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Metodos de Integración

1) $\int e^{3\cos 2x} \operatorname{sen} 2x \, dx$

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du \quad u = g(x)$$

$u = 3\cos(2x)$

$\frac{d}{dx}(3\cos(2x))$

$f = \cos(u)$

$u = 2x$

$3\frac{d}{dx}(\cos(2x))$

$3\frac{d}{du}(\cos(u))\frac{d}{dx}(2x)$

$3(-\operatorname{sen}(u)) \cdot 2 \quad u = 2x$

$3(-\operatorname{sen}(2x) \cdot 2$

$-6\operatorname{sen}(2x)$

$-6\operatorname{sen}(2x) dx = du$

$dx = \frac{du}{-6\operatorname{sen}(2x)}$

$= \int e^u \operatorname{sen}(2x)\left(-\frac{1}{6\operatorname{sen}(2x)}\right) du$

$u = 3\cos(2x)$

$\int -\frac{e^u \cancel{\operatorname{sen}(2x)}}{6\cancel{\operatorname{sen}(2x)}} du$

$= \int -\frac{e^u}{6} du = -\frac{1}{6}\int e^u du = -\frac{1}{6}e^u = -\frac{e^{3\cos(2x)}}{6} + C$

2. $\int \frac{dx}{e^x+1}$

$$\frac{1}{e^x+1} = \frac{1-(e^x+1)}{e^x+1} + 1$$

$$= \frac{1-(e^x+1)}{e^x+1} + 1 = \frac{e^x}{e^x+1}$$

$$\frac{1-(e^x+1)}{e^x+1}$$

$1-(e^x+1)$

$-(e^x+1)$

$-(e^x)-(1)$

$= -e^x-1$

$1-e^x-1$

$= -e^x$

$$= \frac{-e^x}{e^x+1} + 1$$

$$= \int -\frac{e^x}{e^x+1} + 1\,dx$$

$$= -\int \frac{e^x}{e^x+1} + \int 1\,dx$$

$$-\int \frac{e^x}{e^x+1}\,dx \qquad \int 1\,dx$$

$u = e^x+1$

$$\frac{d}{dx}(e^x+1)$$

$$=\frac{d}{dx}(e^x)+\frac{d}{dx}(1)$$

$$du=e^x dx$$

$$dx=\frac{du}{e^x}$$

$$=\int \frac{e^x}{u}\,\frac{1}{e^x}\,du$$

$$\frac{e^x}{u}\cdot\frac{1}{e^x}$$

$$\frac{\cancel{e^x}}{u\cancel{e^x}}$$

$$=\frac{1}{u}$$

$$=\int \frac{1}{u}\,du \qquad u=e^x+1$$

$$\int 1\,dx$$

$$=\ln(|u|) \qquad =x$$

$$=\ln|e^x+1|$$

$$=-\ln|e^x+1|+x+c$$

3. $\int \sqrt{x^2-2x^4}\,dx$

x^2-2x^4

$x^4 = x^2x^2$

$x^2(1-2x^2)$

$= \sqrt{x^2(1-2x^2)}$

$= \sqrt{x^2}\sqrt{1-2x^2}$

$= \int \sqrt{x^2}\sqrt{1-2x^2}\,dx$

$= \int x\sqrt{1-2x^2}\,dx$

Integración por sustitución $u = 1-2x^2$

$= \frac{d}{dx}(1-2x^2)$

$= 0-4x$

$du = -4x\,dx$

$dx = \frac{du}{-4x}$

$= \int x\sqrt{u}\left(-\frac{1}{4x}\right)du$

$\int -\frac{\cancel{x}\sqrt{u}}{4\cancel{x}}\,du$

$\int -\frac{\sqrt{u}}{4}\,du$

$-\frac{1}{4}\int u^{1/2}\,du$

$-\frac{1}{4}\cdot\frac{u^{3/2}}{3/2}$

$-\frac{1}{4}\cdot\frac{(1-2x^2)^{3/2}}{3/2}$

$-\frac{(1-2x^2)^{3/2}}{6}$

$= -\frac{1}{6}(1-2x^2)^{3/2}+C$

DD MM AA

4 $\int x \cot x^2 \, dx$

Integración por sustitución

$u = x^2$

$\frac{d}{dx}(x^2)$

$= 2x$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \int x \cot(u) \frac{du}{2x}$

$\cancel{x} \cot(u) \frac{1}{2\cancel{x}}$

$\frac{\cot(u)}{2}$

$\int \frac{\cot(u)}{2} du$

$= \frac{1}{2} \int \cot(u)\, du$

$V = \text{sen}(u) \qquad u = x^2$

$= \frac{1}{2} \int \frac{\cos(u)}{\text{sen}(u)} du$

$= \frac{1}{2} \int \frac{\cos(u)}{V} \cdot \frac{1}{\cos(u)} dv$

$\frac{d\,\text{sen}(u)}{du}$

$= \frac{1}{2} \int \frac{1}{V} dv$

$= \cos(u)$

$= \frac{1}{2} \ln|V|$

$dv = \cos(u)\, du$

$= \frac{1}{2} \ln|\text{sen}(x^2)| + C$

$du = \frac{dv}{\cos(u)}$

5 $$\int \frac{1}{1+\cos x}\, dx$$

$$u = \tan\left(\frac{x}{2}\right)$$

$$\cos(x) = \frac{1-u^2}{1+u^2}$$

$$dx = \frac{2}{1+u^2}\, du$$

$$= \int \frac{1}{1+\frac{1-u^2}{1+u^2}} \cdot \frac{2}{1+u^2}\, du$$

$$1 + \frac{1-u^2}{1+u^2}$$

$$= \cancel{\frac{(1+u^2)}{(1+u^2)}} + \frac{1-u^2}{1+u^2}$$

$$= \frac{1}{1} + \frac{1-u^2}{1+u^2}$$

$$= \frac{(1+u^2)+(1-u^2)}{1+u^2}$$

$$= \frac{2}{1+u^2}$$

$$= \frac{1}{\frac{2}{1+u^2}}$$

$$= \frac{\cancel{1+u^2}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{1+u^2}}$$

