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A- Methods

The Methods section outlines the procedures followed to produce a comprehensive and organized collection of economic indicators that will underpin the description of the main economic developments in the Turkish economy from year 1998 to the second quarter of 2024. The procedures involve operations to organize the economic indicators in a way that enhances readability and facilitates access and manipulation, as well as operations to derive additional economic indicators and change metrics, and to examine the economic indicators' development. While the first class of operations was implemented using both Microsoft Excel and Python (version 3.10.9), the second class was executed predominantly by means of the Pandas (version 1.5.3) and Plotly (version 5.9.0) libraries in Python. To enhance readability and facilitate access to and manipulation of the economic indicators, we categorized them into data categories and ameliorated the dataset structures by employing multi-indexing and condensing columns' titles. To derive economic indicators and change metrics, we established and adhered to theoretical frameworks. Finally, to study the development of the economic indicators, we utilized the computed metrics as well as the visualization capabilities offered by the Plotly library in Python.

The first subsection of the Methods section provides a brief description of the datasets we downloaded from different sources on the internet and presents our approach to organizing them into data categories. The second subsection introduces the adopted notational framework and presents fundamental calculations and concepts frequently employed throughout the Methods section. Finally, each of the remaining nine subsections encompasses a personalized treatment by data category. Within each subsection, we expound the operations performed on the economic series and conclude with a reference to the final composition of the data category and the data visualizations.

A-I- Data Presentation

In this subsection, we present the configuration adopted to organize the datasets, which we downloaded from different sources on the internet, such as the Turkish Statistical institute's website (TUIK), the Central Bank of the Republic of Turkey's (CBRT) website, and the International Monetary Fund's (IMF) International Financial Statistics dataset (IFS). The configuration relies on the definition of data categories that host closely related datasets, representing strongly linked aspects of the Turkish economy. For instance, both the Main Labor Force Indicators and the Labor Input Indices datasets provide information about the labor market, and thus they can be grouped under a "Labor Market" data category. In this subsection, we exhibit the initial composition of each data category and briefly describe the figuring datasets, while the final composition will be referenced at the end of the corresponding data category subsection.

A-I-1. Data Category 1: GDP and Final Expenditure Components

The GDP and Final Expenditure Components data category contains annual and quarterly datasets of volume and nominal measures of the Turkish gross domestic product (GDP) by expenditure approach. The datasets were downloaded from TUIK's website and have the following specifications: Nominal series are in thousands of Turkish Lira (TRY); volume series are chain-linked, with annual overlap for quarterly series, Laspeyres volume indices with reference year 2009.

We formed one parent dataset per data frequency. Each parent dataset contains sub-datasets that are characterized by a unique combination of: nature of the series—nominal values or volume measures—and adjustments applied to the series—unadjusted, seasonally adjusted, calendar adjusted, or seasonally and calendar adjusted.

For the annual parent dataset, the list of sub-datasets is:

- Unadjusted volume indices of GDP by expenditure approach
- Calendar Adjusted volume indices of GDP by expenditure approach
- Unadjusted values at current prices of GDP by expenditure approach

We also included, in the annual parent dataset, unadjusted values at current prices of the Turkish GDP per Capita expressed in TRY and United States dollars (USD), as well as one series for the mid-year population size of Turkey.

Annual series cover the period 1998-2023.

For the quarterly parent dataset, the list of sub-datasets is:

- Unadjusted volume indices of GDP by expenditure approach
- Seasonally adjusted volume indices of GDP by expenditure approach
- Seasonally and calendar adjusted volume indices of GDP by expenditure approach
- Unadjusted values at current prices of GDP by expenditure approach
- Seasonally adjusted values at current prices of GDP by expenditure approach

Quarterly series cover the period from 1998 to the 2nd Quarter of 2024.

A-II- Notational Framework and Basic Methods and Concepts

This subsection is divided into two sub-subsections. In the first one, we establish the notational framework which we will adopt throughout the Methods section. The aim of the notational framework is to set a system of rules and symbols that will organize and simplify the communication of technical concepts. This system will also evolve in subsequent subsections to adapt to the requirements of each data category. In the second sub-subsection, we present basic concepts and calculations, which are useful to the process of deriving additional economic indicators and calculating change metrics for most data categories. This sub-subsection, besides being a preparatory introduction to the technical content of the

Methods section, will save us from the inconvenience of repetition during the treatments by data category.

A-II-1. Notational Framework

The objective of this sub-subsection is to introduce the notations utilized within the Methods section, as well as to create a basis for the development of new notations during the treatments by data category.

Let X be an economic variable.

a) Flow and stock variables

To differentiate between flow and stock variables, we use different letter symbols for each. For example, for flow variables, nominal values are referred to as C, and volume measures as K. Whereas, for stock indicators, we use N and V, respectively. For some data categories, we will deviate from these variables or add new ones to enhance clarity and representation.

b) Specifications

We use the subscript of X to provide specifications on X. For instance, if X is a value at current prices of GDP expressed in TRY, we will write $X_{GDP,TRY}$. The structure of the subscript will be determined for each data category within the corresponding subsection.

c) Time Periods

We use the superscript of X to refer to the accounting time period. A time period is specified by a tuple of the form (d, m, s, y), which reads: day $1 \leq d \leq 31$ of month $1 \leq m \leq 3$ of quarter $1 \leq s \leq 4$ of year y. We can move to lower frequencies by removing items starting from the left side of the tuple (d, m, s, y). For instance, we write the monthly value of X as $X^{(m,s,y)}$, and the annual value as X^{y}. Sometimes, when the data frequencies are clearly stated, we might simplify the notation to X^{t}. This is helpful when dealing with expressions involving current and past/future periods (e.g. the period preceding (s, y) is either $(s-1, y)$

or $(4, y-1)$; both cases are covered when using t and $t-1$) or when generalizing expressions for different time frequencies. We can also write $X^{d,..,t}$ to explicitly indicate elements from the time tuple (d, m, s, y) while maintaining the simplified form for the remaining elements.

d) Indices, Measures of Change, and Volume Measures

For indices, measures of change, and volume measures, we add "$b \rightarrow$" before the accounting time period in the superscript to refer to the reference/base period. Therefore, we write $X^{b \rightarrow t}$ to indicate an index value, a measure of change, or a volume measure in period t with reference/base period b. For volume measures, respectively indices, when $b = y - 1$ and y is the year of t, then we are referring to volume measures at previous year prices, respectively indices from previous year.

A-II-2. Basic Concepts and Calculations

In this sub-subsection, we present some basic concepts and calculations that are used when deriving additional economic indicators and change metrics for most data categories. We establish the theoretical framework for the calculation of economic ratios, for the measuring of temporal change in the economic variables, for the computation of both contributions to the change and shares in aggregate economic variables when certain conditions are met, and for price deflation.

A-II-2.1. Economic Ratios

An economic ratio is the ratio of two economic variables. In this report, we will specifically use the term to refer to ratios that measure the relative values of two economic variables. Therefore, we will require that both variables are expressed in the same unit of measure. For example, we will often calculate the ratio of a nominal variable X to nominal GDP. If X is expressed in a foreign currency, then either X or GDP should be converted. The outcome of each option depends on the conversion method. In fact, if the exchange rate used to convert X is the same as the exchange rate used to convert GDP, then both options will yield the same result. However, in many situations, this is not the case. For example, if X is an end-of-period stock value, then we would use the end-of-period exchange rate to convert it. On the other hand, GDP measures a flow, and thus it is typically converted using an average period exchange rate. To address this issue, we will often calculate two versions of the ratio based on each of the two options. Nevertheless, attention is yet to be paid when interpreting the results of each. To distinguish between each ratio, we will call the ratio based on the conversion of X: Ratio to GDP, whereas we will call the ratio based on the conversion of GDP: Ratio to Converted GDP.

Another consideration when calculating economic ratios is to determine the time span of the involved economic variables. For example, we will sometimes define the ratio of the quarterly economic variables X and Y in two ways, depending on the time span of Y. The first definition, which we call Ratio to Y, involves the values of X and Y from the same quarter (Equation (1)). The second definition, which we call Ratio to Y(4Q), involves the quarterly value of X and the aggregate value of Y over the successive four quarters ending with the quarter of X (Equation (2)). On the other hand, for annual variables Z and W, we will always work with one definition, which we call Ratio to W, involving the values from the same year (Equation (3)).

$$\begin{cases} (Ratio\ to\ Y)^{(s,y)} = \dfrac{X^{(s,y)}}{Y^{(s,y)}} & (1) \\ \left(Ratio\ to\ Y(4Q)\right)^{t} = \dfrac{X^{t}}{\sum_{p=0}^{3} Y^{t-p}}, \qquad t = (s,y) & (2) \\ (Ratio\ to\ Y)^{y} = \dfrac{Z^{y}}{W^{y}} & (3) \end{cases}$$

A-II-2.2. Measures of Change

In order to measure the evolution of the economic variable X, we compare its current value to past values. We perform this comparison using different measures of change, which we categorize by their type—growth rate or simple difference—and by the base period of comparison—Year-on-Year, Quarter-on-Quarter, or Year-over-Year.

a) Growth Rate

The growth rate of X (also called rate of change, appreciation rate if X is an exchange rate, and inflation rate if X is a price level) in period t_1 relative to the base period t_0 is given by Equation (4) (where we supposed that $X^{t_0} \neq 0$).

$$g_X^{t_0 \to t_1} = \frac{X^{t_1} - X^{t_0}}{|X^{t_0}|} \tag{4}$$

b) Simple Difference

The simple difference in X (also called nominal difference if X is a nominal variable, and percentage point difference if X is a percentage) in period t_1 relative to the base period t_0 is given by Equation (5).

$$\Delta_X^{t_0 \to t_1} = X^{t_1} - X^{t_0} \tag{5}$$

c) Base Period of Comparison

For annual variables, the base period of comparison is usually the previous year. We call the related measure of change a Year-on-Year (Y-on-Y) change. For quarterly variables, we distinguish between Quarter-on-Quarter (Q-on-Q) change and Year-over-Year (Y-o-Y) change. In the first, the base period of comparison is the previous quarter. In the second, the base period of comparison is the same quarter from the previous year.

A-II-2.3. Contributions to Change: The Case of Additivity

Let X and $\{X_i\}_{1 \le i \le m}$ be distinct economic variables, such that X is linked to $\{X_i\}_{1 \le i \le m}$ by the additive relationship represented by Equation (6). This additive relationship enables us to express the measures of change in X presented in Subheading A-II-2.2, "Measures of Change," as a weighted sum of the corresponding measures of change in $\{X_i\}_{1 \le i \le m}$. We will call each term of this sum a “contribution to the change in X.” Based on the categorization of measures of change in Subheading A-II-2.2, we distinguish between two types of contributions to change: contributions to growth and contributions to the simple difference. Furthermore, depending on the base period of comparison, each type of contribution is categorized into contribution to Y-on-Y, Q-on-Q, or Y-o-Y change.

$$\forall t, \qquad X^t = \sum_{i=1}^{m} (-1)^{f(X_i)} * X_i^t \tag{6}$$

Where f is a function that assigns to each element of $\{\mathrm{X_i}\}_{1 \leq i \leq m}$ a value of 0 or 1.

a) Contributions to Growth

The growth rate of X from period t_0 to period t_1 can be decomposed into growth rates of the $\{X_i\}_{1 \leq i \leq m}$ as follows (we suppose that $X^{t_0} \neq 0$).

