Introduction to Probability Models in Marketing

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* Slide thanks to Professor Eva Ascarza

The Data

ID	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
100001	1	0	0	0	0	0	0	?	?	?	?	?
100002	1	0	0	0	0	0	0	?	?	?	?	?
100003	1	0	0	0	0	0	0	?	?	?	?	?
100004	1	0	1	0	1	1	1	?	?	?	?	?
100005	1	0	1	1	1	0	1	?	?	?	?	?
100006	1	1	1	1	0	1	0	?	?	?	?	?
100007	1	1	0	1	0	1	0	?	?	?	?	?
100008	1	1	1	1	1	1	1	?	?	?	?	?
100009	1	1	1	1	1	1	0	?	?	?	?	?
100010	1	1	1	1	1	0	0	?	?	?	?	?
...												
111102	1	1	1	1	1	1	1	?	?	?	?	?
111103	1	0	1	1	0	1	1	?	?	?	?	?
111104	1	0	0	0	0	0	0	?	?	?	?	?

Modelling the Transaction Stream

- Let random variable $X(n)$ denote the # transactions across n consecutive transaction opportunities.
- The customer buys at any given transaction opportunity with probability p:

$$P(X(n) = x \mid p) = \binom{n}{x} p^x (1-p)^{n-x} .$$

- Purchase probabilities (p) are distributed across the population according to a beta distribution:

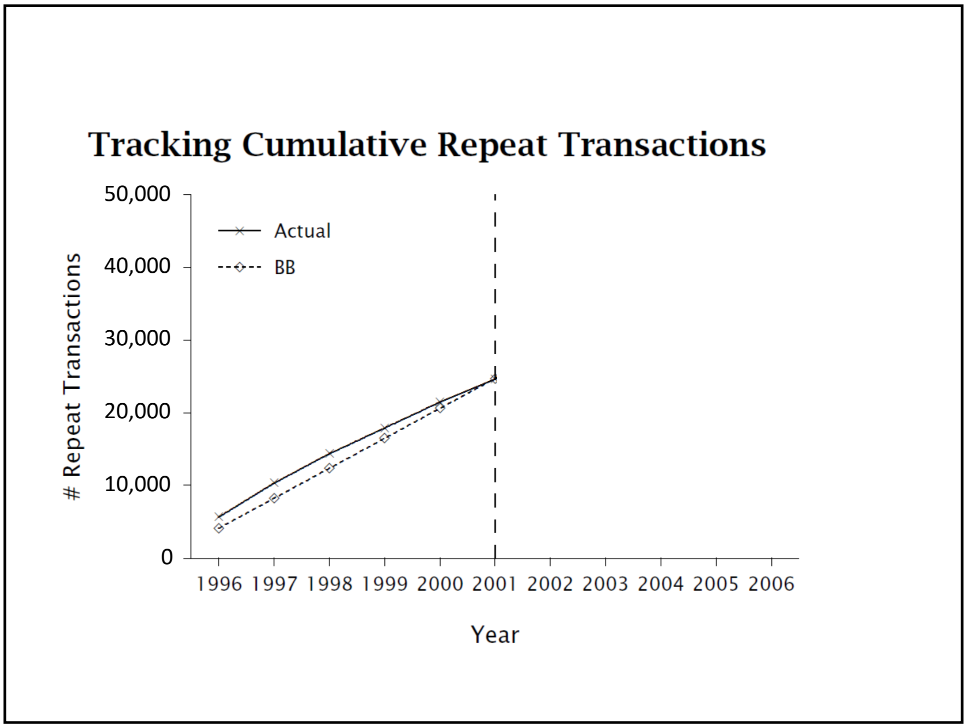
$$g(p \mid \alpha, \beta) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)} .$$