$$= 1$$

$$= \int 1\, du$$

$$= u$$

$$\boxed{= \tan\left(\frac{x}{2}\right) + C}$$

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6. $\int \frac{1}{x\sqrt{4x^2-9}}\,dx$

$\sqrt{bx^2-a}$ $\qquad a=2,\ b=3$

$x=\frac{\sqrt{a}}{\sqrt{b}}\sec(u)$

$\frac{d}{du}\left(\frac{3}{2}\sec(u)\right)$

$\frac{3}{2}\frac{d}{du}\sec(u)$

$=\frac{3}{2}\sec(u)\tan(u)$

$dx=\frac{3}{2}\sec(u)\tan(u)\,du$

$$\int \frac{1}{\frac{3}{2}\cancel{\sec(u)}\sqrt{4\left(\frac{3}{2}\sec(u)\right)^2-9}}\cdot\frac{3}{2}\cancel{\sec(u)}\tan(u)\,du$$

$$\frac{3\tan(u)}{\frac{3}{\cancel{2}}\sqrt{4\left(\frac{3}{2}\sec(u)\right)^2-9}\cdot\cancel{2}}$$

$3\sqrt{4\left(\frac{3}{2}\sec(u)\right)^2-9}$

$3\cdot 3\sqrt{(\sec(u))^2-1}$

$9\sqrt{\tan^2(u)}$

$9\tan(u)$

$=\frac{3\tan(u)}{9\tan(u)}$

$=\frac{1}{3}=\int\frac{1}{3}\,du=\frac{1}{3}\int du=\frac{1}{3}u$ $\boxed{=\frac{1}{3}\operatorname{arcsec}\left(\frac{2}{3}\right)x+c}$

7. $\int \frac{x}{x^4+3}\,dx$

$u = x^2$

$\frac{d}{dx}(x^2)$

$= 2x$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \int \frac{\cancel{x}}{x^4+3} \cdot \frac{du}{2\cancel{x}}$

$\int \frac{1}{2(x^4+3)}\,du$

$= \frac{1}{2}\int \frac{1}{(u^2+3)}\,du$ $\qquad u = \sqrt{3}\,v$

$a = 3 \quad b = 1$ $\qquad x = \frac{\sqrt{a}}{\sqrt{b}}u$

$x = \sqrt{3}\,u$

$\frac{d}{dv}(\sqrt{3}\,v)$

$du = \sqrt{3}\,dv$

$\frac{1}{2}\int \frac{1}{(\sqrt{3}\,v)^2+3}\sqrt{3}\,dv$

$= \frac{1}{2}\int \frac{\sqrt{3}}{3v^2+3}$ $\quad \int \frac{\sqrt{3}}{3(v^2+1)} = \int \frac{1}{\sqrt{3}\,(v^2+1)}$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{3}}\cdot\int\frac{1}{v^2+1}\,dv \qquad v=\frac{u}{\sqrt{3}} \qquad u=x^2$$

$$\int\frac{1}{v^2+1}\,dv=\arctan(v)$$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{3}}\cdot\arctan\left(\frac{x^2}{\sqrt{3}}\right)$$

$$\frac{1}{2\sqrt{3}}\arctan\left(\frac{x^2}{\sqrt{3}}\right)+c$$

8 $$\int\frac{dx}{\sqrt{20+8x-x^2}}$$

$$=-(x^2-8x-20+(-4)^2-(-4)^2)$$

$-x^2+8x+20$ $\qquad =-((x-4)^2-20-(-4)^2)$

$-(x^2-8x-20)$ $\qquad =-(x-4)^2+36$

$2a=-8$

$a=-\frac{8}{2}$ $\qquad =\int\frac{1}{\sqrt{-(x-4)^2+36}}\,dx$

$a=-4$ $\qquad u=x-4$

$$\frac{d}{dx}(x-4)$$

$$\frac{d}{dx}(x)-\frac{d}{dx}(4)$$

$$=\int\frac{1}{\sqrt{-u^2+36}}\,du \qquad =1-0$$

$du=1dx$

$\frac{d}{dv}(6\,\text{sen}(v))$ $\qquad dx=du$

$$6\frac{d}{dv}\,\text{sen}(v)=6\cos(v)$$

$$\frac{1}{\sqrt{-(6\operatorname{sen}(v))^2+36}} \cdot 6\cos(v)\,dv$$

$$= 6 \int \frac{\cos(v)}{6\sqrt{-\operatorname{sen}^2(v)+1}}$$

$v = \operatorname{arcsen}\left(\frac{1}{6}u\right)$

$$= \frac{\cos(v)}{\sqrt{\cos^2(v)}}$$

$u = x-4$

$$= \frac{\cos(v)}{\cos(v)}$$

$$= 1$$

$$= \int 1\,dv = 1v = \operatorname{arcsen}\left(\frac{1}{6}(x-4)\right)+C$$

9.
$$\int \frac{2x+3}{9x^2-12x+8}\,dx$$

$$= \int \frac{2x}{9x^2-12x+8}\,dx + \int \frac{3}{9x^2-12x+8}\,dx$$

$$2\int \frac{x}{9x^2-12x+8}\,dx \qquad 9x^2-12x+8 = 9\left(x-\frac{2}{3}\right)^2+4$$