We decompose the numerator in X's growth rate expression (Equation (4)) using the additive relationship given by Equation (6). Hence,

$$g_X^{t_0 \to t_1} = \sum_{i=1}^{m} (-1)^{f(X_i)} * \frac{X_i^{t_1} - X_i^{t_0}}{|X^{t_0}|}$$

We multiply and divide by $\left|X_i^{t_0}\right|$ (supposing that $X_i^{t_0} \neq 0$),

$$g_X^{t_0 \to t_1} = \sum_{i=1}^{m} (-1)^{f(X_i)} * \frac{\left|X_i^{t_0}\right|}{|X^{t_0}|} * \frac{X_i^{t_1} - X_i^{t_0}}{\left|X_i^{t_0}\right|}$$

Therefore,

$$g_X^{t_0 \to t_1} = \sum_{i=1}^{m} (-1)^{f(X_i)} * \frac{\left|X_i^{t_0}\right|}{|X^{t_0}|} * g_{X_i}^{t_0 \to t_1} \tag{7}$$

$(-1)^{f(X_i)} * \frac{\left|X_i^{t_0}\right|}{|X^{t_0}|} * g_{X_i}^{t_0 \to t_1}$ or simply $(-1)^{f(X_i)} * \frac{X_i^{t_1} - X_i^{t_0}}{|X^{t_0}|}$ is the contribution of X_i to the growth of X between the periods t_0 and t_1.

b) Contribution to the Simple Difference

Following the same approach as for growth rates, we arrive to the fact that the simple difference in X from period t_0 to period t_1 can be decomposed as follows.

$$\Delta_X^{t_0 \to t_1} = \sum_{i=1}^{m} (-1)^{f(X_i)} * \Delta_{X_i}^{t_0 \to t_1} \tag{8}$$

$(-1)^{f(X_i)} * \Delta_{X_i}^{t_0 \to t_1}$ is the contribution of X_i to the simple difference in X between t_0 and t_1.

We will often transform the above contribution to a share by dividing it by the simple difference in X (Equation (9)) (we suppose that $\Delta_X^{t_0 \to t_1} \neq 0$).

$$1 = \sum_{i=1}^{m} (-1)^{f(X_i)} * \frac{\Delta_{X_i}^{t_0 \to t_1}}{\Delta_X^{t_0 \to t_1}} \tag{9}$$

$(-1)^{f(X_i)} * \frac{\Delta_{X_i}^{t_0 \to t_1}}{\Delta_X^{t_0 \to t_1}}$ is X_i's share in the simple difference in X between t_0 and t_1.

A-II-2.4. Shares in Aggregates

Let X and $\{X_i\}_{1 \le i \le m}$ be distinct economic variables, such that X is linked to $\{X_i\}_{1 \le i \le m}$ by the additive relationship represented by Equation (6). Therefore, supposing that $X^t \neq 0$, we can derive shares for the $\{X_i\}_{1 \le i \le m}$ in X as follows.

$$\forall t, \qquad 1 = \sum_{i=1}^{m} (-1)^{f(X_i)} * \frac{X_i^t}{X^t}$$

$(-1)^{f(X_i)} * \frac{X_i^t}{X^t}$ is X_i's share in X in period t.

A-II-2.5. Price Deflation

Let X be a nominal economic variable and P a general price index. We can deflate the variable X by dividing it by the price index P. The resulting volume estimate, X_{real}, is referred to as X "in real terms".

$$\forall t, \qquad X_{real}^t = \frac{X^t}{P^t}$$

An index series with reference year b can be derived by dividing the series in real terms by its annual value in year b for annual series and by its average quarterly value in year b for quarterly series.

A-III- Treatment of Data Category 1: GDP and Final Expenditure Components

In this subsection, we present eight operations carried out on the GDP and Final Expenditure Components data category. We formed the data category's initial parent datasets consisting of the downloaded datasets. We identified additional final expenditure aggregates and computed growth rates and implicit price deflators for GDP and the final expenditure aggregates. Moreover, we calculated additive contributions to the growth of and shares in main aggregates such as GDP. Finally, we decomposed volume and price series into trend and cyclical components and computed volume estimates for GDP per Capita and Change in Stocks.

We divide this subsection into ten sub-subsections. We begin with a sub-subsection outlining the formation of the initial parent datasets. Next, we reserve a sub-subsection for the definition of basic concepts and economic variables relevant to this subsection. Afterward, for each of the remaining seven operations listed in the previous paragraph, we dedicate a sub-subsection, where we establish a theoretical framework in light of the 2008 System of National Accounts (2009), the Handbook on Quarterly National Accounts by Eurostat (2013), and the Quarterly National Accounts Manual by IMF (2017), and where we demonstrate the application to the data category. Finally, we conclude with a sub-subsection addressing the final composition of the data category and the data visualizations.

A-III-1. The Formation of Initial Parent Datasets

The first operation conducted on the GDP and Final Expenditure Components data category consists of data preparation actions that standardize the structure of the downloaded datasets (see Sub-subsection A-I-1, "Data Category 1: GDP and Final Expenditure Components") before consolidating them in parent datasets by data frequency. The objective of this operation is to smooth access to and manipulation of the downloaded economic indicators during the execution of subsequent operations, as well as to facilitate the storage of newly computed economic indicators and metrics.

The first data preparation actions were performed directly on the downloaded Excel files. The downloaded datasets (see Sub-subsection A-I-1, “Data Category 1: GDP and Final Expenditure Components”) were one-by-one transformed into simpler tabular datasets with standardized column names based on the labels of GDP by Expenditure Approach series (except for the GDP per capita dataset, whose column names indicate the currencies in which GDP per capita is measured and distinguish one column for the size of the mid-year population). For annual datasets, a single row index indicates the year, while for quarterly datasets a double-index indicates the year in the first column and the quarter in the second.

The simplified datasets were loaded into Python for further preprocessing. Annual simplified datasets were consolidated into one parent dataset (except the GDP per capita dataset, which was added later), which at this stage consisted of one sub-dataset named “Accounts.” The “Accounts” dataset has a two-levels column index. The first level separates between three sub-datasets—unadjusted values at current prices, unadjusted volume indices, and calendar adjusted volume indices—within which the second level differentiates between the individual GDP by Expenditure Approach series. In a similar fashion, quarterly simplified datasets were consolidated into one parent dataset, which at this stage featured one sub-dataset named “Accounts.” The quarterly “Accounts” dataset has a three-level column index. The first level divides it into two main sub-datasets: value at current prices and volume index datasets. The second level separates the value at current prices dataset into two sub-datasets and the volume index dataset into three sub-datasets based on adjustments: unadjusted and seasonally adjusted for values at current prices, and unadjusted, seasonally adjusted, and seasonally and calendar adjusted for volume indices. Finally, the third level differentiates between the individual GDP by Expenditure Approach series. Subsequent operations on the

GDP and Final Expenditure Components data category produced new datasets, which were added alongside the “Accounts” dataset in the corresponding parent dataset.

The first actions which yielded the simplified datasets were executed manually. Therefore, depending on the downloaded datasets’ licenses of use, either the full simplified datasets or their structures (column headings and row indices) are published with this report. On the other hand, the consolidation actions were executed by programming in Python. The corresponding code is included in the attached “Data Processing” code file within Section I, “GDP and Final Expenditure Components,” under Subheading 1.1, “Data Preparation,” of Heading 1, “Annual Data,” for annual series, and under Subheading 2.1, “Data Preparation,” of Heading 2, “Quarterly Data,” for quarterly series.

A-III-2. Basic Concepts and Variable Definitions

In this sub-subsection, we provide preliminary definitions of basic concepts related to GDP by the expenditure approach. Additionally, we define economic variables used in subsequent sub-subsections, using notations in line with the notational framework outlined in Sub-subsection A-II-1, "Notational Framework."

A-III-2.1. Basic Concepts

a) Elementary Transactions in Products

Adhering to the definitions presented in the manuals cited in this subsection’s introduction, we define an elementary transaction in products as the aggregate of the individual Price*Quantity transactions (hereinafter referred to as individual transactions), whereby the same buyer—a domestic institutional sector or the rest of the world—pays in exchange for quantities (including quality effects) of the same homogeneous product—good or service— for the same use (purpose)—intermediate consumption, final consumption, capital formation, or export. In summary, we define an elementary transaction in products by a tuple of the form $(Buyer, Product, Use)$. We can use additional attributes such as the seller—total economy or the rest of the world—to distinguish imported products, for example.

For a given accounting period (e.g. quarter or year), an elementary transaction in products is characterized by an average price and a total quantity, which are based on the prices and quantities of the individual transactions occurring within that period. The value at current prices of the elementary transaction is calculated as the product of the average price and the total quantity. On the other hand, the volume measure at previous year prices is obtained by multiplying the total quantity in the accounting period by the average price of the previous year.

We refer to an elementary transaction in products, whose purpose is a final use—final consumption, capital formation, or export—as an elementary final expenditure transaction. By definition, every individual transaction is accounted for in at most one elementary final expenditure transaction.

b) Final Expenditure Components and Aggregates

Aggregating multiple elementary final expenditure transactions yields a final expenditure component. For example, Final Consumption Expenditures of Households is the aggregate of elementary final expenditure transactions where the buyer is the households sector and the purpose is final consumption. Moreover, final expenditure components can be further aggregated to form higher-order aggregates. For instance, combining Final Consumption Expenditures of Households and Final Consumption Expenditures by NPISH results in what we refer to as Final Private Consumption Expenditures.

Implicit in our definition is the assumption of a comprehensive and temporally invariant set of subcomponent elementary final expenditure transactions for each final expenditure component. In fact, we assume that the management of which transactions are relevant in a given period and how they are accounted for is handled implicitly by prices and quantities (for example, a transaction can be eliminated by assigning a null quantity). This simplification does not affect the validity of our theoretical frameworks in their role of underpinning the operations conducted on the GDP and Final Expenditure Components data category, while sparing us the need to model the complex practices of managing new and obsolete products.

We say that two final expenditure components or aggregates are disjoint if their subcomponent sets of elementary final expenditure transactions are disjoint.

A-III-2.2. Variable Definitions and Notations

We define the set S of final expenditure components. For each element i of S, we define the following economic variables: (Notes: $t = (s, y)\ or\ t = y$; for simplicity, we hereinafter often refer to the final expenditure components as 'components' and the elementary final expenditure transactions as 'transactions'; all monetary values are assumed to be in domestic currency)

- C_i^t: Annual/Quarterly value at current prices for component i in period t
- $K_i^{y-1\to t}$: Annual/Quarterly volume measure at previous year prices for component i in period t
- $Q_i^{y-1\to t}$: Annual/Quarterly Laspeyres volume index from previous year for component i in period t
- $K_i^{b\to t}$: Annual/Quarterly chain-linked Laspeyres volume measure in monetary terms, with reference year b, for component i in period t (for quarterly measures, the annual overlap technique is used)
- $Q_i^{b\to t}$: Annual/Quarterly chain-linked Laspeyres volume index, with reference year b, for component i in period t (for quarterly indices, the annual overlap technique is used)
- $PP_i^{b\to t}$: Annual/Quarterly chain-linked Paasche price index, with reference year b, for component i in period t (implicit price deflator)
- R_i: The set of subcomponent transactions for component i.

- n_i: The number of transactions for component i (the cardinality of R_i)
- Each transaction in R_i is completely specified by a tuple of the form (i, j) with $1 \leq j \leq n_i$ and has the following attributes:
  - $\bar{P}_{i,j}^t$: The (weighted) average price in period t for transaction j of component i
  - $U_{i,j}^t$: The total quantity in period t for transaction j of component i
  - $Tr_{i,j}^t$: The number of individual transactions (Price*Quantity) in transaction j of component i in period t
  - In period t, each individual transaction included in transaction j is completely specified by a tuple of the form (i, j, p) with $1 \leq p \leq Tr_{i,j}^t$ and has the following attributes:
    - $P_{i,j,p}^t$: The unit price for the individual transaction p of transaction j of component i in period t
    - $U_{i,j,p}^t$: The quantity measure for the individual transaction p of transaction j of component i in period t

We suppose that the time periods start in year 0 and end in year y_{end}, and we define year b such that $0 \leq b \leq y_{end}$.

A-III-3. The Identification of Additional Final Expenditure Aggregates

The objective of this operation is to identify the following final expenditure aggregates for the Turkish economy: Final Private Consumption Expenditure, Final Domestic Consumption Expenditure, and Final Domestic Demand Expenditure. The identification of an aggregate consists of determining its value at current prices and chain-linked volume index series. While the aggregation of values at current prices is straightforward, that of chain-linked volume indices is more complex due to the lack of additivity in chain-linked measures.

This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The sub-subsubsections treat each data frequency—annual and quarterly—separately. The Theoretical Framework opens with a brief description of the aggregation of values at current prices. Next, it provides a definition of the Laspeyres volume index, chain-linking, and unchaining before breaking down and explaining the process of aggregating chain-linked Laspeyres volume indices. Furthermore, the Theoretical Framework for quarterly series contains a discussion of the temporal consistency property. The Application to the Data Category demonstrates the employment of the established theoretical framework to achieve the operation's objectives.

