Simulation

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Introduction

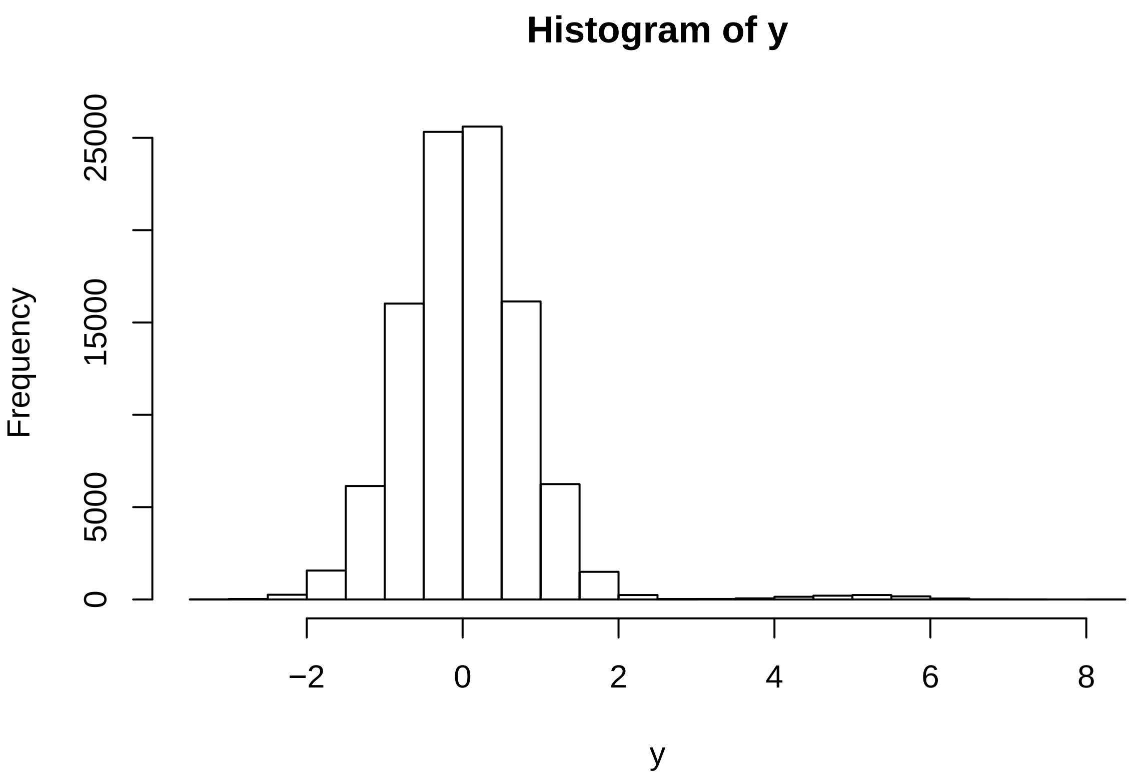
We would like to study if under the setting of outcome depending sampling the conditional likelihood approach is still a valid approach to inference. To study this, I began writing a simulation.

Model

The model I study in my simulation is,

$$Y_{ij} = \beta_0 + \beta X_i + U_i + \epsilon_{ij}.$$

Which is simply a linear model with subject-specific intercept. In my simulation I made X_i a bernoulli random variable with $P(X_i = 1) = 1 - P(X_i = 0) = 0.01$. Where $X_i = 1$ denotes a case and $X_i = 0$ denotes a control. Also, I made X_i fixed within-subject (which may be obvious from notation). However, the errors within-subject are positively correlated with each other. Code for the simulation is shown below, but here is a sample of the histogram of Y values for the population for one repetition



Clearly, there appears to be 2 subpopulations here, where one has many more members than the other. In my simulated data, I make $\beta_0 = 0$ and $\beta = 5$.

Simulation

The goal here is to convince myself that if we sample based off the sum of outcomes for a given subject, (here I made the number of measurements per subject equal to 5) we still get a consistent estimate of for beta. What I did here, was generate a "population" of subjects (number of subjects equals 20,000, each subject has 5 measurements) from the model above. Then I aggregate the data by taking the sum of all measurements for each subject. Then I sample 50 subjects from each group based off that sum. With the sampled data, I fit the model given above and record the estimated value of beta. I repeat this process many times and create a histogram for estimated values for the fixed effect. The code and resulting histogram are shown below.

```
library(nlme)
set.seed(1104)

pop.m<-20000 # number of clusters
pop.n<- 5 # number within clusters
case.prob<-.01 #probability of case in underlying population
beta<-5 #slope for indicator
sigma<- .5 #overall standard deviation in the linear model
tau_e<-0.8 #error correlation
reps=10000


beta.est<-vector(length = reps)
for(i in 1:reps){
x<-rbinom(pop.m,1,prob=case.prob)
x<-rep(x,each=pop.n)

u<-rnorm(pop.m,mean = 0, sd=sqrt(sigma*tau_e)) #cluster samples
u1<-rep(u,each=pop.n) # repeat each cluster sample n times
estar<-rnorm(pop.m*pop.n,mean = 0, sd=sqrt(sigma*(1-tau_e))) # samples within each cluster
err<-u1+estar #total error

y<-beta*x+err
dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
agg.dat<-aggregate(y~id, dat, sum) # sum y by id
case.samp<-sample(agg.dat$id[agg.dat$y>15],50) #sample cases
control.samp<-sample(agg.dat$id[agg.dat$y<15],50)# sample controls
samp<-c(case.samp, control.samp)
samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids

fit<-lme(y~x, data = samp.dat, random = ~1|id)
beta.est[i]<-fixed.effects(fit)[2]
}

hist(beta.est)### histogram looks good
```

Looking back at my notes, I'm not fully sure what to do from here. I have a note that says subtract off the group means of the X's and Y's. That would give me something like that within-subject part of the model. But what do I do with that? look at the estimate for the fixed effects?