$$= 2\cdot\int \frac{x}{9\left(x-\frac{2}{3}\right)^2+4}\,dx$$

$u = x - \frac{2}{3}$

$$\frac{d}{dx}\left(x-\frac{2}{3}\right)$$

$du = 1dx$

$$\frac{d}{dx}(x) - \frac{d}{dx}\left(\frac{2}{3}\right)$$

$dx = du$

$$1-0=1$$

$$\int \frac{x}{9u^2+4}\,du \qquad u = x - \frac{2}{3}$$

$$x = u + \frac{2}{3}$$

$$\int \frac{u+\frac{2}{3}}{9u^2+4}\,du$$

$$= \frac{\frac{3u+2}{3}}{9u^2+4}$$

$$= \frac{3u+2}{3(9u^2+4)}$$

$$2.\ \frac{1}{3}\int \frac{3u+2}{3(9u^2+4)}\,du$$

$$\frac{2}{3}\left(\int \frac{3u}{3(9u^2+4)}\,du + \int \frac{2}{3(9u^2+4)}\,du\right)$$

$V = 9u^2+4$

$$3\int \frac{u}{3(9u^2+4)}\,du$$

$$3\int \frac{1}{18v}\,dv$$

$$\frac{3}{18}\int \frac{1}{v}\,dv$$

$$\frac{3}{18}\ln|v|$$

$$\frac{3}{18}\ln|9u^2+4|$$

$$\frac{1}{6}\ln|9u^2+4|$$

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$$\int \frac{2}{9u^2+4}\,du$$

$$= 2\cdot\frac{1}{3}\left(\frac{1}{6}\ln|9u^2+4| + \frac{1}{3}\arctan\left(\frac{3}{2}u\right)\right)$$

$u = x - \frac{2}{3}$

$$\frac{2}{3}\left(\frac{1}{6}\ln\left|9\left(x-\frac{2}{3}\right)^2+4\right| + \frac{1}{3}\arctan\left(\frac{3}{2}\left(x-\frac{2}{3}\right)\right)\right)$$

$$= \frac{1}{9}\left(\ln|9x^2-12x+8| + 2\arctan\left(\frac{3x-2}{2}\right)\right)$$

$$\int \frac{3}{9x^2-12x+8}\,dx$$

$$= 3\int \frac{1}{9x^2-12x+8}\,dx$$

$$= 3\int \frac{1}{9\left(x-\frac{2}{3}\right)^2+4}\,dx \qquad u = x-\frac{2}{3}$$

$dx = du$

$$= \int \frac{1}{9u^2+4}\,du$$

$u = \frac{2}{3}v$

$$3\int \frac{1}{9u^2+4}\,du \qquad 3\int \frac{1}{6(v^2+1)}\,dv$$

$$\frac{1}{2}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{1}{2}\arctan(v)$$

$v = \frac{3}{2}u$

$\frac{1}{2}\arctan\left(\frac{3}{2}\left(x-\frac{2}{3}\right)\right)$

$\frac{1}{2}\arctan\left(\frac{3}{2}x-1\right)$

$= \frac{1}{9}\left(\ln|9x^2-12x+8| + 2\arctan\left(\frac{3x-2}{2}\right)\right) + \frac{1}{2}\arctan\left(\frac{3}{2}x-1\right)$

$+C$

Escaneado con CamScanner

10. $\int \frac{dx}{\sqrt{x}\,(1-\sqrt{x})}$ $\qquad u = 1-\sqrt{x}$

$\frac{d}{dx}(1-\sqrt{x})$

$\frac{d}{dx}(1) - \frac{d}{dx}(\sqrt{x})$

$0 - \frac{1}{2\sqrt{x}}$

$du = -\frac{1}{2\sqrt{x}}\,dx$

$dx = -2\sqrt{x}\,du$

$= \int \frac{1}{\cancel{\sqrt{x}}\,u}(-2\cancel{\sqrt{x}})\,du$

$= \int \frac{-2}{u}\,du$

$= -2\int \frac{1}{u}\,du$

$u = 1-\sqrt{x}$

$= -2\ln|u|$

$= -2\ln|(1-\sqrt{x})| + C$

11 $\int x^2 \ln(x)\,dx$

$u = \ln(x)$
$v' = x^2$

$u' = \frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$v = \int x^2 dx = \frac{x^3}{3}$

$= \ln(x)\,\frac{x^3}{3} - \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}2}}{3} dx$

$= \frac{1}{3} x^3 \ln(x) - \int \frac{x^2}{3}$

$= \frac{1}{3} x^3 \ln(x) - \frac{1}{3}\int x^2$

$= \frac{1}{3} x^3 \ln(x) - \frac{1}{3}\,\frac{x^3}{3}$

$= \frac{1}{3} x^3 \ln(x) - \frac{x^3}{9} + C$

12.. $\int \arcsin(x)\,dx$

$u = \arcsin(x)$
$v' = 1$

$u' = \frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$v = \int 1\,dx = x$