Modelling the Transaction Stream

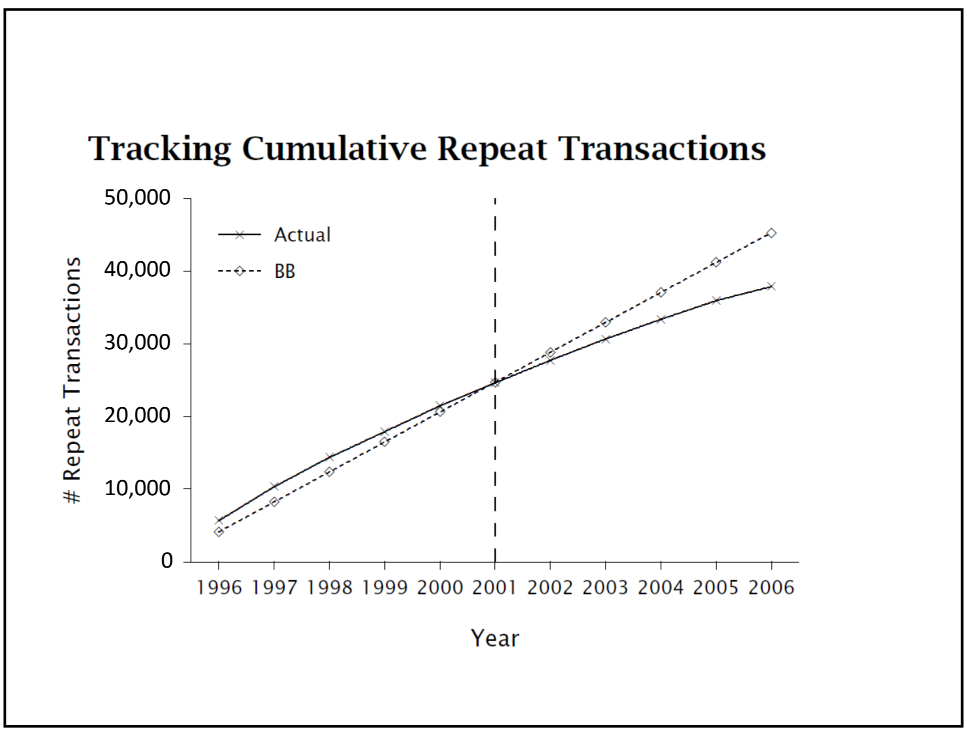
The distribution of transactions for a randomly-chosen individual is given by:

$$\begin{aligned} P(X(n) = x \mid \alpha, \beta) &= \int_0^1 P(X(n) = x \mid p) g(p \mid \alpha, \beta)\, dp \\ &= \binom{n}{x} \frac{B(\alpha + x, \beta + n - x)}{B(\alpha, \beta)} . \end{aligned}$$

which is the beta-binomial (BB) distribution.



Tracking Cumulative Repeat Transactions
50,000
40,000
30,000
20,000
10,000
0
Repeat Transactions
Actual
BB
1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006
Year



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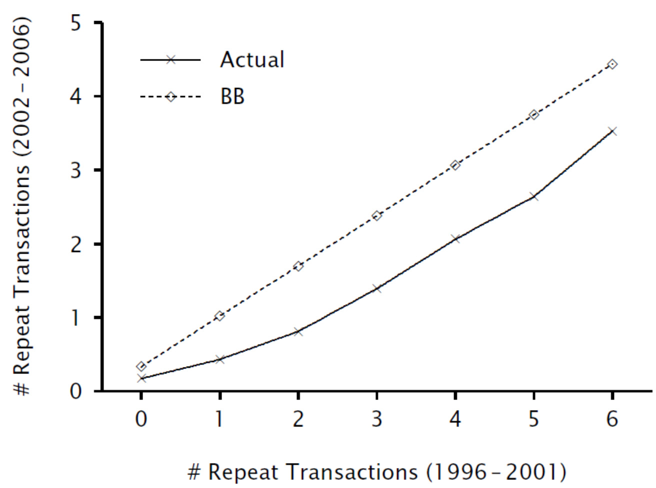
Conditional Expectations



The Data

ID	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sara	1	0	0	0	0	0	0	?	?	?	?	?
100002	1	0	0	0	0	0	0	?	?	?	?	?
100003	1	0	0	0	0	0	0	?	?	?	?	?
Mary	1	0	1	0	1	1	1	?	?	?	?	?
100005	1	0	1	1	1	0	1	?	?	?	?	?
100006	1	1	1	1	0	1	0	?	?	?	?	?
100007	1	1	0	1	0	1	0	?	?	?	?	?
Bob	1	1	1	1	1	1	1	?	?	?	?	?
100009	1	1	1	1	1	1	0	?	?	?	?	?
Pete	1	1	1	1	1	0	0	?	?	?	?	?
...												
111102	1	1	1	1	1	1	1	?	?	?	?	?
Chris	1	0	1	1	0	1	1	?	?	?	?	?
111104	1	0	0	0	0	0	0	?	?	?	?	?

