RWTH Aachen Institute for Theoretical Solid State Physics

Computational Physics (SS 2020)

Prof. Dr. Riccardo Mazzarello **Sheet 3 (20 points)**

Submission: 30 April, 2020, 10:00 a.m.

Important remarks

- Please hand in your solutions in teams of two students.
- Provide a *single* PDF-file containing the written solutions of all exercises, i. e. analytical derivations, numeric results, their interpretation, etc. If it contains a scan, please make sure it is well readable. Plots can either be embedded into this file or, alternatively, given separately (e. g. as PNG-files) and referred to in the PDF.
- Use a current version of Python 3 for your implementations.
- Write readable and *commented* code with self-explanatory variable names.
- Please hand in your solutions (PDF-file with answers, source code files, and, if not embedded in the PDF, PNG-files of plots) as a **single** ZIP-file named

  Surname1-MatNum1-Surname2-MatNum2-Sheet3.zip,

  where MatNum is your matriculation number, by uploading it to RWTH Gigamove (`https://gigamove.rz.rwth-aachen.de`). Please send the download link using your RWTH address to

  manconi@physik.rwth-aachen.de

  and write in the object of the email

  CP Sheet3- Surname1-MatNum1-Surname2-MatNum2.

1. **Stability of the modified Euler method** (4 points)

   In the lecture, the region of absolute stability of the Euler method and the implicit Euler method was derived. Here, derive the stability condition for the modified Euler method in terms of an inequality containing the variable $\tau\lambda$, and plot the region of stability in the complex plane of this variable (hint: `RegionPlot` in Mathematica/WolframAlpha). Which time steps τ fulfill the stability criterion for $0 > \lambda \in \mathbb{R}$?

2. **Driven pendulum** (10 points)

   Use the **4th-order Runge-Kutta method** to analyze the driven pendulum described by

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -k\sin(x) - \gamma\frac{\mathrm{d}x}{\mathrm{d}t} + Q\sin(\Omega t) \tag{1}$$

   Set $k = 1$, $\gamma = 0.5$, $\Omega = 2/3$ and as initial conditions use x(0) = 1, dx/dt(0) = 0. Use the time step $\tau = T_0/200$ where $T_0 = 2\pi/\Omega$. Concretely, do the following:

- Implement the 4th-order Runge-Kutta method.
- Write a small test code that prints the values of x and v at time $t_{20} = T_0/10$ for (i) $Q = 0.5$.
- Compute the trajectories for (ii) $Q = 0.9$ and (iii) 1.2 from $t = 0$ to $t = t_{8000}$, wrap them into the interval $[-\pi, \pi)$ and plot them.
- Compare the trajectories for the different Q qualitatively: What kind of motion do you observe in each case?

3. **Power spectrum of the driven pendulum** (6 points)

   Write a program that calculates the power spectrum of the driven pendulum using the periodogram $P_l(f)$. Determine $P_l(f)$ for each of the three cases (i)–(iii) you considered in exercise 2.
   You can eg. calculate $N = 2^{16}$ successive data points for $x(t)$, and use these to determine $P_l(f)$. In each case, plot $\log P_l(f)$ for the range $0 < \nu < 6\nu_0$, where $\nu_0 = \frac{\Omega}{2\pi}$ is the frequency of the driving force. Discuss your results.

   Use the libraries `scipy.fftpack.fft` and `scipy.signal` for your implementation.