605-HW7

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1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y .

$$Distribution_Y \quad = \quad \frac{Options \quad of \quad the \quad Set \quad Y}{All \quad Options}$$

First we count the way to have values of X between k and a minimum value y.

$$Options \quad of \quad the \quad Set \quad Y \quad = \quad (k-y)^n - (k-y)$$

And there are k^n ways to have n X values between 1 to k.

$$All \quad Options \quad = \quad k^n$$

$$Distribution_Y \quad = \quad \frac{(k-y)^n - (k-y)}{k^n}$$

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

We will use the equation for probability equation for geometric distributions:

$$P_{geometric} \quad = \quad (1-p)^{n-1}p$$

given p (failure probability per year) as 1/10 and n as 9 (number of years until failure).

```
n.a <- 9
p.a <- 1/10

Prob.geom <- 0


for(i in 1:n.a){
  Prob.geom <- Prob.geom + p.a * ( (1-p.a)^(i-1) )

}
Prob.geom
```

```
## [1] 0.6125795
```

```
cat("The probability of failure after 8 years is:", Prob.geom)
```

```
## The probability of failure after 8 years is: 0.6125795
```

```
mean.a <- p.a^(-1)
SD.a <- sqrt((1-p.a)/(p.a^2))
cat("Using a Geometric distribution the expected value is", mean.a, " and the standard deviation is ", S
```

```
## Using a Geometric distribution the expected value is 10  and the standard deviation is  9.486833
```

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

$$P_{Exponential} \quad \approx \quad e^{-\lambda t}$$

```
lambda.d <- 1/10
t.d <- 8
Prob.Expo <- (2.71828)^(-lambda.d*t.d)
cat("The probability that the machine will fail after 8 years using a exponential distribution is:", Pr
```

```
## The probability that the machine will fail after 8 years using a exponential distribution is: 0.4493
```

```
mean.b <- 1/lambda.d
SD.b <- mean.b
cat("Using a Exponential distribution the expected value is", mean.b, " and the standard deviation is "
```

```
## Using a Exponential distribution the expected value is 10  and the standard deviation is  10
```

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

$$P \quad = \binom{n}{k} p^k (1-p)^{(n-k)}$$

```
n.c <- 8
k.c <- 0
p.c <- 1/10
Prob.Binom <- pbinom(k.c, n.c, p.c)
cat("The probability that the machine will fail after 8 years using a Binomial distribution is:", Prob.
```

```
## The probability that the machine will fail after 8 years using a Binomial distribution is: 0.4304672
```

```
mean.c <- n.c * p.c
SD.c <- sqrt(n.c * p.c - (Prob.Binom))
cat("Using a Binomial distribution the expected value is", mean.c, " and the standard deviation is ", SD
```

```
## Using a Binomial distribution the expected value is 0.8  and the standard deviation is  0.6078921
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

$$P_{Poisson} \quad \approx \quad \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

```
lambda.d <- 1/10
r.d <- 0
t.d <- 8
Prob.poisson <- ( (lambda.d*t.d)^r.d / (factorial(r.d)) ) * (2.71828)^(-lambda.d*t.d)
cat("The probability that the machine will fail after 8 years using a Poisson distribution is:", Prob.po
```

```
## The probability that the machine will fail after 8 years using a Poisson distribution is: 0.4493292
```

```
mean.d <- lambda.d * t.d
SD.d <- sqrt(mean.d)
cat("Using a Poisson distribution the expected value is", mean.d, " and the standard deviation is ", SD
```

```
## Using a Poisson distribution the expected value is 0.8  and the standard deviation is  0.8944272
```