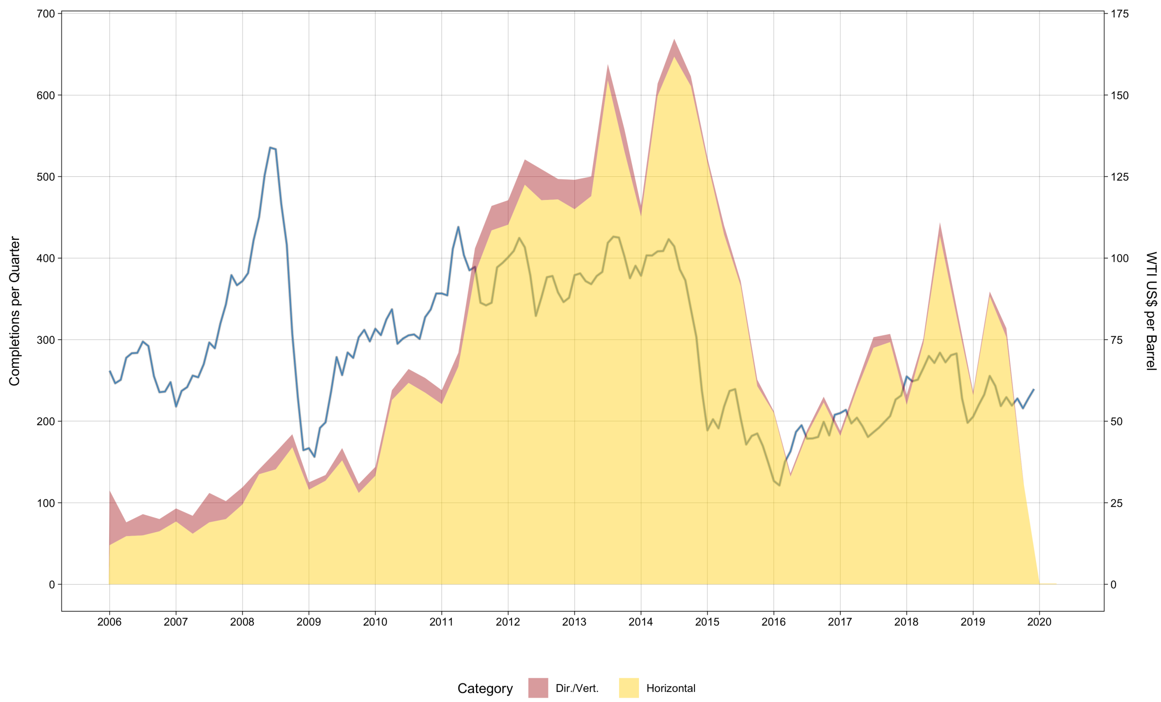

Figure 1: Time Series of the Number of Well Completions in North Dakota

Note: This figure shows the time series of the number of well completions in North Dakota. Horizontal wells have been strictly dominant in that area. The solid line in the figure is the monthly per-barrel spot prices for West Texas Intermediate at Cushing, Oklahoma. The figure suggests that the spot prices positively correlate with horizontal well completions in North Dakota.

2.2 Empirical Analysis

2.2.1 Correlation between Oil Prices and Horizontal Drilling in North Dakota

Figure 1 shows how well completions in North Dakota evolved between 2009 and 2020. As clearly illustrated, well completions, which were driven mainly by horizontal wells, dramatically increased from the beginning of 2010.

According to the figure, it is evident that drilling horizontal wells in North Dakota is closely correlated with oil prices, especially after 2009. On the whole, oil prices significantly increased between 2009 and 2010 and remained high until mid-2014. Then, there was a sharp plunge in oil prices from mid-2014 to the end of 2015, and horizontal well drilling declined too. When oil prices gradually climbed between 2016 and 2020, North Dakota's drilling activities also recovered. To summarize, oil prices and the number of horizontal drilling in North Dakota seem to be positively correlated. Importantly, such a positive correlation between oil prices and horizontal drilling in the state suggests that fracking firms' drilling decisions strongly depend on oil prices. In Section 2.2.2, we show that their drilling decisions are linked with oil prices through the geological features of well sites.

2.2.2 The Role of Geological Quality in Horizontal Drilling

Estimation of Unobservable Geological Characteristics of Horizontal Wells — Not all well-specific information on geological features is available to econometricians. The NDGS geological survey data only include estimates of four different measurements of geological properties at a given location. Because the geospatial data was published to the public in 2008, it is likely that fracking firms, whose objective is to maximize their profits, have already exploited the contents of the maps. As discussed in Agerton (2020), learning about the spatial distribution of deposits by drilling wells is one of three economic factors that govern firms' where-to-drill decisions. So, it is reasonable to suppose that firms have private information about the Bakken area's spatial distribution of geological characteristics, which is not accessible to researchers.

The geological characteristics observed only by firms play two different roles in their drilling decisions. First, firms choose whether to drill a location based on its resource quality. Thus, the sample of wells we observe is not random: it has been selected based on unobservable (to us) resource quality. Second, firms' choice of inputs during hydraulic fracturing of each well may be correlated with the unobservable resource quality. For these reasons, accounting for resource quality is critical in modeling firms' decisions and production functions.

Following Herrnstadt, Kellogg and Lewis (2020), we employ Robinson's partially linear model to determine the unobservable quality of the horizontal wells completed between 2009 and 2018. We first specify the oil production from a horizontal well as

$$\log(y_i) \;=\; \log(\boldsymbol{X}_i)'\boldsymbol{\beta} \;-\; \lambda\left(longitude_i, latitude_i\right) \;+\; \epsilon_i. \tag{1}$$

In this specification, y_i are horizontal well i's cumulative oil production at its cumulative production month 24.[9] The covariate vector $\boldsymbol{X}_i$ for well i includes hydraulic fracturing inputs (i.e., fluid volume, proppant weight, and length of horizontal drilling), cumulative producing days, and observable geological characteristics (i.e., thickness, total organic contents, and thermal maturity). The term $\lambda(longitude_i, latitude_i)$ is a nonparametric function of each well's coordinates and captures well i's unobservable resource quality. Lastly, ϵ_i is a productivity shock not correlated with resource quality.

To operationalize model (1), we estimate the following partially linear model:

$$\log(y_i) \;-\; \widehat{m}_{y_i} \;=\; \left(\log(\boldsymbol{X}_i) \;-\; \widehat{\boldsymbol{m}}_{\boldsymbol{X}_i}\right)'\boldsymbol{\beta} \;+\; \epsilon_i. \tag{2}$$

Here, $\widehat{m}_{y_i}$ are predictions from a non-parametric regression of $\log(y_i)$ on well i's coordinates $(longitude_i, latitude_i)$. The $\widehat{m}_{y_i}$ terms are smoothed means. Differencing these means out serves the same role as the within-transformation in a fixed effects model. In fact, if one used a uniform kernel function, $\widehat{m}_{y_i}$, within discrete cells, the estimator would be mathematically identical to fixed effect estimation with spatial fixed effects for each well. Predic-

[9] That is, unobservable geological features are estimated cross-sectionally in our estimation.

tions $\widehat{\boldsymbol{m}}_{\boldsymbol{X}_i}$ are obtained from different nonparametric regressions whose dependent and independent variables are $\log(\boldsymbol{X}_i)$ and $(longitude_i, latitude_i)$, respectively. The values of primary interest $\widehat{\lambda}_i$ (i.e., the unobservable geological quality of horizontal well i) are estimated as follows[10]:

$$\widehat{\lambda}_i \;=\; \widehat{m}_{y_i} \;-\; \widehat{\boldsymbol{m}}'_{\boldsymbol{X}_i}\widehat{\boldsymbol{\beta}} \tag{3}$$

In our analysis, we define low- and high-quality locations by dividing our sample of well sites into those with $\widehat{m}_{y_i}$ below and above the median. Even though $\widehat{m}_{y_i}$ is estimated from only higher-quality locations with observed drilling, and therefore biased upward, the ordinal ranking of well sites should be affected less. Figure 9 shows the spatial distribution of the estimated quality.

Simultaneous Drilling of Horizontal Wells with Heterogeneous Geological Qualities — Figure 2, summarizing the estimated geological qualities of horizontal wells in scatter plots, clearly demonstrates that horizontal wells with a range of geological qualities were drilled simultaneously in the Bakken region of North Dakota. And as shown in Figure 3, simultaneous drilling is observed even at the firm level.

Simultaneous drilling of both low- and high-quality resources contradicts the well-known least-cost-first extraction rule in nonrenewable resource extraction.[11] According to the rule derived from the canonical Hotelling model, deposits of an exhaustible resource should be exploited in strict order, beginning with the lowest cost deposit.[12] Because a larger estimate of the unobservable geological quality implies higher ultimate oil production, if the rule holds, wells with larger estimates, thus having lower per-barrel marginal cost, should be first extracted. Those figures, however, do not show the theoretical prediction at all.

The High Sensitivity of Drilling Low-quality Horizontal Wells to Negative Price Shocks — In addition to the simultaneous drilling of horizontal wells with heterogeneous qualities, Figure 2 demonstrates an interesting point: the responsiveness of low-quality well drilling to sharp oil price declines from mid-2014 to the end of 2015. The high sensitivity of drilling activities for low-quality horizontal wells to negative price shocks during the period is also pronounced even at the firm level, as illustrated in Figure 3.

Figure 4 shows that drilling associated with held-by-production did not drive the relationship between oil prices and low-quality well drilling, especially between mid-2014 and the end of 2015. The upper panel in the figure illustrates the by-quality time series of the number of horizontal wells drilled that are supposed to be drilling related to Held-By-Production (HBP). In our empirical analysis, we assume that for each of the sections into which horizontal wells in our sample were drilled, the purpose of the first drilling in that section was just HBP.[13] The relatively high drilling rate of low-quality wells, especially between mid-2011 and mid-2013, seems to

[10] For details of Robinson's difference estimator, refer to *9.7.3 Partially Linear Model* in Cameron and Trivedi (2005).

[11] The cost that matters here is the marginal cost. And the marginal cost consists of two distinct costs: the marginal cost of drilling a new well and the marginal cost of extracting oil from existing wells.

[12] See *Selection 7 Depletion and Economic Theory* in Herfindahl and Brooks (2015).

[13] In the Public Land Survey System, a *section*, which is one of 36 sections in a township, is a one-mile-square area.

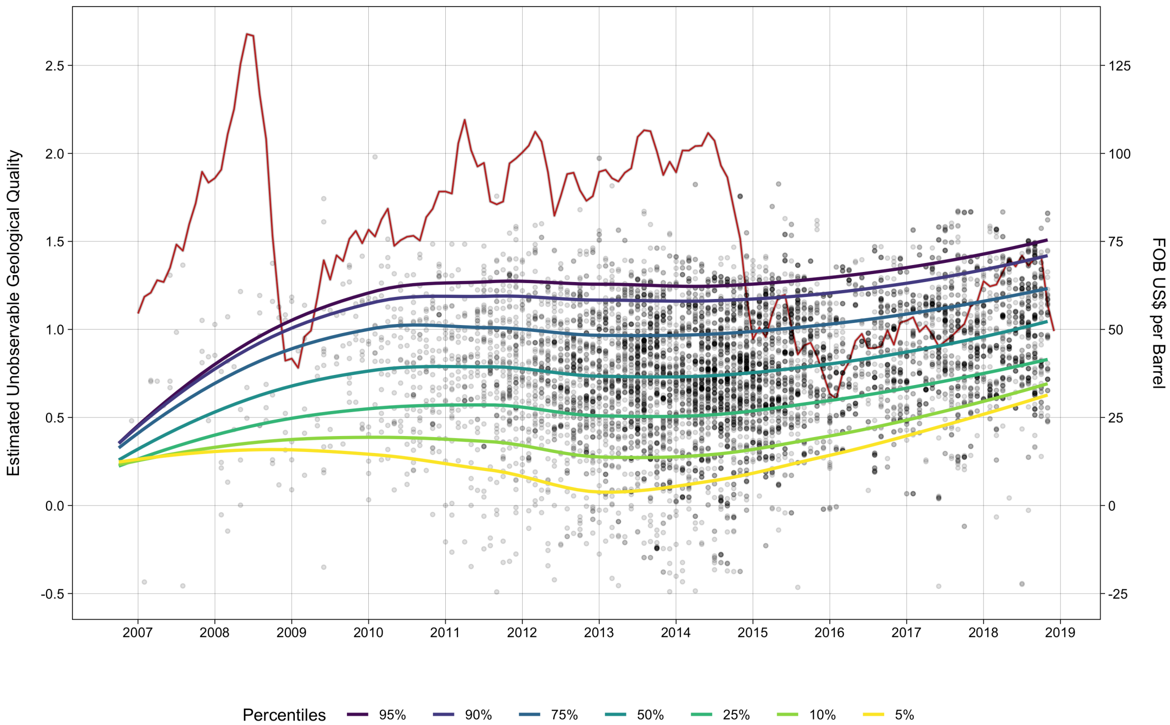

Figure 2: Simultaneous Drilling of Horizontal Wells with Heterogeneous Geological Quality

Note: This figure indicates the estimated geological feature for each horizontal well, depicted as a dot. Those dots definitely suggest the simultaneous drilling of horizontal wells with heterogeneous geological quality. In the figure, percentiles of the estimates, with the 95% confidence interval of each, are also presented. The solid red line is the time series of the monthly per-barrel spot prices for West Texas Intermediate at Cushing, Oklahoma. Oil prices plunged significantly between 2014 and 2015 and rose gradually. The percentile lines skewed upward, especially lower ones, as of the second half of 2014.