$= \arcsin(x)(x) - \int \frac{1}{\sqrt{1-x^2}} x\,dx$

$$= x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx \qquad u = 1 - x^2$$

$$\frac{du}{dx} = -2x$$

$$= \int \frac{x}{\sqrt{u}} \cdot \left(-\frac{1}{2x}\right) du$$

$$du = -2x\,dx$$

$$= \int -\frac{1}{2\sqrt{u}}\,du \qquad dx = \frac{du}{-2x}$$

$$= -\int \frac{1}{2\sqrt{u}}\,du$$

$$= -\sqrt{u} \qquad = -\sqrt{1-x^2}$$

$$= x \arcsin(x) - \left(-\sqrt{1-x^2}\right) + C$$

$$= x \arcsin(x) + \sqrt{1-x^2} + C$$

13. $\int x\sqrt{1+x}\,dx$

$$u = 1 + x$$

$$\frac{d}{dx}(1+x)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(x) \qquad \frac{du}{dx} = 1$$

$$du = dx$$

$$0 + 1 = 1$$

$$= \int x\sqrt{u}\,du \qquad u = 1 + x$$

$$x = u - 1$$

$$= \int (u-1)\sqrt{u}\,du$$

Escaneado con CamScanner

$(u-1)\sqrt{u}$

$= u\sqrt{u} - \sqrt{u}$

$= u \cdot u^{1/2} - \sqrt{u}$

$= u^{3/2} - \sqrt{u}$

$= u^{3/2} - \sqrt{u}$

$= \int u^{3/2} - \sqrt{u}\, du$

$= \int u^{3/2} du - \int \sqrt{u}\, du = \dfrac{u^{(3/2)+1}}{(3/2)+1} - \dfrac{u^{(1/2)+1}}{(1/2)+1}$

$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}$ $\quad u = 1+x$

$= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C$

14. $\int x^3 e^{2x} dx$ $\quad u = x^3 \quad v' = e^{2x}$

$u' = \frac{d}{dx}(x^3)$

$u' = 3x^2$

$\int e^{2x} dx$

$u = 2x$

$= \int e^u \frac{1}{2} du$

$= \frac{1}{2}\int e^u du$

$= \frac{1}{2} e^u du = \frac{1}{2} e^{2x} + C$

Escaneado con CamScanner

$$= x^3 \frac{1}{2}e^{2x} - \int 3x^2 \frac{1}{2}e^{2x}dx$$

$$= \frac{1}{2}e^{2x}x^3 - \int \frac{3}{2}e^{2x}x^2 dx$$

$u = x^2 \quad v' = e^{2x}$

$u' = \frac{d}{d(x)}(x^2)$

$$= \frac{1}{2}e^{2x}x^3 - \frac{3}{2}\int e^{2x}x^2 dx$$

$u' = 2x$

$$= x^2 \frac{1}{2}e^{2x} - \int 2x \frac{1}{2}e^{2x}dx$$

$v = \int e^{2x}dx = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}x^2 - \int e^{2x}x\,dx$$

$$= \frac{3}{2}\left(\frac{1}{2}e^{2x}x^2 - \int e^{2x}x\,dx\right)$$

$$= \int e^{2x}x\,dx$$

$u = 2x \quad \frac{du}{dx} = 2$

$$= \int \frac{e^u u}{4} du$$

$du = 2dx$

$$= \frac{1}{4}\int e^u u\,du$$

$dx = \frac{du}{2}$

$$= \frac{1}{4}\left(e^u du - \int e^u du\right)$$

$$= \frac{1}{4}\left(e^u u - e^u\right)$$

$$= \frac{1}{4}\left(e^{2x} \cdot 2x - e^{2x}\right)$$

$$= \frac{3}{2}\left(\frac{1}{2}e^{2x}x^2 - \frac{1}{4}\left(e^{2x} \cdot 2x - e^{2x}\right)\right)$$

$$\boxed{= \frac{1}{2}e^{2x}x^3 - \frac{3}{2}\left(\frac{1}{2}e^{2x}x^2 - \frac{1}{4}\left(e^{2x}2x - e^{2x}\right)\right) + c}$$

Escaneado con CamScanner

15. $\int \text{sen}(\ln x)\,dx$ $\quad u = \text{sen}(\ln(x))$

$v' = 1$

$\frac{d}{dx}(\text{sen}(\ln(x)))$

$= \frac{d}{du}(\text{sen}(u))\frac{d}{dx}(\ln(x))$ $\quad u = \ln(x)$

$f = \text{sen}(u)$

$= \cos(u) \cdot \frac{1}{x}$

$= \cos(u)\frac{1}{x}$

$= \cos(\ln(x))\frac{1}{x}$

$= \frac{\cos(\ln(x))}{x}$

$v = \int 1\,dx = x$

$= \text{sen}(\ln(x))x - \int \frac{\cos(\ln(x))}{\cancel{x}}\cancel{x}\,dx$

$u = \ln(x)$

$= x\,\text{sen}(\ln(x)) - \int \cos(\ln(x))\,dx$

$\frac{du}{dx} = \frac{1}{x}$

$- \int \cos(\ln(x))\,dx$

$du = \frac{dx}{x}$

$- \int \cos(u)\,x\,du$

$dx = x\,du$

$- \int \cos(u)e^u\,du$

$\frac{d}{du}(e^u) = e^u$

$= e^u\,\text{sen}(u) - \int e^u\,\text{sen}(u)\,du$

$v = \int \cos(u)\,du = \text{sen}(u)$

$u = e^u$

$v' = \text{sen}(u)$

$$u' = \frac{d}{du}(e^u) = e^u$$

$$v = \int \operatorname{sen}(u)\,du = -\cos(u) \qquad V = \int e^u \cos(u)\,du$$