A-III-3.1. Theoretical Framework

We suppose that we want to identify the aggregate A of the disjoint final expenditure subcomponents $\{i\}_{1 \le i \le m} \subset S$ for the years from 0 to y_{end}.

A-III-3.1.1. Annual Series

We suppose that for each of the subcomponents $\{i\}_{1 \le i \le m}$, we have annual series of both values at current prices and chain-linked Laspeyres volume indices with reference year b, covering the years 0 to y_{end}. Furthermore, we suppose that the elements of the series are positive.

a) Aggregation of Values at Current Prices

Values at current prices are additive (see Annex F-I). Therefore, since $\{i\}_{1 \le i \le m}$ are disjoint,

$$\forall y, \qquad C_A^y = \sum_{i=1}^{m} C_i^y \tag{10}$$

b) Laspeyres Volume Indices

The annual Laspeyres volume index for component i in year y, $Q_i^{y-1 \to y}$, is defined in Equation (11).

$$Q_i^{y-1 \to y} = \frac{K_i^{y-1 \to y}}{C_i^{y-1}} = \frac{\sum_{j=1}^{n_i} \bar{P}_{i,j}^{y-1} * U_{i,j}^{y}}{\sum_{j=1}^{n_i} \bar{P}_{i,j}^{y-1} * U_{i,j}^{y-1}} \tag{11}$$

Where,

$$\bar{P}_{i,j}^{y-1} = \frac{\sum_{p=1}^{Tr_{i,j}^{y-1}} P_{i,j,p}^{y-1} * U_{i,j,p}^{y-1}}{\sum_{p=1}^{Tr_{i,j}^{y-1}} U_{i,j,p}^{y-1}}$$

We refer to $Q_i^{y-1\to y}$ as Laspeyres volume index from previous year.

c) Chain-Linking

The series of annual Laspeyres volume indices from previous year for component i $\left(Q_i^{y-1\to y}\right)_{1\le y\le y_{end}}$ can be chain-linked to reference year 0 using Formula (12).

$$Q_i^{0\to y} = \begin{cases} \prod_{k=1}^{y} Q_i^{k-1\to k} & for\ y \ge 1 \\ 1 & for\ y = 0 \end{cases} \tag{12}$$

In year 0, the value of $Q_i^{-1\to 0}$ is not required to define the chain-linked index, as a value of 1 is assigned to the reference year.

The resulting chain-linked Laspeyres volume index series $\left(Q_i^{0\to y}\right)_{0\le y\le y_{end}}$ can be re-referenced to year b by dividing its elements by the index value in year b as shown below.

$$\forall 0 \le y \le y_{end}, \qquad Q_i^{b\to y} = \frac{Q_i^{0\to y}}{Q_i^{0\to b}} \tag{13}$$

The annual series of chain-linked Laspeyres volume indices with reference year b, $\left(Q_i^{\mathrm{b}\to y}\right)_{0\le y\le y_{end}}$, can be multiplied by 100 and presented on a scale of 100 or expressed in monetary terms by multiplying it by the value at current prices of component i in reference year b, C_i^b. We refer to the expression in monetary terms as: annual chain-linked Laspeyres volume measure in monetary terms with reference year b, $K_i^{\mathrm{b}\to y}$.

d) Unchaining

By the term "unchaining," we refer to the reverse process of chain-linking. Unchaining transforms back a chain-linked Laspeyres volume index series to its corresponding series of Laspeyres volume index from previous year.

For $y \geq 1$, the numerator in the right-hand side of Equation (13) can be decomposed as follows,

$$Q_i^{0\to y} = Q_i^{0\to y-1} * Q_i^{y-1\to y}$$

Therefore, remarking that $\frac{Q_i^{0\to y-1}}{Q_i^{0\to b}} = Q_i^{b\to y-1}$, Equation (13) can be rearranged in the following way,

$$\forall 1 \leq y \leq y_{end}, \qquad Q_i^{y-1\to y} = \frac{Q_i^{b\to y}}{Q_i^{b\to y-1}} \tag{14}$$

The unchaining of component i's annual chain-linked Laspeyres volume index series, $\left(Q_i^{b\to y}\right)_{0\leq y\leq y_{end}}$, is achieved through Equation (14) and results into the series of annual Laspeyres volume index from previous year, $\left(Q_i^{y-1\to y}\right)_{1\leq y\leq y_{end}}$.

e) Aggregation of Chain-Linked Laspeyres Volume Indices

The chain-linked Laspeyres volume indices of the disjoint final expenditure subcomponents $\{i\}_{1\leq i\leq m}$ cannot be aggregated directly due to the lack of additivity of chain-linked volume measures. However, we can circumvent this limitation by taking advantage of the additivity of the volume measures at previous year prices (this property is proved in Annex F-II-1.a). In fact, to calculate the annual chain-linked Laspeyres volume indices for aggregate A, we go through the following process:

- Step 1: Unchaining the subcomponents' chain-linked Laspeyres volume indices. For each subcomponent i, we unchain the chain-linked Laspeyres volume index series $\left(Q_i^{b\to y}\right)_{0\leq y\leq y_{end}}$ using Equation (14). This results into series of Laspeyres volume indices from previous year covering the years 1 to y_{end}, $\left(Q_i^{y-1\to y}\right)_{1\leq y\leq y_{end}}$.
- Step 2: Aggregating the subcomponents' Laspeyres volume indices from previous year. For each year y from 1 to y_{end}, we aggregate the volume indices from previous year of subcomponents $\{i\}_{1\leq i\leq m}$, which were obtained in Step 1, using Equation (15) (derived in

Annex F-II-1.b). This results into the series of Laspeyres volume indices from previous year for aggregate A covering the years 1 to y_{end}, $\left(Q_A^{y-1\to y}\right)_{1\le y\le y_{end}}$.

$$\forall 1 \le y \le y_{end}, \qquad Q_A^{y-1\to y} = \sum_{i=1}^{m} \frac{C_i^{y-1}}{C_A^{y-1}} * Q_i^{y-1\to y} \tag{15}$$

- Step 3: Chain-linking the aggregate Laspeyres volume indices from previous year. We obtain the chain-linked Laspeyres volume indices, with reference year 0, for aggregate A, $\left(Q_A^{0\to y}\right)_{0\le y\le y_{end}}$, by chain-linking the aggregate Laspeyres volume indices from previous year, $\left(Q_A^{y-1\to y}\right)_{1\le y\le y_{end}}$, to year 0, as shown in Formula (12). The resulting indices are then re-referenced to year b using Equation (13), and they cover years 0 to y_{end}, $\left(Q_A^{b\to y}\right)_{0\le y\le y_{end}}$.

A-III-3.1.2. Quarterly Series

We suppose that for each of the subcomponents $\{i\}_{1\le i\le m}$, we have quarterly series of both values at current prices and chain-linked, with annual overlap, Laspeyres volume indices with reference year b, covering the periods from the first quarter of year 0 to the last quarter of year y_{end}. Furthermore, we suppose that the elements of each series are positive.

a) Aggregation of Values at Current Prices

Values at current prices are additive (see Annex F-I). Therefore, since $\{i\}_{1\le i\le m}$ are disjoint,

$$\forall y, \forall 1 \le s \le 4, \qquad C_A^{(s,y)} = \sum_{i=1}^{m} C_i^{(s,y)} \tag{16}$$

b) Laspeyres Volume Indices

The quarterly Laspeyres volume index for component i in quarter s of year y, $Q_i^{y-1\to(s,y)}$, is defined in Equation (17).

$$Q_i^{y-1\to(s,y)} = \frac{K_i^{y-1\to(s,y)}}{\frac{1}{4} * \sum_{s=1}^{4} C_i^{(s,y-1)}} = \frac{\sum_{j=1}^{n_i} \bar{P}_{i,j}^{y-1} * U_{i,j}^{(s,y)}}{\frac{1}{4} * \left(\sum_{s=1}^{4} \sum_{j=1}^{n_i} \bar{P}_{i,j}^{(s,y-1)} * U_{i,j}^{(s,y-1)}\right)} \tag{17}$$

Where,

$$\begin{cases} \bar{P}_{i,j}^{(s,y-1)} = \dfrac{\sum_{p=1}^{Tr_{i,j}^{(s,y-1)}} P_{i,j,p}^{(s,y-1)} * U_{i,j,p}^{(s,y-1)}}{\sum_{p=1}^{Tr_{i,j}^{(s,y-1)}} U_{i,j,p}^{(s,y-1)}} \\ \bar{P}_{i,j}^{y-1} = \dfrac{\sum_{s=1}^{4} \bar{P}_{i,j}^{(s,y-1)} * U_{i,j}^{(s,y-1)}}{\sum_{s=1}^{4} U_{i,j}^{(s,y-1)}} \end{cases}$$

c) Chain-Linking: Annual Overlap Technique

The series of quarterly Laspeyres volume indices from previous year, $\left(Q_i^{y-1\to(s,y)}\right)_{1\le y\le y_{end}, 1\le s\le 4}$, for component i can be chain-linked with annual overlap to reference year 0 using Formula (18).

$$Q_i^{0\to(s,y)} = \begin{cases} \left(\prod_{k=1}^{y-1} G_i^{k-1\to k}\right) * Q_i^{y-1\to(s,y)} & for\ y \ge 2 \\ Q_i^{y-1\to(s,y)} & for\ y = 1 \end{cases} \tag{18}$$

Where $\left(G_i^{k-1\to k}\right)$ are the annual links,

$$G_i^{k-1\to k} = \frac{\sum_{s=1}^{4} \sum_{j=1}^{n_i} \bar{P}_{i,j}^{k-1} * U_{i,j}^{(s,k)}}{\sum_{s=1}^{4} \sum_{j=1}^{n_i} \bar{P}_{i,j}^{(s,k-1)} * U_{i,j}^{(s,k-1)}} = \frac{\sum_{s=1}^{4} K_i^{k-1\to(s,k)}}{\sum_{s=1}^{4} C_i^{(s,k-1)}}$$

Where $\bar{P}_{i,j}^{k-1}$ and $\bar{P}_{i,j}^{(s,k-1)}$ are as defined in the previous item b, "Laspeyres Volume Indices."

Based on the definition of $Q_i^{k-1\to(s,k)}$ in Equation (17), the annual links can be written as, (see Annex F-III-a)

$$G_i^{k-1\to k} = \frac{1}{4} * \sum_{s=1}^{4} Q_i^{k-1\to(s,k)} \tag{19}$$

Due to the unavailability of volume indices from previous year in year 0, a chain-linked Laspeyres volume index could not be determined for the quarters of year 0.

The resulting chain-linked Laspeyres volume index series $\left(Q_i^{0\rightarrow(s,y)}\right)_{1\leq y\leq y_{end},1\leq s\leq 4}$ can be re-referenced to year b by dividing its elements by the value of the chain-linked annual link in year b, $G_i^{0\rightarrow b} = \left(\prod_{k=1}^{b} G_i^{k-1\rightarrow k}\right)$, as shown below.

$$\forall 1 \leq y \leq y_{end}, \forall 1 \leq s \leq 4, \qquad Q_i^{b\rightarrow(s,y)} = \frac{Q_i^{0\rightarrow(s,y)}}{G_i^{0\rightarrow b}} \tag{20}$$

The quarterly series of chain-linked Laspeyres volume indices with reference year b, $\left(Q_i^{\mathrm{b}\rightarrow(s,y)}\right)_{1\leq y\leq y_{end},1\leq s\leq 4}$, can be multiplied by 100 and presented on a scale of 100 or expressed in monetary terms by multiplying it by the average quarterly value at current prices of component i in reference year b. We refer to the expression in monetary terms as: quarterly chain-linked Laspeyres volume measure in monetary terms with reference year b, $K_i^{\mathrm{b}\rightarrow(s,y)}$.

d) Unchaining: The Case of Annual Overlap

By the term "unchaining," we refer to the reverse process of chain-linking. Unchaining transforms back a chain-linked Laspeyres volume index series to its corresponding series of Laspeyres volume index from previous year.