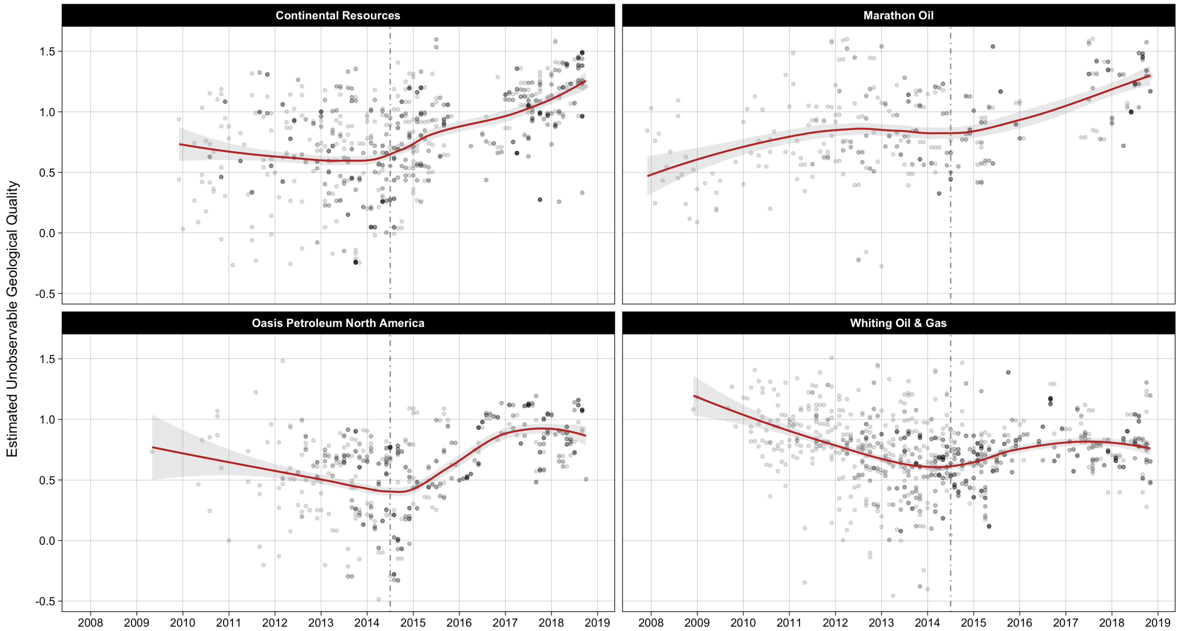

Figure 3: High Sensitivity of Firm-Level Low-Quality Well Drilling to the Negative Price Shocks in 2014-15

Note: This figure shows how the four firms' drilling activity changed over time. Each dot in the figure indicates an individual well's estimated geological feature. The red line, with the 95% confidence interval, in each panel demonstrates the average quality of horizontal wells drilled in a month. As illustrated, the firms significantly reduced drilling low-quality well locations since mid-2014, corresponding to the beginning of the oil price plunge.

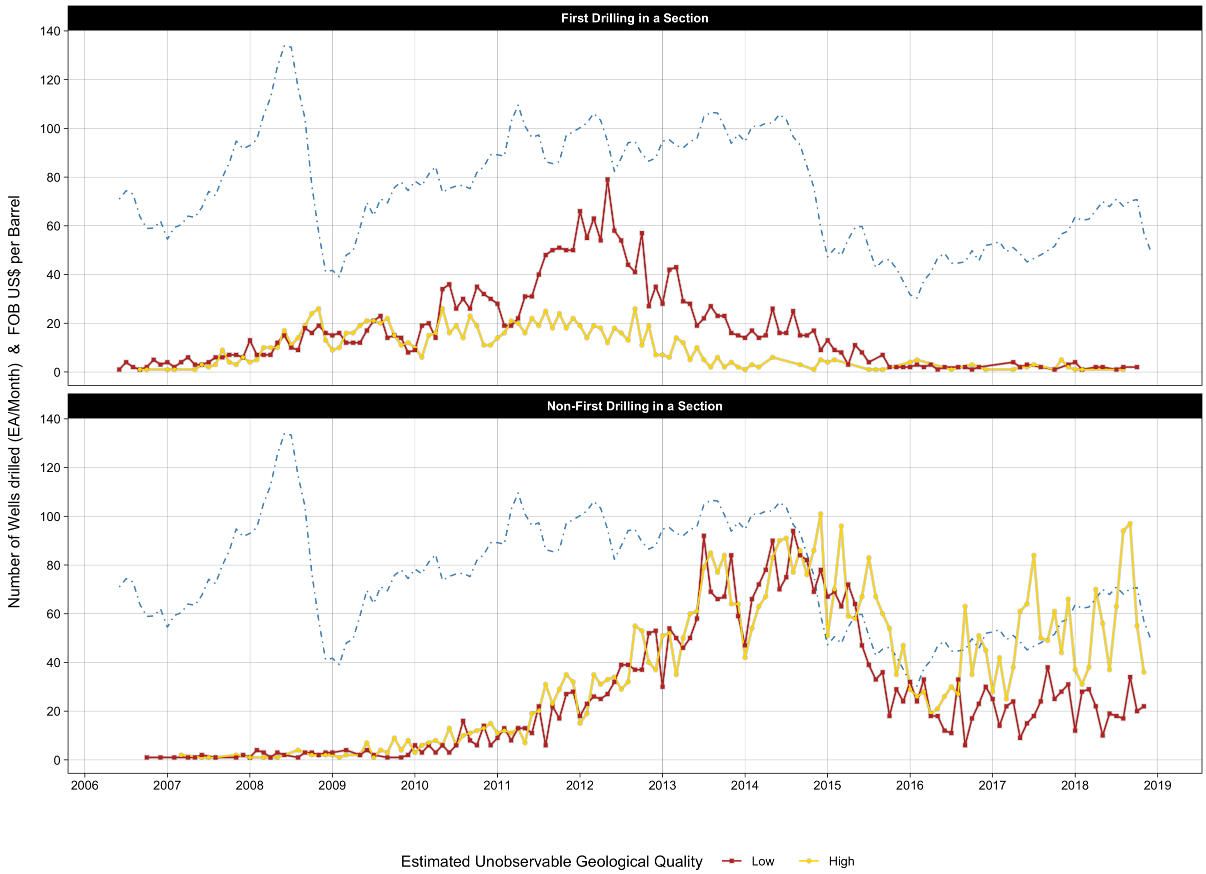

Figure 4: Held-by-Production vs. Non-Held-by-Production Horizontal Well Drilling

Note: This figure depicts the drilling of horizontal wells classified into two quality levels. The upper panel shows how the first drilling in each section, regarded as held-by-production drilling, has evolved. There was only a limited number of held-by-production drilling between 2015 and 2019. The lower panel indicates all subsequent drilling in sections. The collapse in oil prices between mid-2014 and 2015 made post-held-by-production drilling decrease. Drilling of low-quality well locations showed a more significant reduction, especially in 2015. High-quality sites were drilled more than low-quality ones during the period of oil price recovery from 2016 to 2019. In each panel, the dot-dashed line is the time series of the monthly per-barrel spot prices for West Texas Intermediate at Cushing, Oklahoma.

be consistent with the empirical result of Herrnstadt, Kellogg and Lewis (2020): firms bound to a lease contract including use-it-or-lose-it requirements tend to drill low-productivity well locations just before the first lease expires.

The evolving pattern of drilling for each of the three quality levels presented in the lower panel of Figure 4, regarded as post-held-by-production drilling, shows completely different movements from those in the upper panel. Until 2014, horizontal wells of heterogeneous quality were drilled equally and at the same growth rate. But drilling of low-productivity horizontal wells more sensitively reacted to negative price shocks between mid-2014 and 2015, compared to drilling medium- and high-quality wells. The new theoretical approach, required to rationalize the simultaneous drilling of wells with heterogeneous qualities, needs to explain the high sensitivity of low-quality well drilling.

3 A DCDP Model in Continuous Time for Drilling Decisions in Oil and Gas Extraction

This section delves into the analysis of two distinct economic frameworks. In Section 3.1, the optimal drilling and extraction model developed in Anderson, Kellogg and Salant (2018) is reformulated by introducing heterogeneity in resource quality. And it is shown that the recast model cannot justify the empirically observed simultaneous drilling of well locations with varying quality levels. In the subsequent portions of this section, a continuous-time Discrete Choice Dynamic Programming (DCDP) model for oil and gas extraction is developed, successfully articulating the simultaneous drilling of horizontal wells with heterogeneous quality.

3.1 A Limitation of AKS-style Model

The theoretical framework for optimal oil drilling and extraction delineated in Anderson, Kellogg and Salant (2018) (AKS) can be augmented by integrating heterogeneity in the quality of well locations. Suppose that the fracking firm owns well sites of different qualities, indexed by $g \in \{L(ow), H(igh)\}$, and that a homogeneous good (i.e., oil) is yielded from the sites in which new horizontal wells are drilled. Furthermore, suppose that the unit price of the output, $\bar{p}$, is determined exogenously due to the firm's total production being negligible in comparison to the global market for the output. The maximization problem of the firm owning a continuum of infinitesimal well locations with disparate qualities can be articulated as follows:

$$\max_{d^g(t),\ g \in \{L,H\}} \int_0^\infty e^{-rt} \left\{ \bar{p} \sum_g \alpha^g d^g(t) \ - \ C\left(\sum_g d^g(t) \right) \right\} dt \tag{4}$$

subject to

$$\dot{R}^g(t) \ = \ -d^g(t), \quad R_0^g \ = \ R^g(0) \text{ given,} \tag{5}$$

$$d^g(t) \ \geqslant \ 0, \quad R^g(t) \ \geqslant \ 0. \tag{6}$$

In this formulation, state variables $R^g(t)$ denote the measure of undrilled well sites at a given time t. We simplify the AKS model by assuming that firms always produce at their production capacity constraint. This assumption reduces the complexity of the model. Control variables $d^g(t)$ represent the rate at which new horizontal wells are drilled at time t. α^g are the quantity of oil production from the marginally drilled well. Here, we assume that $\alpha^H > \alpha^L$. $C(\cdot)$, indicating the total instantaneous cost of drilling, is solely a function of the drilling rates.[14] Of note, in this formulation for $C(\cdot)$, we assume that locations are perfect substitutes on the cost side. And the profit obtained at time t is discounted at the interest rate r.

The current-value Hamiltonian-Lagrangian of the firm's problem is

$$\begin{aligned} \mathcal{H} \; &= \; \bar{p}\sum_g \alpha^g d^g(t) \; - \; C\left(\sum_g d^g(t)\right) \\ &+ \; \sum_g \pi^g(t)\left(-d^g(t)\right) \\ &+ \; \sum_g \lambda_1^g(t) d^g(t) \; + \; \sum_g \lambda_2^g(t) R^g(t), \end{aligned} \tag{7}$$

where π_R^g are costate variables for the state variables R^g. $\lambda_j, \; j \in \{1,2\}$ are the shadow cost of each constraint.

For a given quality level g, two necessary conditions characterize the firm's optimal rate of drilling:

$$d^g(t) \; \geqslant \; 0, \;\; \alpha^g \bar{p} \; - \; C'\left(\sum_g d^g(t)\right) \; - \; \pi^g(t) \; + \; \lambda_1^g(t) \; \leqslant \; 0, \;\; C.S., \tag{8}$$

$$\dot{\pi}^g(t) \; = \; r\pi^g(t) \; - \; \lambda_2^g(t). \tag{9}$$

When horizontal wells with heterogeneous quality are drilled simultaneously (i.e., for each g, $d^g(t) > 0$, which leads to $\lambda_1^g(t) = 0$), necessary condition (8) implies that the shadow price on the resource constraint at time t equals the profit on the marginal well:

$$\pi^g(t) \; = \; \alpha^g \bar{p} \; - \; C'\left(\sum_g d^g(t)\right). \tag{10}$$

In addition, when both types of horizontal well sites are not fully exhausted (i.e., for each g, $R^g(t) > 0$, which in turn $\lambda_2^g(t) = 0$), necessary condition (9) means that the shadow value of the marginal undrilled well at time t grows at the rate of r:

$$\dot{\pi}^g(t) \; = \; r\pi^g(t). \tag{11}$$

The necessary conditions collectively suggest that the simultaneous drilling of horizontal wells with heterogeneous quality cannot be justified in the AKS framework when $d^g(t) > 0$ and $R^g(t) > 0$, which hold before all

[14] Regarding the total cost of oil production, we follow the assumption made in Anderson, Kellogg and Salant (2018): per-barrel extraction costs from existing wells are negligible.

available well sites are developed. The following stems from equation (10):

$$\pi^H(t) \; - \; \pi^L(t) \; = \; (\alpha^H - \alpha^L)\bar{p}. \tag{12}$$

This relationship implies that the difference in the shadow value between high- and low-quality well locations is simply a revenue difference at any time t. And as shown below, differentiating equation (10) with respect to time implies that for a given $C''(\cdot)$, $\dot{d}^g(t)$ and $d^g(t)$ determine the value of the time derivative of $\pi_t^g(t), \; g \in \{L, H\}$ and that $\dot{\pi}_t^L(t)$ and $\dot{\pi}_t^H(t)$ have the same value at a given time t:

$$\dot{\pi}^L(t), \; \dot{\pi}^H(t) \; = \; -\left(\sum_g \dot{d}^g(t)\right) C'' \left(\sum_g d^g(t)\right). \tag{13}$$

Here, based on equation (11), the relationship between the time derivatives for two distinct qualities indicates that $\pi^L(t) = \pi^H(t)$ holds at all time t. However, this equality contradicts equation (12) because $\alpha^H > \alpha^L$. In other words, the simultaneous drilling of horizontal wells with heterogeneous quality does not hold in the AKS framework.

