a) $x_1(t), x_2(t) \in \mathbb{R}, \mathbb{C}$, su distancia media se expresa a partir de la potencia media de su diferencia

$$d^2(x_1, x_2) = \overline{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt$$

$$x_1(t) = A \cdot e^{j\omega_0 t} \quad , \quad x_2(t) = B e^{j \cdot 5 \cdot \omega_0 t}$$

con $\omega_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$,

Determinar la distancia entre las señales

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T |x_1(t) - x_2(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left( \int_0^T |x_1(t)|^2 dt - 2 \int_0^T x_1(t) x_2(t) \, dt + \int_0^T |x_2(t)|^2 dt \right)$$

Ya que $\int |x(t)|^2 dt = \int x(t) \cdot x^*(t) \, dt$

Entonces:

$$\int_0^T |x_1(t)|^2 dt = \int_0^T x_1(t) \cdot x_1^*(t) \, dt$$

$$= \int_0^T A \cdot e^{j\omega_0 t} \cdot A e^{-j\omega_0 t} dt = A^2 \int_0^T e^0 \, dt = A^2 \cdot t \Big|_0^T$$

$$= A^2 \cdot T = \overline{P}_{x_1}$$

Al igual que $\int_0^T |x_2(t)|^2 dt = B^2 \cdot T = \overline{P}_{x_2}$

$$-2 \int_0^T x_1(t) \cdot x_2(t) \, dt = -2 \int_0^T A \cdot e^{j\omega_0 t} \cdot B e^{j5\omega_0 t} dt$$

$$= -2AB \int_0^T e^{j6\omega_0 t} dt \quad , \quad u = j6\omega_0 t \Rightarrow du = j6\omega_0 dt \Rightarrow dt = \frac{du}{j6\omega_0}$$

$$= \frac{-2AB}{j6\omega_0} \int_0^T e^u \, du = \frac{-2AB}{j6\omega_0} \cdot e^{j6\omega_0 t} \Big|_0^T$$

$$= \frac{-2AB}{j6\omega_0} \cdot \left(e^{j6\omega_0 T} - \cancel{e^0}^{1}\right) = \frac{-AB}{j3\frac{2\pi}{T}} \cdot \left(e^{j6\cdot\frac{2\pi}{\cancel{T}}\cancel{T}} - 1\right)$$

$$= -\frac{AB\cdot T}{j6\pi} \cdot \left(e^{j12\pi} - 1\right)$$

Reemplazando en:

$$\lim_{T\to\infty} \frac{1}{T}\left(\int_0^T |x_1(t)|^2 dt - 2\int_0^T x_1(t)\cdot x_2(t)\, dt + \int_0^T |x_2(t)|^2 dt\right)$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\left(A^2\cdot\cancel{T} - \frac{AB\cancel{T}}{j6\pi}\left(e^{j12\pi} - 1\right)\cdot\cancel{T} + B^2\cancel{T}\right)$$

$$= A^2 - \frac{AB}{j6\pi}\left(e^{j12\pi} - 1\right) + B^2$$

b) Convirtiendo la señal continua $x(t)$ a tiempo discreto tenemos:

$$t \rightarrow n \cdot T_s = \frac{n}{f_s}$$

entonces $\quad x(t) \rightarrow x[n]$

$$x(t) = 3 \cdot \cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$$

$$\rightarrow x[n] = 3\cos\left(\frac{1000\pi \cdot n}{5000}\right) + 5 \cdot sen\left(\frac{2000\pi n}{5000}\right) + 10\cos\left(\frac{11000 t n}{5000}\right)$$

$$= 3\cos\left(\frac{\pi}{5} n\right) + 5\,sen\left(\frac{2\pi}{5} n\right) + \underbrace{10\cos\left(\frac{11\pi}{5} t n\right)}_{\text{Fuera de rango copia}}$$

$$\frac{11\pi}{5} - 2\pi = \frac{\pi}{5}$$

$$x[n] = 13\cos\left(\frac{\pi}{5} n\right) + 5\,sen\left(\frac{2\pi}{5} n\right) \;//$$

Comprobando la quasiperiodicidad:

$\Omega_1 = \pi/5 \qquad \frac{\Omega_1}{\Omega_2} = \frac{\pi/5}{2\pi/5} = \frac{1}{2} \in \mathbb{Q}$

$\Omega_2 = 2\pi/5$

$\omega_1 = 1000\pi \qquad \omega_1/\omega_2 = \frac{1000\pi}{2000\pi} = \frac{1}{2} \in \mathbb{Q}$

$\omega_2 = 2000\pi \qquad \omega_1/\omega_3 = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in \mathbb{Q}$

$\omega_3 = 11000\pi$

$\omega_2/\omega_3 = \frac{2000\pi}{11000\pi} = \frac{2}{11} \in \mathbb{Q}$

Por lo tanto $x(t)$ es quasiperiódica.

c) Para la nueva señal de entrada $x(t)$ se comprueba su quasiperiodicidad.

$$x(t) = 20\,(\cos(t/3) + \cos(t/4))\ [A]$$

$$\omega_1 = t/3 = 2\pi/T_1 \Rightarrow T_1 = 2\pi/\omega_1 = 2\pi/(1/3) = 6\pi$$

$$\omega_2 = t/4 = 2\pi/T_2 \Rightarrow T_2 = 2\pi/\omega_2 = 2\pi/(1/4) = 8\pi$$

Calculando MCM $(T_1, T_2)$:

$$\begin{array}{cc|c} 6\pi & 8\pi & 2\pi \\ 3 & 4 & 2 \\ 3 & 2 & 2 \\ 3 & 1 & 3 \\ 1 & 1 & \end{array} \Rightarrow MCM = 24\pi = T$$

Comprobando que sea quasiperiódica

$$\frac{\omega_1}{\omega_2} = \frac{6\pi}{8\pi} = \frac{3}{4} \in \mathbb{Q} \rightarrow \text{es quasiperiódica}$$