For $y \geq 1$, the numerator in the right-hand side of Equation (20) can be decomposed as follows, with $G_i^{0\rightarrow 0} = 1$,

$$Q_i^{0\rightarrow(s,y)} = G_i^{0\rightarrow y-1} * Q_i^{y-1\rightarrow(s,y)}$$

Therefore, with $G_i^{b\rightarrow y-1} = \frac{G_i^{0\rightarrow y-1}}{G_i^{0\rightarrow b}}$, Equation (20) can be rearranged in the following way,

$$\forall 1 \leq y \leq y_{end}, \forall 1 \leq s \leq 4, \qquad Q_i^{y-1\rightarrow(s,y)} = \frac{Q_i^{b\rightarrow(s,y)}}{G_i^{b\rightarrow y-1}} \tag{21}$$

The chain-linked annual link, $G_i^{b\rightarrow y-1}$, can be derived from the quarterly chain-linked volume indices $Q_i^{b\rightarrow(s,y-1)}$ using Equation (22) (See Annex F-III-b).

$$G_i^{b \to y-1} = \frac{1}{4} * \sum_{s=1}^{4} Q_i^{b \to (s,y-1)} \tag{22}$$

The unchaining of component i's quarterly chain-linked Laspeyres volume index series, $\left(Q_i^{b \to (s,y)}\right)_{0 \le y \le y_{end}, 1 \le s \le 4}$, is achieved through Equation (21) and results into the series of quarterly Laspeyres volume indices from previous year, $\left(Q_i^{y-1 \to (s,y)}\right)_{1 \le y \le y_{end}, 1 \le s \le 4}$.

e) Aggregation of Chain-Linked Laspeyres Volume Indices

To calculate the quarterly chain-linked Laspeyres volume indices for aggregate A, we go through a process similar to the one adopted for annual series:

- Step 1: Unchaining the subcomponents' chain-linked Laspeyres volume indices. For each component i, we unchain the chain-linked Laspeyres volume index series $\left(Q_i^{b \to (s,y)}\right)_{0 \le y \le y_{end}, 1 \le s \le 4}$ using Equations (21). The annual links utilized to unchain are obtained using Equation (22). The unchaining results into series of quarterly Laspeyres volume indices from previous year covering years $1 \le y \le y_{end}$, $\left(Q_i^{y-1 \to (s,y)}\right)_{1 \le y \le y_{end}, 1 \le s \le 4}$.
- Step 2: Aggregating the subcomponents' Laspeyres volume indices from previous year. For each year y from 1 to y_{end}, we aggregate the volume indices from previous year of subcomponents $\{i\}_{1 \le i \le m}$, which were obtained in Step 1, using Equation (23) (see Annex F-II-2.b). This results into series of Laspeyres volume indices from previous year for aggregate A covering the years 1 to y_{end}, $\left(Q_A^{y-1 \to (s,y)}\right)_{1 \le y \le y_{end}, 1 \le s \le 4}$.

$$\forall 1 \le y \le y_{end}, \forall 1 \le s \le 4, \qquad Q_A^{y-1 \to (s,y)} = \sum_{i=1}^{m} \frac{\sum_{s=1}^{4} C_i^{(s,y-1)}}{\sum_{s=1}^{4} C_A^{(s,y-1)}} * Q_i^{y-1 \to (s,y)} \tag{23}$$

- Step 3: Deriving annual links for aggregate A. The annual links for aggregate A can be obtained by applying Equation (19) to the quarterly Laspeyres volume indices from previous year of aggregate A obtained in the previous step.
- Step 4: Chain-linking the aggregate Laspeyres volume indices from previous year. We derive the chain-linked Laspeyres volume indices, with reference year 0, for aggregate A, $\left(Q_A^{0\to(s,y)}\right)_{1\le y\le y_{end},1\le s\le 4}$, by chain-linking the aggregate Laspeyres volume indices from previous year, $\left(Q_A^{y-1\to(s,y)}\right)_{1\le y\le y_{end},1\le s\le 4}$, using the annual links derived in the previous step, as described in Formula (18). The resulting indices are then re-referenced to year b using Equation (20), and they cover years $1 \le y \le y_{end}$, $\left(Q_A^{b\to(s,y)}\right)_{1\le y\le y_{end},1\le s\le 4}$.

f) Temporal Consistency

Temporal consistency between a quarterly series and its annual counterpart occurs when the temporal aggregation of the quarterly series yields the annual one. For instance, quarterly and annual Laspeyres volume indices are temporally consistent when the annual averages of the quarterly indices match the annual indices. On the other hand, values at current prices, respectively volume measures at previous year prices, are temporally consistent when the annual sums of the quarterly values, respectively measures, are equal to the annual values, respectively measures.

In this paragraph, we assume that the quarterly and annual values at current prices and volume measures at previous year prices are temporally consistent and draw conclusions on the quarterly Laspeyres volume indices. In fact, a direct implication of this assumption is that the quarterly and annual Laspeyres volume indices from previous year are also temporally consistent (see Annex F-IV-b). Furthermore, based on Equation (19), this implies that the annual links used in the annual overlap technique are equal to the annual Laspeyres volume indices from previous year. Therefore, the chain-linked annual links match the annual chain-linked Laspeyres volume indices. Combining the previous result and Equation (22) leads to the fact that the quarterly and annual chain-linked Laspeyres volume indices are also temporally consistent (when the annual overlap technique is used). To summarize, in Annex F-IV, we show that the temporal consistency of values at current prices and volume measures at previous year prices implies that the Laspeyres volume indices from previous year are also temporally consistent, which is equivalent to the chain-linked Laspeyres volume indices being temporally consistent (when the annual overlap technique is used).

System (24) expresses the previously discussed implications when we assume that the values at current prices and volume measures at previous year prices are temporally consistent over the years $\{y \geq 0\}$ and $\{y \geq 1\}$, respectively. System (24)'s equations are mutually equivalent (the proof is provided in the second paragraph of Annex F-IV-b).

$$\begin{cases} \forall y \geq 1, & \frac{1}{4} * \sum_{s=1}^{4} Q_i^{y-1\to(s,y)} = Q_i^{y-1\to y} \\ \forall y \geq 1, & G_i^{y-1\to y} = Q_i^{y-1\to y} \\ \forall y \geq 0, & G_i^{b\to y} = Q_i^{b\to y} \\ \forall y \geq 0, & \frac{1}{4} * \sum_{s=1}^{4} Q_i^{b\to(s,y)} = Q_i^{b\to y} \end{cases} \tag{24}$$

We expect the temporal consistency assumptions to hold and their implications to follow. However, when adjustments are applied to the quarterly series, such as seasonal adjustments, these properties might no longer be directly satisfied.

A-III-3.2. Application to the Data Category

We utilized the above theoretical framework to identify the following Turkish final expenditure aggregates $\{A_x\}_{1\leq x\leq 3}$ based on the respective final expenditure sub-aggregates (or subcomponents, as referred to in the theoretical framework) $\{S_x\}_{1\leq x\leq 3}$:

- Final Private Consumption Expenditure A_1: Aggregate of sub-aggregates S_1—Final Consumption Expenditure of Households and Final Consumption Expenditure of NPISH.

- Final Domestic Consumption Expenditure A_2: Aggregate of sub-aggregates S_2—Final Consumption Expenditure of Households, Final Consumption Expenditure of NPISH, and Government Final Consumption Expenditure.
- Final Domestic Demand Expenditure A_3: Aggregate of sub-aggregates S_3—Final Consumption Expenditure of Households, Final Consumption Expenditure of NPISH, Government Final Consumption Expenditure, and Gross Fixed Capital Formation.

The above aggregates can also be defined using a nested structure, as Final Private Consumption Expenditure is a sub-aggregate of Final Domestic Consumption Expenditure, which, in turn, is a sub-aggregate of Final Domestic Demand Expenditure.

For each aggregate A_x, the sub-aggregates S_x are disjoint as assumed in the theoretical framework. Moreover, for each sub-aggregate, we have both annual and quarterly series of values at current prices and chain-linked, with annual overlap for quarterly series, Laspeyres volume indices with reference year 2009, both of which are positive and cover the period from 1998 to the 2nd Quarter of 2024 ((Q2,2024)), inclusive. Ideally, we want the identification process to yield the same series covering the same period for aggregates $\{A_x\}_{1\leq x\leq 3}$. However, we know from the theoretical framework that this objective is not completely attainable for quarterly volume index series due to the absence of data from 1997. Let $S_T = \bigcup_{x=1}^{3} S_x$.

A-III-3.2.1. Annual Series

For each final expenditure sub-aggregate in S_T, the annual value at current prices series is available in unadjusted form. On the other hand, there are two versions of the annual volume index series: unadjusted and calendar adjusted.

a) The Aggregation of Values at Current Prices

The annual unadjusted values at current prices for the final expenditure sub-aggregates in S_T cover the period from 1998 to 2023. Therefore, applying Equation (10) to each set of sub-aggregates in $\{S_x\}_{1\leq x\leq 3}$, we calculated annual unadjusted values at current prices for aggregates $\{A_x\}_{1\leq x\leq 3}$ covering the same period.

b) The Aggregation of Chain-Linked Laspeyres Volume Indices

Both versions—unadjusted and calendar adjusted—of the annual chain-linked Laspeyres volume indices with reference year 2009 for sub-aggregates S_T cover the years 1998 to 2023. We applied the aggregation process, which is presented in Item e, "Aggregation of Chain-Linked Laspeyres Volume Indices," of Sub-subsubsection A-III-3.1, "Theoretical Framework," to each set of sub-aggregates $\{S_x\}_{1\leq x\leq 3}$ and for each version of the series. This resulted in both unadjusted and calendar adjusted annual chain-linked Laspeyres volume index series with reference year 2009 for aggregates $\{A_x\}_{1\leq x\leq 3}$ covering the period 1998 to 2023, $\left(Q_{A_x}^{2009\to y}\right)_{1998\leq y\leq 2023}$, which we converted to a scale of 100.

The application of the aggregation process is straightforward. However, we should mention that in the second step—Step 2: Aggregating the subcomponents' Laspeyres volume indices from previous year—for the calendar adjusted series, we employed unadjusted values at current prices instead of calendar adjusted ones in the aggregation equation (15). The reason for this is the unavailability of calendar adjusted values at current prices. In fact, it can be inferred from the following statement by the data provider (TUIK) in the quarterly GDP series metadata that the values at current prices do not contain calendar effects: "As most of GDP in Current Prices and Volume Indices series contain seasonal effects some of the volume indices also contain calendar effects" (2024). If our inference is correct, then the calendar adjusted series of values at current prices are the same as the unadjusted ones.

c) Dataset Management and Code Implementation

The computed series were inserted into the "Accounts" dataset of the annual parent dataset, within the sub-dataset that corresponds to the nature of the series—volume index or value at current prices—and the adjustments applied to the series.

The aggregation processes and the insertion of the series into the annual parent dataset were implemented using Python, primarily with the Pandas library. The corresponding code is included in the attached "Data Processing" code file under Subheading 1.2, "Identification of Additional Final Expenditure Aggregates," of Heading 1, "Annual Data," within Section I, "GDP and Final Expenditure Components."

A-III-3.2.2. Quarterly Series

For each final expenditure sub-aggregate in S_T, the quarterly series of values at current prices is available in both unadjusted and seasonally adjusted forms. On the other hand, there are three versions of the quarterly chain-linked Laspeyres volume index series: unadjusted, seasonally adjusted, and seasonally and calendar adjusted. However, we will limit our consideration to the unadjusted and seasonally and calendar adjusted versions.

a) Aggregation of Values at Current Prices

We applied Equation (16) to each set of sub-aggregates $\{S_x\}_{1 \le x \le 3}$ and for each version of the value at current prices series covering the period (Q1,1998) to (Q2,2024). This resulted in both unadjusted and seasonally adjusted quarterly series of values at current prices for aggregates $\{A_x\}_{1 \le x \le 3}$ covering the same period.

b) Aggregation of Chain-Linked Laspeyres Volume Indices

Our aim is to compute quarterly unadjusted and quarterly seasonally and calendar adjusted chain-linked Laspeyres volume indices for aggregates $\{A_x\}_{1 \le x \le 3}$ by aggregating the volume indices of the respective sub-aggregates $\{S_x\}_{1 \le x \le 3}$ (The derivation of seasonally and calendar adjusted volume indices for $\{A_x\}_{1 \le x \le 3}$ by aggregating seasonally and calendar adjusted volume indices of the sub-aggregates is in line with the TUIK's adoption of the indirect seasonal adjustment approach). Before going through the aggregation process, which is described in Item e, "Aggregation of Chain-Linked Laspeyres Volume Indices," of Sub-subsubsection A-III-3.1, "Theoretical Framework," we ran a temporal consistency check for sub-aggregates S_T's values at current prices and chain-linked Laspeyres volume indices. The results of this test deepened our knowledge of the series' characteristics and enabled us to exploit most of the annual series of sub-aggregates S_T and aggregates $\{A_x\}_{1\leq x\leq 3}$ during the process of aggregating quarterly series. We provide below an overview of the steps followed to compute the quarterly volume indices starting from the temporal consistency check.