Adding a constraint on extraction capacity, as is done in Holland (2003), can suggest that simultaneous extraction of different resource qualities is optimal. However, in the framework, setting the upper bound of an extraction-related constraint seems too arbitrary and complicates drawing implications from necessary conditions. From the empirical perspective, it is also intractable to quantify the margin for market-wide, or firm-wide, extraction capacity for each drilling decision. Furthermore, empirical estimation of the model from microeconomic data on drilling and production is, in general, too demanding. Those difficulties call for a new theoretical approach that thoroughly explains our empirical findings for drilling decisions made by fracking firms in North Dakota.

The need leads us to develop a continuous-time Discrete Choice Dynamic Programming (DCDP) model that formulates a firm's drilling decision on a particular well site as an optimal stopping problem. Under this new theoretical framework, we can rationalize the simultaneous drilling of horizontal wells with heterogeneous resource quality without specifying any capacity constraint. Moreover, our DCDP model yields empirically testable predictions about how firms' drilling and production activities vary with oil prices. In the next section, we will present the basic elements and assumptions of our DCDP framework.

3.2 Setup

In our DCDP framework, in which time is continuous and indexed by $t \in [0, \infty)$, we assume a continuum of infinitesimally small potential drilling sites in which horizontal wells will be developed. Let R_t denote the measure of undrilled well sites at time t. In the AKS framework, R_t is the remaining reserves at time t. Therefore, the two can be equal but do not have to be. Without loss of generality, we fix the initial level of potential well sites in the market to be the value of one (i.e., $R_0 = 1$). For simplicity, it is also assumed that only one horizontal well is drilled into an infinitesimal site.

Each infinitesimal well site is owned by a single fracking firm indexed by an integer scalar $i = 1, 2, \cdots$.[15] A Poisson arrival process with rate parameter λ_a governs firms' drilling opportunity. In other words, for the firm i, an opportunity to drill arrives at rate λ_a. When a drilling opportunity arrives at time t, the firm i makes a choice at time t, denoted a_{it}, between two alternatives in a discrete choice set $\mathcal{A} = \{0, 1\}$:

$$a_{it} = \begin{cases} 0 & \text{if the firm decides not to drill a well in the site} \\ 1 & \text{if the firm decides to drill a well in the site.} \end{cases} \tag{14}$$

As implied, a_{it} is a well-site-level control variable. If the choice made by the firm i at time t is $a_{it} = 0$, the firm has another drilling opportunity that comes later. In other words, $a_{it} = 0$ is a costless continuation choice. On the other hand, when $a_{it} = 1$, the firm i exits the market after drilling a horizontal well into the site and producing oil from it.

The oil production from the horizontal well drilled into site i at time t is assumed to occur only during the same period:

$$q_t = \alpha a_{it}, \tag{15}$$

where α is the amount of oil produced from a well. For simplicity, we assume that α is a constant across locations. This formulation can certainly be modified to allow for production decline over time. However, we mainly focus on firms' investment and drilling decisions, so we abstract away from production declines. In an empirical exercise, account for the fact that a well produces for multiple periods by assuming that firms sell their production forward and receive the present value of revenue at the time they drill and complete the well.

The linear cost of drilling a horizontal well into the site i is assumed. That is,

$$c_t = c a_{it}. \tag{16}$$

For simplicity, it is also assumed that the drilling cost for the marginal well is uniform across well sites (i.e., c is the same for all potential well locations.). In addition, as in Section 3.1, we take the assumption of the negligible extraction costs.

In our theoretical framework, firms' site-level drilling decisions can be easily aggregated at the market level.

[15] In this paper, we use i to denote a potential well site or a firm interchangeably.

It is natural to define aggregate drilling in the market at time t, denoted D_t, as follows:

$$D_t \;\equiv\; \int_{\lambda_a R_t} a_{it}\; di. \tag{17}$$

In this definition, $\lambda_a R_t$ indicates a group of potential sites into which a horizontal well can be drilled when a drilling opportunity arrives at time t.[16] If each potential well site has the same probability of drilling a horizontal well into it at time t (denoted Pr_t), then D_t can be expressed as follows[17]:

$$D_t \;=\; \lambda_a R_t Pr_t. \tag{18}$$

We further assume that each period, measure E locations are also exogenously discovered and added to the set of potential sites. We ignore the exploration-associated costs in our formulation. So, the evolution path of the remaining well sites is governed by the following relationship:

$$\dot{R}_t \;\equiv\; -D_t \;+\; E \;=\; -\lambda_a R_t Pr_t \;+\; E. \tag{19}$$

In addition, because we assume that α is uniform across potential sites, aggregate oil production in the market at time t, denoted Q_t, is simply proportional to D_t:

$$Q_t \;\equiv\; \alpha D_t \;=\; \alpha \lambda_a R_t Pr_t. \tag{20}$$

Oil prices are discretized. For a given oil production Q, the following inverse demand function determines the oil price[18]:

$$p_k \;=\; p_{0,k} \;-\; \bar{p}_1 Q. \tag{21}$$

Here, k is an integer scalar index $k = 1, 2, \cdots, K$, by which every available demand level $p_{0,k}$ in a finite state space $\mathcal{X}$ is enumerated.[19] Moreover, $p_{0,k}$ and $\bar{p}_1$ are non-negative. Oil prices determined by the function can vary due to a finite-state Markov jump process.[20] The process is a jump process on $\mathcal{X}$. Parameters $\lambda_{k\ell}$ that indicate the rates at which particular exogenous state transitions from k to $\ell \neq k$ occur govern this process. This formulation allows oil prices to evolve endogenously in a smooth way, and it also accommodates stochastic

[16] This interpretation implies $\lambda_a \leqslant 1$.

[17] In the expression, $\lambda_a Pr_t$ indicates the probability of drilling a potential well site in the next instant, given it has not been drilled to time t. That is, $\lambda_a Pr_t$ is the hazard rate at time t, denoted h_t. Therefore, it is natural that $D_t = R_t h_t$.

[18] For simplicity, we omit the t subscript in the inverse demand function.

[19] In other words, $\mathcal{X}$ is a discrete state space for $p_{0,k}$.

[20] A Markov jump process with finite states is a stochastic process that has discrete movements governed by a Poisson arrival process. For details, see Doytchinov and Irby (2010).

price jumps. It is also assumed that all firms in the market take oil prices as given in a competitive equilibrium.

An infinitesimal well site provides two different types of payoffs. First, each undrilled well site generates a continuous and constant flow payoff (denoted f_t) that could be zero or negative. Second, choosing an action in the choice set $\mathcal{A}$ when a drilling opportunity arrives at time t yields an instantaneous payoff. An additively separable payoff function, denoted $U(\boldsymbol{X}_t, \epsilon_t)$, represents the instantaneous payoff:

$$U(\boldsymbol{X}_t, \epsilon_t) \;=\; \begin{cases} 0 \;+\; \epsilon_{0t} & \text{if } \; a_t = 0 \\ \psi(\boldsymbol{X}_t) \;+\; \epsilon_{1t} & \text{if } \; a_t = 1. \end{cases} \tag{22}$$

In the payoff function, $\boldsymbol{X}_t$ is a vector of relevant state and control variables. In addition, $\psi(\cdot)$ and ϵ_{at} indicate a choice-specific instantaneous payoff from oils produced from the site and choice-specific cost shocks, respectively. Of note, contrary to the flow payoff, the instantaneous payoff is applicable only to potential well locations under a drilling opportunity.

The choice-dependent instantaneous payoff $\psi(\cdot)$ is defined differently for two maximization problems, the social planner's and firm's problems, which will be discussed in the following sections. Specifically, in the social planner's problem, the instantaneous payoff is the net benefit achieved by consuming oils produced from drilled well sites in the market:

$$\begin{aligned} \psi(Q_t, D_t) \;&=\; u(Q_t) \;-\; c(D_t) \\ &=\; u(\alpha R_t Pr_t) \;-\; c(R_t Pr_t). \end{aligned} \tag{23}$$

Here, $u(\cdot)$ is the total utility obtained from oil consumption at time t, whereas $c(\cdot)$ is the total drilling cost at time t. For the case of $Q_t = 0$, $u(\cdot)$ and $c(\cdot)$ are normalized to zero.[21] Of note, $\boldsymbol{X}_t = (Q_t, D_t)$ in this maximization problem.[22] On the other hand, in the firm's problem, the instantaneous payoff is simply the net profit from oil production. So, if the firm i's choice is $a_{it} = 1$ when a drilling opportunity arrives at time t, then

$$\psi(p_k) \;=\; \alpha p_k \;-\; c. \tag{24}$$

Here, the firm i is supposed to be in state k. As shown, $\boldsymbol{X}_t = (p_k)$ in the firm's problem.

In the payoff function, ϵ_{at}, a component of the payoff of an alternative a, is an idiosyncratic cost shock at time t.[23] In our context, ϵ_{at}, which relies on the choice of a decision maker (e.g., the social planner or the firm), can be perceived as a composite cost element that affects the decision maker's choice at time t between drilling today and drilling in the future and that varies over time. For example, for the firm i, ϵ_{at} could include capacity-constraint-induced costs, whose value varies with a_{it}. With the interpretation of ϵ_{at}, it is not required

[21] From equation (20), it is clear that $Q_t = 0$ implies $D_t = 0$.
[22] In the social planner's problem, the state and control variables are R_t and Pr_t, respectively. For details, see Section 3.3.
[23] We can regard $\epsilon_{a,t}$ as an element of the unobservable state vector $\boldsymbol{\epsilon}_t$. In our case that $\mathcal{A} = \{0, 1\}$, $\boldsymbol{\epsilon}_t = (\epsilon_{0t},\ \epsilon_{1t})$.

to specifically model a set of constraints at time t in our framework.

The choice-specific cost shock ϵ_{at} drives the decision maker's drilling decision at time t. Because the shock is observable only by it, without ϵ_{at}, the (observable) state variable cannot perfectly explain the choice at time t in our model. Intuitively, when the decision maker decides whether to drill a well into a given well site or not at time t, $a_t = 1$ will be the optimal choice if the value of the payoff function conditional on $a_t = 1$ is greater than or equal to that conditional on $a_t = 0$. Mathematically,

$$\begin{aligned} \psi(\boldsymbol{X}_t) \ + \ \epsilon_{1t} \ &\geqslant \ \epsilon_{0t} \\ \epsilon_{1t} \ - \ \epsilon_{0t} \ &\geqslant \ -\psi(\boldsymbol{X}_t). \end{aligned} \tag{25}$$

The decision rule implies that the magnitude of $\epsilon_{1t} - \epsilon_{0t}$ determines the optimal choice.

Since ϵ_{at} is not observable, utilizing the decision rule directly is infeasible. However, according to Aguirregabiria and Magesan (2013), the expected value of ϵ_{at} conditional on alternative a_t being chosen under the decision rule can be expressed with Pr_t. To be specific, when $a_t = 1$, the conditional expected value of ϵ_{1t}, denoted e_{1t}, is given as follows[24]:

$$e_{1t} \ \equiv \ E[\epsilon_{1t} \mid a_t = 1] \ = \ \sigma\left(\gamma \ - \ \ln(Pr_t)\right). \tag{26}$$

Here, γ is Euler's constant. And it is assumed that ϵ_{at} follow the Type 1 Extreme Value (T1EV) distribution with the location parameter 0 and the scale parameter σ and are independently and identically distributed. The expected value allows our analytical as well as empirical analysis of drilling decisions to be tractable without observing ϵ_{at}. Throughout this paper, we keep the assumption about the distribution of the cost shocks.

3.3 Social Planner's Problem and Necessary Conditions

In this section, using the continuous-time DCDP framework, we develop the social planner's problem. When an opportunity to drill potential well sites arrives, the social planner makes two decisions. First, the planner must choose, via the choice of Pr_t, the aggregate quantity of drilling. Second, the planner must also decide which locations will be drilled. Each site can be indexed by $\epsilon_{1t} - \epsilon_{0t}$, and so selecting which sites to drill can be thought of as selecting which indices to drill. Because there is a continuum of potential well locations, determining the optimal policy (i.e., the optimal Pr_t) is simply to choose a threshold that does not depend on the specific set of cost shocks realized at time t. Potential well locations with indices above the threshold drill, whereas those below it wait to drill. The one-to-one mapping between the threshold index and the choice of Pr_t is characterized by equation (25).