Updated Simulation

Discrete X

I've updated the model I'm studying, now we have,

$$Y_{ij} = \beta_0 + \beta X_{ij} + U_i + \epsilon_{ij}.$$

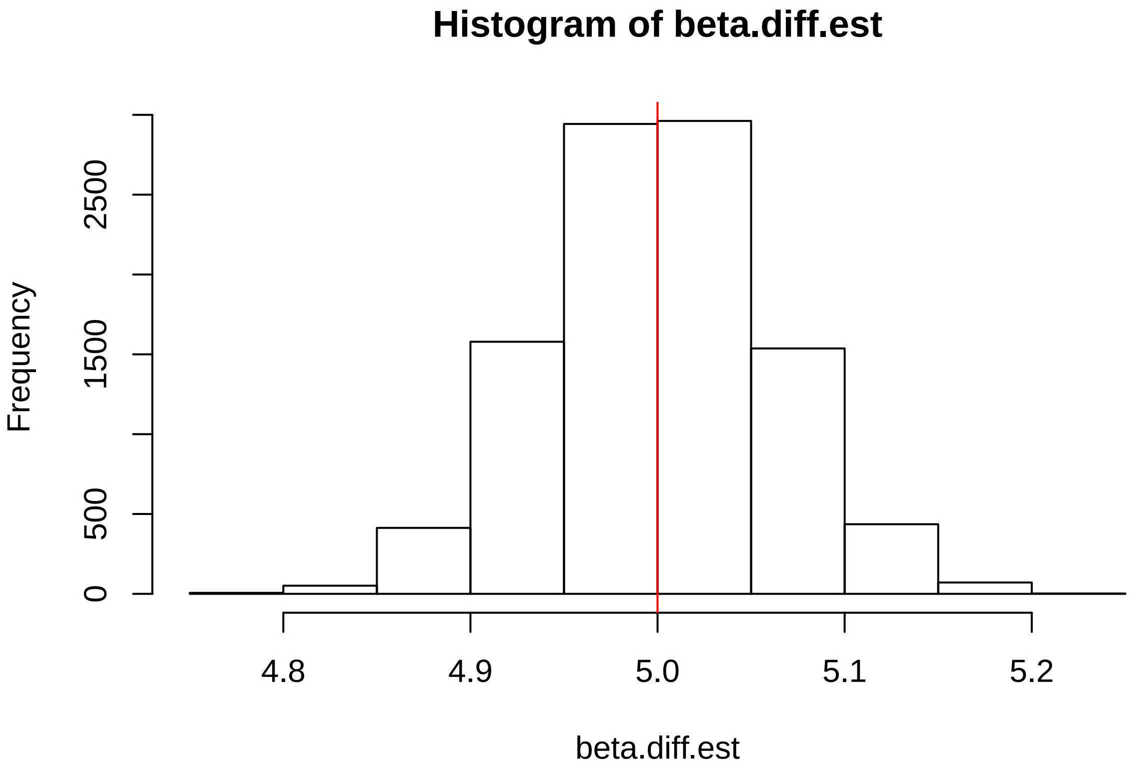
Where I set $\beta_0 = 0$ and $\beta = 5$. The X_{ij} terms are generated as follows: 1.) We generate $X^*_{ij} \sim N(U_i, 1)$, 2.) We then make the terms into a binary random variable by coding $X^*_{ij} > 2.5$ as 1 and 0 otherwise. The rest of the simulation is the same as above except at the end we fit a model of the form,

$$Y_{ij} = \beta_0 + \beta_1 \bar{X}_{i.} + \beta_2 \left(X_{ij} - \bar{X}_{i.} \right) + U_i + \epsilon_{ij}$$

```
####ODS Conditional Likelihood Simulation: Jacob M. Maronge 01/09/17

#### Inspired by the paper: Separating between- and within-cluster covariate effects by using conditiona
#### By John M. Neuhaus and Charles E. McCulloch

library(nlme)
set.seed(1104)

pop.m<-20000 # number of clusters
pop.n<- 5 # number within clusters
beta<-5 #slope for indicator
sigma<- 1 #overall standard deviation in the linear model
tau_e<-0.8 #error correlation
reps=10000


beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)
beta.x_ibar.est<-vector(length = reps)
beta.x_ibar.cov.prob<-vector(length = reps)

for(i in 1:reps){
u<-rnorm(pop.m,mean = 0, sd=sqrt(sigma*tau_e)) #cluster samples
u1<-rep(u,each=pop.n) # repeat each cluster sample n times
estar<-rnorm(pop.m*pop.n,mean = 0, sd=sqrt(sigma*(1-tau_e))) # samples within each cluster
err<-u1+estar #total error

x<-rnorm(u1,1)
x<-as.numeric(x>2.5)
y<-beta*x+err
dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
agg.dat<-aggregate(y~id, dat, sum) # sum y by id
case.samp<-sample(agg.dat$id[agg.dat$y>12],50) #sample cases
control.samp<-sample(agg.dat$id[agg.dat$y<12],50)# sample controls
samp<-c(case.samp, control.samp)
samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
samp.dat$x_ibar<-x_ibar
samp.dat$diff<-samp.dat$x-samp.dat$x_ibar # calculate x_ij-x_ibar
```

```
fit<-lme(y~diff+x_ibar, data = samp.dat, random = ~1|id)
beta.diff.est[i]<-fixed.effects(fit)[2]
beta.diff.cov.prob[i]<-(intervals(fit)$fixed[2,1]<=beta&intervals(fit)$fixed[2,3]>=beta)
beta.x_ibar.est[i]<-fixed.effects(fit)[3]
beta.x_ibar.cov.prob[i]<-(intervals(fit)$fixed[3,1]<=beta&intervals(fit)$fixed[3,3]>=beta)
}
```