Model Development

A customer's relationship with a firm has two phases: he is "alive" (A) for some period of time, then becomes permanently inactive ("dies", D).

- While "alive", the customer buys at any given transaction opportunity with probability p:

$$P(Y_t = 1 \mid p, \text{alive at } t) = p$$

- A "living" customer becomes inactive at the beginning of a transaction opportunity with probability θ

$$\Rightarrow P(\text{alive at } t \mid \theta) = P(\underbrace{AA\ldots A}_{t} \mid \theta) = (1-\theta)^t$$

Model Development

Consider the following transaction pattern:

1996	1997	1998	1999	2000	2001
1	0	0	1	0	0

- The customer must have been alive in 1999 (and therefore in 1996–1998)
- Three scenarios give rise to no purchasing in 2000 and 2001

1996	1997	1998	1999	2000	2001
A	A	A	A	D	D
A	A	A	A	A	D
A	A	A	A	A	A

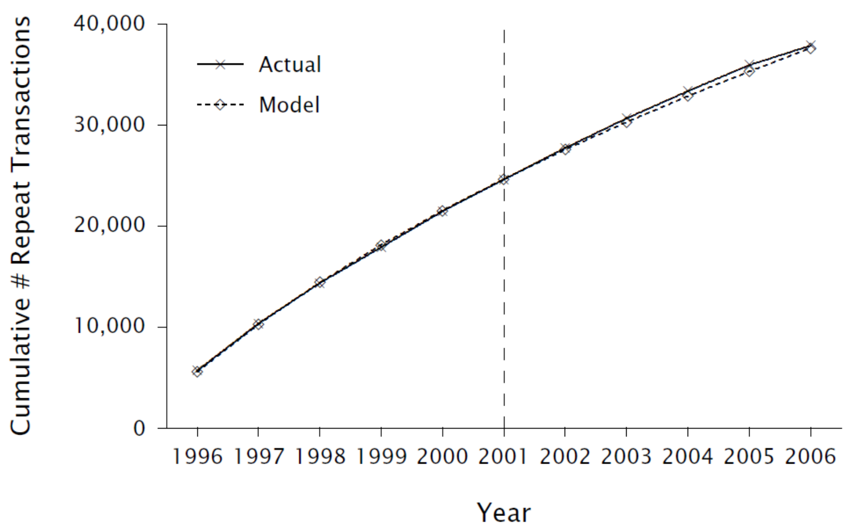
Model Development

Removing the conditioning on the latent traits p and θ,

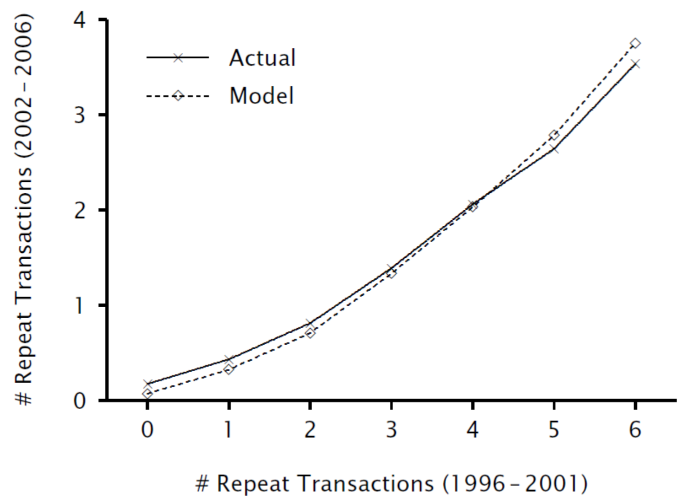
$$\begin{aligned}
L(\alpha,\beta,\gamma,\delta \mid x, t_x, n) &= \int_0^1 \int_0^1 L(p,\theta \mid x, t_x, n) g(p \mid \alpha,\beta) g(\theta \mid \gamma,\delta)\, dp\, d\theta \\
&= \frac{B(\alpha+x,\beta+n-x)}{B(\alpha,\beta)} \frac{B(\gamma,\delta+n)}{B(\gamma,\delta)} \\
&\quad + \sum_{i=0}^{n-t_x-1} \frac{B(\alpha+x,\beta+t_x-x+i)}{B(\alpha,\beta)} \frac{B(\gamma+1,\delta+t_x+i)}{B(\gamma,\delta)}
\end{aligned}$$

... which is (relatively) easy to code-up in Excel.

