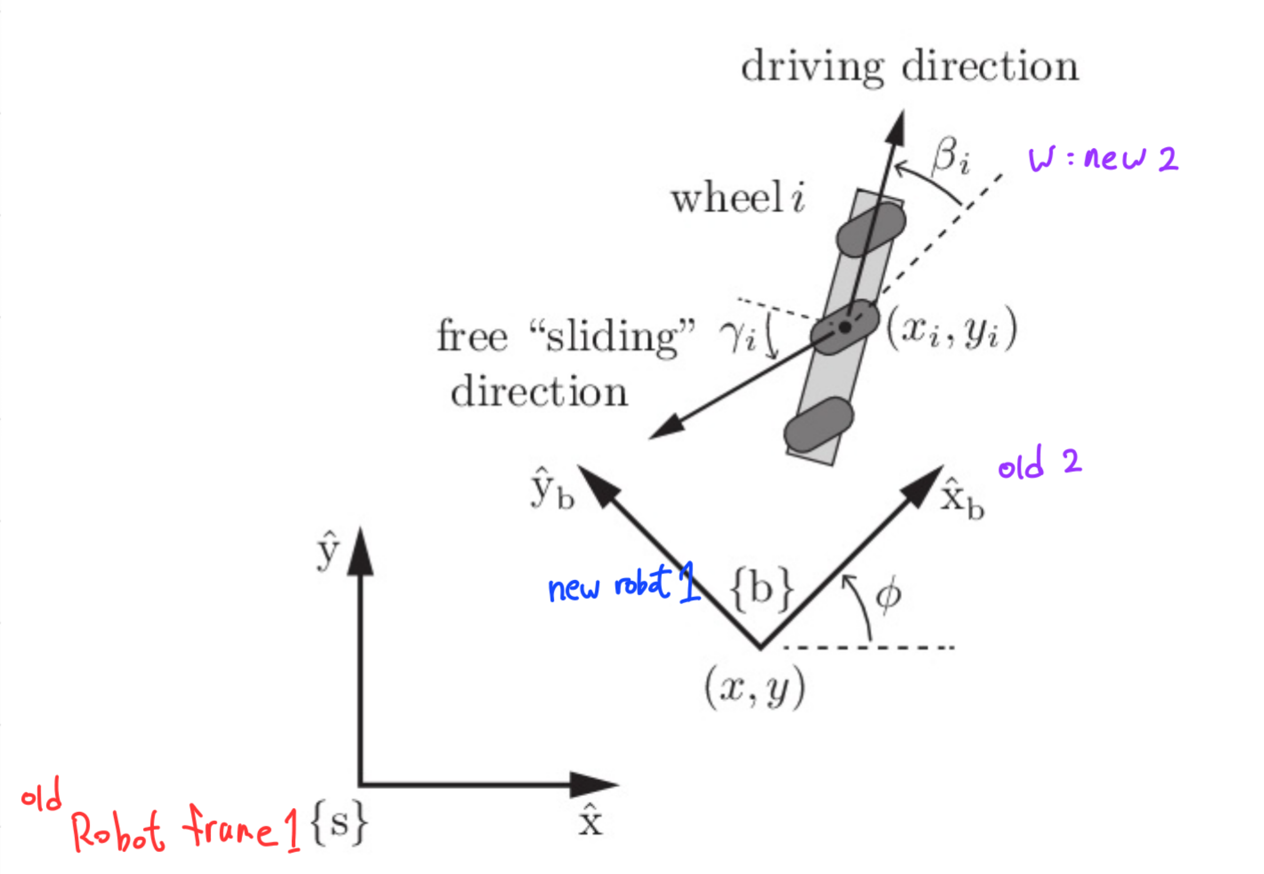


$$u_i = \frac{1}{r_i}\begin{bmatrix} 1 & \tan\gamma_i \end{bmatrix} \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}}_{\text{↳ wheel velocity }(w)}$$

– one mecanum wheel of kinematics

How to get v_x, v_y?

