Computational Physics – Exercise 7: How to solve Maxwell's equations numerically? II

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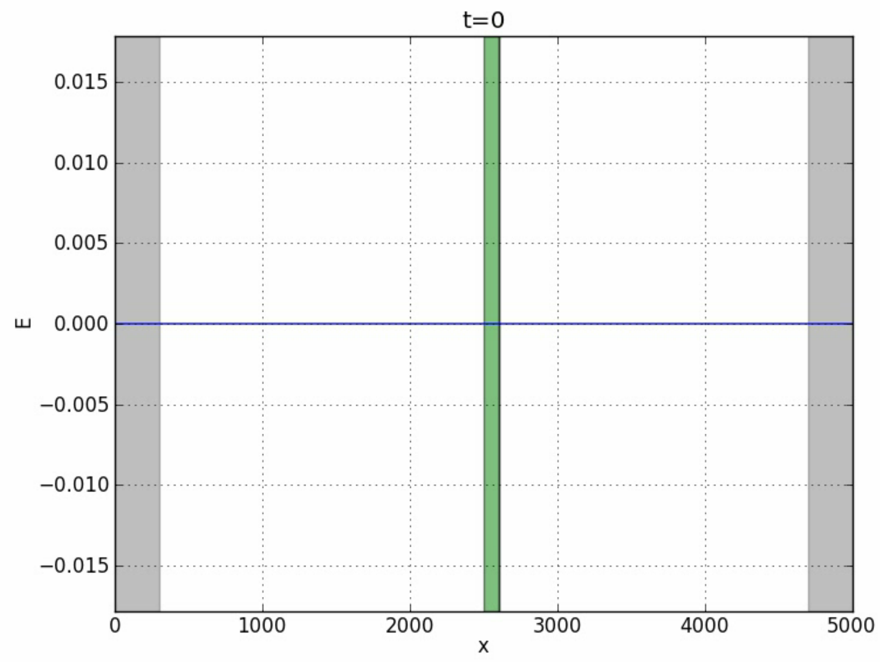
Exercise

Simulation of transmission and reflection of light by a glass plate with the Yee algorithm.



Exercise

- Parameters:
  - Wavelength (sets the length scale): $\lambda = 1$
  - Number of grid points per wavelength: 50
  - Spatial resolution: $\Delta = \lambda / 50 = 0.02$
  - Temporal resolution: $\tau = 0.9\Delta, \tau = 1.05\Delta$
    (Courant condition!)
  - Length of simulation box: $X = 100\lambda = L\Delta \Rightarrow L = 5000$
  - Source frequency: $f = v / \lambda = 1 / \lambda = 1 \Rightarrow \omega = 2\pi f = 2\pi$
  - Number of time steps: $m = 10000$

Exercise

- Materials:
  - Matched boundary layers for reflectionless absorption of the EM waves at the boundary

$$\sigma(x) = \sigma^*(x) = \begin{cases} 1 & \text{if} \quad 0 \le x \le 6\lambda \\ 0 & \text{if} \quad 6\lambda < x < L\Delta - 6\lambda \\ 1 & \text{if} \quad L\Delta - 6\lambda \le x \le L\Delta \end{cases}$$

  - Gray areas in the picture

Exercise

- Materials:
  - Glass layer of thickness 2λ placed in the middle of the system (green area in the picture)
  - Index of refraction of glass: $n = 1.46$

$$\varepsilon(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x < L\Delta / 2 \\ n^2 & \text{if} \quad L\Delta / 2 \leq x < L\Delta / 2 + 2\lambda \\ 1 & \text{if} \quad L\Delta / 2 + 2\lambda \leq x \leq L\Delta \end{cases}$$

