605 - Week 10 Assigment

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Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability .4 and loses A dollars with probability .6. Find the probability that he wins 8 dollars before losing all of his money if:

First, we need to find the transition matrix for the following problem in the form below:

$$\mathbf{P} = \left(\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right)$$

> Here I is an r -by-r indentity matrix, 0 is an r -by- t zero matrix, $\mathbf{R}$ is a nonzero t -by-r matrix, and $\mathbf{Q}$ is an t -by-t matrix.

The we can use these submatrices to find N the fundamental Matrix:

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$$

> Definition 11.3 For an absorbing Markov chain $\mathbf{P}$, the matrix $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$ is called the fundamental matrix for $\mathbf{P}$. The entry n_{ij} of $\mathbf{N}$ gives the expected number of times that the process is in the transient state s_j if it is started in the transient state s_i .

Finally, we can calculate the absortion probability matrix B:

> Theorem 11.6 Let b_{ij} be the probability that an absorbing chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i. Let $\mathbf{B}$ be the matrix with entries b_{ij}. Then $\mathbf{B}$ is an t -by-r matrix, and
> $\mathbf{B} = \mathbf{NR}$
> where $\mathbf{N}$ is the fundamental matrix and $\mathbf{R}$ is as in the canonical form.

(a) he bets 1 dollar each time (timid strategy).

The probability matrix P for the timid approach is:

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]  0.0  0.4  0.0  0.0  0.0  0.0  0.0  0.6  0.0
##  [2,]  0.6  0.0  0.4  0.0  0.0  0.0  0.0  0.0  0.0
##  [3,]  0.0  0.6  0.0  0.0  0.4  0.0  0.0  0.0  0.0
##  [4,]  0.0  0.0  0.6  0.0  0.4  0.0  0.0  0.0  0.0
##  [5,]  0.0  0.0  0.0  0.6  0.0  0.4  0.0  0.0  0.0
##  [6,]  0.0  0.0  0.0  0.0  0.6  0.0  0.4  0.0  0.0
##  [7,]  0.0  0.0  0.0  0.0  0.0  0.6  0.0  0.0  0.4
##  [8,]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0
##  [9,]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0
```

The timid matrix Q is:

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]  0.0  0.4  0.0  0.0  0.0  0.0  0.0
## [2,]  0.6  0.0  0.4  0.0  0.0  0.0  0.0
## [3,]  0.0  0.6  0.0  0.0  0.4  0.0  0.0
## [4,]  0.0  0.0  0.6  0.0  0.4  0.0  0.0
## [5,]  0.0  0.0  0.0  0.6  0.0  0.4  0.0
## [6,]  0.0  0.0  0.0  0.0  0.6  0.0  0.4
## [7,]  0.0  0.0  0.0  0.0  0.0  0.6  0.0
```

The timid matrix R is:

```
##      [,1] [,2]
## [1,]  0.6  0.0
## [2,]  0.0  0.0
## [3,]  0.0  0.0
## [4,]  0.0  0.0
## [5,]  0.0  0.0
## [6,]  0.0  0.0
## [7,]  0.0  0.4
```

The fundamental matrix N is:

```
##       [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]
## [1,] 1.593 0.988 0.585 0.316 0.527 0.277 0.111
## [2,] 1.482 2.470 1.462 0.790 1.316 0.693 0.277
## [3,] 1.316 2.193 2.777 1.501 2.501 1.316 0.527
## [4,] 1.216 2.026 2.567 2.927 3.212 1.690 0.676
## [5,] 1.066 1.777 2.251 2.567 4.278 2.252 0.901
## [6,] 0.842 1.403 1.777 2.026 3.377 3.093 1.237
## [7,] 0.505 0.842 1.066 1.216 2.026 1.856 1.742
```

The absortion probability matrix B is:

```
##       [,1]  [,2]
## [1,] 0.956 0.044
## [2,] 0.889 0.111
## [3,] 0.790 0.211
## [4,] 0.730 0.270
## [5,] 0.640 0.360
## [6,] 0.505 0.495
## [7,] 0.303 0.697
```

```
## The timid probability to go from 1 dollar to 8 is: 0.044
```

(b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). *The probability matrix P for the bold approach is:*

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0  0.4  0.0  0.6  0.0
## [2,]    0  0.0  0.4  0.6  0.0
## [3,]    0  0.0  0.0  0.6  0.4
## [4,]    0  0.0  0.0  1.0  0.0
## [5,]    0  0.0  0.0  0.0  1.0
```

The bold matrix Q is:

```
##      [,1] [,2] [,3]
## [1,]    0  0.4  0.0
## [2,]    0  0.0  0.4
## [3,]    0  0.0  0.0
```

The bold matrix R is:

```
##      [,1] [,2]
## [1,]  0.6  0.0
## [2,]  0.6  0.0
## [3,]  0.6  0.4
```

The fundamental matrix N is:

```
##      [,1] [,2] [,3]
## [1,]    1  0.4 0.16
## [2,]    0  1.0 0.40
## [3,]    0  0.0 1.00
```

The absortion probability matrix B is:

```
##       [,1]  [,2]
## [1,] 0.936 0.064
## [2,] 0.840 0.160
## [3,] 0.600 0.400
```

```
## The bold probability to go from 1 dollar to 8 is: 0.064
```

(c) Which strategy gives Smith the better chance of getting out of jail?

The bold strategy leads to a higher probability 0.064 to reach 8 dollars starting from 1 dollar. Compared to the lower probability of 0.044 for the timid strategy.