- Step 0: Temporal consistency check. The temporal consistency check features the comparison of both unadjusted and seasonally adjusted quarterly values at current prices to annual unadjusted ones, as well as the comparison of both unadjusted, and seasonally and calendar adjusted quarterly chain-linked Laspeyres volume indices to each of unadjusted and calendar adjusted annual indices. In summary, the results show that both unadjusted and seasonally adjusted values at current prices are temporally consistent with the annual unadjusted values. For quarterly unadjusted volume indices, temporal consistency holds with respect to annual unadjusted indices, except for GDP in year 1998. On the other hand, for quarterly seasonally and calendar adjusted series, the property of temporal consistency with respect to annual calendar adjusted series is observed for all series, except for GDP and Government Final Consumption Expenditure during the years 1998 to 2008. A more detailed summary of these findings is provided in Annex G-1. While the temporal inconsistencies observed in GDP volume index series do not affect the aggregation process because GDP is not part of it, the temporal inconsistencies in seasonally and calendar adjusted Government Final Consumption Expenditure volume index series are relevant to the process, as the series is used to calculate seasonally and calendar adjusted versions of both Final Domestic Consumption Expenditure and Final Domestic Demand Expenditure volume index series.
- Step 1: Unchaining the sub-aggregates' chain-linked Laspeyres volume indices. We unchained sub-aggregates S_T's quarterly unadjusted and seasonally and calendar adjusted

chain-linked Laspeyres volume indices using Equation (21). We know from Item f, "Temporal Consistency," of Sub-subsubsection A-III-3.1, "Theoretical Framework," that the temporal consistency of chain-linked Laspeyres volume indices is equivalent to the annual links, which are used in the annual overlap technique, being equal to the annual Laspeyres volume indices (System (24)). Hence, based on the temporal consistency results, the annual links used to unchain quarterly unadjusted series were directly obtained from the corresponding annual unadjusted chain-linked Laspeyres volume index series. Moreover, the annual links used to unchain quarterly seasonally and calendar adjusted series were directly obtained from the corresponding annual calendar adjusted chain-linked Laspeyres volume index series, except for seasonally and calendar adjusted Government Final Consumption Expenditure. This latter's annual links were derived using Equation (22). In conclusion, Step 1 yielded unadjusted and seasonally and calendar adjusted Laspeyres volume indices from previous year for sub-aggregates S_T covering the period from (Q1,1999) to (Q2,2024).

- Step 2: Aggregating the sub-aggregates' Laspeyres volume indices from previous year. For each aggregate A_x, we computed unadjusted and seasonally and calendar adjusted volume indices from previous year covering the period from (Q1,1999) to (Q2,2024) by applying Equation (23) to the corresponding version of sub-aggregates S_x's volume indices from previous year, which were obtained in Step 1. Thanks to the temporal consistency of both quarterly seasonally adjusted and quarterly unadjusted values at current prices with respect to annual unadjusted ones, and to the absence of calendar effects in values at current prices as explained in the application to annual series, the aggregation weights utilized in Equation (23) are equal to those used to aggregate annual indices in Equation (15).

- Step 3: Deriving annual links for aggregates $\{A_x\}_{1\leq x\leq 3}$. The annual links will be used in Step 4 to chain-link, with annual overlap, aggregates $\{A_x\}_{1\leq x\leq 3}$'s volume indices from previous year, which we derived in Step 2, to the reference period 2009. We used the fact that the combined temporal consistency of Laspeyres volume indices and values at current prices transfers from sub-aggregates to aggregates (see Annex F-IV-c) to determine whether aggregates $\{A_x\}_{1\leq x\leq 3}$'s Laspeyres volume indices are necessarily temporally consistent. Since the seasonally and calendar adjusted chain-linked Laspeyres volume index series for Government Final Consumption Expenditure is the only series exhibiting temporal inconsistencies, we know that all aggregates $\{A_x\}_{1\leq x\leq 3}$'s chain-linked Laspeyres volume index series are necessarily temporally consistent, except the seasonally and calendar adjusted versions of both Final Domestic Consumption Expenditure and Final Domestic Demand Expenditure, whose sub-aggregates include the seasonally and calendar adjusted Government Final Consumption Expenditure. Consequently, following the same reasoning as the one used in Step 1 to derive annual links, the annual links needed to chain-link aggregates $\{A_x\}_{1\leq x\leq 3}$'s unadjusted series were directly obtained from the corresponding annual unadjusted chain-linked Laspeyres volume index series, which we calculated in the application to annual series. Moreover, the annual links needed to chain-link Final Private Consumption Expenditure's seasonally and calendar adjusted series were directly obtained from the corresponding annual calendar adjusted chain-linked Laspeyres volume index series, which we calculated in the application to annual series. On the other hand, the annual links needed to chain-link the seasonally and calendar adjusted series of Final Domestic Consumption Expenditure and Final Domestic Demand Expenditure were derived using Equation (19) applied to the corresponding quarterly Laspeyres volume indices from previous year, which we calculated in Step 2. The calculated annual links were then chain-linked to the reference year 2009 using Formula (12) and Equation (13).

- Step 4: Chain-linking aggregates $\{A_x\}_{1\leq x\leq 3}$'s Laspeyres volume indices from previous year. The chain-linking, with annual overlap, of the Laspeyres volume indices from previous year, $\left(Q_{A_x}^{y-1\to(s,y)}\right)_{(1999,Q1)\ to\ (Q2,2024)}$, using the annual links $\left(G_{A_x}^{2009\to y}\right)_{1998\leq y\leq 2023}$ derived in Step 3, is as described by the below combination of Formula (18) and Equation (20).

$$\forall(Q1,1999) \leq (s,y) \leq (Q2,2024), \qquad Q_{A_x}^{2009\to(s,y)} = G_{A_x}^{2009\to y-1} * Q_{A_x}^{y-1\to(s,y)}$$

  Step 4 yielded unadjusted and seasonally and calendar adjusted chain-linked Laspeyres volume indices for aggregates $\{A_x\}_{1\leq x\leq 3}$ covering the period (Q1,1999) to (Q2,2024). As explained in the theoretical framework, because of the unavailability of volume indices from previous year in year 1998, which, in this case, is due to the unavailability of chain-linked volume indices for the sub-aggregates in year 1997, a chain-linked Laspeyres volume index could not be determined in the quarters of year 1998 for aggregates $\{A_x\}_{1\leq x\leq 3}$.

c) Dataset Management and Code Implementation

The computed series were inserted into the "Accounts" dataset of the quarterly parent dataset, within the sub-dataset that corresponds to the nature of the series—volume index or values at current prices—and the adjustments applied to the series.

The temporal consistency check, the aggregation processes, and the insertion of the series into the parent dataset were implemented using Python, primarily with the Pandas library. The corresponding code is included in the attached "Data Processing" code file under Subheading 2.2, "Identification of Additional Final Expenditure Aggregates," of Heading 2, "Quarterly Data," within Section I, "GDP and Final Expenditure Components."

A-III-4. The Calculation of Growth Rates

The objective of this operation is to measure the growth of GDP, GDP per capita, and the final expenditure components. This is achieved through the calculation of the growth rates of their values at current prices and volume indices.

This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework provides the definition of growth rates as well as their categorization depending on the base period of comparison. On the other hand, the Application to the Data Category outlines the operation's implementation.

A-III-4.1. Theoretical Framework

We already established the theoretical framework for the calculation of growth rates in Sub-subsubsection A-II-2.2, "Measures of Change."

A-III-4.2. Application to the Data Category

We computed annual and quarterly growth rates for all available versions of values at current prices and volume indices of GDP, the final expenditure aggregates, and GDP per capita.

a) Categories of Growth Rates

For annual series, we calculated Year-on-Year growth rates. Meanwhile, for quarterly series we calculated both Quarter-on-Quarter and Year-over-Year growth rates. However, we keep in mind that the Quarter-on-Quarter growth rates are more suitable for seasonally and calendar adjusted series, and that the Year-over-Year growth rates are useful when measuring growth in unadjusted series.

b) Temporal Coverage

Based on the temporal coverage of the value at current prices and volume index series, the growth rate series have the following temporal coverage:

- Year-on-Year growth rates cover the years from 1999 to 2023.

- Quarter-on-Quarter growth rates cover the period from (Q2,1998) to (Q2,2024), except for the growth rates of the aggregate volume indices calculated in Sub-subsection A-III-3, “The Identification of Additional Final Expenditure Aggregates,” for which the starting period is (Q2,1999).
- Year-over-Year growth rates cover the period from (Q1,1999) to (Q2,2024), except for the growth rates of the aggregate volume indices calculated in Sub-subsection A-III-3, “The Identification of Additional Final Expenditure Aggregates,” for which the starting period is (Q1,2000).

c) Dataset Management and Code Implementation

The Calculation of annual growth rates (except GDP per capita growth rates) resulted in the creation of a new dataset with the same structure as the annual “Accounts” dataset (see A-III-1, “The formation of Initial Parent Datasets”). The new dataset was named “Growth Rate” and integrated into the annual parent dataset. On the other hand, the calculation of quarterly growth rates yielded two datasets, one for Q-on-Q growth rates and the other for Y-o-Y growth rates. Both datasets have the same structure as the quarterly “Accounts” dataset (see A-III-1, “The formation of Initial Parent Datasets”) and were integrated into the quarterly parent dataset under the names: “Growth Rate QonQ” and “Growth Rate YoY”. For GDP per capita series, the corresponding growth rate series were added to the GDP per capita dataset, which is added to the annual parent dataset in a subsequent operation (see Item c of A-III-9.2).

The computation of growth rates and the integration of the resulting datasets into the parent datasets were implemented using Python, primarily with the Pandas library. The code for annual growth rates and quarterly growth rates is included in the attached “Data Processing” code file, under Subheading 1.3 and Subheading 2.3, respectively, “Calculation of Growth Rates,” within Section I, “GDP and Final Expenditure Components.”

A-III-5. The Calculation of Implicit Price Deflators

The objective of this operation is to calculate the implicit price deflators for GDP and the final expenditure components, as well as to measure their evolution. This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework provides the definition of the implicit price deflator and some measures of inflation. On the other hand, the Application to the Data Category offers an overview of the operation's implementation.

A-III-5.1. Theoretical Framework

The annual implicit price deflator for component i is defined using annual values at current prices and chain-linked Laspeyres volume indices in the following way.

$$\forall y \geq 0, \qquad P_i^{b\to y} = \frac{\frac{C_i^y}{C_i^b}}{Q_i^{b\to y}}$$

We show in Annex F-V-1.2.a that the above price index is equal to the annual chain-linked Paasche price index, with reference year b, for component i, $PP_i^{b\to y}$.

$$\forall y \geq 0, \qquad PP_i^{b\to y} = \frac{\frac{C_i^y}{C_i^b}}{Q_i^{b\to y}} = \frac{C_i^y}{K_i^{b\to y}} \tag{25}$$

Similarly, the quarterly implicit price deflator for component i is defined using quarterly values at current prices and chain-linked Laspeyres volume indices in the following way.

$$\forall y \geq 0, \forall 1 \leq s \leq 4, \qquad P_i^{b\to(s,y)} = \frac{\frac{C_i^{(s,y)}}{\frac{1}{4} * \sum_{s=1}^{4} C_i^{(s,b)}}}{Q_i^{b\to(s,y)}}$$

We show in Annex F-V-1.2.b that the above price index is equal to the quarterly chain-linked, with annual overlap, Paasche price index, with reference year b, for component i, $PP_i^{b\to(s,y)}$.

$$\forall y \geq 0, \forall 1 \leq s \leq 4, \qquad PP_i^{b\to(s,y)} = \frac{\dfrac{C_i^{(s,y)}}{\frac{1}{4} * \sum_{s=1}^{4} C_i^{(s,b)}}}{Q_i^{b\to(s,y)}} = \frac{C_i^{(s,y)}}{K_i^{b\to(s,y)}} \tag{26}$$

The evolution of the implicit price deflators can be measured by calculating their growth rates, which are referred to as inflation rates, as described in Sub-subsubsection A-II-2.2, "Measures of Change."