b^{new}

old

Robot frame S

$$u_i = \frac{1}{r_i}\begin{bmatrix} 1 & \tan\gamma_i \end{bmatrix} \begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \mathcal{V}_b$$

$$u_i = \frac{1}{r_i}\begin{bmatrix} 1 & \tan\gamma_i \end{bmatrix} \begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix}}_{\Downarrow} \begin{bmatrix} \dot{\phi} \\ \boxed{\begin{matrix} \dot{x} \\ \dot{y} \end{matrix}} \end{bmatrix}$$

$\#$ v_x, v_y

$$\mathcal{V}^{new} = \underbrace{R^{T}_{z,\phi}}_{} \mathcal{V}^{old} \quad \Leftarrow \quad p^{old} = \underbrace{R_{z,\phi}}_{} p^{new}$$

$$\text{↳ } R^T = \begin{bmatrix} c\phi & s\phi \\ -s\phi & c\phi \end{bmatrix} \qquad \text{↳ } R^{old}_{new} = \begin{bmatrix} c\phi & -s\phi \\ s\phi & c\phi \end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{r_i} & \frac{\tan\gamma_i}{r_i}\end{bmatrix}\underbrace{\begin{bmatrix}\cos\beta_i & \sin\beta_i\\ -\sin\beta_i & \cos\beta_i\end{bmatrix}}_{R_z^T \beta}\underbrace{\begin{bmatrix}-y_i & 1 & 0\\ x_i & 0 & 1\end{bmatrix}}\underbrace{\boxed{\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & \sin\phi\\ 0 & -\sin\phi & \cos\phi\end{bmatrix}}}_{b}\begin{bmatrix}\dot\phi\\ \dot x\\ \dot y\end{bmatrix}$$

b: old → w: new

Center of robot frame to Center of each wheel trans. matrix

$$\begin{bmatrix}\frac{1}{r_i} & \frac{\tan\gamma_i}{r_i}\end{bmatrix}\underbrace{\begin{bmatrix}\cos\beta_i & \sin\beta_i\\ -\sin\beta_i & \cos\beta_i\end{bmatrix}}_{\text{rotating frome }\{b\}\text{ to }\{w\}}\underbrace{\begin{bmatrix}-y_i & 1 & 0\\ x_i & 0 & 1\end{bmatrix}}_{\text{translating to the wheel } i \text{ center}}\underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & \sin\phi\\ 0 & -\sin\phi & \cos\phi\end{bmatrix}}_{\text{going frame }\{s\}\text{ to }\{b\}}\begin{bmatrix}\dot\phi\\ \dot x\\ \dot y\end{bmatrix}$$

$$u_i = \underbrace{\underbrace{\frac{1}{r_i}}_{\text{wheel radius}}\underbrace{\begin{bmatrix}1 & \tan\gamma_i\end{bmatrix}}_{\text{component in driving direction}}\underbrace{\begin{bmatrix}\cos\beta_i & \sin\beta_i\\ -\sin\beta_i & \cos\beta_i\end{bmatrix}}_{\text{linear velocity at wheel, in wheel frame}}\underbrace{\begin{bmatrix}-y_i & 1 & 0\\ x_i & 0 & 1\end{bmatrix}}_{\text{linear velocity at wheel, in }\{b\}}}_{1 \times 3 \text{ vector.}}\mathcal{V}_b$$

angular velocity

$$\mathcal{V}_b = \begin{bmatrix}\omega_{bz}\\ \boxed{\begin{matrix}v_{bx}\\ v_{by}\end{matrix}}\end{bmatrix}$$ body twist.

linear velocity

$$h_i(\phi) = \frac{1}{r_i \cos\gamma_i} \begin{bmatrix} x_i \sin(\beta_i + \gamma_i) - y_i \cos(\beta_i + \gamma_i) \\ \cos(\beta_i + \gamma_i + \phi) \\ \sin(\beta_i + \gamma_i + \phi) \end{bmatrix}^T$$

$$\boxed{\underset{1\times 1}{u_i} = \underset{1\times 3}{h_i(\phi)} \cdot \underset{3\times 1}{\dot{q}} = \begin{bmatrix} h_1(\phi) \\ \vdots \\ h_m(\phi) \end{bmatrix} \underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{V_d}}$$

Inverse Velocity eq's.