The goal of the social planner is to maximize welfare in the market. In this maximization problem, for a

[24] In other words, e_{at} indicate the mean of the cost shock conditional on choice a_t.

given Pr_t, the market's welfare obtained from potential well sites at time t is defined as follows[25]:

$$\begin{aligned} W_t^{sp} \equiv\ & R_t f_t \\ & + u(\alpha\lambda_a R_t Pr_t) - c(\lambda_a R_t Pr_t) \\ & + \lambda_a R_t \Big\{ Pr_t \cdot \sigma\big(\gamma - \ln(Pr_t)\big) + (1 - Pr_t) \cdot \sigma\big(\gamma - \ln(1 - Pr_t)\big) \Big\} \end{aligned} \tag{27}$$

As shown, the welfare of the market at time t consists of three components.[26] The first line indicates the flow payoff received from undrilled well locations at time t. The second line means the choice-specific instantaneous payoff, which is presented as equation (23). The last line suggests the payoff related to choice-specific cost shocks.[27] Note that regarding the choice-dependent cost shocks, their expected value is used and that only the terms in the last two lines, which depend on the social planner's drilling decisions, are associated with λ_a.

In the continuous-time DCDP framework, the social planner's welfare problem is given by

$$W^{sp*} = \max_{\{Pr_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-rt} W_t^{sp}\ dt \tag{28}$$

subject to

$$\dot{R}_t = -\lambda_a R_t Pr_t + E, \quad R_0 = R(0) = 1 \text{ given,} \tag{29}$$

$$R_t \geqslant 0, \quad 0 < Pr_t < 1. \tag{30}$$

As shown, W_t^{sp} is discounted at the rate of interest r.

Under the assumption of an interior solution (i.e., $Pr_t \in (0, 1)$), the current-value Hamiltonian-Lagrangian of the social planner's problem is given by

$$\begin{aligned} \mathcal{H}^{sp} =\ & R_t f_t \\ & + u(\alpha\lambda_a R_t Pr_t) - c(\lambda_a R_t Pr_t) \\ & + \lambda_a R_t \big\{ Pr_t \cdot \sigma\big(\gamma - \ln(Pr_t)\big) + (1 - Pr_t) \cdot \sigma\big(\gamma - \ln(1 - Pr_t)\big) \big\} \\ & + \pi_t(-\lambda_a R_t Pr_t + E). \end{aligned} \tag{31}$$

[25] Using D_t, we can re-write the definition: $W_t^{sp} = R_t f_t + u(\alpha D_t) - c(D_t) + D_t \sigma\big(\gamma - \ln(Pr_t)\big) + (\lambda_a R_t - D_t)\sigma\big(\gamma - \ln(1 - Pr_t)\big)$.

[26] W_t^{sp} can be interpreted differently. To be specific, W_t^{sp} is the sum of the flow payoff from the undrilled well sites at time t (i.e., $R_t f_t$), the net (expected) payoff obtained from the marginally drilled well site at time t $\big($i.e., $u(\alpha\lambda_a R_t Pr_t) - c(\lambda_a R_t Pr_t) + \lambda_a R_t Pr_t \sigma\big(\gamma - \ln(Pr_t)\big)\big)$, and the (expected) gains from the well sites that are available but decided not to drill at time t $\big($i.e., $\lambda_a R_t (1 - Pr_t)\sigma\big(\gamma - \ln(1 - Pr_t)\big)\big)$.

[27] The last line in equation (27) can be rewritten using e_0 and e_1:

$$\lambda_a R_t \big\{ Pr_t \cdot \sigma\big(\gamma - \ln(Pr_t)\big) + (1 - Pr_t) \cdot \sigma\big(\gamma - \ln(1 - Pr_t)\big) \big\} = \lambda_a R_t \big\{ Pr_t \cdot e_1 + (1 - Pr_t) \cdot e_0 \big\}.$$

The necessary conditions of the current-value Hamiltonian-Lagrangian are as follows:

$$\lambda_a R_t \big\{ \alpha u'(\alpha \lambda_a R_t Pr_t) \; - \; c'(\lambda_a R_t Pr_t) \; - \; \sigma \ln(Pr_t) \; + \; \sigma \ln(1 - Pr_t) \; - \; \pi_t \big\} \; \leqslant \; 0, \;\; 0 \; < \; Pr_t \; < \; 1, \tag{32}$$

$$\dot{\pi}_t \; = \; r\pi_t \; - \; \big\{ f_t \; + \; \lambda_a \sigma \big(\gamma \; - \; \ln(1 - Pr_t) \big) \big\}, \tag{33}$$

$$\lim_{t \to \infty} e^{-rt} (R_t \pi_t) \; = \; 0. \tag{34}$$

If $R_t > 0$, necessary condition (32) yields the following expression for the costate variable π_t, which is a function of R_t and Pr_t:

$$\begin{aligned} \pi_t \; &= \; \alpha u'(\alpha \lambda_a R_t Pr_t) \; - \; c'(\lambda_a R_t Pr_t) \; - \; \sigma \ln(Pr_t) \; + \; \sigma \ln(1 - Pr_t) \\ &= \; \big\{ \alpha u'(\alpha \lambda_a R_t Pr_t) \; - \; c'(\lambda_a R_t Pr_t) \; + \; \sigma \big(\gamma \; - \; \ln(Pr_t) \big) \big\} \; - \; \sigma \big(\gamma \; - \; \ln(1 - Pr_t) \big). \end{aligned} \tag{35}$$

In addition, substituting condition (35) into condition (33) yields the following Euler equation that governs the dynamics of the social planner's welfare maximization problem over time:

$$\begin{aligned} &\frac{\dot{\pi}_t \; + \; \big\{ f_t \; + \; \lambda_a \sigma \big(\gamma \; - \; \ln(1 - Pr_t) \big) \big\}}{r} \\ &= \; \alpha u'(\alpha \lambda_a R_t Pr_t) \; - \; c'(\lambda_a R_t Pr_t) \; + \; \sigma \big(\gamma \; - \; \ln(Pr_t) \big) \; - \; \sigma \big(\gamma \; - \; \ln(1 - Pr_t) \big). \end{aligned} \tag{36}$$

We will discuss the implications of necessary conditions (33) and (35) later.

The social planner's problem has a unique steady state (R_{ss}, π_{ss}) such that $\dot{R}_{ss}, \; \dot{\pi}_{ss} = 0$. From necessary conditions (29) and (33), R_t and π_t must satisfy the following equations at steady state:

$$\begin{cases} R_{ss} \; = \; \dfrac{E}{\lambda_a Pr_{ss}} \\ \pi_{ss} \; = \; \dfrac{f_t \; + \; \lambda_a \sigma \left(\gamma \; - \; \ln(1 - Pr_{ss}) \right)}{r}. \end{cases} \tag{37}$$

On top of the equations, necessary condition (35) has to hold at the steady state simultaneously. Solving the system of three equations, we can uniquely identify (R_{ss}, π_{ss}), including the value of the control variable at the steady state (i.e., Pr_{ss}). As implied by the first equation in (37), R_{ss} is non-zero positive if $E \neq 0$. Using a Taylor series approximation, $\dot{R}_t$ and $\dot{\pi}_t$ can be linearized near the steady state (R_{ss}, π_{ss}), and this linearization process shows that the steady state of the infinite-horizon maximization problem is a saddle point, which is demonstrated in Figure 5.[28]

[28] Regarding the saddle property, see *9.5 Steady states in autonomous infinite-horizon problems* in Leonard and Long (1992). And derivation details are provided in A.1.1.

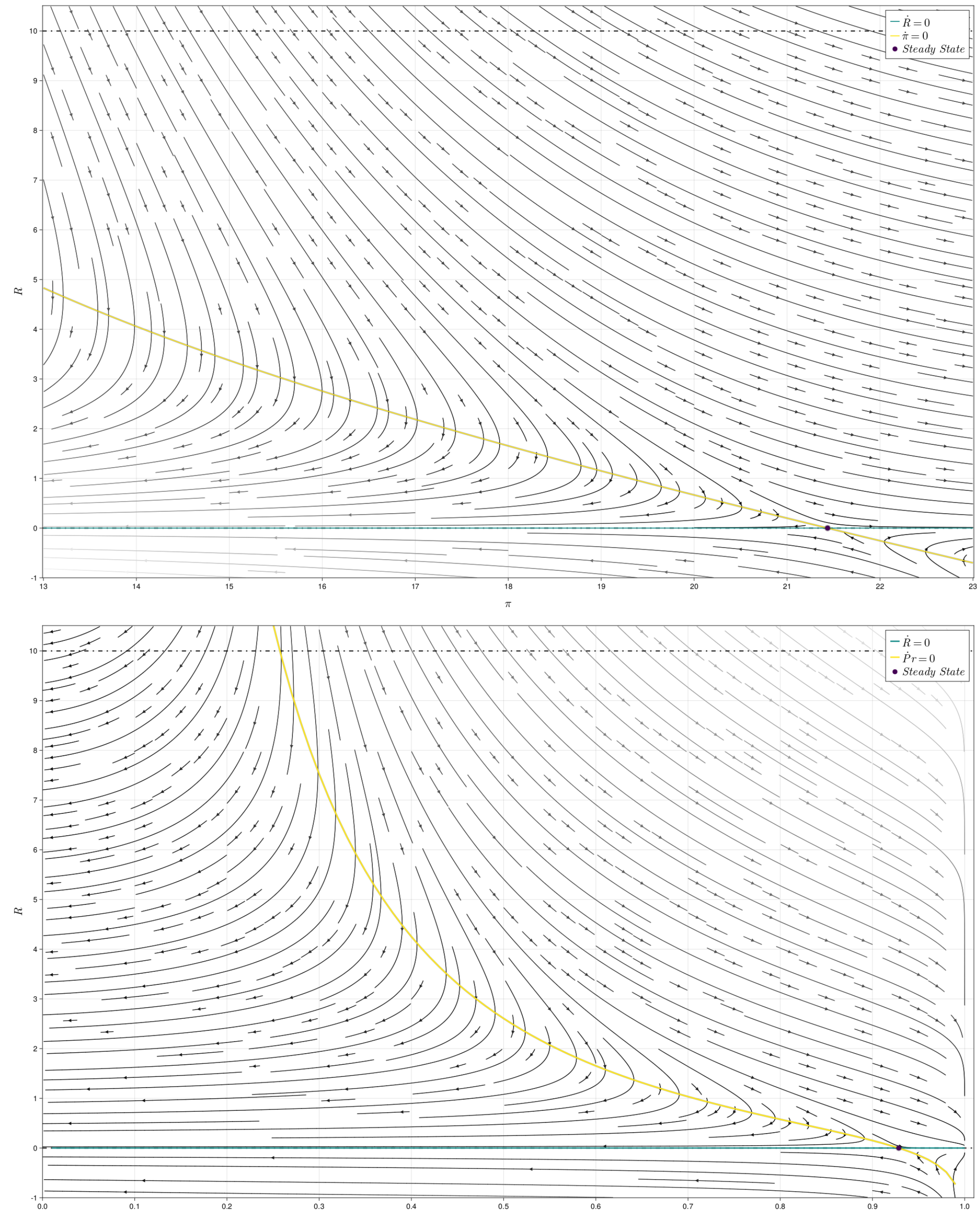

Figure 5: Phase Diagrams for the Social Planner's Problem

Note: This figure demonstrates two phase diagrams for the social planner's problem. The upper diagram illustrates that the steady state is exactly a saddle point. This figure assumes that a dispersion parameter of $\sigma = 1$, an interest rate of $r = 0.15$, an initial number of well sites of $R_0 = 10$, additional well sites of $E = 0$, and exogenously given constant oil prices of $p = 50$. Also, a linear cost function of $c(D_t) = 1 + 2D_t$ is assumed.

3.3.1 Implications of Necessary Conditions

Necessary conditions (33) and (35) have important economic implications of the optimal path of well sites' depletion. Firstly, necessary condition (33) demonstrates significant implications of π_t's growth over time. We can re-express this condition as follows[29]:

$$\dot{\pi}_t = r\left\{\pi_t - \frac{f_t + \lambda_a\sigma\big(\gamma - \ln(1-Pr_t)\big)}{r}\right\}. \tag{38}$$

This expression clearly indicates that π_t grows slower than the rate of interest r. The necessary condition also suggests that π_t increases concavely with Pr_t and converges in the limit, unlike the exponential growth of the shadow price in Hotelling's framework.[30]

Secondly, necessary condition (35) directly provides us with what π_t means. In this condition, the three terms in the curly bracket collectively mean the net benefit from the output (i.e., oils) produced from the marginally drilled well site at time t. The last term among them is the expected value of the cost shock for the marginal well location at time t when the social planner decides to drill it (i.e., e_{1t}). The remaining term in this condition represents the opportunity cost of drilling the marginal site at time t.[31] Hence, the necessary condition indicates that the costate variable π_t implies the net shadow value of the marginally drilled well site in the current-value term at time t.

Necessary condition (33) also enables us to understand what π_t means from a different perspective. We can re-write this necessary condition as follows:

$$\pi_t = e^{rt}\left[\lim_{T\to\infty}\int_t^T e^{-r\tau}\Big\{f_t + \lambda_a\sigma\big(\gamma - \ln(1-Pr_\tau)\big)\Big\}\,d\tau\right]. \tag{39}$$

This equation implies that π_t is the marginally undrilled well site's aggregate future payoff (i.e., the sum of the flow payoff and the expected value of ϵ_{0t}'s from time t and beyond), as the current value at time t, if the well site will remain undeveloped. Therefore, it is clear that drilling well locations is economic depletion in our framework, as extracting an exhaustible resource is in Hotelling's model.

From the above discussions, necessary conditions (35) and (39) collectively suggest that on the optimal path of drilling, the marginal undeveloped well site will be drilled at time t if the net gains from drilling it at time t equal the undrilled site' aggregate future gains from time t and beyond. In other words, at the margin, drilling a horizontal well today is an optimal choice if its value is indifferent to the value of simply holding it forever. Indeed, this implication holds under the Hotelling framework.