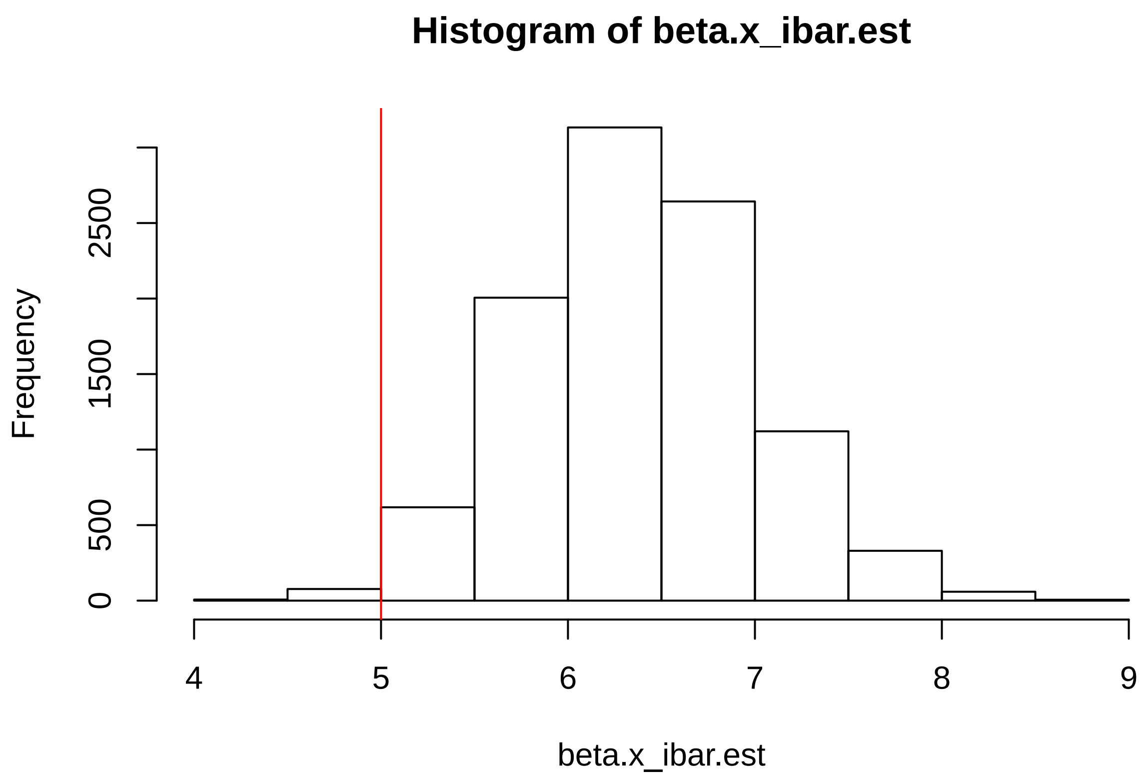
```
hist(beta.diff.est)
abline(v = 5, col = "red")
```




```
mean(beta.diff.cov.prob)
```

```
## [1] 0.9498
```

```
hist(beta.x_ibar.est)
abline(v = 5, col = "red")
```




```r
mean(beta.x_ibar.cov.prob)
```

```
## [1] 0.454
```