$$= -e^u \cos(u) - \int -e^u \cos(u)\,du$$

$$= e^u \operatorname{sen}(u) - \left(-e^u \cos(u) - \int -e^u \cos(u)\,du\right)$$

$$V = e^u \operatorname{sen}(u) - (-e^u \cos(u) - (-V))$$

$$V = e^u \operatorname{sen}(u) - (-e^u \cos(u) - (-V))$$

$$V = e^u \operatorname{sen}(u) + e^u \cos(u) - V$$

$$V + V = e^u \operatorname{sen}(u) + e^u \cos(u) - V + V$$

$$2V = e^u \operatorname{sen}(u) + e^u \cos(u)$$

$$\frac{\cancel{2}V}{\cancel{2}} = \frac{e^u \operatorname{sen}(u)}{2} + \frac{e^u \cos(u)}{2}$$

$$V = \frac{e^u \operatorname{sen}(u)}{2} + \frac{e^u \cos(u)}{2}$$

$$\int e^u \cos(u)\,du = \frac{e^u \operatorname{sen}(u)}{2} + \frac{e^u \cos(u)}{2}$$

$$= \frac{e^u \operatorname{sen}(u)}{2} + \frac{e^u \cos(u)}{2} \qquad u = \ln(x)$$

$$= \frac{e^{\ln(x)} \operatorname{sen}(\ln(x))}{2} + \frac{e^{\ln(x)} \cos(\ln(x))}{2}$$

$$= \frac{x \operatorname{sen}(\ln(x))}{2} + \frac{x \cos(\ln(x))}{2}$$

$$= x \operatorname{sen}(\ln(x)) - \left(\frac{1}{2} x \operatorname{sen}(\ln(x)) + \frac{1}{2} x \cos(\ln(x))\right)$$

$= x \,\text{sen}(\ln(x)) - \frac{1}{2} x \,\text{sen}(\ln(x)) - \frac{1}{2} x \cos(\ln(x))$

$$\boxed{= \frac{1}{2} \text{sen}(\ln(x)) - \frac{1}{2} x \cos(\ln(x)) + C}$$

16. $\int e^{3x} \,\text{sen} 2x \,dx$

$u = e^{3x}$
$v' = \text{sen}(2x)$

$u' = \frac{d}{dx}(e^{3x}) = e^{3x} 3$

$u = 3x$

$\frac{d}{du}(e^u) = e^u$

$\frac{d}{dx}(3x) = 3$

$= e^u \cdot 3 \quad = 3e^{3x}$

$v = \int \text{sen}(2x) dx \qquad u = 2x$

$\frac{du}{dx} = 2 \quad du = 2dx$

$dx = \frac{du}{2}$

$\frac{1}{2} \int \text{sen}(u) du$

$\frac{1}{2}(-\cos(u))$

$-\frac{1}{2}\cos(2x)$

$= e^{3x}\left(-\frac{1}{2}\cos(2x)\right) - \int e^{3x} 3\left(-\frac{1}{2}\cos(2x)\right)dx$

$= -\frac{1}{2} e^{3x} \cos(2x) - \left(-\frac{3}{2} \int e^{3x} \cos(2x) dx\right)$

$u = e^{3x}$
$v' = \cos(2x)$

$$u' = \frac{d}{dx}(e^{3x}) = e^{3x}3$$

$$v = \int \cos(2x)dx \qquad u = 2x$$

$$\frac{du}{dx} = 2$$

$$\int \cos(u)\frac{1}{2}du \qquad du = 2dx$$

$$dx = \frac{du}{2}$$

$$= \int \cos(u)\frac{1}{2}du$$

$$= \frac{1}{2}\operatorname{sen}(u)$$

$$= \frac{1}{2}\operatorname{sen}(2x)$$

$$= e^{3x}\frac{1}{2}\operatorname{sen}(2x) - \int e^{3x}\cdot 3\cdot\frac{1}{2}\operatorname{sen}(2x)dx$$

$$= \frac{1}{2}e^{3x}\operatorname{sen}(2x) - \int \frac{3}{2}e^{3x}\operatorname{sen}(2x)dx$$

$$-\frac{1}{2}e^{3x}\cos(2x) - \left(-\frac{3}{2}\left(\frac{1}{2}e^{3x}\operatorname{sen}(2x) - \int \frac{3}{2}e^{3x}\operatorname{sen}(2x)dx\right)\right)$$

$$= \int e^{3x}\operatorname{sen}(2x)dx$$

$$\text{sea } \int e^{3x}\operatorname{sen}(2x)dx = u$$

$$u = -\frac{1}{2}e^{3x}\cos(2x) - \left(-\frac{3}{2}\left(\frac{1}{2}e^{3x}\operatorname{sen}(2x) - \frac{3}{2}u\right)\right)$$

$$u = \frac{2e^{3x}\cos(2x)}{13} + \frac{3e^{3x}\operatorname{sen}(2x)}{13} = \int e^{3x}\operatorname{sen}(2x)dx$$