[29] Because $\dot{\pi}_t$ must be zero at the steady state, this equation implies that $\pi_t = \big\{f_t + \sigma\big(\gamma - \ln(1-Pr_t)\big)\big\}/r$ at the steady state. It is clear that both f_t and Pr_t must be a constant at the steady state. Based on this observation, we can draw the same expression for π_t at the steady state from equation (39): $\pi_t = e^{rt}\big\{f_t + \sigma\big(\gamma - \ln(1-Pr_t)\big)\big\}\int_t^\infty e^{-r\tau}\,d\tau = \big\{f_t + \sigma\big(\gamma - \ln(1-Pr_t)\big)\big\}/r$.

[30] From the beginning of drilling (i.e., $t=0$), R_t decreases. If Pr_t is maintained at the initial level, the rate of drilling will quickly converge to zero. So, Pr_t must continuously grow to keep drilling. In equation (38), the increase in Pr_t leads to the reduction in $\dot{\pi}_t$. Moreover, as discussed earlier, Pr_t approaches to Pr_{ss}.

[31] If the social planner decides not to drill a horizontal well into the marginal well site at time t, then the expected value of ϵ_{0t} conditional on $a_t = 0$ (i.e., e_{0t}) is the only gain the social planner gets from the decision.

In addition, the costate variable π_t can be expressed in terms of W_t^{sp}. Specifically, π_t, which indicates the net shadow value of the marginally drilled well site as the current value at time t, is the marginal welfare with respect to D_t as shown below:

$$\frac{\partial W_t^{sp}}{\partial D_t} = \alpha u'(\alpha D_t) - c'(D_t) + \sigma\big(\gamma - \ln(Pr_t)\big) - \sigma\big(\gamma - \ln(1 - Pr_t)\big). \tag{40}$$

That is, the marginally drilled site's shadow value is the same as the marginal welfare of drilling.

The value of σ, which is the dispersion parameter of the I.I.D. T1EV cost shocks, provides two interesting implications. First, the social planner's problem reverts to the Hotelling model of the optimal extraction of a nonrenewable resource when σ goes to zero. Taking limits to zero for necessary conditions (33) and (35) with the assumption of $f_t = 0$ yields the followings, which are identical to the two necessary conditions in Hotelling's classic model of depletion[32]:

$$\begin{cases} \lim\limits_{\sigma \to 0} \dot{\pi}_t = r\pi_t \\ \lim\limits_{\sigma \to 0} \pi_t = \alpha u'(\alpha \lambda_a R_t Pr_t) - c'(\lambda_a R_t Pr_t). \end{cases} \tag{41}$$

Intuitively, the limiting case that σ takes the value of zero means that the drilling decision for the marginal well location depends only on drilling costs and the interest rate r, which are not stochastic, unlike cost shocks ϵ_{at}'s.

Second, the magnitude of σ determines the rate of drilling, and also production. To be specific, an increase in the magnitude of σ reduces the drilling rate. When the value of σ grows, the importance of the cost shocks increases relative to the observable components in the payoff function (i.e., $\psi(\boldsymbol{X}_t)$). In other words, as more payoff comes from the cost shocks, the option value of each well location increases. Therefore, a larger value of σ makes the social planner wait for a better cost shock, which in turn, delays well drilling.

Transversality condition (34) rules out too aggressive depletion of well sites. Note that the transversality condition holds even when $R_t \neq 0$.

3.4 Firm's Problem

In this section, we develop the firm's problem under the settings of our DCDP model in continuous time. Following Arcidiacono et al. (2016), we formulate the value function for a particular potential well site owned by the firm i that is forward-looking and discounts future payoffs at rate $\rho \in (0, \infty)$. Specifically, when the site is in state k, its value function is given as follows[33]:

$$-\dot{V}_{ik} + \left(\rho + \lambda_a + \sum_{\ell \neq k} \lambda_{k\ell}\right) V_{ik} = f_{ik} + \lambda_a E\Big[\max_{a \in \mathcal{A}} \{V_{ik} + \psi_{iak} + \epsilon_{iak}\}\Big] + \sum_{\ell \neq k} \lambda_{k\ell} V_{i\ell}. \tag{42}$$

[32] We take the assumption of zero flow payoff because the payoff element is not introduced in Hotelling's theoretical model.
[33] Detailed derivation is presented in A.1.2.

For the site, the value function V_{ik} represents the present discounted value of all payoffs obtained from starting at state k and behaving optimally in all subsequent periods.[34] Here, it is assumed that the firm i's drilling decisions have no impact on the price of oil in the market.[35] $\dot{V}_{ik}$ is the time derivative of V_{ik}. The three terms in the round bracket on the left-hand side are the sum of the discount factor and the rates at which the state can change. The right-hand side consists of the flow payoff, the expected value relying on the firm's decisions, and the rate-weighted value related to exogenous state transitions. The expectation is for the joint distribution of ϵ_{i0k} and ϵ_{i1k}. While adding in the T1EV cost shocks is tedious, it also allows us to reconcile an empirical, firm-level model with aggregate time paths of resource extraction.

For a given opportunity to choose an action $a \in \mathcal{A}$, the probability of drilling a horizontal well into the potential site conditional on state k, denoted Pr_k, can be defined as follows[36]:

$$Pr_k \ \equiv \ \Pr\Big[\ \psi_{i1k} + \epsilon_{i1k} \ \geqslant \ V_{ik} + \psi_{i0k} + \epsilon_{i0k} \ | \ k \ \Big]. \tag{43}$$

As shown in Arcidiacono et al. (2016), for each action $a \in \mathcal{A}$, the second term on the right-hand side of equation (42) is

$$\lambda_a E\Big[\max_{a\in\mathcal{A}} \{V_{ik} + \psi_{iak} + \epsilon_{iak}\} \Big] \ = \ \underbrace{\lambda_a \left\{V_{ik} + \sigma\big(\gamma - \ln(1 - Pr_k)\big)\right\}}_{\text{if } \ a\,=\,0} \ = \ \underbrace{\lambda_a \left\{\psi_{i1k} + \sigma\big(\gamma - \ln(Pr_k)\big)\right\}}_{\text{if } \ a\,=\,1}, \tag{44}$$

where the choice-specific instantaneous payoff function for $a = 1$ (i.e., ψ_{i1k}) is the function (24).

Under the assumption that there is no exogenous demand shock, some algebraic manipulation on the value function (42), with the expressions in (44), yields the Euler equation that drives the dynamics of the firm's optimal drilling decisions[37]:

$$\frac{\dot{V}_{ik} \ + \ f_{ik} \ + \ \lambda_a\sigma\big(\gamma \ - \ \ln(1 - Pr_k)\big)}{\rho} \ = \ \alpha p_k \ - \ c \ + \ \sigma\big(\gamma \ - \ \ln(Pr_k)\big) \ - \ \sigma\big(\gamma \ - \ \ln(1 - Pr_k)\big). \tag{45}$$

The left-hand side of this equation represents the sum of the instantaneous change in V_{ik} and the firm's expected payoff when deciding not to drill a horizontal well into the site as the current value at the time point of decision. In the equation, the right-hand side, which is equal to V_{ik} as shown in A.1.3, is the firm's payoff if it chooses to drill a horizontal well into the site, including the opportunity cost of the decision. In other words, V_{ik} is the well location's net shadow value as the current value, which is represented as π_t in the necessary conditions for the social planner's problem. Based on the relationship between V_{ik} and π_t, it is evident that the Euler equation (45) drawn from the firm i's well-site-level decisions coincides with the Euler equation (36) of the social planner's

[34] Although V_{ik} varies over time, we omit the t subscript for simplicity.
[35] In other words, the assumption implies that the resulting state of taking action a by the firm i, denoted $\ell(i, a, k)$, is k.
[36] For given values of parameters, we can compute the value of each Pr_k, $k = 1, 2, \cdots, K$, by using value function iterations.
[37] See A.1.3 for details.

welfare maximization problem.[38]

4 Equilibrium Dynamics with Oil Prices

This section examines how the time paths for optimal drilling and production vary with oil prices. We first return to the social planner model in Section 3.3, allowing for endogenous prices but also assuming no aggregate uncertainty about prices. Using the Euler equation and Implicit Function Theorem, we predict the impact of a small, unanticipated change in prices on the drilling probability. We then use a numerical simulation to illustrate how the optimal drilling and production paths respond to unexpected demand shocks. We also compute the equilibrium paths under two different scenarios for oil prices: exogenous and endogenous oil prices. Finally, based on the firm's problem developed in Section 3.4, we investigate the heterogeneous impacts of expected demand shocks on drilling well sites of different quality.

4.1 Impacts of Unexpected Demand Shocks

The Implicit Function Theorem (IFT) allows us to predict the impact of sudden demand shocks on the equilibrium paths for drilling and production. Applying the IFT to equation (45) (i.e., the Euler equation of the firm's problem) yields

$$\frac{\partial Pr_k}{\partial p_k} = \left\{ \frac{\rho Pr_k(1 - Pr_k)}{\sigma(\lambda_a Pr_k + \rho)} \right\} \frac{\partial \psi_{i1k}}{\partial p_k}. \tag{46}$$

This resulting equation suggests that an unexpected positive price shock will lead to a higher drilling rate. The equation also shows that drilling is more responsive to prices when the relative importance of cost shocks, which is captured by the magnitude of σ, is small.

Figure 6 depicts how the paths of drilling probability, drilling, reserves, and oil price respond to two unanticipated demand shocks.[39] The first negative demand shock causes drilling probability discontinuously decreases. Due to the reduction in drilling probability, drilling demonstrates a discontinuous decrease too. Moreover, the negative demand shock also reduces the depletion rate of the remaining well locations. As shown in the last panel in the figure, the oil price jumps down on impact after the negative demand shock, then gradually rises.[40] The later positive demand shock induces the opposite reactions in the equilibrium paths.

[38] There are two differences between the Euler equations. First, the rate of interest r is used in the social planner's problem, whereas the discount rate ρ is utilized in the firm's problem. Second, we exploit two different choice-specific instantaneous payoff functions, which are demonstrated in (23) and (24), in the two dynamic optimization problems. Of note, both of the choice-specific instantaneous payoff functions indicate the net benefit obtained from the output of drilling activities.

[39] Given parameter values utilized for this simulation, the condition for the positive relationship between drilling probability and unexpected price shocks in (46) (i.e., $\partial \psi_{i1k}/\partial p_k - (1/\rho)(\partial f_{ik}/\partial p_k) > 0$) holds.

[40] Drilling, and thus production, rapidly diminishes for a while after $t = 0$. Then, its rate of change gradually decreases, and drilling eventually converges to a lower bound. In other words, the time path of drilling has a convex profile. Because equation (21) determines the oil price at time t, the time path for the endogenous oil price is a concave curve.

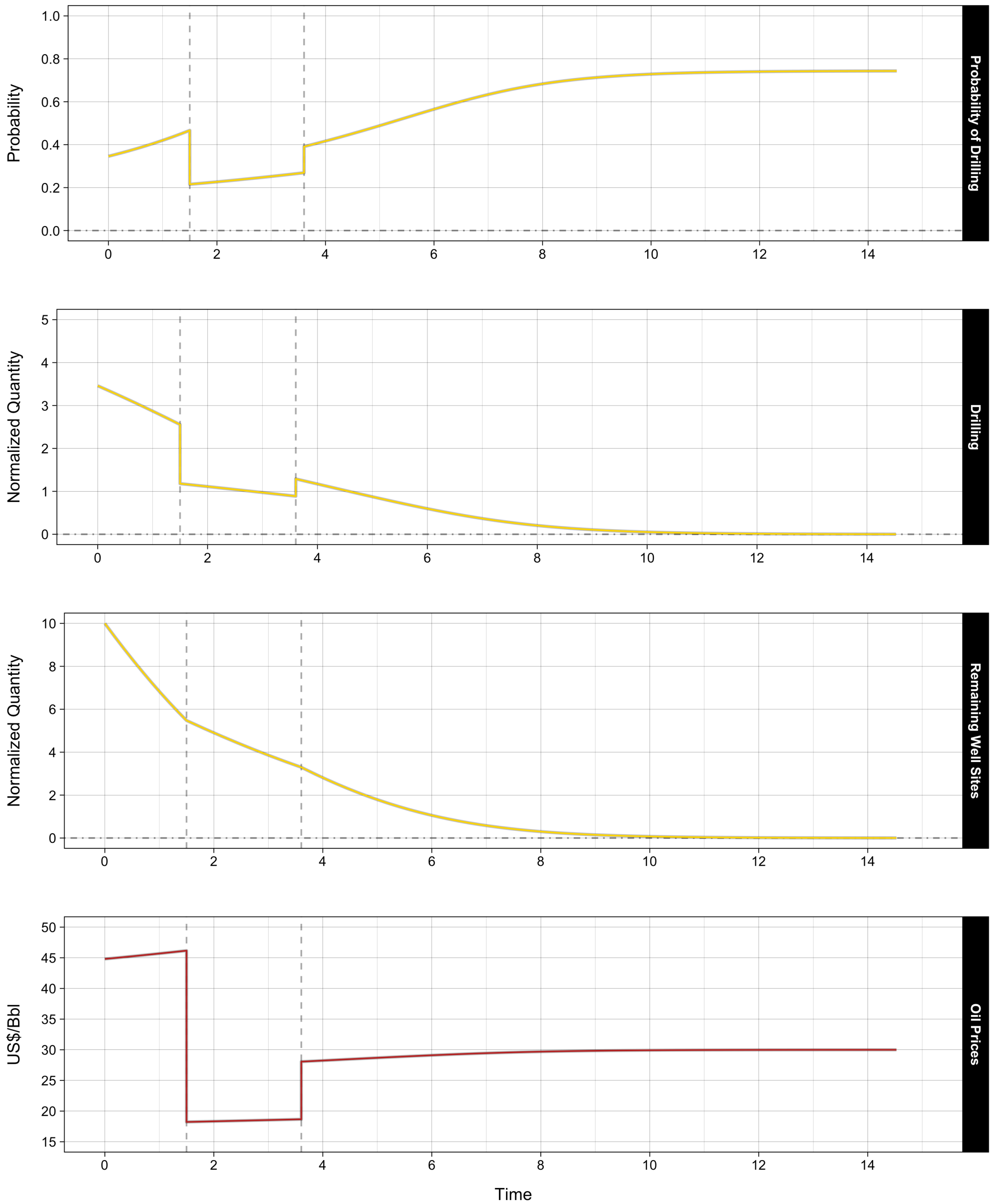

Figure 6: Equilibrium Paths under Unexpected Demand Shocks

Note: This figure shows the equilibrium paths of drilling probability, drilling, and the remaining well sites, which are obtained from a simulation for two unanticipated demand shocks. For these simulation results, we assume the identical parameter values and cost function utilized to draw the phase diagrams in Figure 5.

