605-Wk14-HW

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Taylor Functions

Function and Series	First Few Terms	Interval of Convergence
$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{x^{2n-1}}{(2n-1)!}$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots$	$(-1, 1]$
$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$(-\infty, \infty)$
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$	$1 + x + x^2 + x^3 + \cdots$	$(-1, 1)$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.

We can use the library *rSymPy* to symbolically calculate the Taylor expansio of the series above. Below we compare how closely the Taylor expansion value compares to the numerical value evaluated at 0.8

$$\frac{1}{1-x}$$

```r
for (n in 1:10) {
    sympy("var('p')")
    sympy("var('x')")  # or sympy('x = Symbol('x', real=True)')
    equation <- paste("p=series(1/(1-x), x, 0,", n, ")")  #'p=series(1/(1-x), x, 0, 10)'
    xt <- sympy(equation)  # expand about 0 to 10th order
    # xt <- sympy('p=series(1/(1-x), x, 0, 10)') # expand about 0
    # to 10th order Remove order information
    xt0 <- sympy("p.removeO()")
    # Test results
    x <- 1/3
    T1 <- eval(parse(text = xt0))  # Evaluate the result, xt0
    T2 <- 1/(1 - x)  # The correct value
    Difference <- (T1 - T2)
    cat("\nAt", n, " terms, The Taylor expansion is", xt0, ". \n Evaulating at 0, the difference between
        Difference, "\n")
    rm(x)
}
```

```
##
## At 1  terms, The Taylor expansion is 1 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.5
##
## At 2  terms, The Taylor expansion is 1 + x .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.1666667
##
```

```
## At 3  terms, The Taylor expansion is 1 + x + x**2 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.05555556
##
## At 4  terms, The Taylor expansion is 1 + x + x**2 + x**3 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.01851852
##
## At 5  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.00617284
##
## At 6  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 + x**5 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.002057613
##
## At 7  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 + x**5 + x**6 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.0006858711
##
## At 8  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 + x**5 + x**6 + x**7 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -0.0002286237
##
## At 9  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 + x**5 + x**6 + x**7 + x**8 .
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -7.62079e-05
##
## At 10  terms, The Taylor expansion is 1 + x + x**2 + x**3 + x**4 + x**5 + x**6 + x**7 + x**8 + x**9
##  Evaulating at 0, the difference between the Taylor expansion and the built-in function is
##  -2.540263e-05
```

e^x

```r
for (n in 1:10) {
    sympy("var('p')")
    sympy("var('x')")  # or sympy('x = Symbol('x', real=True)')
    equation <- paste("p=series(exp(x), x, 0,", n, ")")  #'p=series(1/(1-x), x, 0, 10)'
    xt <- sympy(equation)  # expand about 0 to 10th order
    # xt <- sympy('p=series(1/(1-x), x, 0, 10)') # expand about 0
    # to 10th order Remove order information
    xt0 <- sympy("p.removeO()")
    # Test results
    x <- 0.8
    T1 <- eval(parse(text = xt0))  # Evaluate the result, xt0
    T2 <- exp(x)  # The correct value
    Difference <- (T1 - T2)
    cat("\nAt", n, " terms, The Taylor expansion is", xt0, ". \n Evaluating at 0.8, the difference betwe
        Difference, "\n")
    rm(x)
}
```

```
##
## At 1  terms, The Taylor expansion is 1 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -1.225541
##
## At 2  terms, The Taylor expansion is 1 + x .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.4255409
##
## At 3  terms, The Taylor expansion is 1 + x + x**2/2 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.1055409
##
## At 4  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.0202076
##
## At 5  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.003140928
##
## At 6  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.0004102618
##
## At 7  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 + x**6/720 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -4.617294e-05
##
## At 8  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 + x**6/720 + x**7/5
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -4.562778e-06
##
```

```
## At 9  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 + x**6/720 + x**7/5
##  Evaulating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -4.017623e-07
##
## At 10  terms, The Taylor expansion is 1 + x + x**2/2 + x**3/6 + x**4/24 + x**5/120 + x**6/720 + x**7/
##  Evaulating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -3.189423e-08
```

$\ln(1+x)$

```r
for (n in 1:10) {
    sympy("var('p')")
    sympy("var('x')")  # or sympy('x = Symbol('x', real=True)')
    equation <- paste("p=series(log(1+x), x, 0,", n, ")")  #'p=series(1/(1-x), x, 0, 10)'
    xt <- sympy(equation)  # expand about 0 to 10th order
    # xt <- sympy('p=series(1/(1-x), x, 0, 10)') # expand about 0
    # to 10th order Remove order information
    xt0 <- sympy("p.removeO()")
    # Test results
    x <- 0.8
    T1 <- eval(parse(text = xt0))  # Evaluate the result, xt0
    T2 <- log(1 + x)  # The correct value
    Difference <- (T1 - T2)
    cat("\nAt", n, " terms, The Taylor expansion is", xt0, ". \n Evaluating at 0.8, the difference betw
        Difference, "\n")
    rm(x)
}
```

```
##
## At 1  terms, The Taylor expansion is 0 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.5877867
##
## At 2  terms, The Taylor expansion is x .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  0.2122133
##
## At 3  terms, The Taylor expansion is x - x**2/2 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.1077867
##
## At 4  terms, The Taylor expansion is x - x**2/2 + x**3/3 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  0.06288
##
## At 5  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.03952
##
## At 6  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 + x**5/5 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  0.026016
##
## At 7  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 + x**5/5 - x**6/6 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.01767466
##
## At 8  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 + x**5/5 - x**6/6 + x**7/7 .
##  Evaluating at 0.8, the difference between the Taylor expansion and the built-in function is
##  0.01228465
##
```

```
## At 9  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 + x**5/5 - x**6/6 + x**7/7 - x**8/8
##  Evaulating at 0.8, the difference between the Taylor expansion and the built-in function is
##  -0.008686871
##
## At 10  terms, The Taylor expansion is x - x**2/2 + x**3/3 - x**4/4 + x**5/5 - x**6/6 + x**7/7 - x**8/
##  Evaulating at 0.8, the difference between the Taylor expansion and the built-in function is
##  0.00622621
```