$$= -\frac{2e^{3x}\cos(2x)}{13} + \frac{3e^{3x}\operatorname{sen}(2x)}{13} + C$$

17 $\int \cos^5(x)dx$ $\qquad \cos^2(x) = 1 - \text{sen}^2(x)$

$$= \int (\cos^2(x))^2 \cos(x)dx$$

$$= \int (1 - \text{sen}^2(x))^2 \cos(x)\,dx$$

$u = \text{sen}(x)$

$\int (1-u^2)^2 du$ $\qquad \frac{du}{dx} = \cos(x)$

$du = \cos(x)dx$

$dx = \frac{du}{\cos(x)}$

$$= \int (1-u^2)^2 \cancel{\cos(x)} \frac{1}{\cancel{\cos(x)}} du$$

$$= \int (1-u^2)^2 du$$

$$= \int 1 - 2u^2 + u^4 du$$

$$= \int 1du - \int 2u^2 du + \int u^4 du$$

$$= u - \frac{2u^3}{3} + \frac{u^5}{5} \qquad u = \text{sen}(x)$$

$$= \text{sen}(x) - \frac{2(\text{sen}(x))^3}{3} + \frac{(\text{sen}(x))^5}{5} + C$$

18. $\int \text{sen}^3 x \cos^2 x dx$ $\qquad \text{sen}^2(x) = 1 - \cos^2(x)$

$= \int \text{sen}^2(x)\text{sen}(x) \cos^2(x)\, dx$ $\qquad u = \cos(x)$

$$= \int (1 - \cos^2(x))\, \text{sen}(x) \cos^2(x) dx$$

$\frac{du}{dx} = -\text{sen}(x)$ $\quad du = -\text{sen}(x)dx$ $\quad dx = -\frac{du}{\text{sen}(x)}$

$$= \int (1-u^2)\, \cancel{\operatorname{sen}(x)}\, u^2 \left(-\frac{1}{\cancel{\operatorname{sen}(x)}}\right) du$$

$$= \int -u^2(1-u^2)$$

$$= \int -u^2 + u^4\, du$$

$$= -\int u^2\, du + \int u^4\, du$$

$$= -\frac{u^3}{3} + \frac{u^5}{5} \qquad u = \cos(x)$$

$$= -\frac{\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C$$

19. $\int e^{3x} x^2 \operatorname{sen} x\, dx$

$\int$	d
x^2	$e^{3x}\operatorname{sen}x$
$\frac{x^3}{3}$	$3e^{3x}\cos x$
$\frac{x^4}{12}$	$-9e^{3x}\operatorname{sen}x$
$\frac{x^5}{60}$	$-27e^{3x}\cos x$
$\frac{x^6}{360}$	$81e^{3x}\operatorname{sen}x$

$\int e^{3x} x^2 \operatorname{sen} x\, dx$

$$= \frac{1}{3} e^{3x} x^3 \operatorname{sen} x - \frac{1}{4} e^{3x} x^4 \cos x - \frac{3}{20} e^{3x} x^5 \operatorname{sen} x + \frac{3}{40} e^{3x} \cos x + C$$

$$= e^{3x}\left(\operatorname{sen} x\left(\frac{1}{3}x^3 - \frac{3}{20}x^5\right) + \cos\left(-\frac{1}{4}x^4 + \frac{3}{40}x^6\right)\right) + C$$

DD MM AA

20. $\int \tan^3 2x \sec^3 2x \, dx$

$= \int \tan^2(2x) \tan(2x) \sec^3(2x) dx$ $\quad \tan^2(x) = \sec^2(x) - 1$

$= \int (-1+\sec^2(2x)) \tan(2x) \sec^3(2x) dx$

$u = \sec(2x)$

$\frac{du}{dx} = \sec(2x)\tan(2x)2$

$du = 2\sec(2x)\tan(2x)dx$

$dx = \frac{du}{2\sec(2x)\tan(2x)}$

$= \int (-1+u^2) \cancel{\tan(2x)} u^3 \frac{du}{\sec(2x)\cancel{\tan(2x)} \cdot 2}$

$= \int \frac{(u^2-1)}{2} u^2 du$

$= \int \frac{u^4}{2} - \frac{u^2}{2} du$

$= \int \frac{u^4}{2} du - \int \frac{u^2}{2} du$

$= \frac{u^5}{10} - \frac{u^3}{6}$

$= \frac{\sec^5(2x)}{10} - \frac{\sec^3(2x)}{6} + C$

Escaneado con CamScanner

Integrales Impropias

1. $\int_1^{\infty} \frac{1}{(2x+1)^3}\,dx$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2dx$

$dx = \frac{du}{2}$

$= \int \frac{1}{u^3} \cdot \frac{1}{2}\,du$

$= \int \frac{1}{2u^3}\,du$

$= \frac{1}{2}\int \frac{1}{u^3}\,du$

$= \frac{1}{2}\int u^{-3}\,du$

$= \frac{1}{2} \cdot \frac{u^{-2}}{-2}$

$= -\frac{u^{-2}}{4}$

$= -\frac{(2x+1)^{-2}}{4}$

$= -\frac{1}{4(2x+1)^2} + C$

$\lim_{x\to 1} \left(-\frac{1}{4(2x+1)^2}\right) = -\frac{1}{4(2(1)+1)^2} = -\frac{1}{36}$

$\lim_{x\to \infty} \left(-\frac{1}{4(2x+1)^2}\right)^{\to 0} = 0 \qquad = 0 - \left(-\frac{1}{36}\right)$

Convergente $\boxed{= \frac{1}{36}}$

2. $\int_2^{\infty} \frac{1}{x\ln(x)}\,dx$ $\quad u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$ $\quad du = \frac{dx}{x}$