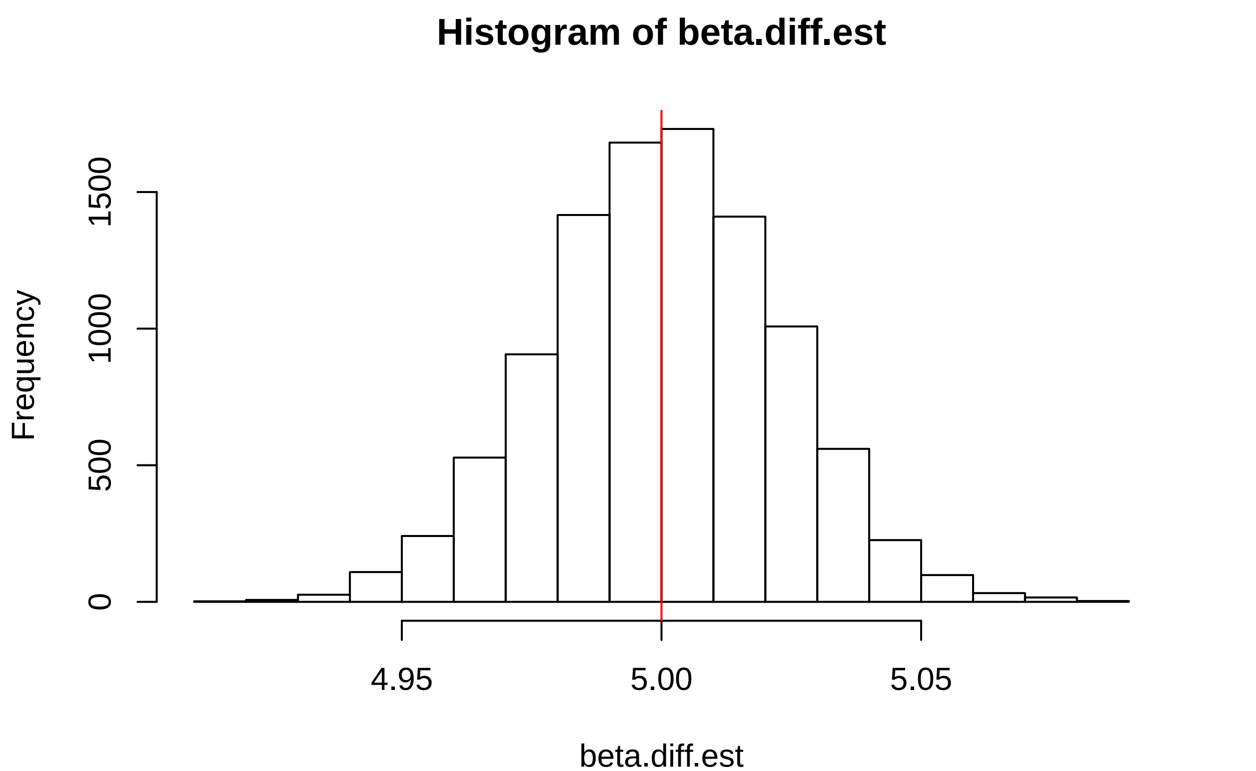
Continuous X

Here I generate the same model as above, except I don't convert the X's to discrete values.

```r
library(nlme)
set.seed(1104)

pop.m<-20000 # number of clusters
pop.n<- 5 # number within clusters
beta<-5 #slope for indicator
sigma<- 1 #overall standard deviation in the linear model
tau_e<-0.8 #error correlation
reps=10000


beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)
beta.x_ibar.est<-vector(length = reps)
beta.x_ibar.cov.prob<-vector(length = reps)

for(i in 1:reps){
u<-rnorm(pop.m,mean = 0, sd=sqrt(sigma*tau_e)) #cluster samples
u1<-rep(u,each=pop.n) # repeat each cluster sample n times
estar<-rnorm(pop.m*pop.n,mean = 0, sd=sqrt(sigma*(1-tau_e))) # samples within each cluster
err<-u1+estar #total error
```

```r
x<-rnorm(u1,1)
y<-beta*x+err
dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
agg.dat<-aggregate(y~id, dat, sum) # sum y by id
case.samp<-sample(agg.dat$id[agg.dat$y>12],50) #sample cases
control.samp<-sample(agg.dat$id[agg.dat$y<12],50)# sample controls
samp<-c(case.samp, control.samp)
samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
samp.dat$x_ibar<-x_ibar
samp.dat$diff<-samp.dat$x-samp.dat$x_ibar # calculate x_ij-x_ibar

fit<-lme(y~diff+x_ibar, data = samp.dat, random = ~1|id)
beta.diff.est[i]<-fixed.effects(fit)[2]
beta.diff.cov.prob[i]<-(intervals(fit)$fixed[2,1]<=beta&intervals(fit)$fixed[2,3]>=beta)
beta.x_ibar.est[i]<-fixed.effects(fit)[3]
beta.x_ibar.cov.prob[i]<-(intervals(fit)$fixed[3,1]<=beta&intervals(fit)$fixed[3,3]>=beta)
}
```

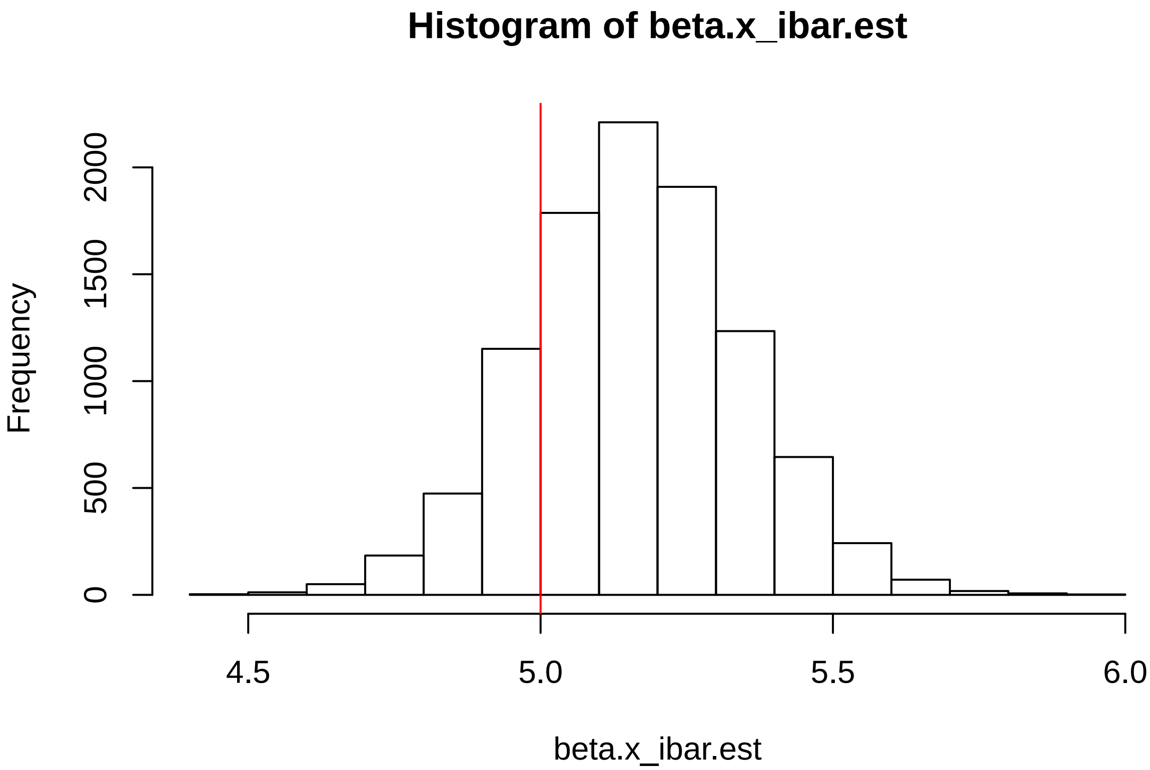
```r
hist(beta.diff.est)
abline(v = 5, col = "red")
```




```r
mean(beta.diff.cov.prob)
```

```
## [1] 0.9471
```

```r
hist(beta.x_ibar.est)
abline(v = 5, col = "red")
```




```r
mean(beta.x_ibar.cov.prob)
```

```
## [1] 0.8693
```

