Homework 5 – Probabilistic Models

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Empirical Models in Marketing

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Question 1 Model Construction. I analyzed Vespur with the beta-geometric/beta-Bernoulli (BG/BB) model to capture the rider behavior in discrete-time, non-contractual settings. The heterogeneity in the alive consumers' purchase probabilities follows a beta distribution with probability distribution function

$$f(p|\alpha,\beta) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)}$$

The heterogeneity of dropout rates of living customers follows a beta distribution with probability distribution function

$$f(\theta|\gamma,\delta) = \frac{\theta^{\gamma-1}(1-\theta)^{\delta-1}}{B(\gamma,\delta)}$$

Given that the transaction probability p and the dropout probability θ vary independently across customers, the likelihood of a randomly chosen customer with purchase history (x, t_x, n) is

$$L(\alpha,\beta,\gamma,\delta|x,t_x,n) = \frac{B(\alpha+x,\beta+n-x)}{B(\alpha,\beta)}\frac{B(\gamma,\delta+n)}{B(\gamma,\delta)} + \sum_{i=0}^{n-t_x-1}\frac{B(\alpha+x,\beta+t_x-x+i)}{B(\alpha,\beta)}\frac{B(\gamma+1,\delta+t_x+i)}{B(\gamma,\delta)}$$

From the perspective of Consumer Lifetime Value, the present value of the expected future transaction stream with purchase history (x, t_x, n) is defined as Discounted Expected Residual Transactions (DERT)

$$DERT(d|\alpha,\beta,\gamma,\delta,x,t_x,n) = \frac{B(\alpha+x+1,\beta+n-x)}{B(\alpha,\beta)}\frac{B(\gamma,\delta+n+1)}{B(\gamma,\delta)(1+d)} \times \frac{{}_2F_1(1,\delta+n+1;\gamma+\delta+n+1;1/(1+d))}{L(\alpha,\beta,\gamma,\delta|x,t_x,n)}$$

where ${}_2F_1()$ is the Gaussian hypergeometric function.

Question 2 Parameter Estimation and Interpretstion. Given the Vespur rider history data, the estimated α is 1.295, the estimated β is 0.837, the estimated γ is 0.463, the estimated δ is 2.656.

1. Based on BG/BB model, the company should expect 1541 transactions for the first week of October.
2. For a consumer who used the service in every week (during both August and September), the probability that she will make a transaction in the first week of October is 86.7%.
3. For a consumer who made 4 transactions in August (every week in August) but none in September, the probability that she will make a transaction in the first week of October is 3.3%.
4. From the perspective of consumer lifetime value, the ANNUAL discount rate is 10% and the following 1000 periods (weeks) are used for calculating DERT.
    a For a user who only had the first free ride, the number of discounted expected residual transactions is 2.075.
    b For a user who had two rides, with the last ride occurring in the last week of August, the number of discounted expected residual transactions is 13.419.
    c For a user who user who had a ride on every single week during both August and September, the number of discounted expected residual transactions is 99.008.