$dx = x\,du$

$\int \frac{1}{xu} x\,du$

$= \int \frac{1}{u}\,du$

$= \ln(|u|)$

$= \ln|\ln(x)| + c$

$\lim_{x \to 2} \ln|\ln(x)|$

$= \ln|\ln(2)|$

$\lim_{x \to \infty} \ln|\ln(\infty)| = \infty \to$ Divergente

3. $\int_2^6 \frac{y}{\sqrt{y-2}}\,dx$ $\quad u = \sqrt{x-2}$

$\int_2^6 \frac{x}{\sqrt{x-2}}\,dx$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{x-2}}$

$du = \frac{dx}{2\sqrt{x-2}}$

$= \int \frac{x}{u} \cdot 2\sqrt{x-2}\,du$

$dx = 2\sqrt{x-2}\,du$

$= \int \frac{x}{\cancel{u}} 2\cancel{u}\,du$

$$= \int 2x\,du \qquad u = \sqrt{x-2}$$

$$(u)^2 = (\sqrt{x-2})^2$$

$$= \int 2(u^2+2)\,du \qquad u^2 = x-2$$

$$u^2 + 2 = x$$

$$= 2\int u^2 + 2\,du$$

$$\int u^2\,du = \frac{u^3}{3}$$

$$\int 2\,du = 2u$$

$$= 2\left(\frac{u^3}{3} + 2u\right)$$

$$\lim_{u \to 2} \frac{2u^3}{3} + 2u \qquad \lim_{u \to 0} \frac{2u^3}{3} + 2u = 0$$

$$= 2\left(\frac{(2)^3}{3} + 2(2)\right)$$

$$= \frac{16}{3} + 8$$

$$= \frac{16}{3} + \frac{24}{3}$$

$$= \frac{40}{3}$$

Convergente

DD MM AA

4. $\int_{-1}^{1} \frac{1}{x^2-2x} dx$

(x^2-2x)

$x(x-2)$

$x=0 \quad x=2$

$= \int_{-1}^{0} \frac{1}{x^2-2x} dx + \int_{0}^{1} \frac{1}{x^2-2x} dx$

$2c=-2 \quad c=-1$

$= \int_{-1}^{0} \frac{1}{x^2-2x} dx$

x^2-2x

$x^2-2x+(-1)^2-(-1)^2$

$= \int_{-1}^{0} \frac{1}{(x-1)^2-1} dx$

$x^2-2x+(-1)^2=(x-1)^2$

$=(x-1)^2-1$

$u=x-1$

$\frac{d}{dx}(x-1)$

$\frac{d}{dx}(x) - \frac{d}{dx}(1)$

$\int \frac{1}{u^2-1} du$

$x=-1 \quad u=-2$

$=1-0$

$x=0 \quad u=-1$

$\frac{du}{dx}=1$

$= \int_{-2}^{-1} \frac{1}{u^2-1} du$

$du=dx$

$= -\int_{-2}^{-1} \frac{1}{-u^2+1} du$

$= -\left[\frac{\ln|u+1|}{2} - \frac{\ln|u-1|}{2}\right]_{-2}^{-1}$

Divergente

$\lim_{u \to -2^+} \left(\frac{1}{2}(\ln|u+1|-\ln|u-1|)\right) = -\frac{1}{2}$

$\lim_{u \to -1^-} \left(\frac{1}{2}(\ln|u+1|-\ln|u-1|)\right) = -\infty$

5 $\int_1^{\infty} \frac{\tan^{-1}x}{x^2}\,dx$

$u = \arctan(x)$

$\int_1^{\infty} \frac{\arctan(x)}{x^2}\,dx$

$u' = \frac{d}{dx}(\arctan(x)) = \frac{1}{x^2+1}$

$V = \int \frac{1}{x^2}\,dx$

$V = \int x^{-2}\,dx$

$V = \frac{x^{-1}}{-1}$

$V = -x^{-1}$

$V = -\frac{1}{x}$

$= \arctan(x)\left(-\frac{1}{x}\right) - \int \frac{1}{x^2+1}\left(-\frac{1}{x}\right)dx$

$= -\frac{\arctan(x)}{x} - \int -\frac{1}{x(x^2+1)}\,dx$

$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$

$dx = \frac{du}{2x}$

$-\left(\int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx\right)$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

D.D MM AA

$= \int \frac{x}{u} \cdot \frac{1}{2x}\,du$

$= \int \frac{1}{2u}\,du$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|$

$= \frac{1}{2}\ln|x^2+1|$

$0-\left(-\frac{\pi}{4}-\frac{1}{2}\ln(2)\right)$

Convergente

$= -\frac{\arctan(x)}{x} + \ln|x| - \frac{1}{2}\ln|x^2+1| + C$

$\lim_{x \to 1}\left(-\frac{\arctan(x)}{x} + \ln|x| - \frac{1}{2}\ln|x^2+1|\right) = -\frac{\pi}{4} - \frac{1}{2}\ln(2)$

$\lim_{x \to \infty}\left(-\frac{\arctan(x)}{x} + \ln|x| - \frac{1}{2}\ln|x^2+1|\right) = 0$

6 $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

$u = -x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$dx = -\frac{du}{2x}$