Logistic regression simulation 02/05/18

Discrete X

Since we've done the normal case, we decided to study a logistic model of the form,

$$\text{logit}(Y_{ij}) = \beta_0 + \beta X_{ij} + U_i.$$

Where I set $\beta_0 = -1.5$ and $\beta = 3$. The U_i terms are distributed as $U_i \sim N(0, 1/2)$. The X_{ij} terms are generated as follows: 1.) We generate $X^*_{ij} \sim N(U_i, 1)$, 2.) We then make the terms into a binary random variable by coding $X^*_{ij} > 2.5$ as 1 and 0 otherwise. Then we generate values Z_{ij} where,

$$Z_{ij} = \beta_0 + \beta X_{ij} + U_i.$$

Next, we take,

$$p_{ij} = \frac{1}{1 + e^{-Z_{ij}}}.$$

Finally, we generate our Y_{ij} by taking,

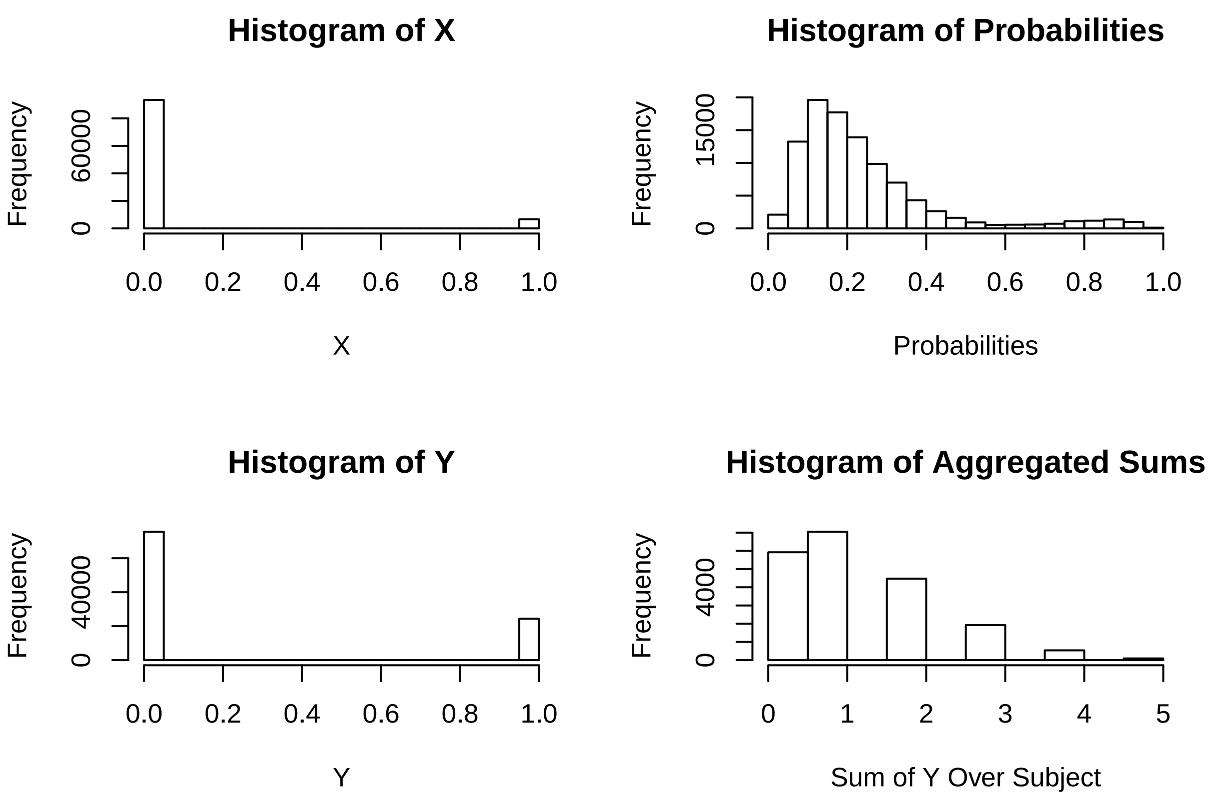
$$Y_{ij} = Bern(p_{ij}).$$

We then fit a model of the form,

$$\text{logit}(Y_{ij}) = \beta_0 + \beta_1 \bar{X}_{i.} + \beta_2 \left(X_{ij} - \bar{X}_{i.}\right) + U_i.$$

Below, I show some plots from one repetition of my simulation.

```
## Loading required package: Matrix
```



```r
library(lme4)
set.seed(1104)

pop.m<-80000 # number of clusters
pop.n<- 5 # number within clusters
beta1<-3 #slope for indicator
beta0<--1.5 #intercept terms
tau_sq<-.5 #variance of random intercept
reps<-10000

beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)
beta.x_ibar.est<-vector(length = reps)
beta.x_ibar.cov.prob<-vector(length = reps)
beta.int.est<-vector(length = reps)
beta.int.cov.prob<-vector(length = reps)
for(i in 1:reps){
  u<-rnorm(pop.m,mean = 0, sd=sqrt(tau_sq)) #cluster samples
  u1<-rep(u,each=pop.n) # repeat each cluster sample n times
  x<-rnorm(u1,1)
  x<-as.numeric(x>2.5)
  z<-beta0+beta1*x+u1
  pr<-1/(1+exp(-z))
  y<-rbinom(n=pop.m*pop.n,size = 1,prob = pr )
  dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
  agg.dat<-aggregate(y~id, dat, sum) # sum y by id
  case.samp<-sample(agg.dat$id[agg.dat$y>=4],250) #sample cases
  control.samp<-sample(agg.dat$id[agg.dat$y<4],250)# sample controls
  samp<-c(case.samp, control.samp)
  samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
  samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
  x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
  samp.dat$x_ibar<-(x_ibar)
  samp.dat$diff<-(samp.dat$x-samp.dat$x_ibar) # calculate x_ij-x_ibar

  fit.logit<-glmer(y~diff+x_ibar+(1|id),data = samp.dat,
                   family = binomial(link = "logit"),
                   glmerControl(optimizer = c("bobyqa","Nelder_Mead")) )
  beta.int.est[i]<-coef(summary(fit.logit))[1,1]
  beta.int.cov.prob[i]<-((coef(summary(fit.logit))[1,1]-qt(.975, df=2000)*coef(summary(fit.logit))[1,2]
  beta.diff.est[i]<-coef(summary(fit.logit))[2,1]
  beta.diff.cov.prob[i]<-((coef(summary(fit.logit))[2,1]-qt(.975, df=2000)*coef(summary(fit.logit))[2,2
  beta.x_ibar.est[i]<-coef(summary(fit.logit))[3,1]
  beta.x_ibar.cov.prob[i]<-((coef(summary(fit.logit))[3,1]-qt(.975, df=2000)*coef(summary(fit.logit))[3

  }
```

```r
hist(beta.int.est,main = "Histogram of intercept estimates",xlim = c(-1.5,0))
abline(v = -1.5, col = "red")
mean(beta.int.cov.prob)
hist(beta.diff.est, main = "Coefficient of (x_ij-x_ibar) estimates")
abline(v = 3, col = "red")
mean(beta.diff.cov.prob)
hist(beta.x_ibar.est, main = "Coefficient of x_ibar Estimates")
```

