

a)  $x_1(t), x_2(t) \in \mathbb{R}, \mathbb{C}$ , su distancia media se expresa a partir de la potencia media de su diferencia

$$d^2(x_1, x_2) = \overline{P}_{x_1 - x_2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

$$x_1(t) = A \cdot e^{j\omega_0 t}, \quad x_2(t) = B e^{j5 \cdot \omega_0 t}$$

$$\text{con } \omega_0 = \frac{2\pi}{T}; \quad T, A, B \in \mathbb{R}^+,$$

Determinar la distancia entre las señales

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |x_1(t) - x_2(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left( \int_0^T |x_1(t)|^2 dt - 2 \int_0^T x_1(t) x_2(t) dt + \int_0^T |x_2(t)|^2 dt \right)$$

$$\text{Ya que } \int_0^T |x(t)|^2 dt = \int_0^T x(t) \cdot x^*(t) dt$$

Entonces:

$$\int_0^T |x_1(t)|^2 dt = \int_0^T x_1(t) \cdot x_1^*(t) dt$$

$$= \int_0^T A \cdot e^{j\omega_0 t} \cdot A e^{-j\omega_0 t} dt = A^2 \int_0^T e^0 dt = A^2 \cdot t \Big|_0^T$$

$$= A^2 \cdot T = \overline{P}_{x_1} //$$

$$\text{Al igual que } \int_0^T |x_2(t)|^2 dt = B^2 \cdot T = \overline{P}_{x_2} //$$

$$- 2 \int_0^T x_1(t) \cdot x_2(t) dt = -2 \int_0^T A \cdot e^{j\omega_0 t} \cdot B e^{j5\omega_0 t} dt$$

$$= -2AB \int_0^T e^{j6\omega_0 t} dt, \quad u = j6\omega_0 t \Rightarrow du = j6\omega_0 dt \Rightarrow dt = \frac{du}{j6\omega_0}$$

$$= \frac{-2AB}{j6\omega_0} \int_0^T e^u du = \frac{-2AB}{j6\omega_0} \cdot e^{j6\omega_0 t} \Big|_0^T$$

$$= \frac{-2AB}{j6\omega_0} \cdot (e^{j6\omega_0 T} - 1) = \frac{-AB}{j3 \frac{2\pi}{T}} \cdot (e^{j6 \cdot \frac{2\pi}{T} \cdot T} - 1)$$

$$= -\frac{AB \cdot T}{j6\pi} \cdot (e^{j12\pi} - 1)$$

Reemplazando:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \left( \int_0^T |x_1(t)|^2 dt - 2 \int_0^T x_1(t) \cdot x_2(t) dt + \int_0^T |x_2(t)|^2 dt \right)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left( A^2 \cdot T - \frac{ABT}{j6\pi} (e^{j12\pi} - 1) + B^2 T \right)$$

$$= A^2 - \frac{AB}{j6\pi} (e^{j12\pi} - 1) + B^2$$

b) convirtiendo la señal continua  $x(t)$  a tiempo discreto tenemos:

$$t \rightarrow n \cdot T_s = \frac{n}{f_s}$$

entonces  $x(t) \rightarrow x[n]$

$$x(t) = 3 \cdot \cos(1000\pi t) + 5 \sin(2000\pi t) + 10 \cos(11000\pi t)$$

$$\rightarrow x[n] = 3 \cos\left(\frac{1000\pi \cdot n}{5000}\right) + 5 \sin\left(\frac{2000\pi n}{5000}\right) + 10 \cos\left(\frac{11000\pi n}{5000}\right)$$

$$= 3 \cos\left(\frac{\pi}{5} n\right) + 5 \sin\left(\frac{2\pi}{5} n\right) + 10 \cos\left(\frac{11\pi}{5} n\right)$$

$$\frac{11\pi}{5} - 2\pi = \frac{\pi}{5}$$

Fuera de rango  
copia.

$$x[n] = 13 \cos\left(\frac{\pi}{5} n\right) + 5 \sin\left(\frac{2\pi}{5} n\right) //$$

Comprobando la quasiperiodicidad:

$$\Omega_1 = \pi/5$$

$$\Omega_2 = 2\pi/5$$

$$\frac{\Omega_1}{\Omega_2} = \frac{\pi/5}{2\pi/5} = \frac{1}{2} \in \mathbb{Q}$$

$$\omega_1 = 1000\pi$$

$$\omega_2 = 2000\pi$$

$$\omega_3 = 11000\pi$$

$$\omega_1/\omega_2 = \frac{1000\pi}{2000\pi} = \frac{1}{2} \in \mathbb{Q}$$

$$\omega_1/\omega_3 = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in \mathbb{Q}$$

$$\omega_2/\omega_3 = \frac{2000\pi}{11000\pi} = \frac{2}{11} \in \mathbb{Q}$$

Por lo tanto  $x(t)$  es quasiperiódica.

Una vez que se encontró que la señal no fue representada correctamente, se procede a discretizar con la nueva frecuencia de muestreo:

$$t = nT_s = \frac{n}{f_s} = n/11000$$

entonces  $x(t) \rightarrow x[n]$

$$x[n] = 3 \cos\left(\frac{1000\pi n}{11000}\right) + 5 \cdot \sin\left(\frac{2000\pi n}{11000}\right) + 10 \cos\left(\frac{4000\pi n}{11000}\right)$$

$$x[n] = 3 \cos\left(\frac{\pi}{11} n\right) + 5 \sin\left(\frac{2\pi}{11} n\right) + 10 \cos(\pi n) //$$



c) Para la nueva señal de entrada  $x(t)$  se comprueba su quasiperiodicidad.

$$x(t) = 20 (\cos(t/3) + \cos(t/4)) \text{ [A]}$$

$$\omega_1 = t/3 = 2\pi/T_1 \Rightarrow T_1 = 2\pi/\omega_1 = 2\pi/(1/3) = 6\pi$$

$$\omega_2 = t/4 = 2\pi/T_2 \Rightarrow T_2 = 2\pi/\omega_2 = 2\pi/(1/4) = 8\pi$$

Calculando MCM( $T_1, T_2$ ):

$6\pi$	$8\pi$	$2\pi$
3	4	2
3	2	2
3	1	3

$$\Rightarrow \text{MCM} = 24\pi = T //$$

Comprobando que sea quasiperiódica

$$\frac{\omega_1}{\omega_2} = \frac{6\pi}{8\pi} = \frac{3}{4} \in \mathbb{Q} \rightarrow \text{es quasiperiódica}$$