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

↳ number of wheel

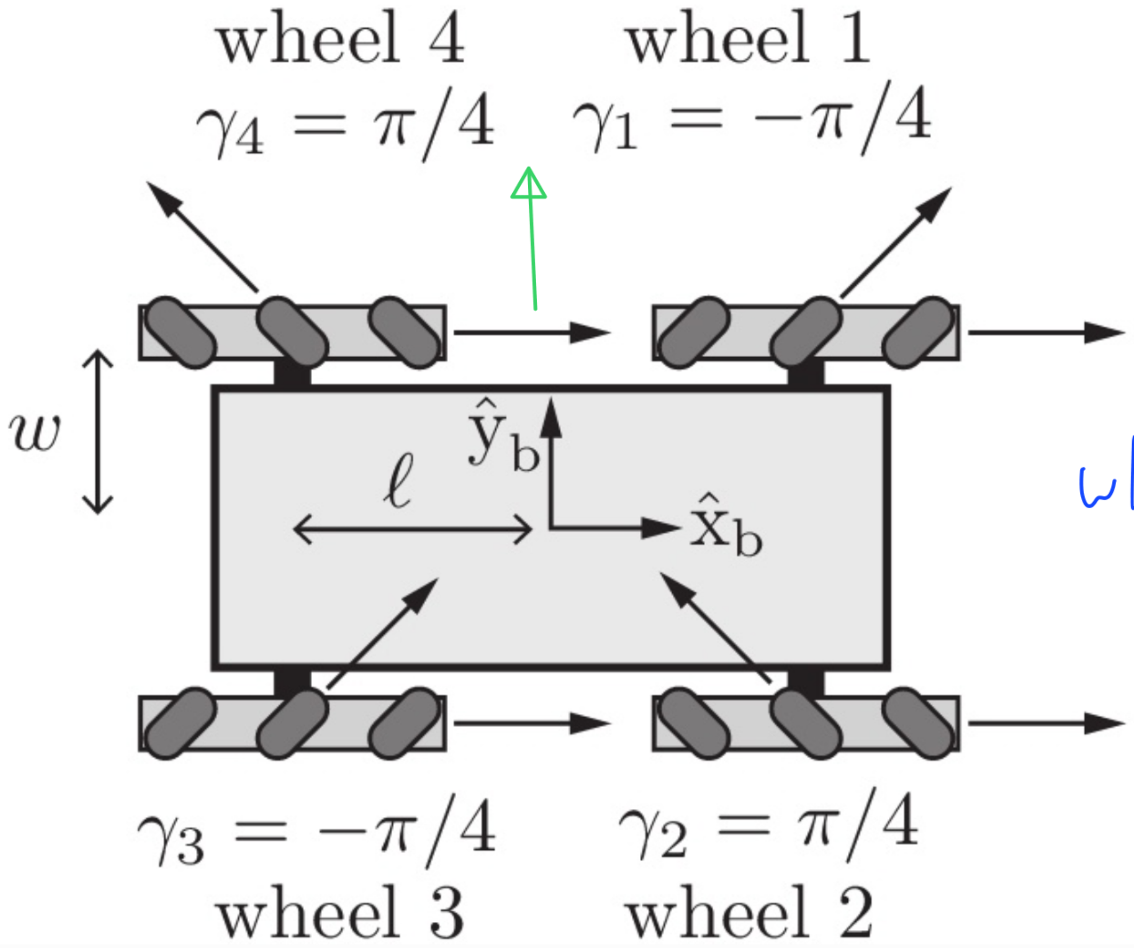
$$\dot{q} = H^{-1} \vec{u}$$

Ex) wheel 3 → $h_i(\phi)$ 3×3 ⇒ it can easily Inverse

But, whee 4 → $h_i(\phi)$ 4×3 ⇒ it can't easily Inverse ⇒ Pseudo Inverse.

