605-Week9

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11. The price of one share of stock in the Pilsdorf Beer Company (see Exercise 8.12) is given by Y_n on the n th day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 100,$ estimate the probability that Y_{365} is:

(a) ≥ 100

We can solve this problem with the pnorm function from R:

$$\Pr(X \leq x) = F(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

that given a number x it computes the probability that a normally distributed random number will be less than that number.

```
mean <- 0
x <- 100
var <- 1/4
diff <- x - 100
sd<- sqrt(var)
n <- 365-1
q <- diff/sqrt(n)
p <- pnorm(q, mean, sd, lower.tail = FALSE) %>% round(2)
cat("The probability that that Y365 is more or equal than", x, "is", p)
```

```
## The probability that that Y365 is more or equal than 100 is 0.5
```

(b) ≥ 110

```
mean <- 0
x <- 110
var <- 1/4
diff <- x - 100
sd<- sqrt(var)
n <- 365-1
q <- diff/sqrt(n)
p <- pnorm(q, mean, sd, lower.tail = FALSE) %>% round(2)
cat("The probability that that Y365 is more or equal than", x, "is", p)
```

```
## The probability that that Y365 is more or equal than 110 is 0.15
```

(c) ≥ 120

```r
mean <- 0
x <- 120
var <- 1/4
diff <- x-100
sd<- sqrt(var)
n <- 365-1
q <- diff/sqrt(n)
p <- pnorm(q, mean, sd, lower.tail = FALSE) %>% round(2)
cat("The probability that that Y365 is more or equal than", x, "is", p)
```

```
## The probability that that Y365 is more or equal than 120 is 0.02
```

***Calculate the expected value and variance of the binomial distribution using the moment generating function.**

Let's start with the binomial distribution:

$$b(x; n, p) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \quad \text{with} \quad q = 1 - p$$

The moment generating function is of the form:

$$M_x(t) = \sum_{x=0}^{n} e^{xt} \frac{n!}{x!(n-x)!} p^x q^{n-x}$$
$$M(t) = \left(q + pe^t\right)^n$$

We can then calculate the mean as the first derivative of the moment generating function:

$$M'(t) = n\left[1 - p + pe^t\right]^{n-1}\left(pe^t\right)$$

solving we find:

$$\mu = np$$

and the variance as the second derivative of the moment generating function:

$$M''(t) = n\left[1 - p + pe^t\right]^{n-1}\left(pe^t\right) + \left(pe^t\right) n(n-1)\left[1 - p + pe^t\right]^{n-2}\left(pe^t\right)$$

solving we find:

$$\sigma^2 = np(1-p)$$

Calculate the expected value and variance of the exponential distribution using the moment generating function.

Let's start with the exponential distribution:

$$f(x) = \frac{1}{\theta} e^{-x/\theta}$$

The moment generating function is of the form:

$$M_x(t) = E\left(e^{tX}\right) = \int_0^{\infty} e^{tx}\left(\frac{1}{\theta}\right) e^{-x/\theta} dx$$

$$M(t) = \left[(1-p) + pe^t\right]^n$$

We can then calculate the mean as the first derivative of the moment generating function:

$$M'(t) = n\left[pe^t + (1-p)\right]^{n-1} pe^t$$

solving we find:

$$\mu = n(n-1)p^2 + np$$

and the variance as the second derivative of the moment generating function:

$$M''(t) = n(n-1)\left[pe^t + (1-p)\right]^{n-2} p^2 e^{2t} + n\left[pe^t + (1-p)\right]^{n-1} pe^t$$

solving we find:

$$\sigma^2 = \frac{1-p}{p^2}$$