$$\mu(x) = 1$$

Exercise

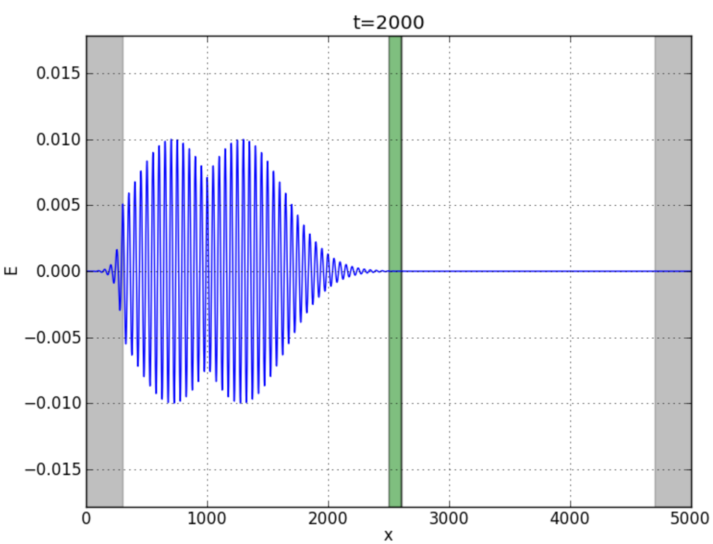
- Current source at $x_S = 20\lambda \Leftrightarrow i_S = x_S / \Delta = 1000$
- To create a nice wave packet, we turn on the source slowly and we also turn it of slowly

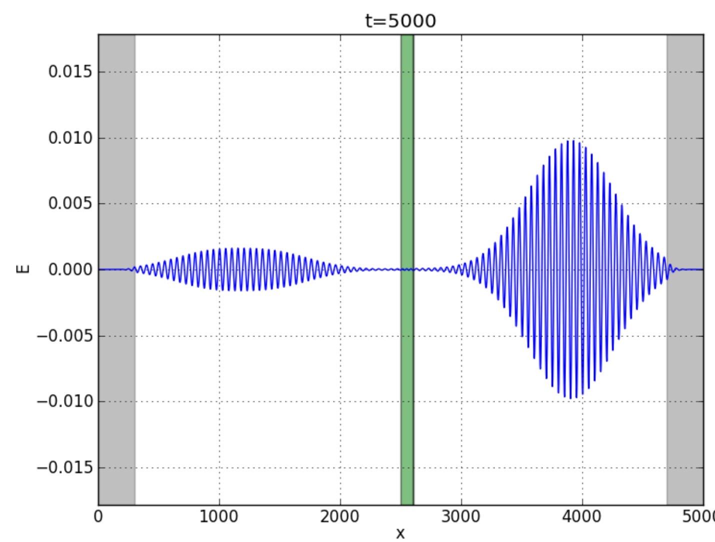
$$J_S(i_S, t) = \sin(2\pi t f)e^{-((t-30)/10)^2}$$

where $f = 1$ is the frequency of the current source

Exercise

- Plot the *E*-field for various numbers of time steps

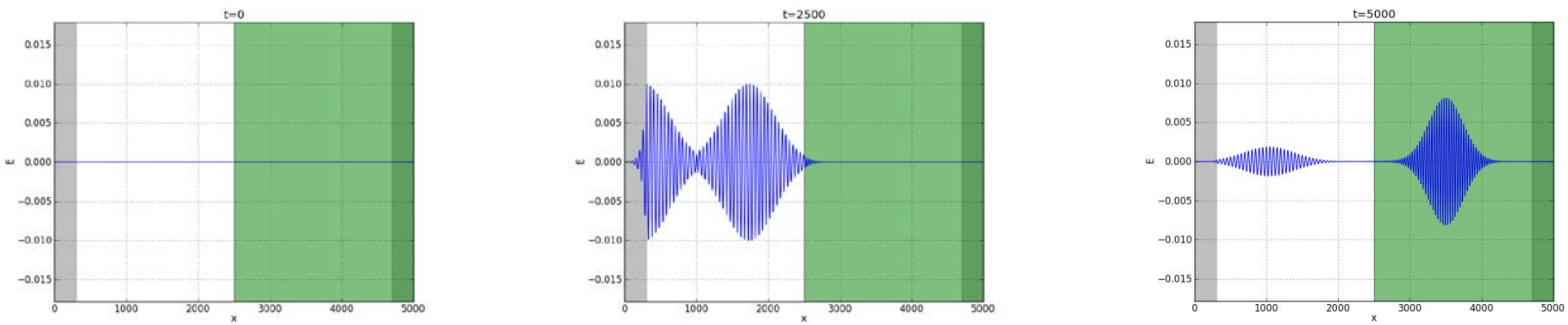




- What happens for $\tau = 1.05\Delta$?