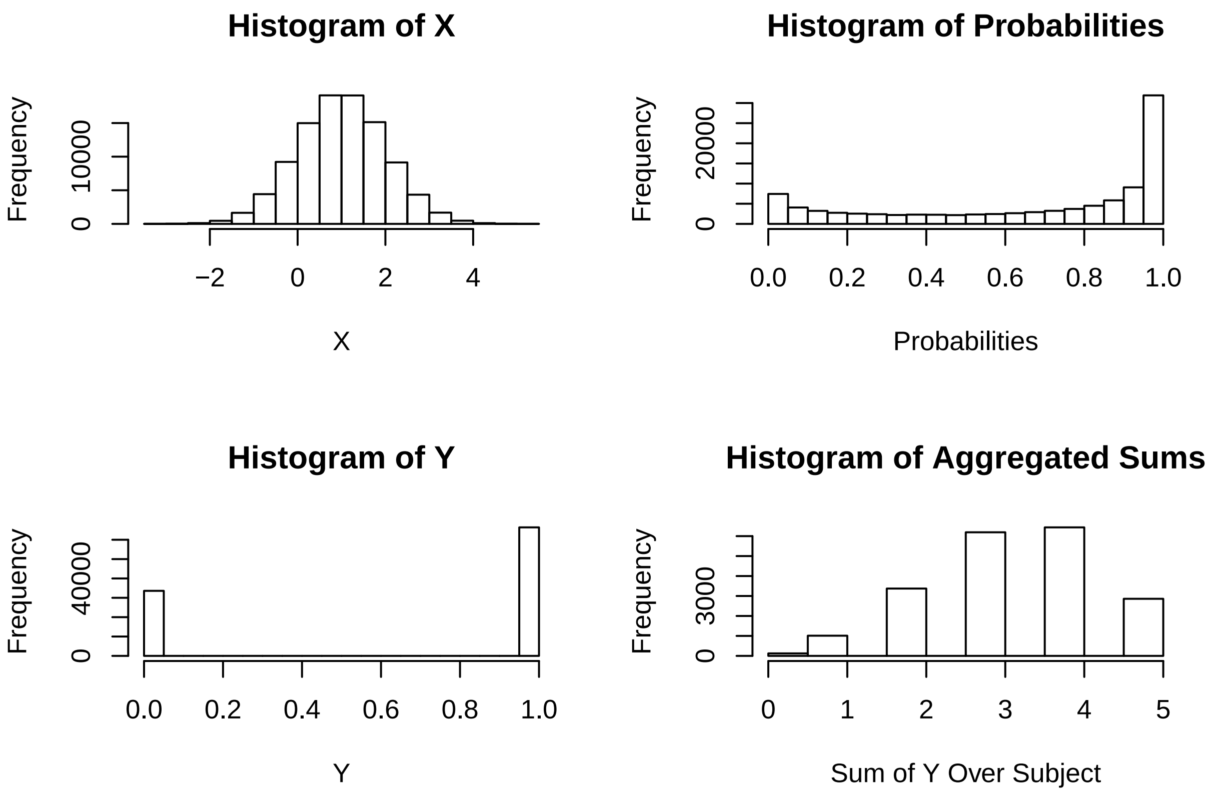
```
abline(v = 3, col = "red")
mean(beta.int.cov.prob)
```

The vertical red lines are drawn at the true value for each parameter. The intercept term appears to be biased, as well as the between-subject effect, and possibly also the within-subject effect.

Continuous X

Here I generate the same model as above, except I don't convert the X's to discrete values.

Below, I show some plots from one repetition of my simulation.




```
library(lme4)
set.seed(1104)

pop.m<-80000 # number of clusters
pop.n<- 5 # number within clusters
beta1<-3 #slope for indicator
beta0<--1.5 #intercept terms
tau_sq<-.5 #variance of random intercept
reps<-10000

beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)
beta.x_ibar.est<-vector(length = reps)
beta.x_ibar.cov.prob<-vector(length = reps)
beta.int.est<-vector(length = reps)
beta.int.cov.prob<-vector(length = reps)
for(i in 1:reps){
```

```
  u<-rnorm(pop.m,mean = 0, sd=sqrt(tau_sq)) #cluster samples
  u1<-rep(u,each=pop.n) # repeat each cluster sample n times
  x<-rnorm(u1,1)
  z<-beta0+beta1*x+u1
  pr<-1/(1+exp(-z))
  y<-rbinom(n=pop.m*pop.n,size = 1,prob = pr )
  dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
  agg.dat<-aggregate(y~id, dat, sum) # sum y by id
  case.samp<-sample(agg.dat$id[agg.dat$y>=4],250) #sample cases
  control.samp<-sample(agg.dat$id[agg.dat$y<4],250)# sample controls
  samp<-c(case.samp, control.samp)
  samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
  samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
  x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
  samp.dat$x_ibar<-(x_ibar)
  samp.dat$diff<-(samp.dat$x-samp.dat$x_ibar) # calculate x_ij-x_ibar

  fit.logit<-glmer(y~diff+x_ibar+(1|id),data = samp.dat,
                   family = binomial(link = "logit"),
                   glmerControl(optimizer = c("bobyqa","Nelder_Mead")) )
  beta.int.est[i]<-coef(summary(fit.logit))[1,1]
  beta.int.cov.prob[i]<-((coef(summary(fit.logit))[1,1]-qt(.975, df=2000)*coef(summary(fit.logit))[1,2]
  beta.diff.est[i]<-coef(summary(fit.logit))[2,1]
  beta.diff.cov.prob[i]<-((coef(summary(fit.logit))[2,1]-qt(.975, df=2000)*coef(summary(fit.logit))[2,2
  beta.x_ibar.est[i]<-coef(summary(fit.logit))[3,1]
  beta.x_ibar.cov.prob[i]<-((coef(summary(fit.logit))[3,1]-qt(.975, df=2000)*coef(summary(fit.logit))[3

}
```