A-III-5.2. Application to the Data Category

We calculated annual and quarterly implicit price deflators for GDP and the final expenditure aggregates and measured their inflation.

a) Categories of Implicit Price Deflators

For annual series, we computed both unadjusted implicit price deflators, which are based on unadjusted volume indices and values at current prices, and calendar adjusted implicit price deflators, which are based on calendar adjusted volume indices and unadjusted values at current prices (the justification for the use of annual unadjusted values at current prices is provided in A-III-3.2.1, in the second paragraph of Item b). The computations were based on the direct application of Equation (25).

For quarterly series, we computed unadjusted implicit price deflators, which are based on unadjusted volume indices and values at current prices, seasonally adjusted implicit price deflators, which are based on seasonally adjusted volume indices and values at current prices, and seasonally and calendar adjusted implicit price deflators, which are based on seasonally and calendar adjusted volume indices and seasonally adjusted values at current prices (the justification for the use of quarterly seasonally adjusted values at current prices is the same as the justification for the use of annual unadjusted values at current prices provided in A-III-3.2.1, in the second paragraph of Item b). The computations were based on Equation (26), where, using the temporal consistency of values at current prices (see Annex G-I-2), we replaced $\sum_{s=1}^{4} C_i^{(s,b)}$ by the corresponding annual value C_i^b.

b) Inflation Rates

For annual implicit price deflators, we calculated Y-on-Y inflation rates. On the other hand, for quarterly implicit price deflators, we calculated both Q-on-Q and Y-o-Y inflation rates. However, we keep in mind that Q-on-Q inflation rates are more suitable for seasonally and calendar adjusted prices, and that Y-o-Y inflation rates are useful when assessing changes in unadjusted prices.

c) Temporal Coverage

Based on the temporal coverage of the value at current prices and volume index series, the implicit price deflator series have the following temporal coverage:

- Annual implicit price deflators cover the years 1998-2023.
- Quarterly implicit price deflators cover the period from (Q1,1998) to (Q2,2024), except for the implicit price deflators of the aggregates identified in Sub-subsection A-III-3, "The Identification of Additional Final Expenditure Aggregates," for which the temporal coverage is from (Q1,1999) to (Q2,2024).

Based on the temporal coverage of the implicit price deflators, the inflation rates have the following temporal coverage:

- Y-on-Y inflation rates cover the years from 1999 to 2023.
- Q-on-Q inflation rates cover the period from (Q2,1998) to (Q2,2024), except for the inflation rates based on the implicit price deflators of the aggregates identified in Sub-subsection A-III-3, "The Identification of Additional Final Expenditure Aggregates," for which the starting period is (Q2,1999).

- Y-o-Y inflation rates cover the period from (Q1,1999) to (Q2,2024), except for the inflation rates based on the implicit price deflators of the aggregates identified in Sub-subsection A-III-3, "The Identification of Additional Final Expenditure Aggregates," for which the starting period is (Q1,2000).

d) Dataset Management and Code Implementation

The calculation of annual implicit price deflators and their Y-on-Y inflation rates resulted in the formation of two datasets with a two-level column index. One dataset is for unadjusted series and the other for calendar adjusted series. The first level divides each dataset into individual sub-datasets for GDP and each final expenditure component. Within each sub-dataset, the second level separates between the implicit price deflator series and the corresponding Y-on-Y inflation rate series. The new datasets were added to the annual parent dataset under the names "Price Deflator" and "Price Deflator Cal Adjusted", respectively.

The calculation of quarterly implicit price deflators and their inflation rates resulted in the formation of one dataset with a three-level column index. The first level separates the dataset into three sub-datasets based on adjustment: unadjusted, seasonally adjusted, or seasonally and calendar adjusted. The second level divides each of the sub-datasets into individual sub-datasets for GDP and each final expenditure component. Finally, the third level distinguishes between the implicit price deflator series, the Q-on-Q inflation rate series, and the Y-o-Y inflation rate series. The new dataset was added to the quarterly parent dataset under the name "Price Deflator".

The computation of implicit price deflators and inflation rates as well as the integration of the resulting datasets into the parent datasets were implemented using Python, primarily with the Pandas library. The code for annual and quarterly series is included in the attached "Data Processing" code file, under Subheading 1.4 and Subheading 2.4, respectively, “Calculation of Implicit Price Deflators,” within Section I, “GDP and Final Expenditure Components.”

A-III-6. The Calculation of Additive Contributions to Growth

The objective of this operation is to measure the contributions of the relevant final expenditure components to the growth of the following aggregates: GDP, Final Domestic Consumption Expenditure, and Final Domestic Demand Expenditure. For this purpose, the growth rates of the aggregates' chain-linked Laspeyres volume measures in monetary terms and values at current prices are decomposed into additive contributions from the relevant components. While decomposing growth rates of values at current prices is relatively straightforward, decomposing growth rates of chain-linked Laspeyres volume measures in monetary terms is more involved due to the lack of additivity in chain-linked measures.

This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework presents the expressions for additive contributions to growth, with references to Annex F, where they are derived. The Application to the Data Category highlights key aspects of the operation's implementation.

A-III-6.1. Theoretical Framework

We assume that the aggregate A and the final expenditure components $\{i\}_{1\leq i\leq m}$ are distinct and linked through the following additive relationships.

$$\forall t, \quad \begin{cases} C_A^t = \sum_{i=1}^{m} (-1)^{f(i)} * C_i^t \\ K_A^{y-1\rightarrow t} = \sum_{i=1}^{m} (-1)^{f(i)} * K_i^{y-1\rightarrow t} \end{cases} \qquad (27)$$

Where $t = y$ or $t = (s, y)$, and thus $y - 1$ is the previous year. f is a function which assigns to each element of $\{i\}_{1\leq i\leq m}$ a value of 0 or 1.

Our assumption includes the case where $\{i\}_{1 \le i \le m}$ are disjoint final expenditure components whose aggregate is A (in which case $f = 0$). We used this more relaxed assumption to cover the calculation of additive contributions to the growth of aggregates obtained by both summing and netting out components, such as GDP.

We want to calculate the additive contributions of the components $\{i\}_{1 \le i \le m}$ to the growth in aggregate A's values at current prices and chain-linked Laspeyres volume measures in monetary terms (referred to as chain-linked Laspeyres volume measures in this sub-subsubsection).

A-III-6.1.1. Annual Series

We suppose that for the aggregate A and each one of the components $\{i\}_{1 \le i \le m}$, we have annual series of values at current prices, chain-linked Laspeyres volume measures with reference year b, and their corresponding implicit price deflators, each covering the years 0 to y_{end}. Furthermore, we suppose that the elements of aggregate A's series are positive.

a) Contributions to the Growth Rate of Values at Current Prices

Based on our assumption (1st equation in System (27)) and the result established in Item a, "Contributions to Growth," of Sub-subsubsection A-II-2.3, "Contributions to Change: The Case of Additivity," the contributions of component i to the annual growth rate of aggregate A's values at current prices are given by the bellow expression.

$$\forall 1 \le y \le y_{end}, \qquad (-1)^{f(i)} * \frac{C_i^{y} - C_i^{y-1}}{C_A^{y}} \tag{28}$$

b) Contributions to the Growth Rate of Chain-Linked Laspeyres Volume Measures

The contributions of component i to the annual growth rate of aggregate A's chain-linked Laspeyres volume measure with reference year b are given by the below expression (see Annex F-VI-1).

$$\forall 1 \le y \le y_{end}, \qquad (-1)^{f(i)} * \frac{K_i^{b\to y} - K_i^{b\to y-1}}{K_A^{b\to y-1}} * \frac{PP_i^{b\to y-1}}{PP_A^{b\to y-1}} \tag{29}$$

A-III-6.1.2. Quarterly Series

We suppose that for the aggregate A and each one of the components $\{i\}_{1\le i\le m}$, we have quarterly series of values at current prices, chain-linked Laspeyres volume measures with reference year b, and their corresponding implicit price deflators, covering the period from the first quarter of year 0 to the last quarter of year y_{end}. Furthermore, we suppose that the elements of aggregate A's series are positive.

a) Contributions to the Growth Rate of Values at Current Prices

Based on our assumption (1st equation of System (27)) and the result established in Item a, "Contributions to Growth," of Sub-subsubsection A-II-2.3, "Contributions to Change: The Case of Additivity," the contributions of component i to the Q-on-Q and Y-o-Y growth rate of aggregate A's values at current prices are given by the bellow expressions.

$$\begin{cases} \forall t \in T_1, & (-1)^{f(i)} * \dfrac{C_i^t - C_i^{t-1}}{C_A^{t-1}} & for\ Q - on - Q\ Growth \\ \forall t \in T_2, & (-1)^{f(i)} * \dfrac{C_i^t - C_i^{t-4}}{C_A^{t-4}} & for\ Y - o - Y\ Growth \end{cases} \tag{30}$$

Where T_1 and T_2 represent the periods from $(Q2,0)$ to $(Q4, y_{end})$ and from $(Q1,1)$ to $(Q4, y_{end})$, respectively. t represents a quarter, $t-1$ is the preceding quarter, and $t-4$ is the same quarter from the previous year.

b) Contributions to the Quarter-on-Quarter Growth Rate of Chain-Linked Laspeyres Volume Measures

The contributions of component i to the Q-on-Q growth rate of aggregate A's chain-linked Laspeyres volume measure with reference year b are given by the below expressions (see Annex F-VI-2.1).

- If $s = 2,3,4$, then $\forall 1 \le y \le y_{end}$,

$$(-1)^{f(i)} * \frac{K_i^{b\to(s,y)} - K_i^{b\to(s-1,y)}}{K_A^{b\to(s-1,y)}} * \frac{PG_i^{b\to y-1}}{PG_A^{b\to y-1}} \tag{31}$$

If $s = 1$, then $\forall 2 \leq y \leq y_{end}$,

$$(-1)^{f(i)} * \left(\frac{K_i^{b\to(1,y)} - K_i^{b\to(4,y-1)}}{K_A^{b\to(4,y-1)}} * \frac{PG_i^{b\to y-1}}{PG_A^{b\to y-1}} + \left(\frac{K_i^{b\to(4,y-1)}}{K_A^{b\to(4,y-1)}} - \frac{\sum_{s=1}^{4} K_i^{b\to(s,y-1)}}{\sum_{s=1}^{4} K_A^{b\to(s,y-1)}} \right) * \left(\frac{PG_i^{b\to y-1}}{PG_A^{b\to y-1}} - \frac{PG_i^{b\to y-2}}{PG_A^{b\to y-2}} \right) \right) \tag{32}$$

Where $PG_i^{b\to y}$ refers to the annual chain-linked Paasche price index with reference year b that is based on temporally aggregating quarterly values (see Item b of Annex F-V-1.1). When quarterly values at current prices and chain-linked Laspeyres volume indices are temporally consistent with their annual counterparts, then $PG_i^{b\to y} = PP_i^{b\to y}$ (see the conclusion of the proof in Item b of Annex F-V-2).

c) Contributions to the Year-over-Year Growth Rate of Chain-Linked Laspeyres Volume Measures

The contributions of component i to the Y-o-Y growth rate of aggregate A's chain-linked Laspeyres volume measure with reference year b are given by the below expression (see Annex F-VI-2.2).

$\forall 2 \leq y \leq y_{end}, \forall 1 \leq s \leq 4$,

$$(-1)^{f(i)} * \left(\frac{K_i^{b\to(s,y)} - K_i^{b\to(s,y-1)}}{K_A^{b\to(s,y-1)}} * \frac{PG_i^{b\to y-1}}{PG_A^{b\to y-1}} + \left(\frac{K_i^{b\to(s,y-1)}}{K_A^{b\to(s,y-1)}} - \frac{\sum_{s=1}^{4} K_i^{b\to(s,y-1)}}{\sum_{s=1}^{4} K_A^{b\to(s,y-1)}} \right) * \left(\frac{PG_i^{b\to y-1}}{PG_A^{b\to y-1}} - \frac{PG_i^{b\to y-2}}{PG_A^{b\to y-2}} \right) \right) \tag{33}$$

Where $PG_i^{b\to y}$ is as defined in previous Item b.

