605Discussion3

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C25† Find the eigenvalues, eigenspaces, algebraic and geometric multiplicities for the 3 × 3 identity matrix I3. Do your results make sense?

The 3 × 3 identity matrix I3 is:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To compute the eigenvalues we start with the condition $det(A - \lambda I) = 0$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix}$$

and $det \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix}$ is equal to:

$$= (1-\lambda) det \begin{pmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix} - 0 \cdot det \begin{pmatrix} 0 & 0 \\ 0 & 1-\lambda \end{pmatrix} + 0 \cdot det \begin{pmatrix} 0 & 1-\lambda \\ 0 & 0 \end{pmatrix}$$

The two left components are zero and we are only left with $= (1-\lambda)(1-\lambda)^2$.

Therefore:

$$det\left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) : \quad (-\lambda + 1)^3$$

With the eigenvalue of λ with a multiplicity of 3 (3 repetitions).

Finding the eigenvectors of a identity matrix does not require any calculations. If A is the identity matrix, every vector has Ax = x. All vectors are eigenvectors of I . All eigenvalues "lambda" are equal 1. The set of all vectors that satisfy this condition in the case of a 3x3 matrix are:

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The eigenvalues of a diagonal matrix are the elements along the diagonal. While the eigenvectors for correspond to the set of column vectors in the identity matrix.