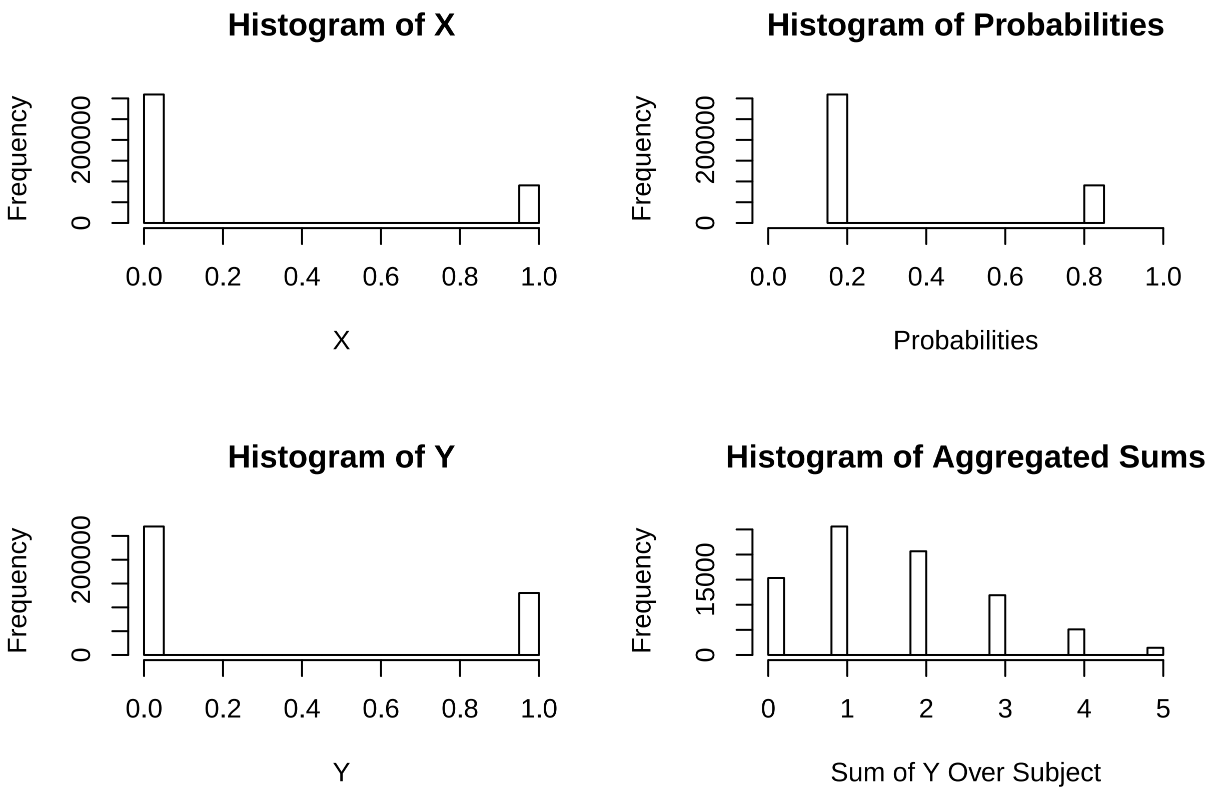
```
hist(beta.int.est,main = "Histogram of intercept estimates",xlim = c(-1.5,0))
abline(v = -1.5, col = "red")
mean(beta.int.cov.prob)
hist(beta.diff.est, main = "Coefficient of (x_ij-x_ibar) estimates")
abline(v = 3, col = "red")
mean(beta.diff.cov.prob)
hist(beta.x_ibar.est, main = "Coefficient of x_ibar Estimates")
abline(v = 3, col = "red")
mean(beta.int.cov.prob)
```

Conditional logistic regression

Discrete X

Below, I show some plots from one repetition of my simulation.

```
##
##     0     1     2     3     4     5
## 15329 25583 20653 11903  5101  1431
```




```r
set.seed(1104)
library(survival)
pop.m<-80000 # number of clusters
pop.n<- 5 # number within clusters
beta1<-3 #slope for indicator
beta0<--1.5 #intercept terms
tau_x<-0.5
sigma<-1
reps<-10000

beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)

for(i in 1:reps){
  v<-rnorm(pop.m,mean = 0, sd=sqrt(sigma*tau_x)) #cluster samples
  v1<-rep(v,each=pop.n) # repeat each cluster sample n times
  xstar<-rnorm(pop.m*pop.n,mean = 0, sd=sqrt(sigma*(1-tau_x))) # samples within each cluster
  x<-v1+xstar #total x
  x<-as.numeric(x>.75)
  z<-beta0+beta1*x
  pr<-1/(1+exp(-z))
  y<-rbinom(n=pop.m*pop.n,size = 1,prob = pr )
  dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
  agg.dat<-aggregate(y~id, dat, sum) # sum y by id
  table(agg.dat$y)
  case.samp<-sample(agg.dat$id[agg.dat$y>=4],250) #sample cases
  control.samp<-sample(agg.dat$id[agg.dat$y<4],250)# sample controls
  samp<-c(case.samp, control.samp)
  samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
```