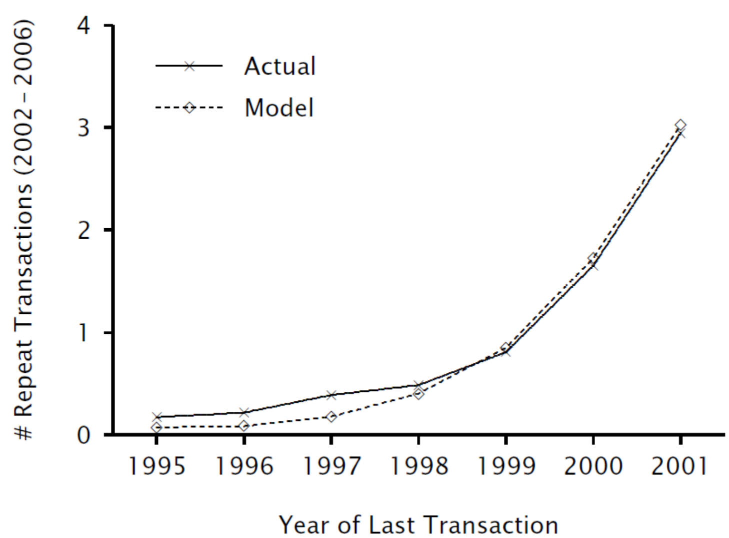
Tracking Cumulative Repeat Donations



Conditional Expectations by Frequency



Conditional Expectations by Recency



Expected # Transactions in 2002 – 2006

Bob (R:6, F:6) is expected to donate 3.75 times out of 5 opportunities between 2002 and 2006, surprisingly low given his 100% donation rate

Mary (R:6, F:4) and **Pete** (R:4, F:4) made the same number of transactions, but their recency is different. **Mary** is expected to donate almost 5 times more than **Pete**.

Sarah, with very low R and F, is lapsed and/or a very light donor

# Rpt Trans (1996-2001)	Year of Last Transaction						
	1995	1996	1997	1998	1999	2000	2001
0	0.07						
1		0.09	0.31	0.59	0.84	1.02	1.15
2			0.12	0.54	1.06	1.44	1.67
3				0.22	1.03	1.80	2.19
4					0.58	2.03	2.71
5						1.81	3.23
6							3.75

Probability of being alive in 2002

Bob (R:6, F:6) is expected to donate 3.75 times out of 5 opportunities between 2002 and 2006, surprisingly low given his 100% donation rate

Mary (R:6, F:4) and **Pete** (R:4, F:4) made the same number of transactions, but their recency is different. **Mary** is expected to donate almost 5 times more than **Pete**.

Sarah, with very low R and F, is lapsed and/or a very light donor (hard to tell)

# Rpt Trans (1996-2001)	Year of Last Transaction						
	1995	1996	1997	1998	1999	2000	2001
0	0.11						
1		0.07	0.25	0.48	0.68	0.83	0.93
2			0.07	0.30	0.59	0.80	0.93
3				0.10	0.44	0.77	0.93
4					0.20	0.52	0.93
5						1.81	0.93
6							0.93

Summary of BGBB model

- Noncontractual setting
  - Transactions: Binary process
  - "Death" process: Discrete time
- Data Generating Process
- Conditional expectations
- P(alive)

Fader, Peter S., Bruce G.S. Hardie, and Jen Shang (2010), "Customer-Base Analysis in a Discrete-Time Noncontractual Setting," *Marketing Science*, 29 (6), 1086-1108.

Different Business Settings

Noncontractual setting with "infinite" time periods

- Transactions
  - Binomial → Poisson
  - BB → NBD

- "Death" process
  - Geometric → Exponential
  - BG → EG (Pareto of second kind)

Different Business Settings

What if "death" was observed?

What if individual transaction behavior changes over time?

Remember…

- Behavior predicts behavior (Duh!)
- Data generating process – likelihood
- Do you observe "death"?
- Validate your model!
- You know how to:
  - Build BB → BG/BB → BG/NBD
- How would you…
  - Add other behaviors?
  - Incorporate dynamics?