It cannot give me exactly value
It gives me minimum error

mecanum wheel $\beta_i = 0$



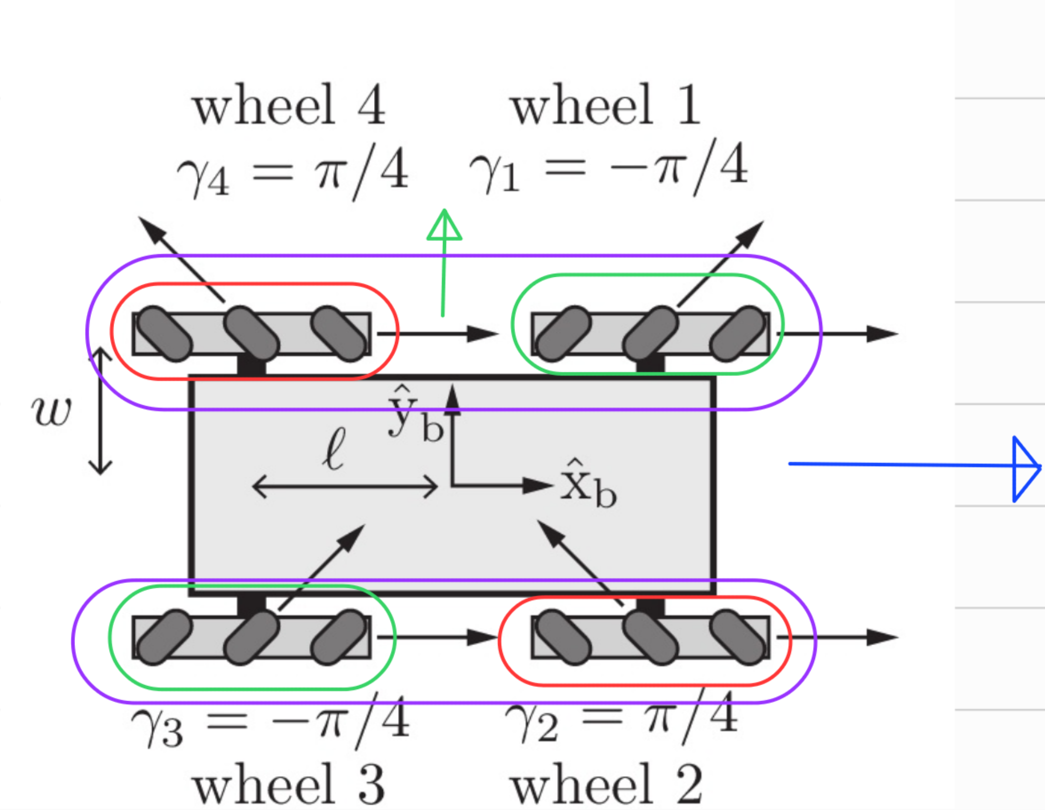
wheel 4

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = H(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} -\ell - w & 1 & -1 \\ \ell + w & 1 & 1 \\ \ell + w & 1 & -1 \\ -\ell - w & 1 & 1 \end{bmatrix} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

U_m O $\longrightarrow$ W_{bz}, V_{bx}, V_{by} ?

Pseudo Inverse $\Rightarrow$ error

(It means sliding)



$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = H(0)\mathcal{V}_b = \underbrace{\boxed{\frac{1}{r}\begin{bmatrix} -\ell - w & 1 & -1 \\ \ell + w & 1 & 1 \\ \ell + w & 1 & -1 \\ -\ell - w & 1 & 1 \end{bmatrix}}}_{H(0)} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

if) $\begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix} \Rightarrow$ forward $\Rightarrow u_1 = u_2 = u_3 = u_4$

$\boxed{\text{Same rpm.}}$

$\begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} \Rightarrow$ Sideway $\Rightarrow$

$u_1 = u_3$

$u_2 = u_4$

$u_1 = -u_2$

$$\begin{bmatrix} w \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{Rotate} \Rightarrow \begin{matrix} u_1 = u_4 \\ u_2 = u_3 \\ u_1 = -u_2 \end{matrix}$$