$= \int xe^{u}\left(-\frac{1}{2x}\right)du$

$= \int -\frac{e^{u}}{2}\,du$

$= -\frac{1}{2}\int e^{u}\,du$

$= -\frac{1}{2}e^{u}$

$= -\frac{1}{2}e^{-x^2} + C$

$\lim_{x \to \infty} -\frac{1}{2}e^{-x^2} = 0$

Convergente

$\lim_{x \to -\infty} -\frac{1}{2}e^{-x^2} = 0$

7. $\int_1^{\infty} \frac{1}{v^2+2v-3} dv$

$-\int_1^{\infty} \frac{1}{-v^2-2v+3}$

$-\int_1^{\infty} \frac{1}{-(v+1)^2+4} dv \qquad u = v+1$

$-\int_1^{\infty} \frac{1}{-u^2+4} \qquad u = 2w$

$= \int \frac{1}{-(2w)^2+4} 2dw \qquad \frac{d(2w)}{dw} = 2$

$\frac{du}{dw} = 2$

$= -\int \frac{1}{2(-w^2+1)} dw \qquad du = 2dw$

$= -\frac{1}{2}\int \frac{1}{-w^2+1} dw \qquad dw = \frac{2}{du}$

$= -\frac{1}{2}\left(\frac{\ln|w+1|}{2} - \frac{\ln|w-1|}{2}\right) \qquad w = \frac{u}{2} \quad u = v+1$

$= -\frac{1}{4}\left(\ln\left|\frac{v+1}{2}+1\right| - \ln\left|\frac{v+1}{2}-1\right|\right) + C$

$\lim_{v \to 1^+} \left(-\frac{1}{4}\left(\ln\left|\frac{v+1}{2}+1\right| - \ln\left|\frac{v+1}{2}-1\right|\right)\right) = -\infty$

Divergente

$\lim_{v \to \infty} \left(-\frac{1}{4}\left(\ln\left|\frac{v+1}{2}+1\right| - \ln\left|\frac{v+1}{2}-1\right|\right)\right) = 0$

DD MM AA

8 $\int_0^5 \frac{w}{w-2}\,dx$

$\int_0^5 \frac{x}{x-2}\,dx$

$x-2$
$x=2$

$= \int_0^2 \frac{x}{x-2}\,dx + \int_2^5 \frac{x}{x-2}\,dx$

$u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$= \int \frac{x}{u} \cdot 1\,du$

$= \int \frac{x}{u}\,du \qquad x = u+2$

$= \int_{-2}^{0} \frac{u+2}{u}\,du$

$\int_{-2}^{0} \frac{u}{u} + \frac{2}{u}\,du$

$\int_{-2}^{0} 1 + \frac{2}{u}\,du$

$\int_{-2}^{0} 1 + \int_{-2}^{0} \frac{2}{u}\,du$

$u\Big]_{-2}^{0} + 2\ln|u|\Big]_{-2}^{0}$

$\lim\limits_{u\to -2} u + 2\ln|u| = -2 + 2\ln(2)$

$\lim\limits_{u\to 0} u + 2\ln|u| = -\infty$

Divergente

Escaneado con CamScanner

9. $\int_{-1}^{0} \frac{e^{1/x}}{x^3}\,dx \qquad u = \frac{e^{1/x}}{x^3} \qquad v' = 1$

$u' = -\frac{e^{1/x}(3x+1)}{x^5} \qquad v = \int 1\,dx = x$

$= \left[\frac{e^{1/x}}{x^2} - \int -\frac{e^{1/x}(3x+1)}{x^4}\,dx\right]_{-1}^{0}$

$= -\left(\int \frac{3e^{1/x}}{x^3}\,dx + \int \frac{e^{1/x}}{x^4}\,dx\right)$

$= -\left(-3\left(\frac{e^{1/x}}{x} - e^{1/x}\right) + \left(-\frac{e^{1/x}}{x^2} + \frac{2e^{1/x}}{x} - 2e^{1/x}\right)\right)$

$= \frac{3e^{1/x}}{x} - e^{1/x} + \frac{e^{1/x}}{x^2} - \frac{2e^{1/x}}{x}$

$= \left[\frac{e^{1/x}}{x^2} - \left(\frac{3e^{1/x}}{x} - e^{1/x} + \frac{e^{1/x}}{x^2} - \frac{2e^{1/x}}{x}\right)\right]_{-1}^{0}$

$= \left[\frac{-e^{1/x} + e^{1/x}x}{x}\right]_{-1}^{0}$

$\lim_{x \to 1} \left(\frac{-e^{1/x} + e^{1/x}x}{x}\right) = \frac{2}{e}$

$\lim_{x \to 0} \left(\frac{-e^{1/x} + e^{1/x}x}{x}\right) = 0$

$= 0 - \frac{2}{e}$

$= \frac{2}{e}$ Convergente

MM AA

10 $\int_{-\infty}^{0} \frac{1}{3-4x}\,dx$

$u = 3-4x$

$\frac{du}{dx} = -4$

$du = -4dx$

$dx = -\frac{du}{4}$

$= \int \frac{1}{u}\left(-\frac{1}{4}\right) du$

$= \int -\frac{1}{4u}\, du$

$= -\frac{1}{4}\int \frac{1}{u}\, du$

$= -\frac{1}{4}\ln(u) \quad u = 3-4x$

$= -\frac{1}{4}\ln|3-4x| + C$

$\lim_{x\to-\infty} -\frac{1}{4}\ln|3-4x| = -\infty$

Divergente

$\lim_{x\to 0} -\frac{1}{4}\ln(3-4x) = -\frac{1}{4}\ln(3)$