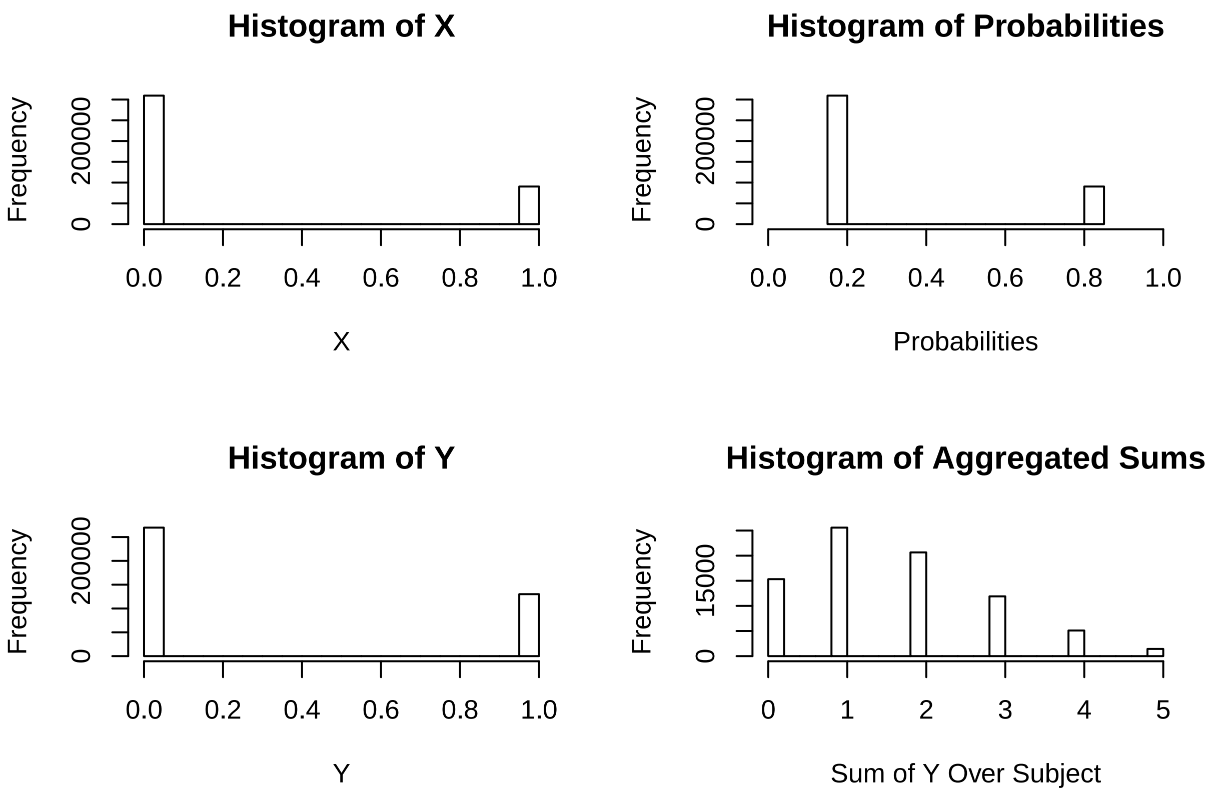
```r
  samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
  x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
  samp.dat$x_ibar<-(x_ibar)
  samp.dat$diff<-(samp.dat$x-samp.dat$x_ibar) # calculate x_ij-x_ibar
  table(samp.dat$diff,samp.dat$y)
  fit.clogit<-clogit(y ~ diff + strata(id), samp.dat)
  beta.diff.est[i]<-coef(fit.clogit)[1]
  ci<-confint(fit.clogit)
  beta.diff.cov.prob[i]<-(ci[1,1]<=beta1&ci[1,2]>=beta1)
}
```

```r
hist(beta.diff.est, main = "Estimates of Coefficient (x_ij-x_ibar)")
abline(v = 3, col = "red")
mean(beta.diff.cov.prob)
```

Continuous X

Below, I show some plots from one repetition of my simulation.

```
##
##     0     1     2     3     4     5
## 15329 25583 20653 11903  5101  1431
```




```r
set.seed(1104)
library(survival)
pop.m<-80000 # number of clusters
pop.n<- 5 # number of within clusters
beta1<-3 #slope for indicator
beta0<--1.5 #intercept terms
```

```r
tau_x<-0.5
sigma<-1
reps<-10000

beta.diff.est<-vector(length = reps)
beta.diff.cov.prob<-vector(length = reps)

for(i in 1:reps){
  v<-rnorm(pop.m,mean = 0, sd=sqrt(sigma*tau_x)) #cluster samples
  v1<-rep(v,each=pop.n) # repeat each cluster sample n times
  xstar<-rnorm(pop.m*pop.n,mean = 0, sd=sqrt(sigma*(1-tau_x))) # samples within each cluster
  x<-v1+xstar #total x
  z<-beta0+beta1*x
  pr<-1/(1+exp(-z))
  y<-rbinom(n=pop.m*pop.n,size = 1,prob = pr )
  dat<-data.frame(y=y,x=x,id=rep(c(1:pop.m),each=pop.n)) #make data
  agg.dat<-aggregate(y~id, dat, sum) # sum y by id
  table(agg.dat$y)
  case.samp<-sample(agg.dat$id[agg.dat$y>=4],250) #sample cases
  control.samp<-sample(agg.dat$id[agg.dat$y<4],250)# sample controls
  samp<-c(case.samp, control.samp)
  samp.dat<-subset(dat,dat$id%in%samp) # get dataframe for sampled ids
  samp.agg.dat<-aggregate(x~id, samp.dat, mean) #calculate means for x
  x_ibar<-rep(samp.agg.dat$x,each=pop.n) # mach means dimensions with dat
  samp.dat$x_ibar<-(x_ibar)
  samp.dat$diff<-(samp.dat$x-samp.dat$x_ibar) # calculate x_ij-x_ibar
  table(samp.dat$diff,samp.dat$y)
  fit.clogit<-clogit(y ~ diff + strata(id), samp.dat)
  beta.diff.est[i]<-coef(fit.clogit)[1]
  ci<-confint(fit.clogit)
  beta.diff.cov.prob[i]<-(ci[1,1]<=beta1&ci[1,2]>=beta1)
}
```

```r
hist(beta.diff.est, main = "Estimates of Coefficient (x_ij-x_ibar)")
abline(v = 3, col = "red")
mean(beta.diff.cov.prob)
```