Exercise

- Make the glass plate very thick, as shown in these pictures



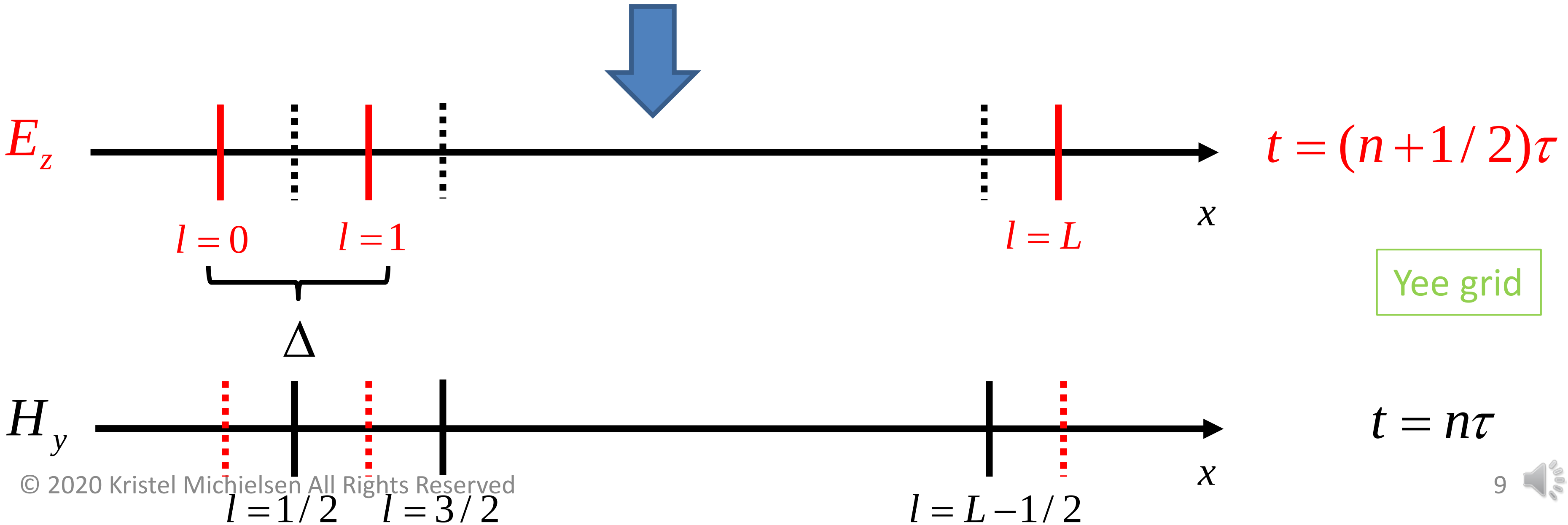
- From the maximum of the incident wave packet and the reflected wave packet, estimate the reflection coefficient of glass

$$R = \left|E^{\text{maximum}}_{\text{reflected}}\right|^2 \Big/ \left|E^{\text{maximum}}_{\text{incident}}\right|^2$$

Exercise: 1D Maxwell equation

Consider the Maxwell equation in 1D

$$\boxed{\begin{aligned}\frac{\partial H_y(x,t)}{\partial t} &= \frac{1}{\mu(x)}\left[\frac{\partial E_z(x,t)}{\partial x} - \sigma^*(x) H_y(x,t)\right] \\ \frac{\partial E_z(x,t)}{\partial t} &= \frac{1}{\varepsilon(x)}\left[\frac{\partial H_y(x,t)}{\partial x} - J_{\text{source}_z}(x,t) - \sigma(x) E_z(x,t)\right]\end{aligned}}$$