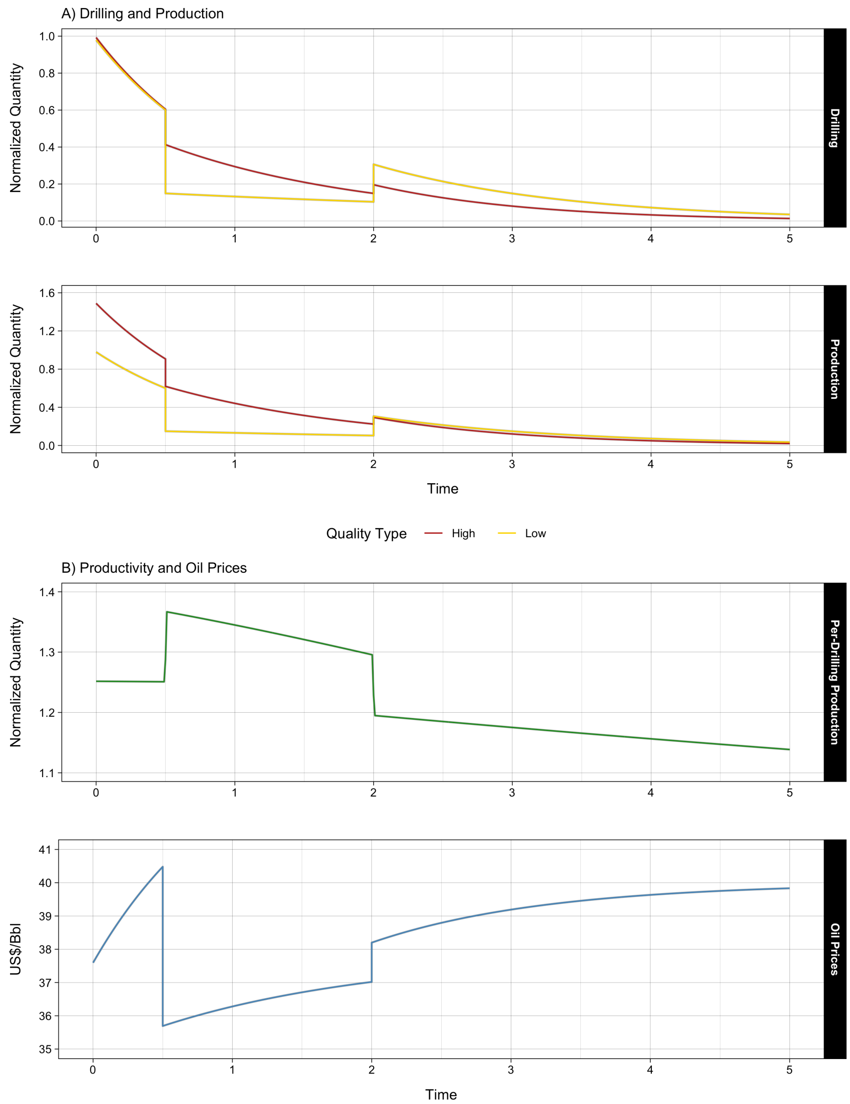

Figure 7: Heterogeneous Impacts of Unexpected Demand Shocks on Drilling and Production

Note: This figure demonstrates how the firm's drilling activity at well sites of heterogeneous quality responds to unexpected demand shocks. As shown in the first panel, the drilling probability of low-quality well sites shows a higher sensitivity to the first negative demand shock. The second panel demonstrates that although the drilling probability of low-quality well locations more sensitively responds to the second positive demand shock, the probability is still lower than that of high-quality well sites. The third panel illustrates the impacts of the demand shocks on oil extraction productivity. Clearly, the first negative shock discontinuously increases per-drilling oil production due to the high sensitivity of low-quality location drilling. To the second positive demand shock, the per-drilling production showed the opposite reaction. For this simulation, we assume that a dispersion parameter of $\sigma = 1$, an interest rate of $r = 0.05$, an initial number of well sites of $R_0^g = 1$, additional well sites of $E^g = 0$, where $g \in \{L, H\}$. Also, it is assumed that a flow payoff function of $f(p_k) = 20 - 0.5p_t$ and that an instantaneous payoff function of $\psi_{iak}(p_k, a) = [\{g_i^L + 1.5(1 - g_i^L)\}p_k - 2]a$.

4.2 Endogenous vs. Exogenous Oil Prices

This section examines the equilibrium path of drilling for each of the endogenous and exogenous oil prices. In the endogenous-price scenario, the market clearing oil price is determined from equation (21). Moreover, the exogenous price means the case of a constant oil price, in which the oil production industry is small relative to the world oil market.[41]

The Euler equation (45) allows us to predict the differences in drilling and production time paths between the two price scenarios. The endogenous price is lower than the exogenous price when there is oil production (i.e., $Q > 0$). In the Euler equation, a lower price, thus a lower instantaneous payoff of drilling (i.e., ψ_{i1k}), implies a lower probability of drilling. In other words, endogenizing the time path of oil prices makes the probability of drilling at time $t = 0$ decrease due to the initial production (i.e., $Q_0 > 0$). Since increasing drilling, and thus production, causes oil prices endogenously determined to fall, there would be no incentive to rapidly raise the rate of drilling. For these reasons, drilling under endogenous oil prices (denoted D_t^{en}) would be small relative to that under exogenous oil prices (denoted D_t^{ex}) for some period after $t = 0$. But at some time point, D_t^{en} would be larger than D_t^{ex} because of the lower level of undrilled reserves in the exogenous-price case. Figure 8, which shows the time paths for drilling and the remaining reserves for each type of oil price, supports the predictions.

4.3 Heterogeneous Impacts of Unanticipated Demand Shocks on Drilling Well Sites of Different Quality

To examine how the firm's drilling activity at well sites of heterogeneous quality responds to unexpected demand shocks, we suppose that a particular well location i has a quality type that falls in one from the quality set $\mathcal{Q} = \{L(ow), H(igh)\}$. Furthermore, we also assume that when the well location is not drilled yet, the choice-specific instantaneous payoff $\psi_{i\alpha k}$ takes the following functional form:

$$\psi_{iak}(p_k, a; \boldsymbol{\theta}_\psi) \ = \ \big[\big\{g_i^L \ + \ (1 - g_i^L)\alpha^H\big\}p_k \ - \ c\big]a, \tag{47}$$

where g_i^L is a binary indicator with the value of one when well location i is a low-quality well site and α^H, which is greater than 1, is the normalized oil production from the site i whose quality type is H.[42]

Figure 7 illustrates the results from a simulation. As discussed in Section 2.2.2, our empirical analysis reveals that fracking firms in North Dakota more significantly reduced drilling at low-quality well sites, more than at the high-quality ones, when experiencing sharp oil price declines. The time paths for drilling and production presented in the figure clearly show that the model-predicted elasticity of drilling s greater on low-quality well sites, just as illustrated in Figure 3.[43] As demonstrated in the third panel of the figure, per-drilling production,

[41] In other words, $\bar{p}_1 = 0$ in equation (21) in the exogenous-price scenario.

[42] In the formulation, we implicitly normalize the oil production from a well location to 1.

[43] The term $Pr_k(1 - Pr_k)$ in equation (46) is a parabolic curve that goes to zero as Pr_k approaches to zero or one and that has its maximum value at $Pr_k = 1/2$. These properties of the term suggest the possibility that the drilling of high-quality well sites is

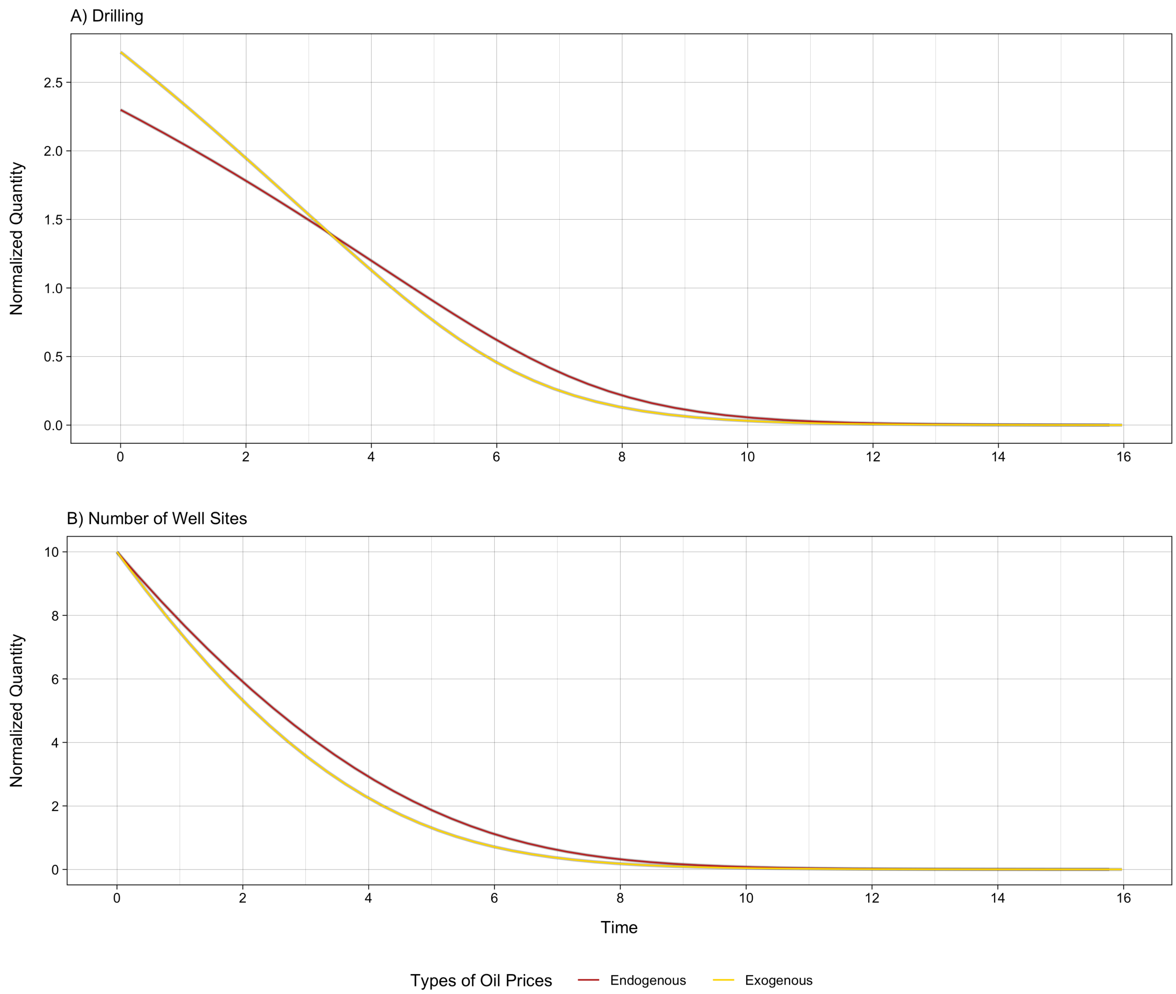

Figure 8: Time Paths for Drilling and Reserves under Endogenous and Exogenous Oil Prices

Note: This figure depicts differences in the time paths for drilling and the remaining well locations under endogenous and exogenous oil prices. This figure takes the assumptions exploited in Figure 5. See the text for details.

i.e., productivity, increases discontinuously due to the negative demand shock. Consequently, as described in the fourth panel, per-drilling oil production is also improved.

5 Conclusion

In this paper, we first present two interesting empirical findings about the drilling decisions of hydraulic fracturing companies in North Dakota. First, the oil producers drilled horizontal wells into sites of heterogeneous quality, not in a strict order, but simultaneously. Second, the drilling of low-quality well sites was more responsive to the significant plunge in oil prices in the second half of 2014 than that of high-quality ones.

less responsive to a demand shock.

