DATA 605 - Discussion Week 2

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9/9/2019

Let's work on Problem C29 from the Determinant chapter:

$$\det \begin{pmatrix} 2 & 3 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

We can reduce this to row echelon form to solve for the determinant.

First let's cancel element [4,2] by $R_4 \leftarrow R_4 - 1 \cdot R_2$

$$\begin{pmatrix} 2 & 3 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

Then let's cancel element [4,3] by $R_4 \leftarrow R_4 - 1 \cdot R_3$

$$\begin{pmatrix} 2 & 3 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 & -5 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

Then let's cancel element [5,4] by $R_5 \leftarrow R_5 + \frac{1}{2} \cdot R_4$

$$\begin{pmatrix} 2 & 3 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 & -5 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

The determinant of the matrix in row echelon form is the diagonal product of the matrix.

$$2 \cdot 1 \cdot 1 \cdot (-2) \left(-\frac{1}{2} \right) = 2$$