Exercise: Yee algorithm

$$\frac{H_y\big|_{l+1/2}^{n+1} - H_y\big|_{l+1/2}^{n}}{\tau} = \frac{1}{\mu_{l+1/2}}\left[\frac{E_z\big|_{l+1}^{n+1/2} - E_z\big|_{l}^{n+1/2}}{\Delta} - \sigma^*_{l+1/2} H_y\big|_{l+1/2}^{n+1/2}\right]$$

$$\frac{E_z\big|_{l}^{n+1/2} - E_z\big|_{l}^{n-1/2}}{\tau} = \frac{1}{\varepsilon_l}\left[\frac{H_y\big|_{l+1/2}^{n} - H_y\big|_{l-1/2}^{n}}{\Delta} - J_{\text{source}_z}\big|_{l}^{n} - \sigma_l E_z\big|_{l}^{n}\right]$$

$$\Downarrow$$

$$H_y\big|_{l+1/2}^{n+1} = H_y\big|_{l+1/2}^{n} + \frac{\tau}{\mu_{l+1/2}}\left[\frac{E_z\big|_{l+1}^{n+1/2} - E_z\big|_{l}^{n+1/2}}{\Delta} - \sigma^*_{l+1/2}\left(\frac{H_y\big|_{l+1/2}^{n+1} + H_y\big|_{l+1/2}^{n}}{2}\right)\right]$$

$$E_z\big|_{l}^{n+1/2} = E_z\big|_{l}^{n-1/2} + \frac{\tau}{\varepsilon_l}\left[\frac{H_y\big|_{l+1/2}^{n} - H_y\big|_{l-1/2}^{n}}{\Delta} - J_{\text{source}_z}\big|_{l}^{n} - \sigma_l\left(\frac{E_z\big|_{l}^{n-1/2} + E_z\big|_{l}^{n+1/2}}{2}\right)\right]$$

Exercise: Yee algorithm

$$H_y\Big|_{l+1/2}^{n+1} = H_y\Big|_{l+1/2}^{n} + \frac{\tau}{\mu_{l+1/2}}\left[\frac{E_z\big|_{l+1}^{n+1/2} - E_z\big|_{l}^{n+1/2}}{\Delta} - \sigma^*_{l+1/2}\left(\frac{H_y\big|_{l+1/2}^{n+1} + H_y\big|_{l+1/2}^{n}}{2}\right)\right]$$

$$E_z\Big|_{l}^{n+1/2} = E_z\Big|_{l}^{n-1/2} + \frac{\tau}{\varepsilon_l}\left[\frac{H_y\big|_{l+1/2}^{n} - H_y\big|_{l-1/2}^{n}}{\Delta} - J_{\text{source}_z}\Big|_{l}^{n} - \sigma_l\left(\frac{E_z\big|_{l}^{n-1/2} + E_z\big|_{l}^{n+1/2}}{2}\right)\right]$$

$$\Downarrow$$

$$\left(1+\frac{\sigma^*_{l+1/2}\tau}{2\mu_{l+1/2}}\right)H_y\Big|_{l+1/2}^{n+1} = \left(1-\frac{\sigma^*_{l+1/2}\tau}{2\mu_{l+1/2}}\right)H_y\Big|_{l+1/2}^{n} + \frac{\tau}{\mu_{l+1/2}}\left[\frac{E_z\big|_{l+1}^{n+1/2} - E_z\big|_{l}^{n+1/2}}{\Delta}\right]$$

$$\left(1+\frac{\sigma_l\tau}{2\varepsilon_l}\right)E_z\Big|_{l}^{n+1/2} = \left(1-\frac{\sigma_l\tau}{2\varepsilon_l}\right)E_z\Big|_{l}^{n-1/2} + \frac{\tau}{\varepsilon_l}\left[\frac{H_y\big|_{l+1/2}^{n} - H_y\big|_{l-1/2}^{n}}{\Delta} - J_{\text{source}_z}\Big|_{l}^{n}\right]$$