We develop a continuous-time Discrete Choice Dynamic Programming (DCDP) to explain the empirical facts that are not captured in the economic models developed in other papers. Our theoretical framework is analytically tractable. Moreover, our economic model, in which choice-specific cost shocks at each decision opportunity allow us to avoid modeling specific constraints (e.g., capacity and transmission constraints), provides us with insightful implications for how the drilling activity of fracking firms operating in North Dakota evolves in response to oil price changes. Furthermore, in our model of optimal drilling, well-level drilling decisions naturally lead to the optimal firm-level path of drilling.

In addition to the analytical advantages, our model can also be estimable empirically. We can easily take our model to detailed well-level drilling and production data. Therefore, utilizing the empirically estimated values of structural parameters, we can perform counterfactual analysis on real-world issues in the oil and gas industry, such as a change in the severance tax rate in North Dakota. In other words, our theoretical framework can bridge Hotelling-style theoretical approaches with empirical research that relies heavily on microeconomic data.

A Appendixes

A.1 Details in Derivations

A.1.1 Linearization near the Steady State of the Social Planner's Problem

Using a Taylor series expansion, $\dot{R}_t$ can be approximated near the steady state (R_{ss}, π_{ss}):

$$\begin{aligned}
\dot{R}_t \;&\approx\; (-\lambda_a R_{ss} Pr_{ss} \;+\; E) \\
&\quad + \frac{\partial}{\partial R_t}(-\lambda_a R_t Pr_t \;+\; E)(R_t \;-\; R_{ss}) \;+\; \frac{\partial}{\partial \pi_t}(-\lambda_a R_t Pr_t \;+\; E)(\pi_t \;-\; \pi_{ss}) \\
&= 0 \;+\; \lambda_a \left(-Pr_t \;-\; \frac{\partial Pr_t}{\partial R_t} R_t\right)(R_t \;-\; R_{ss}) \;+\; \lambda_a \left(-\frac{\partial Pr_t}{\partial \pi_t} R_t\right)(\pi_t \;-\; \pi_{ss}) \\
&= \lambda_a \left(-Pr_t \;-\; \frac{\partial Pr_t}{\partial R_t} R_t\right)(R_t \;-\; R_{ss}) \;+\; \lambda_a \left(-\frac{\partial Pr_t}{\partial \pi_t} R_t\right)(\pi_t \;-\; \pi_{ss}).
\end{aligned}$$

In the same way, the linear approximation of $\dot{\pi}_t$ near the steady state (R_{ss}, π_{ss}) is given by

$$\begin{aligned}
\dot{\pi}_t \;&\approx\; \big(r\pi_{ss} \;-\; \lambda_a \sigma\big(\gamma \;-\; \ln(1 - Pr_{ss})\big)\big) \\
&\quad + \frac{\partial}{\partial R_t}\big(r\pi_t \;-\; \lambda_a \sigma\big(\gamma \;-\; \ln(1 - Pr_t)\big)\big)(R_t \;-\; R_{ss}) \\
&\quad + \frac{\partial}{\partial \pi_t}\big(r\pi_t \;-\; \lambda_a \sigma\big(\gamma \;-\; \ln(1 - Pr_t)\big)\big)(\pi_t \;-\; \pi_{ss}) \\
&= 0 \;+\; \left(-\frac{\lambda_a \sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial R_t}\right)(R_t \;-\; R_{ss}) \;+\; \left(r \;-\; \frac{\lambda_a \sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial \pi_t}\right)(\pi_t \;-\; \pi_{ss}) \\
&= \lambda_a \left(-\frac{\sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial R_t}\right)(R_t \;-\; R_{ss}) \;+\; \lambda_a \left(\frac{r}{\lambda_a} \;-\; \frac{\sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial \pi_t}\right)(\pi_t \;-\; \pi_{ss}).
\end{aligned}$$

From those two approximations, the linearized system near the steady state (R_{ss}, π_{ss}) is

$$\begin{aligned}
\begin{pmatrix} \dot{R}_t \\ \dot{\pi}_t \end{pmatrix} \;&=\; \lambda_a \begin{pmatrix} -Pr_t \;-\; \frac{\partial Pr_t}{\partial R_t} R_t & -\frac{\partial Pr_t}{\partial \pi_t} R_t \\ -\frac{\sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial R_t} & \frac{r}{\lambda_a} \;-\; \frac{\sigma}{1 - Pr_t}\frac{\partial Pr_t}{\partial \pi_t} \end{pmatrix} \begin{pmatrix} R_t \;-\; R_{ss} \\ \pi_t \;-\; \pi_{ss} \end{pmatrix} \\
&= \lambda_a \begin{pmatrix} [1] & [2] \\ [3] & [4] \end{pmatrix} \begin{pmatrix} R_t \;-\; R_{ss} \\ \pi_t \;-\; \pi_{ss} \end{pmatrix}.
\end{aligned}$$

Applying the Implicit Function Theorem to necessary condition (35), we can obtain the followings:

$$\begin{cases}
\dfrac{\partial Pr_t}{\partial R_t} \;=\; -\dfrac{\big(\alpha^2 u''(\lambda_a \alpha R_t Pr_t) - c''(\lambda_a R_t Pr_t)\big)\lambda_a Pr_t}{\big(\alpha^2 u''(\alpha \lambda_a R_t Pr_t) - c''(\lambda_a R_t Pr_t)\big)\lambda_a R_t \;-\; \frac{\sigma}{Pr_t(1 - Pr_t)}} \\
\dfrac{\partial Pr_t}{\partial \pi_t} \;=\; \dfrac{1}{\big(\alpha^2 u''(\alpha \lambda_a R_t Pr_t) - c''(\lambda_a R_t Pr_t)\big)\lambda_a R_t \;-\; \frac{\sigma}{Pr_t(1 - Pr_t)}}
\end{cases}$$

Because $\alpha^2 u''(\alpha \lambda_a R_t Pr_t) - c''(\lambda_a R_t Pr_t) < 0$ in our setting, $\partial Pr_t / \partial R_t < 0$ and $\partial Pr_t / \partial \pi_t < 0$.

In the coefficient matrix,

$$
\begin{aligned}
[1] &: \quad -Pr_t \; - \; \frac{\partial Pr_t}{\partial R_t} R_t \; = \; \frac{\frac{\sigma}{Pr_t(1-Pr_t)}}{\big(\alpha^2 u''(\alpha\lambda_a R_t Pr_t) - c''(\lambda_a R_t Pr_t)\big)\lambda_a R_t \; - \; \frac{\sigma}{Pr_t(1-Pr_t)}} \; < \; 0; \\
[2] &: \quad -\frac{\partial Pr_t}{\partial \pi_t} R_t \; > \; 0; \\
[3] &: \quad -\frac{\sigma}{1-Pr_t}\frac{\partial Pr_t}{\partial R_t} \; > \; 0; \text{ and} \\
[4] &: \quad \frac{r}{\lambda_a} \; - \; \frac{\sigma}{1-Pr_t}\frac{\partial Pr_t}{\partial \pi_t} \; > \; 0.
\end{aligned}
$$

Therefore, the determinant of the coefficient matrix clearly has a negative value (i.e., $[1] \times [4] - [2] \times [3] < 0$).

A.1.2 The Value Function for a Well Site i in state k in Continuous Time

In our framework, the instantaneous Bellman equation can be approximated as follows:

$$
\begin{aligned}
(1+\tau\rho)V_{ik}(t) \; \approxeq \; & \tau f_{ik} \\
& + \; \tau\lambda_a E\Big[\max_{a\in\mathcal{A}} \big\{V_{i,\ell(i,a,k)}(t+\tau) \; + \; \psi_{iak} \; + \; \epsilon_{iak}\big\}\Big] \\
& + \; \sum_{\ell\neq k} \tau\lambda_{k\ell} V_{i\ell}(t+\tau) \\
& + \; \left\{1 \; - \; \tau\left(\lambda_a \; + \; \sum_{\ell\neq k}\lambda_{k\ell}\right)\right\} V_{ik}(t+\tau),
\end{aligned}
$$

where $1+\tau\rho$ is the discount factor for the time increment τ, $\tau\lambda_a$ is the probability that the firm in state k choose an action a in an incremental time period τ, and $\sum_{\ell\neq k}\tau\lambda_{k\ell}$ is the probability of moving from state k to state ℓ. The curly bracket of the fourth line in the expression, therefore, means the probability that the firm remains at state k.

Rearranging terms, dividing by τ, and letting $\tau \to 0$ yield a simpler expression:

$$-\left\{V_{ik}(t+\tau) \;+\; V_{ik}(t)\right\} \;+\; \tau\rho V_{ik}(t) \;+\; \tau\left(\lambda_a \;+\; \sum_{\ell\neq k}\lambda_{k\ell}\right)V_{ik}(t+\tau)$$

$$\approxeq\; \tau f_{ik} \;+\; \tau\lambda_a E\Big[\max_{a\in\mathcal{A}}\left\{V_{i,\ell(i,a,k)}(t+\tau) \;+\; \psi_{iak} \;+\; \epsilon_{iak}\right\}\Big] \;+\; \sum_{\ell\neq k}\tau\lambda_{k\ell}V_{i\ell}(t+\tau)$$

$$-\frac{1}{\tau}\left\{V_{ik}(t+\tau) \;+\; V_{ik}(t)\right\} \;+\; \rho V_{ik}(t) \;+\; \left(\lambda_a \;+\; \sum_{\ell\neq k}\lambda_{k\ell}\right)V_{ik}(t+\tau)$$

$$\approxeq\; f_{ik} \;+\; \lambda_a E\Big[\max_{a\in\mathcal{A}}\left\{V_{i,\ell(i,a,k)}(t+\tau) \;+\; \psi_{iak} \;+\; \epsilon_{iak}\right\}\Big] \;+\; \sum_{\ell\neq k}\lambda_{k\ell}V_{i\ell}(t+\tau)$$

$$-\dot{V}_{ik}(t) \;+\; \left(\rho \;+\; \lambda_a \;+\; \sum_{\ell\neq k}\lambda_{k\ell}\right)V_{ik}(t)$$

$$=\; f_{ik} \;+\; \lambda_a E\Big[\max_{a\in\mathcal{A}}\left\{V_{i,\ell(i,a,k)}(t) \;+\; \psi_{iak} \;+\; \epsilon_{iak}\right\}\Big] \;+\; \sum_{\ell\neq k}\lambda_{k\ell}V_{i\ell}(t).$$

The assumption that the firm i's drilling decisions have no impact on the price of oil in the market (i.e., $\ell(i,a,k) = k$ for any $a \in \mathcal{A}$) yields an even simpler expression:

$$-\dot{V}_{ik}(t) \;+\; \left(\rho \;+\; \lambda_a \;+\; \sum_{\ell\neq k}\lambda_{k\ell}\right)V_{ik}(t)$$

$$=\; f_{ik} \;+\; \lambda_a E\Big[\max_{a\in\mathcal{A}}\left\{V_{ik}(t) \;+\; \psi_{iak} \;+\; \epsilon_{iak}\right\}\Big] \;+\; \sum_{\ell\neq k}\lambda_{k\ell}V_{i\ell}(t).$$

A.1.3 The Euler Equation for the Firm's Problem

The assumption of no exogenous demand shock suggests that $\lambda_{k\ell} = 0$, for all ℓ. Under this assumption, the firm's value function, which is presented in equation (42), is simplified as follows:

$$-\dot{V}_{ik}(t) \;+\; (\rho \;+\; \lambda_a)V_{ik}(t) \;=\; f_{ik} \;+\; \lambda_a E\Big[\max_{a\in\mathcal{A}}\left\{V_{ik}(t) \;+\; \psi_{iak} \;+\; \epsilon_{iak}\right\}\Big]$$

In addition, equation (44) allows yielding the even simpler functional form of the firm's value function. When $a = 0$,

$$V_{ik}(t) \;=\; \frac{\dot{V}_{ik}(t) \;+\; f_{ik} \;+\; \lambda_a\sigma\big(\gamma \;-\; \ln(1-Pr_k)\big)}{\rho}.$$

In the case of $a = 1$,

$$V_{ik}(t) \;=\; \frac{\dot{V}_{ik}(t) \;+\; f_{ik} \;+\; \lambda_a\left\{\psi_{i1k} \;+\; \sigma\big(\gamma \;-\; \ln(Pr_k)\big)\right\}}{\rho \;+\; \lambda_a}.$$

Equating the two equations, with some algebra, allows us having the Euler equation:

$$\frac{\dot{V}_{ik}(t) \ + \ f_{ik} \ + \ \lambda_a \sigma\big(\gamma \ - \ \ln(1 - Pr_k)\big)}{\rho} \ = \ \psi_{i1k} \ + \ \sigma\big(\gamma \ - \ \ln(Pr_k)\big) \ - \ \sigma\big(\gamma \ - \ \ln(1 - Pr_k)\big).$$

Similarly, $V_{ik}(t)$ can be expressed without $\dot{V}_{ik}(t)$ as follows:

$$V_{ik}(t) \ = \ \psi_{i1k} \ + \ \sigma\big(\gamma \ - \ \ln(Pr_k)\big) \ - \ \sigma\big(\gamma \ - \ \ln(1 - Pr_k)\big).$$

A.2 Additional Figure(s) and Table(s)

