Homework 1 – Estimating a Logit Model

Jin Miao

Empirical Models in Marketing

February 6, 2018

Question 1 Model Construction. The indirect utility for each individual household i to choose brand (alternative) j in a particular purchase trip (time) t is

$$U_{ijt} = V_{ijt} + \epsilon_{ijt}$$

where

$$V_{ijt} = \alpha_{ij} + \beta_1 price_{ijt}$$

ϵ_{ijt} are IID Extreme Value Distributed. The Cumulative Distrbution Function of ϵ_{ijt} is

$$F(\epsilon_{ijt}) = e^{-e^{-\epsilon_{ijt}}}$$

Consequently,

$$P(choice_{ij} = 1) = P(V_{i1t} + \epsilon_{i1t} > V_{i2t} + \epsilon_{i2t}) = P(\epsilon_{i1t} - \epsilon_{i2t} > V_{i2t} - V_{i1t})$$

where

$$V_{i2t} - V_{i1t} = \alpha_{i2} + \beta_1 price_{i2t} - (\alpha_{i1} + \beta_1 price_{i1t}) = (\alpha_{i2} - \alpha_{i1}) + \beta_1(price_{i2t} - price_{i1t})$$

The propbability that each individual household i chooses brand (alternative) j in a particular purchase trip (time) t is

$$P(choice_{it} = j) = \frac{e^{V_{ijt}}}{\sum_{k=1}^{2} e^{V_{ikt}}}$$

More specifically, α_{i2} is set as the benchmark level such that

$$P(choice_{it} = 1) = \frac{e^{V_{i1t}}}{e^{V_{i1t}} + e^{V_{i2t}}} = \frac{e^{\alpha_{i1}+\beta_1 price_{i1t}}}{e^{\alpha_{i1}+\beta_1 price_{i1t}} + e^{\beta_1 price_{i2t}}}$$

$$P(choice_{it} = 2) = \frac{e^{V_{i2t}}}{e^{V_{i1t}} + e^{V_{i2t}}} = \frac{e^{\beta_1 price_{i2t}}}{e^{\alpha_{i1}+\beta_1 price_{i1t}} + e^{\beta_1 price_{i2t}}}$$

Question 2 Likelihood. The complete data likelihood is

$$L = \prod_{i=1}^{300} \prod_{t=1}^{10} \prod_{j=1}^{2} P(choice_{it} = j)^{Y_{ijt}}$$

1

where Y_{ijt} is an indicator variable such that

$$Y_{ijt} = \begin{cases} 0, & \text{if brand j was chosen by household i at time t} \\ 1, & \text{if brand j was not chosen by household i at time t} \end{cases}$$

Question 3 Model Estimation. I estimate this model with R. The output is shown as follows:

Table 1: The Coefficients of Multinomial Logit Model

Estimate	SE	T-value
0.040	0.039	1.034
-0.777	0.043	-18.210

Based on these parameters, $-2 \log L = 3771.227$, $AIC = 3775.227$, $BIC = 3787.24$.

Question 4 Result Interpretation. No brand is preferred. There is no significant difference between consumers' inherent preferences for brands because the estimate of α is not significant from 0. Consumers are sensitive to the changes in yogurt prices. When one brand increases its price, the probability for consumers to choose this brand is significantly lower.

Since $\hat{\beta} = -0.777 < 0$, Own-Price Elasticity is $E_{ip_{jt}} = \hat{\beta} p_{ijt}(1 - P(choice_{it} = j)) < 0$ and Cross-Price Elasticity is $E_{ip_{kt}} = -\hat{\beta} p_{ikt}(1 - P(choice_{it} = k)) > 0$. This implies that when one brand increases its price, its market share will go down and its competitor's market share will increase.