Exercise: Yee algorithm

$$H_y\Big|_{l+1/2}^{n+1} = \left(\frac{1-\dfrac{\sigma^*_{l+1/2}\tau}{2\mu_{l+1/2}}}{1+\dfrac{\sigma^*_{l+1/2}\tau}{2\mu_{l+1/2}}} \right) H_y\Big|_{l+1/2}^{n} + \left(\frac{\dfrac{\tau}{\mu_{l+1/2}}}{1+\dfrac{\sigma^*_{l+1/2}\tau}{2\mu_{l+1/2}}} \right) \left[\frac{E_z\Big|_{l+1}^{n+1/2} - E_z\Big|_{l}^{n+1/2}}{\Delta} \right]$$

$$E_z\Big|_{l}^{n+1/2} = \left(\frac{1-\dfrac{\sigma_l\tau}{2\varepsilon_l}}{1+\dfrac{\sigma_l\tau}{2\varepsilon_l}} \right) E_z\Big|_{l}^{n-1/2} + \left(\frac{\dfrac{\tau}{\varepsilon_l}}{1+\dfrac{\sigma_l\tau}{2\varepsilon_l}} \right) \left[\frac{H_y\Big|_{l+1/2}^{n} - H_y\Big|_{l-1/2}^{n}}{\Delta} - J_{\mathrm{source}_z}\Big|_{l}^{n} \right]$$

Update rules

Exercise: Yee algorithm

$$\boxed{\begin{aligned} H_y\Big|_{l+1/2}^{n+1} &= A_{l+1/2}\, H_y\Big|_{l+1/2}^{n} + B_{l+1/2}\left[\frac{E_z\Big|_{l+1}^{n+1/2} - E_z\Big|_{l}^{n+1/2}}{\Delta}\right] \\ E_z\Big|_{l}^{n+1/2} &= C_l\, E_z\Big|_{l}^{n-1/2} + D_l\left[\frac{H_y\Big|_{l+1/2}^{n} - H_y\Big|_{l-1/2}^{n}}{\Delta} - J_{\text{source}_z}\Big|_{l}^{n}\right] \end{aligned}}$$