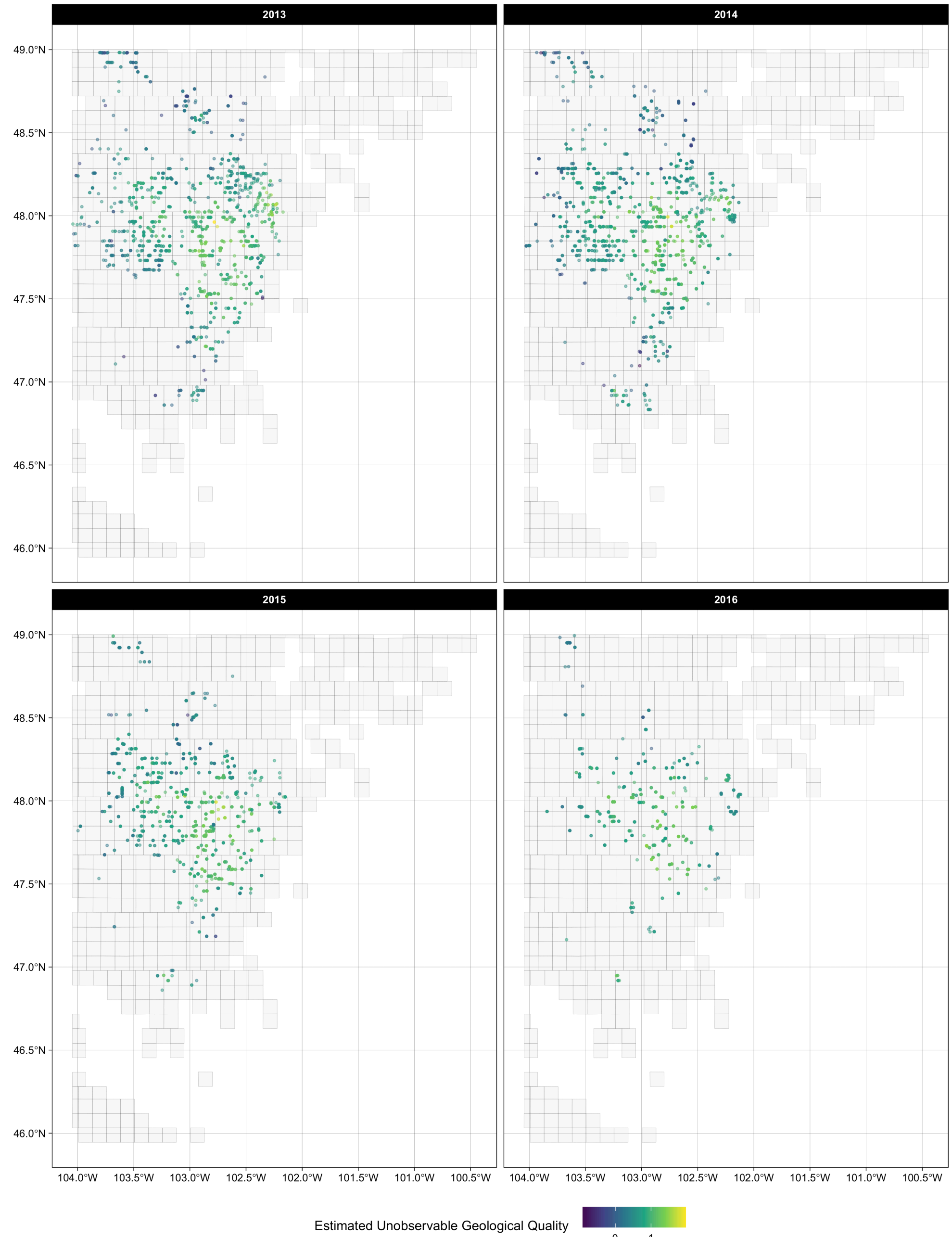

Figure 9: Spatial Distribution of the Estimated Geological Characteristic by Year

Note: This figure illustrates the estimated geologic feature for each horizontal well completed between 2013 and 2016 by year. In this figure, each square is a section in the Public Land Survey System, whereas each dot indicates an individual well's geological characteristic. It is apparent that the share of drilling of horizontal wells with (relatively) small estimates decreased significantly starting in 2014, corresponding to the beginning of the oil price crash.

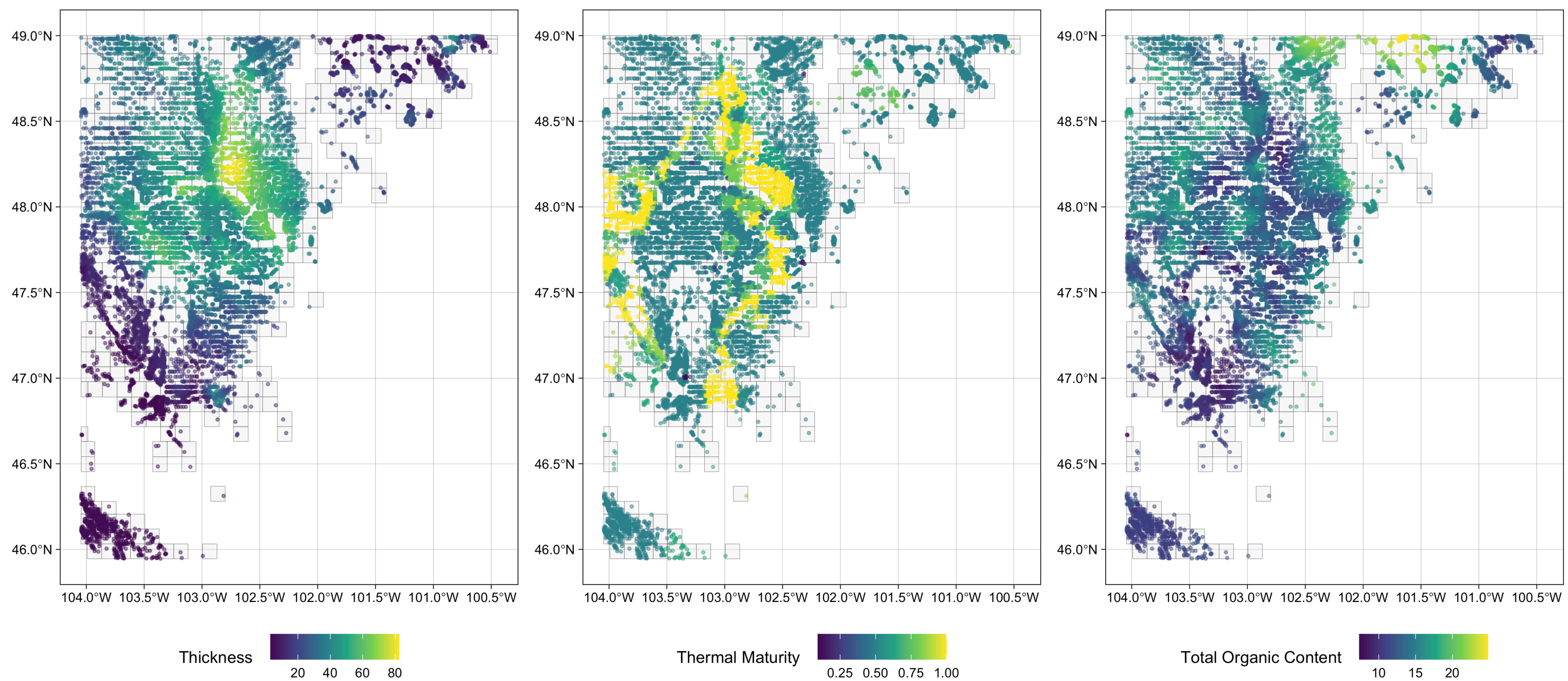

Figure 10: Spatial Distributions of Geological Characteristics

Note: This figure depicts the spatial distributions of three geological features—thickness, thermal maturity, and total organic contents, which are available from the NDGS' geological survey data— for the horizontal wells in our sample. In this figure, each square is a section in the Public Land Survey System, whereas each dot indicates an individual well's geological characteristic.

References

Agerton, Mark. 2020. "Learning where to drill: Drilling decisions and geological quality in the haynesville shale." *Available at SSRN 3583149.*

Aguirregabiria, Victor, and Arvind Magesan. 2013. "Euler Equations for the Estimation of Dynamic Discrete Choice Structural Models." Vol. 31, 3–44. Emerald Group Publishing Limited.

Anderson, Soren T., Ryan Kellogg, and Stephen W. Salant. 2018. "Hotelling under Pressure." *Journal of Political Economy*, 126(3): 984–1026.

Arcidiacono, Peter, Patrick Bayer, Jason R. Blevins, and Paul B. Ellickson. 2016. "Estimation of Dynamic Discrete Choice Models in Continuous Time with an Application to Retail Competition." *The Review of economic studies*, 83(3 (296)): 889–931.

Arrow, Kenneth J., and Sheldon Chang. 1982. "Optimal pricing, use, and exploration of uncertain natural resource stocks." *Journal of environmental economics and management*, 9(1): 1–10.

Boomhower, Judson. 2019. "Drilling Like There's No Tomorrow: Bankruptcy, Insurance, and Environmental Risk." *American Economic Review*, 109(2): 391–426.

Brehm, Paul A., and Eric Lewis. 2021. "Information asymmetry, trade, and drilling: evidence from an oil lease lottery." *The Rand journal of economics*, 52(3): 496–514.

Cameron, A. Colin, and P. K. Trivedi. 2005. *Microeconometrics : Methods and Applications.* Cambridge University Press.

Covert, Thomas R. 2015. "Experiential and Social Learning in Firms: the Case of Hydraulic Fracturing in the Bakken Shale." *Available at SSRN 2481321.*

Doytchinov, Bogdan, and Rachel Irby. 2010. "TIME DISCRETIZATION OF MARKOV CHAINS." *Pi Mu Epsilon journal*, 13(2): 69–82.

Farrow, Scott, and Jeffrey A. Krautkraemer. 1989. "Extraction at the intensive margin: Metal supply and grade selection in response to anticipated and unanticipated price changes." *Resources and energy*, 11(1): 1–21.

Fitzgerald, Timothy. 2015. "Experiential Gains with a New Technology: An Empirical Investigation of Hydraulic Fracturing." *Agricultural and Resource Economics Review*, 44(2): 83–105.

Gaudet, Gérard. 2007. "Natural resource economics under the rule of Hotelling." *The Canadian journal of economics*, 40(4): 1033–1059.

Gilbert, Richard J. 1979. "Optimal Depletion of an Uncertain Stock." *The Review of economic studies*, 46(1): 47–57.

Heaps, Terry. 1985. "The taxation of nonreplenishable natural resources revisited." *Journal of environmental economics and management*, 12(1): 14–27.

Herfindahl, Orris C., and David B. Brooks. 2015. *Resource economics : selected works of Orris C. Herfindahl. Routledge revivals*, Abingdon, Oxon ;:Routledge.

Herrnstadt, Evan M, Ryan Kellogg, and Eric Lewis. 2020. "The Economics of Time-Limited Development Options: The Case of Oil and Gas Leases." National Bureau of Economic Research Report 0898-2937.

Holland, Stephen P. 2003. "Extraction capacity and the optimal order of extraction." *Journal of environmental economics and management*, 45(3): 569–588.

Kellogg, Ryan. 2014. "The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling †." *American Economic Review*, 104(6): 1698–1734.

Lee, Ruiwen. 2019. "Does Self-Reporting Measure Up? Environmental Misreporting in the Bakken." Job Market Paper.

Leonard, Daniel, and Ngo Van Long. 1992. *Optimal control theory and static optimization in economics.* Cambridge ;:Cambridge University Press.

Lewis, Eric. 2019. "Patchwork Policies, Spillovers, and the Search for Oil and Gas." *American Economic Journal: Economic Policy*, 11(1): 380–405.

Muehlenbachs, Lucija, Elisheba Spiller, and Christopher Timmins. 2015. "The Housing Market Impacts of Shale Gas Development." *The American economic review*, 105(12): 3633–3659.

Pindyck, Robert S. 1978. "The Optimal Exploration and Production of Nonrenewable Resources." *The Journal of political economy*, 86(5): 841–861.

Pindyck, Robert S. 1980. "Uncertainty and Exhaustible Resource Markets." *The Journal of political economy*, 88(6): 1203–1225.

Pindyck, Robert S. 1981. "The Optimal Production of an Exhaustible Resource When Price Is Exogenous and Stochastic." *The Scandinavian Journal of Economics*, 83(2): 277–288.

Quyen, N. V. 1991. "Exhaustible Resources: A Theory of Exploration." *The Review of economic studies*, 58(4): 777–789.

Slade, Margaret E. 1982. "Trends in natural-resource commodity prices: An analysis of the time domain." *Journal of environmental economics and management*, 9(2): 122–137.

Stiglitz, Joseph. 1974. "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths." *The Review of economic studies*, 41(5): 123–137.

Sweeney, James L. 1977. "Economics of Depletable Resources: Market Forces and Intertemporal Bias." *The Review of economic studies*, 44(1): 125–141.

Swierzbinski, Joseph E., and Robert Mendelsohn. 1989. "Exploration and Exhaustible Resources: The Microfoundations of Aggregate Models." *International economic review (Philadelphia)*, 30(1): 175–186.

Thille, Henry, and Margaret Emily Slade. 2009. "Whither Hotelling: Tests of the Theory of Exhaustible Resources." *Annual review of resource economics*, 1(1): 239–259.