A-III-6.2. Application to the Data Category

We used the earlier theoretical framework to calculate additive contributions to the growth of the following aggregates $\{A_x\}_{1\leq x\leq 3}$ from the respective sets of final expenditure components $\{S_x\}_{1\leq x\leq 3}$:

- Final Domestic Consumption Expenditure A_1: The set of contributing components S_1 includes Final Consumption Expenditure of Households, Final Consumption Expenditure of NPISH, Final Private Consumption Expenditure, and Government Final Consumption Expenditure.
- Final Domestic Demand Expenditure A_2: The set of contributing components S_2 includes the elements of S_1, Final Domestic Consumption Expenditure, and Gross Fixed Capital Formation.
- GDP A_3: The set of contributing components S_3 includes the standard final expenditure components (defined in Glossary) as well as the final expenditure components identified in Sub-subsection A-III-3.

For Final Domestic Consumption Expenditure and Final Domestic Demand Expenditure, the contributing components can be regrouped into sets of disjoint subcomponents for which the equations of System (27) hold with $f = 0$. On the other hand, the same equations hold by definition for GDP based on the expenditure approach, with the value of the function f being equal to 1 for Imports of goods and services and to 0 for all the other components.

a) Categories of Contributions

For annual series, we calculated Year-on-Year unadjusted contributions to the nominal and volume (based on the chain-linked Laspeyres volume measure) growth of aggregates $\{A_x\}_{1\leq x\leq 3}$. Meanwhile, for quarterly series, we computed Quarter-on-Quarter and Year-over-Year contributions to the growth of aggregates $\{A_x\}_{1\leq x\leq 3}$. The contributions are either unadjusted or seasonally and calendar adjusted for volume series (chain-linked Laspeyres volume measures), and unadjusted or seasonally adjusted for nominal series.

b) Key Practical Aspects

The calculation of the contributions to the nominal growth of aggregates $\{A_x\}_{1\le x\le 3}$ is a direct application of formulas (28) and (30) to the already available series of values at current prices. On the other hand, before applying formulas (29), (31), (32), and (33) to calculate the contributions to the volume growth of aggregates $\{A_x\}_{1\le x\le 3}$, we derived the required series—Laspeyres volume measures in monetary terms and Paasche price indices (implicit price deflators) for each aggregate and component. For unadjusted annual series, the volume measures in monetary terms are obtained by multiplying the chain-linked Laspeyres volume index series by the corresponding unadjusted values at current prices in the reference year 2009. Meanwhile, the Paasche price indices were already computed in a previous operation; see A-III-5, "The Calculation of Implicit Price Deflators." The requirements for quarterly series are more demanding. First, quarterly, respectively annual (based on temporally aggregated quarterly series), volume measures were computed by multiplying the quarterly chain-linked Laspeyres volume index series, respectively their annual average series, by the corresponding rescaled annual unadjusted values at current prices, respectively the annual unadjusted values at current prices, in the reference year 2009 (the use of annual unadjusted values at current prices is justified by their temporal consistency property and the assumption of insignificant calendar effects). To compute the annual chain-linked Paasche price indices (implicit price deflators) based on temporally aggregated quarterly series, we divided the annual unadjusted values at current prices (the previous justification for using annual unadjusted values at current prices applies here as well) by the corresponding annual volume measures in monetary terms that are based on temporally aggregated quarterly series (this is an application of Equation (25) to the temporally aggregated quarterly series). The reason we did not use the annual unadjusted and calendar adjusted implicit price deflators derived in A-III-5 is due to the temporal inconsistencies in some volume measure series, such as Government Final Consumption Expenditure (see Annex G-I-1).

The contribution of the component Change in Stocks to the volume growth of GDP is derived as a residual—the difference between the growth rate of GDP and the aggregate contribution of the components other than Change in Stocks. This is because chain-linked volume measures are not defined for Change in Stocks (see IMF 2017, 8.90 in *Quarterly National Accounts Manual*)

c) Temporal Coverage

Based on the temporal coverage of the value at current prices and volume index series, the contribution to growth series have the following temporal coverage:

- Annual contributions to growth cover the years 1999-2023.
- Quarterly contributions to Q-on-Q nominal growth cover the period from (Q2,1998) to (Q2,2024).
- Quarterly contributions to Y-o-Y nominal growth cover the period from (Q1,1999) to (Q2,2024)
- Quarterly contributions to Q-on-Q volume growth cover the period from (Q2,1999) to (Q2,2024)
- Quarterly contributions to Y-o-Y volume growth cover the period from (Q1,2000) to (Q2,2024)

d) Dataset Management and Code Implementation

The calculation of annual contributions to the growth of aggregates $\{A_x\}_{1\le x\le 3}$ (defined in the introduction to A-III-6.2) resulted in the formation of three datasets, one dataset per aggregate A_x, with a two-level column index. The first level divides each dataset into two sub-datasets, with one corresponding to the contributions to volume growth and the other to the contributions to nominal growth. Within each sub-dataset, the second level separates between the growth rate of aggregate A_x and its decomposition into additive contributions per component $i \in S_x$ ($\{S_x\}_{1 \leq x \leq 3}$ is defined in the introduction to A-III-6.2). The new datasets were added to the annual parent dataset under the names “Contribution to Domestic Consumption Growth,” “Contribution to Domestic Demand Growth,” and “Contribution to GDP Growth.”

The calculation of quarterly contributions to the growth of aggregates $\{A_x\}_{1 \leq x \leq 3}$ resulted in the formation of six datasets—two datasets per aggregate A_x, one for Quarter-on-Quarter growth and the other for Year-over-Year growth. Each dataset has a three-level column index. The first level separates between two sub-datasets, with one corresponding to the contributions to volume growth and the other to the contributions to nominal growth. The second level divides each of the sub-datasets into two sub-datasets based on adjustment: unadjusted or seasonally and calendar adjusted for volume series, and unadjusted or seasonally adjusted for nominal series. Finally, the third level separates between the growth rate of aggregate A_x and its decomposition into additive contributions per component $i \in S_x$. The new datasets were added to the quarterly parent dataset under the names “Contribution to Domestic Consumption Growth QonQ,” “Contribution to Domestic Consumption Growth YoY,” “Contribution to Domestic Demand Growth QonQ,” “Contribution to Domestic Demand Growth YoY,” “Contribution to GDP Growth QonQ,” and “Contribution to GDP Growth YoY.”

The computation of the additive contributions to growth as well as the integration of the resulting datasets into the parent datasets were implemented using Python, primarily with the Pandas library. The code for annual series and quarterly series is included in the attached “Data Processing” code file, under Subheading 1.6, “Calculation of Additive Contributions to Growth,” and Subheadings 2.6, “Calculation of Additive Contributions to QonQ Growth” and

2.7, “Calculation of Additive Contributions to YoY Growth,” respectively, within Section I, “GDP and Final Expenditure Components.”

A-III-7. The Calculation of Shares

The objective of this operation is to compute shares for the relevant final expenditure components in the following aggregates: GDP, Final Domestic Consumption Expenditure, and Final Domestic Demand Expenditure. For this purpose, nominal shares are calculated and decomposed into volume and price components. This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework provides a framework for the calculation of nominal shares as well as their decomposition into volume and price components. Meanwhile, the Application to the Data Category offers an overview of the operation’s implementation.

A-III-7.1. Theoretical Framework

We assume that the aggregate A and the final expenditure components $\{i\}_{1\leq i\leq m}$ are distinct and linked through the following additive relationship. Furthermore, we suppose that the elements of aggregate A’s nominal and volume series are positive.

$$\forall t, \qquad C_A^t = \sum_{i=1}^{m} (-1)^{f(i)} * C_i^t \tag{34}$$

Where $t = y$ or $t = (s, y)$. f is a function which assigns to each element of $\{i\}_{1\leq i\leq m}$ a value of 0 or 1.

Our assumption includes the case where $\{i\}_{1\leq i\leq m}$ are disjoint final expenditure components whose aggregate is A (in which case $f = 0$). We used this more relaxed assumption to cover the calculation of shares in aggregates obtained by both summing and netting out components, such as GDP.

a) Nominal Shares

Based on our assumption (Equation (34)) and the result established in Sub-subsubsection A-II-2.4, "Shares in Aggregates," the nominal shares of component i in aggregate A are given by the bellow expression.

$$\forall t, \qquad (-1)^{f(i)} * \frac{C_i^t}{C_A^t} \tag{35}$$

b) The Decomposition of Nominal Shares into Volume and Price Components

The nominal shares derived in the previous item can be decomposed as follows. For a period $t = y$ or $t = (s, y)$ such that $y \geq 0$, the values at current prices C_i^t and C_A^t can be decomposed using the definition of the Paasche price index (implicit price deflator) presented in Equations (25) and (26), in Sub-subsection A-III-5, "The Calculation of Implicit Price Deflators." Therefore, the nominal share for component i in period t can be expressed as follows.

$$(-1)^{f(i)} * \frac{C_i^t}{C_A^t} = (-1)^{f(i)} * \frac{K_i^{b\to \mathrm{t}}}{K_A^{b\to \mathrm{t}}} * \frac{PP_i^{b\to t}}{PP_A^{b\to t}}$$

We refer to $(-1)^{f(i)} * \frac{K_i^{b\to \mathrm{t}}}{K_A^{b\to \mathrm{t}}}$ as the volume component of the nominal share and to $\frac{PP_i^{b\to t}}{PP_A^{b\to t}}$ as the price component. While it is tempting to interpret the volume component as a volume share, we must remember that the sum across components $\{i\}_{1\leq i\leq m}$ does not amount to one due to the non-additivity of chain-linked volume measures. Nevertheless, we consider the volume component to represent a relative volume, which measures the importance of component i's volume relative to the volume of aggregate A. On the other hand, the price component represents a relative price, which compares the evolution of the price level of component i to that of aggregate A.

A-III-7.2. Application to the Data Category

We used the previous theoretical framework to calculate nominal shares and derive their volume and price components for the sets of final expenditure components $\{S_x\}_{1\leq x\leq 3}$ in the respective aggregates $\{A_x\}_{1\leq x\leq 3}$. The sets $\{A_x\}_{1\leq x\leq 3}$ and $\{S_x\}_{1\leq x\leq 3}$ are as defined in the introduction to A-III-6.2, in Sub-subsection A-III-6, "The Calculation of Additive Contributions to Growth."

a) Categories of Nominal Shares and Their Decomposition

For annual series, we computed shares based on unadjusted values at current prices. For quarterly series, we calculated shares based on each of seasonally adjusted and unadjusted values at current prices. The calculation of the shares' volume and price components is straightforward. However, we should mention that, maintaining the assumption of insignificant calendar effects in the values at current prices (see the second paragraph of Item b in A-III-3.2.1), we chose to use the seasonally and calendar adjusted volume measures and price indices to decompose quarterly seasonally adjusted nominal shares. Meanwhile, for annual unadjusted nominal shares, we utilized unadjusted volume measures and prices indices. The shares of Change in Stocks in GDP could not be decomposed due to the unavailability of chain-linked volume measure and price index series for this component.

b) Temporal Coverage

Based on the temporal coverage of the value at current prices series, the nominal share series have the following temporal coverage:

- Annual nominal shares cover the years 1998–2023.
- Quarterly nominal shares cover the period from (Q1,1998) to (Q2,2024).

Based on the temporal coverage of the chain-linked Laspeyres volume measure and Paasche price index series, the nominal shares' volume and price components (except for the shares of Change in Stocks in GDP) have the following temporal coverage:

- Annual components cover the years 1998-2023.
- Quarterly components for shares not involving the aggregates identified in Sub-subsection A-III-3, either as components (element of S_x) or as aggregate (A_x), cover the period from (Q1,1998) to (Q2,2024).
- Quarterly components for shares involving the aggregates identified in Sub-subsection A-III-3, either as components (element of S_x) or aggregate (A_x), cover the period from (Q1,1999) to (Q2,2024).

c) Dataset Management and Code Implementation

The calculation of annual nominal shares in aggregates $\{A_x\}_{1 \le x \le 3}$ (defined in the introduction to A-III-6.2) as well as their decomposition into volume and price components resulted in the creation of three new datasets, one dataset per aggregate A_x, with a two-level column index. The first level separates each dataset into three sub-datasets: one corresponding to nominal shares, the second to the volume component, and the last to the price component. The second level divides each sub-dataset into individual series for each final expenditure component included in S_x. The new datasets were added to the annual parent dataset under the names "Share in GDP," "Share in Domestic Consumption," and "Share in Domestic Demand."