Update rules

$\varepsilon = \mu = 1$

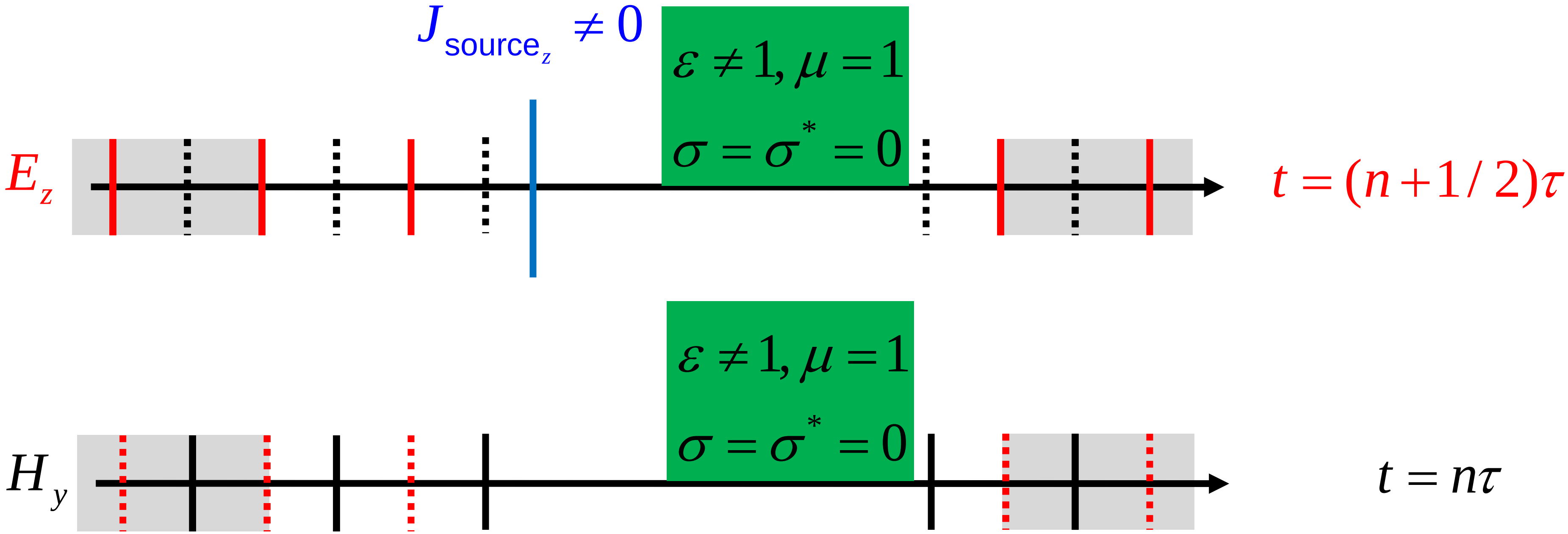
$\sigma = \sigma^* = 1$

$\varepsilon = \mu = 1 \Rightarrow c = 1/\sqrt{\varepsilon\mu} = 1$

$\sigma = \sigma^* = 0$

$\varepsilon = \mu = 1$

$\sigma = \sigma^* = 1$



Length of the line: $L\Delta = X$

Boundary conditions: Absorbing boundaries ($\sigma = \sigma^* = 1$)

