

# GRAPH CONVOLUTIONAL NETWORKS

CMU 11441/11641/11741: ML FOR TEXT & GRAPH MINING

Due date: 12/3/2021, 11:59 PM EST

## Instructions

- Allowed libraries: This assignment involves implementing graph convolutional networks. You are not allowed to use any libraries that implement GCNs out of the box (like Pytorch-geometric). It is allowed to use autodiff libraries like Pytorch/Tensorflow.  
We highly recommend using Python + Pytorch for this assignment.
- Statement of Assurance
  1. Did you receive any help whatsoever from anyone in solving this assignment? No
  2. Did you give any help whatsoever to anyone in solving this assignment? No
  3. Did you find or come across code that implements any part of this assignment? No

## 1 GCN Review (30 points)

Q1. What is the big-O time complexity of the computation expressed in Equation ?? in terms of  $|V|$ ,  $|E|$ ,  $d$ ,  $k$ , and  $L$ ? Your expression should not contain any other term. Assume  $d < k$ .

- We need to do computation for  $L$  layers
- Each computation can be seen as:
  - Matrix product ( $O(k^2)$ ), which needs to be done  $O(|E|/|V|)$  times (average number of neighbors) for each node :  $O(|V| |E|/|V| k^2) = O(|E| k^2)$ .
  - Also need to apply  $\sigma$ , which is  $O(|V|)$
- This totals:  $O(L \times (|E| \times k^2 + |V|))$

Q2. What is the space complexity of the computation expressed in Equation ?? in terms of  $|V|$ ,  $|E|$ ,  $d$ ,  $k$ , and  $L$  (assume intermediate terms are saved)? Your expression should not contain any other term.

Node features (final only):  $O(|V| \times k)$

Considering the Intermediate computations (L Wh products):  $O(L \times |V| \times k)$

Weight matrices:  $O(L \times k^2)$

Adjacency list for neighbors:  $O(|E|)$

Total:  $O(L \times k \times (|V| + k) + |E|)$

## 2 Graph Exploration (20 points)

| Graph             | Karate | Cora  | Citeseer |
|-------------------|--------|-------|----------|
| Max in-degree     | 18     | 169   | 100      |
| Min in-degree     | 2      | 2     | 1        |
| Average in-degree | 5.58   | 4.89  | 3.73     |
| # nodes           | 34     | 2708  | 3312     |
| # edges           | 190    | 13264 | 12384    |
| Node feature dim  | 34     | 1433  | 3703     |

Table 1: Graph statistics

## 3 Node classification

### 3.1 Implementation (60 points)

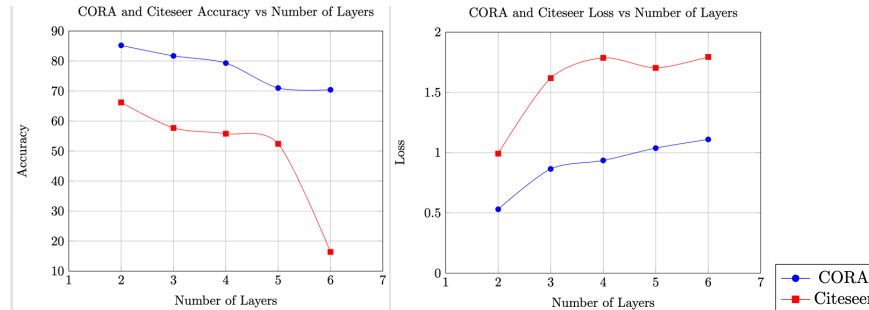
| Graph    | Accuracy % | Loss  |
|----------|------------|-------|
| KARATE   | 100        | 0     |
| CORA     | 85.2       | 0.529 |
| CITeseer | 66.2       | 0.992 |

Table 2: Node classification results

### 3.2 Varying L (20 points)

For both CORA and CITESEER, modify the GNN to include  $L=3,4,5,6$  layers and plot the loss and accuracy vs.  $L$ . Summarize your observations in 2-3 lines.

Test Accuracy and Loss plots for CORA and Citeseer



In general, accuracy lowers and loss increases with  $L$ . This may indicate that more complex models are harder to train, hence more epochs or more data could benefit models with larger  $L$ . Looking at train VS validation loss/acc in the std output, larger models had an harder time generalizing, motivating harder regularization.

### 3.3 Topological features vs. inbuilt features (20 points)

| Graph              | Accuracy % | Loss  |
|--------------------|------------|-------|
| Cora               | 85.2       | 0.529 |
| Cora_topo          | 41.8       | 1.553 |
| Cora_plus_topo     | 66.0       | 0.988 |
| Citeseer           | 66.2       | 0.992 |
| Citeseer_topo      | 25.7       | 1.756 |
| Citeseer_plus_topo | 63.6       | 1.144 |

Table 2: Effect of topological features

Looking at the table, topological features alone have weak performance. Combining them with the original features did not improve the results. This second setup may require more testing and hyperparameter tuning as the input features now have a higher dimension.

## 4 Link prediction

### 4.1 Training data for link prediction (20 points)

A.

| Graph    | # Positive edges | # Negative edges |
|----------|------------------|------------------|
| KARATE   | 190              | 190              |
| CORA     | 13264            | 13264            |
| CITeseer | 12384            | 12384            |

Table 3: Training data statistic for link prediction

B. How is the training data for link prediction created? Please explain in 2-3 lines.

First, split the set of all edges  $(v_i, v_j)$  into train, validation, and test splits. All the edges present in the training set are naturally considered positive instances. Negatives are sampled by randomly shuffling the nodes of the positive edges.

### 4.2 Implementation (80 points)

| Graph    | Accuracy % | Loss  |
|----------|------------|-------|
| KARATE   | 51.34      | 1.008 |
| CORA     | 93.25      | 0.179 |
| CITeseer | 94.95      | 0.152 |

Table 4: Link Prediction Results

## 5 Graph classification

### 5.1 Graph Statistics (10 points)

| Graph            | MUTAG | ENZYMES |
|------------------|-------|---------|
| Num graphs       | 141   | 360     |
| Avg. num nodes   | 18.85 | 33.27   |
| Avg. num edges   | 94.04 | 221.19  |
| Node feature dim | 8     | 22      |

Table 5: Graph statistics for the graph classification datasets

### 5.2 Implementation (90 points)

| Graph             | MUTAG |    |    | ENZYMES |    |    |
|-------------------|-------|----|----|---------|----|----|
|                   | P     | R  | F1 | P       | R  | F1 |
| Mean-pooling      | 63    | 76 | 64 | 34      | 37 | 33 |
| Max-pooling       | 84    | 83 | 83 | 37      | 42 | 37 |
| Last-node pooling | 71    | 77 | 73 | 35      | 36 | 35 |

Table 6: Graph classification results. Please use macro-averages to report the precision, recall, and F1 score for ENZYMES.

## References