The calculation of quarterly nominal shares in aggregates $\{A_x\}_{1 \le x \le 3}$ as well as their decomposition into volume and price components resulted in the formation of three new datasets, one dataset per aggregate A_x, with a three-level column index. The first level separates each dataset into three sub-datasets: one corresponding to nominal shares, the second to the volume component, and the last to the price component. The second level divides each of the sub-datasets into two sub-datasets based on adjustment: unadjusted or seasonally and calendar adjusted (seasonally adjusted, for nominal shares). Finally, the third level divides each sub-dataset into individual series for each final expenditure component included in S_x. The new datasets were added to the quarterly parent dataset under the names "Share in GDP," "Share in Domestic Consumption," and "Share in Domestic Demand."

The computation of the nominal shares, their decomposition into volume and price components, and the integration of the resulting datasets into the parent datasets were implemented using Python, primarily with the Pandas library. The code for annual series and quarterly series is included in the attached "Data Processing" code file, under Subheadings 1.7 and 2.8, respectively, "Calculation of Shares in Main Aggregates," within Section I, "GDP and Final Expenditure Components."

A-III-8. The Decomposition into Trend and Cyclical Components

The objective of this operation is to decompose volume and price series of GDP and the final expenditure components into trend and cyclical components. One method among others to achieve this is the Hodrick-Prescott (HP) filter (Hodrick and Prescott 1997). This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework provides a brief description of the HP filter, while the Application to the Data Category outlines the decomposition process using the filter.

A-III-8.1. Theoretical Framework

The HP filter decomposes a time series y^t into the sum of a trend component and a cyclical component, $y^t = \tau^t + c^t$. The trend component τ^t is found by solving the below minimization problem.

$$\min_{\tau^t} \sum_t (y^t - \tau^t)^2 + \lambda \sum_t [(\tau^{t+1} - \tau^t) - (\tau^t - \tau^{t-1})]^2$$

Where λ is a positive parameter that penalizes variation in the second difference of the trend component τ_t. As the parameter lambda (λ) increases, the solution converges towards a linear trend.

A-III-8.2. Application to the Data Category

We used the HP filter to decompose annual calendar adjusted and quarterly seasonally and calendar adjusted volume and price series for GDP and the available final expenditure components. We set the parameter lambda at a value of 6.25 for annual series (Ravn and Uhlig 2002) and a value of 1600 for quarterly series (Hodrick and Prescott 1997). The filter was then applied to the natural logarithmic transform of the series. The resulting trend components were then transformed back using the exponential function. For example, for the final expenditure component i's volume index series, Q_i^t, the application of the HP filter resulted in the decomposition: $\log(Q_i^t) = {\tau'}_i^t + {c'}_i^t$. Where ${\tau'}_i^t$ is the trend component. The trend component for the original series Q_i^t was derived, and based on that, the cyclical component c_i^t and the percentage deviation from the trend ρ_i^t were calculated as shown in System (36).

$$\begin{cases} \tau_i^t = \exp\left({\tau'}_i^t\right) \\ c_i^t = \dfrac{Q_i^t}{\tau_i^t} \\ \rho_i^t = c_i^t - 1 = \dfrac{Q_i^t - \tau_i^t}{\tau_i^t} \end{cases} \tag{36}$$

Finally, we calculated Y-on-Y, Q-on-Q, and Y-o-Y growth rates for the trend component as described in Sub-subsubsection A-II-2.2, "Measures of Change."

a) Temporal Coverage

Based on the temporal coverage of the volume index and price index series, the decomposition series—trend, cyclical component, and percentage deviation from the trend—have the following temporal coverage:

- Annual series cover the years 1998–2023.

- Quarterly series not involving the aggregates identified in Sub-subsection A-III-3 cover the period from (Q1,1998) to (Q2,2024).
- Quarterly series involving the aggregates identified in Sub-subsection A-III-3 cover the period from (Q1,1999) to (Q2,2024).

Due to the above temporal coverage of the decomposition series, the trend component's growth rate series cover the following time periods:

- Y-on-Y growth rates cover the years 1999-2023.
- For series not involving the aggregates identified in Sub-subsection A-III-3, Q-on-Q and Y-o-Y growth rates cover the periods from (Q2,1998) to (Q2,2024) and from (Q1,1999) to (Q2,2024), respectively.
- For series involving the aggregates identified in Sub-subsection A-III-3, Q-on-Q and Y-o-Y growth rates cover the periods from (Q2,1999) to (Q2,2024) and from (Q1,2000) to (Q2,2024), respectively.

b) Dataset Management and Code Implementation

The decomposition process and the calculation of trend components' growth rates for annual volume and price series resulted in the formation of two datasets with a two-level columns index: one dataset for volume series and the other for price series. The first column level divides each dataset into individual sub-datasets for GDP and each final expenditure component. The second level separates the decomposition series—trend, cyclical component, and percentage deviation from the trend—and the trend components' Y-on-Y growth rate series into individual series. The new datasets were added to the annual parent dataset under the names "Volume Decomp" and "Price Decomp".

The decomposition process and the calculation of trend components' growth rates for quarterly volume and price series resulted in the formation of two datasets with a three-level columns index: one dataset for volume series and the other for price series. The first column level, which indicates that the decomposition is based on adjusted series, is primarily added to maintain a structure for the datasets that is compatible with the structure of the quarterly parent dataset. The second level divides each dataset into individual sub-datasets for GDP and each final expenditure component. Finally, the third level separates the decomposition series—trend, cyclical component, and percentage deviation from the trend—and the trend components' Q-on-Q and Y-o-Y growth rate series into individual series. The new datasets were added to the quarterly parent dataset under the names "Volume Decomp" and "Price Decomp".

The decomposition process, the calculation of trend components' growth rates, and the integration of the resulting datasets into the parent datasets were implemented using Python, primarily with the Pandas library. The code for annual series and quarterly series is included in the attached "Data Processing" code file, under Subheadings 1.5 and 2.5, respectively, "Volume and Price Decomposition into Trend and Cyclical Components," within Section I, "GDP and Final Expenditure Components."

A-III-9. Price Deflation

The objective of this operation is to derive volume estimates for GDP per capita and the Change in Stocks component. This sub-subsection is divided into two sub-subsubsections: Theoretical Framework and Application to the Data Category. The Theoretical Framework presents the theory underlying the deflation process, while the Application to the Data Category outlines key aspects of the operation's execution.

A-III-9.1. Theoretical Framework

The theory underlying the deflation process is sufficiently covered in Sub-subsection A-III-5, "The Calculation of Implicit Price Deflators" and Sub-subsubsection A-II-2.5, "Price Deflation." While Sub-subsection A-III-5 addresses the case where the deflation is based on a price index directly related to the nominal series, Sub-subsubsection A-II-2.5 discusses deflation using a general price index. An illustrative example of the difference between these approaches is the deflation of GDP per capita and the Change in Stocks component using the chain-linked Paasche price index for GDP.

a) GDP per Capita

If the values at current prices and the volume measures at previous year prices for GDP per capita are derived from those of GDP as follows,

$$\forall k, \qquad \begin{cases} C_{GPC}^{k} = \dfrac{C_{GDP}^{k}}{POP^{k}} \\ K_{GPC}^{k-1\to k} = \dfrac{K_{GDP}^{k-1\to k}}{POP^{k}} \end{cases}$$

Where the subscript GPC refers to GDP per capita and POP^{y} is the population size.

Then, we show in Annex F-VII that,

$$\forall y \geq 0, \qquad PP_{GPC}^{\mathrm{b}\to y} = PP_{GDP}^{\mathrm{b}\to y}$$

Therefore, based on the definition of the implicit price deflator in Sub-subsection A-III-5 (Equation (25)), deflating GDP per capita using the chain-linked Paasche price index for GDP, yields the chain-linked Laspeyres volume measure for GDP per capita.

$$\forall y \geq 0, \qquad Q_{GPC}^{b\to y} = \frac{C_{GPC}^{y}}{C_{GPC}^{b}} * \frac{1}{PP_{GDP}^{\mathrm{b}\to y}} \tag{37}$$

b) Change in Stocks

The Change in Stocks component can be deflated using the chain-linked Paasche price index for GDP as described in Sub-subsubsection A-II-2.5. However, the resulting volume estimate is not a chain-linked Laspeyres volume measure, as the chain-linked Paasche price index for GDP is a general price index that is not linked by an accounting relationship, such as those presented in Sub-subsection A-III-5, “The Calculation of Implicit Price Deflators,” to the value at current prices of the Change in Stocks component. Instead, the chain-linked Paasche price index for GDP serves here as an adjustment for changes in purchasing power. Consequently, the resulting volume estimates reflect values at constant purchasing power, which we refer to as Change in Stocks in real terms (see United Nations et al. 2009 [SNA], 2.66).

A-III-9.2. Application to the Data Category

We used the annual and quarterly chain-linked Paasche price index, with reference year 2009, for GDP series to deflate annual value at current prices series for GDP per Capita, as well as both annual and quarterly value at current prices series for Change in Stocks. The resulting series are the annual chain-linked Laspeyres volume index, with reference year 2009, for GDP per Capita series and the quarterly and annual Change in Stocks index in real terms with reference year 2009 series.

a) Categories of Volume series

For annual series, we computed unadjusted volume estimates, which are based on unadjusted values at current prices and implicit price deflators, and calendar adjusted volume estimates, which are based on calendar adjusted implicit price deflators and unadjusted values at current prices (the justification for the use of annual unadjusted values at current prices is provided in the second paragraph of Item b, in A-III-3.2.1).

For quarterly series, we computed unadjusted volume estimates, which are based on unadjusted values at current prices and implicit price deflators, and seasonally and calendar adjusted volume estimates, which are based on seasonally and calendar adjusted implicit price deflators and seasonally adjusted values at current prices (the justification for the use of quarterly seasonally adjusted values at current prices is the same as the justification for the use of annual unadjusted values at current prices).

b) Temporal Coverage

Based on the temporal coverage of the chain-linked Paasche price index for GDP series and the series of values at current prices for GDP per capita and Change in Stocks, the deflated series have the following temporal coverage:

- Annual series cover the years 1998–2023.
- Quarterly series cover the period from (Q1,1998) to (Q2,2024).

c) Dataset Management and Code Implementation

The calculation of annual volume estimates for GDP per capita and the Change in Stocks component resulted in the creation of new annual volume series. For GDP per capita, the volume series as well as the corresponding Y-on-Y growth rates (growth rates are discussed in A-III-4) were added to the GDP per capita dataset, which in turn was added to the annual parent dataset under the name “GDP per Capita.” For the Change in Stocks component, the volume series and the corresponding Y-on-Y growth rate series were directly inserted into the annual parent dataset, within the relevant sub-datasets—the “Accounts” sub-dataset for volume series and the “Growth Rate” sub-dataset for growth rate series.

The calculation of quarterly volume estimates for the Change in Stocks component resulted in the creation of new quarterly volume series. The volume series and the corresponding Q-on-Q and Y-o-Y growth rate series (growth rates are discussed in A-III-4) were directly inserted into the quarterly parent dataset, within the relevant sub-datasets—the “Accounts” sub-dataset for volume series and the “Growth Rate QonQ” or “Growth Rate YoY” sub-datasets for growth rate series.

The computation of volume estimates for GDP per capita and the Change in Stocks component as well as the insertion of the related series into the parent datasets were implemented using Python, primarily with the Pandas library. The code is included in the attached “Data Processing” code file, under Subheading 1.8, “GDP per Capita,” for GDP per capita series, and Subheadings 1.9 and 2.9, “Other,” respectively, for annual and quarterly Change in Stocks series, within Section I, “GDP and Final Expenditure Components.”

A-III-10. Final Data Category Composition and Visualizations

A summary of the final structure and constituents of the GDP and Final Expenditure Components data category is provided in Annex H-I.

Multiple graphs were created for the GDP and Final Expenditure Components data category using the Plotly library in Python. The corresponding code is included in the attached “Data Visualization” code file, in Section I, “GDP and Final Expenditure Components.”