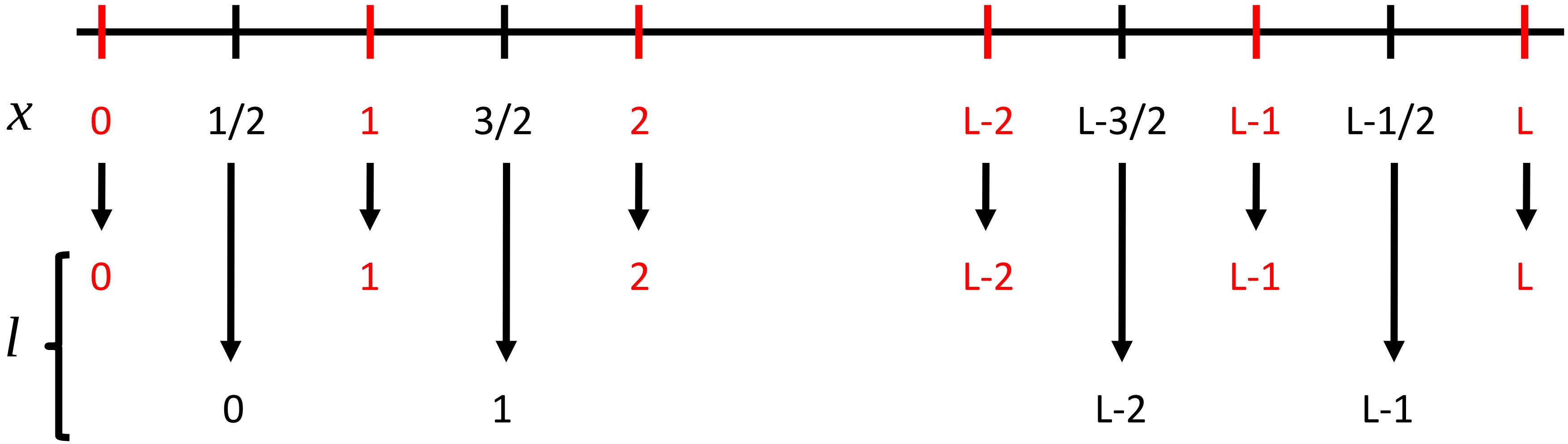
Source: Pulsed source

Boundary conditions: $E_z = E_L = 0$

Exercise: Yee algorithm

E_z, C, D

H_y, A, B



$$E_z : l = 0, \ldots, L; x = l\Delta$$

$$H_y : l = 0, \ldots, L-1; x = (l + 1/2)\Delta$$

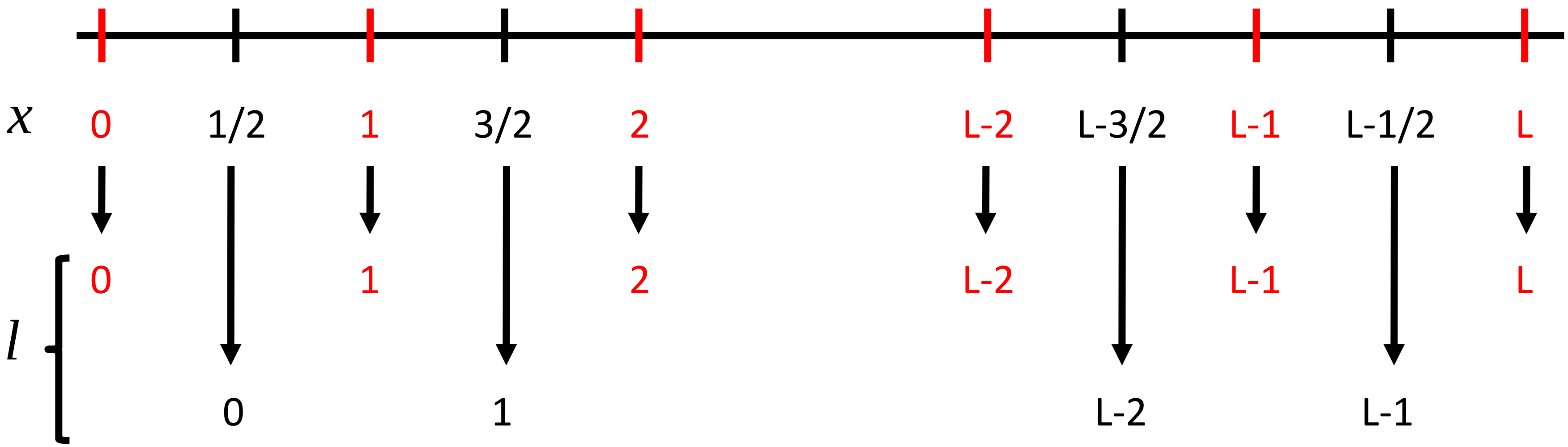
Initialize: $E(0:L) = 0; \quad C(0:L); \quad D(0:L)$

$$H(0:L-1) = 0; \quad A(0:L-1); \quad B(0:L-1)$$

E_z, C, D

H_y, A, B

Exercise: Yee algorithm



Iteration: $E_l^{n+1/2} = \frac{D_l}{\Delta}\left(H_{l+1/2}^{n} - H_{l-1/2}^{n}\right) + C_l E_l^{n-1/2} - \delta_{l,l_S} D_{l_S} J(l_S, n\tau)$

$$H_{l+1/2}^{n+1} = \frac{B_{l+1/2}}{\Delta}\left(E_{l+1}^{n+1/2} - E_l^{n+1/2}\right) + A_{l+1/2} H_{l+1/2}^{n}$$

$$E(1:L-1) = D(1:L-1) * \left[H(1:L-1) - H(0:L-2)\right] / \Delta + C(1:L-1) * E(1:L-1)$$

$$E(l_S) = E(l_S) - D(l_S) J(l_S, n\tau)$$

$$H(0:L-1) = B(0:L-1) * \left[E(1:L) - E(0:L-1)\right] / \Delta + A(0:L-1) * H(0:L-1)$$

Exercise: Yee algorithm

- Implementation of the Yee algorithm
  - Use one array for the $\vec{E}$ field
  - Use one array for the $\vec{H}$ field
  - Use only one vector for the $\vec{E}$ field and for the $\vec{H}$ field and update their elements!
  - Instead of using four separate arrays for $\varepsilon, \mu, \sigma, \sigma^*$ use four arrays for the coefficients A, B, C, D in the Maxwell equation

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Report

- Filename: Report_7_Surname1_Surname2.pdf, where Surname1 < Surname02 (alphabetical order). Example: Report_7_Jin_Lagemann.pdf (Do not use "umlauts" or any other special characters in the names)
- Content of the report:
  - Names + matricle numbers + e-mail addresses + title
  - Introduction: describe briefly the problem you are modeling and simulating (write in complete sentences)
  - Simulation model and method: describe briefly the model and simulation method (write in complete sentences)
  - Simulation results: show figures (use grids, with figure captions !) depicting the simulation results. Give a brief description of the results (write in complete sentences)
  - Discussion: summarize your findings
  - Appendix: Include the listing of the program

Due date: 5 PM, June 23, 2020