Th4 (Taylor) 主要功能：提高精确度

条件：$f(x)$ 在 $x=x_0$ 邻域内 $n+1$ 阶可导

结论：$f(x) = P_n(x) + R_n(x)$

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其中，$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$

$$+ \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$R_n(x) = \begin{cases} \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1} & : L \\ o((x-x_0)^n) & : \text{皮亚诺型} \end{cases}$$

即

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots$$

$$+ \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + R_n(x)$$

If $x_0 = 0$

$$f(x) = f(0) + f'(0) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$+ R_n(x)$ ：麦克劳林公式

记：① $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n)$

② $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

$+ \frac{(-1)^n}{(2n+1)!} x^{2n+1} + o(x^{2n+1})$

③ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$+ \frac{(-1)^n}{(2n)!} x^{2n} + o(x^{2n})$

④ $\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n)$

⑤ $\frac{1}{1+x} = 1 - x + x^2 \cdots + (-1)^n x^n + o(x^n)$

⑥ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \cdots + \frac{(-1)^{n-1}}{n} x^n + o(x^n)$

型六，麦克劳林公式求极限

例1. $\lim\limits_{x \to 0} \frac{x\cos x - \sin x}{x^3}$

解：$\sin x = x - \frac{x^3}{3!} + o(x^3) = x - \frac{x^3}{6} + o(x^3)$

$\cos x = 1 - \frac{x^2}{2!} + o(x^2)$

$$x\cos x = x - \frac{x^3}{2} + o(x^3)$$

$$x\cos x - \sin x = -\frac{x^3}{3} + o(x^3)$$

$$\sim -\frac{1}{3}x^3$$

$\therefore$ 原式 $= -\frac{1}{3}$　用洛必达也可以算出

例2、$\lim\limits_{x\to 0} \dfrac{e^{-\frac{x^2}{2}} - 1 + \frac{x^2}{2}}{x^4}$

解：$e^x = 1 + x + \frac{x^2}{2!} + o(x^2)$

$$\therefore e^{-\frac{x^2}{2}} = 1 - \frac{x^2}{2} + \frac{x^4}{8} + o(x^4)$$

$$e^{-\frac{x^2}{2}} - 1 + \frac{x^2}{2} = \frac{x^4}{8} + o(x^4)$$

$\therefore$ 原式 $= \frac{1}{8}$

⑦ $(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3$

$+ \cdots$

例3 $\lim\limits_{x\to 0}\frac{\sqrt{1+x}+\sqrt{1-x}-2}{x^2}$

解：$\because (1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + o(x^2)$

$\therefore \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$

$\sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$

$\therefore \sqrt{1+x} + \sqrt{1-x} - 2 = -\frac{1}{4}x^2 + o(x^2)$

$\therefore$ 原式 $= -\frac{1}{4}$