Mathematical Models in Marketing
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Fast Polyhedral Adaptive Conjoint Estimation
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Key contributions

- A new adaptive fast question design and estimation algorithms for partial profile conjoint analysis based on polyhedron conceptualization of product design space.
    - Very nice geometric intuition. Fast solution methods based on linear programming / interior point methods.
- Comprehensive testing of the method and comparison to other approaches based on (a) Monte-Carlo simulation test, as well as (b) internal and external validity tests using real-life experiments.

Methods overview

Polyhedron construction

The paper proposes a new method which conceptualizes a product design space as a polyhedron and pairwise comparison questions as hyperplanes that restrict us to the portion of the polyhedron.

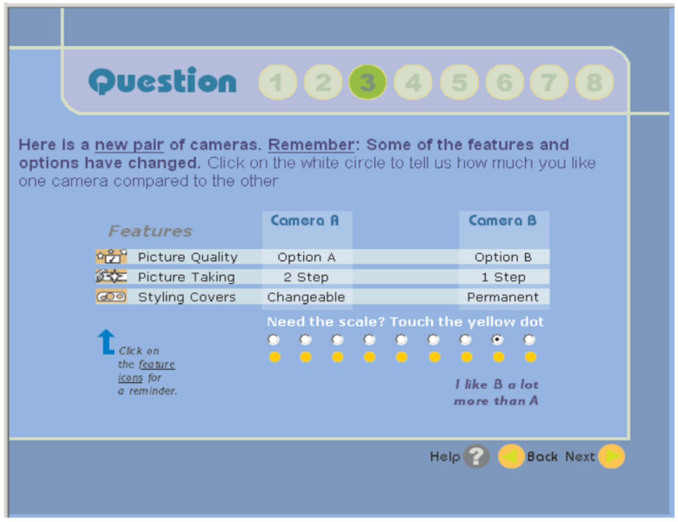
Polyhedron is situated in the multi-dimensional space, where each dimension corresponds to a product attribute (assuming binary levels of attributes), and a point coordinate (u_1, u_2, … , u_p) in a polyhedron indicates a specific combination of additive utilities (partworths) associated with a level of each attribute.

The surface of the polyhedron is defined by a constraint of the type $u_1 + u_2 + \cdots + u_p \leq 100$.

When an attribute has more than two (ordinal) levels (e.g. three), it is split into several (two) binary attributes, and an additional constrain is imposed ($u_m \leq u_h$) to ensure that the medium level has lower path worth than the high attribute.

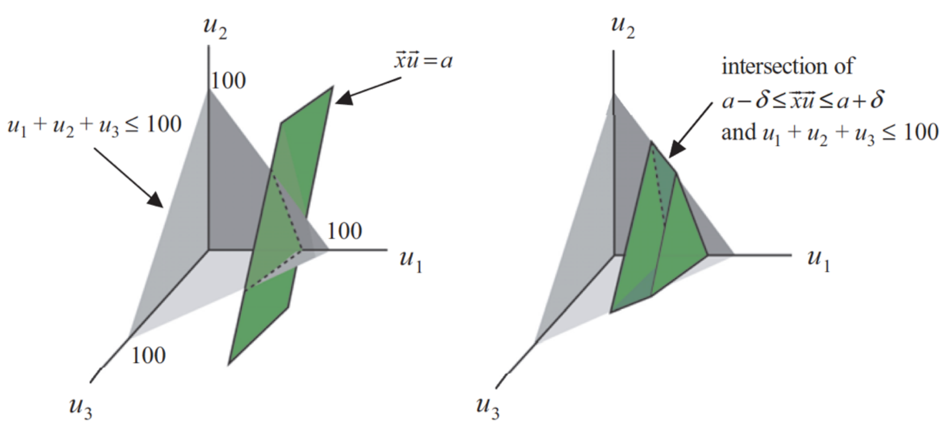
The goal is to find a point in the polyhedron that, based on the questions answered, captures the most desired levels of each attribute to the user.

__Answer to a question as a hyperplane__



The procedure is based on metric-paired-comparison questions, where user indicates which partial product profile z_l or z_r he prefers, and by how much a. Each profile is a binary vector with a length p - number of attributes (assuming binary attributes). We define $x = z_l - z_r$ as a vector describing differences between profiles. Let u be the vector of path-worth coordinates in the polyhedron, then $x^T u = a$ defines a hyperplane through a polyhedron, reducing the dimensionality of the polyhedron by 1, and constraining us to a lower set of the combinations of desirable pathworths.

Figure 2 Respondent's Answers Affect the Feasible Region



(a) (b)

Notes. (a) Metric rating without error. (b) Metric rating with error.

Because the user may err, instead of strict equality $x^T u = a$, we can use inequality constraint $a - \delta \leq x^T u \leq a + \delta$ where $\delta > 0$ is some error.

Procedure

Now that we have the above conceptualization, it is easy to come up with the optimization procedure – with two desiderata:

- Based on the reduced set of points in the polyhedron we want to find the best-guess balanced design.

Finding the Analytic Center

The analytic center is the point in P that maximizes the geometric mean of the distances from the point to the faces of P. We find the analytic center by solving (OPT1).

$$\begin{aligned} \max \quad & \sum_{j=1}^{p+r'} \ln(u_j), \\ \text{subject to} \quad & X\vec{u} = \vec{a}, \qquad \vec{u} > \vec{0}. \end{aligned} \qquad \text{(OPT1)}$$

- At each step, we want to ask such a profile comparison question that will give us the smallest set possible of attribute combinations.

This is achieved by finding the tightest ellipse around the current feasible polyhedron (the analytic center design is placed approximately at the center of the ellipse).

Here we need to realize that the smallest set of such attributes will be likely found when the answered-question hyperplane intersects the longest axis of the ellipsoid. This is equivalent to finding x (profile difference vector) that is parallel to the longest axis of the ellipsoid. It can be obtained after solving another simple optimization problem – see appendix.

After some number of questions there will be no more feasible space in the ellipsoid. Then we can either increase the error in the hyperplane inequality, or ask questions at random.

Experiments

Experiments are discussed in detail in the paper – see table 8. **First,** the analytic center algorithm and others such as Hierarchical Bayes are tested using Monte Carlo simulation experiments under different levels of consumer heterogeneity and response errors. The comparative analysis of different algorithms is given. Polyhedron method works best with small number of questions. **Second,** methods are also tested in term of predictive ability of choices of real people and in the case of a real product launch. The findings are aligned with Monte Carlo results. The conjoint output was correlated with market launch outcome.

Future directions

- Can we avoid reversion to random question selection?
- Improve performance under presence of accurate self-explicated questions (e.g., directly rating feature importance).

Table 8 Detailed Summary of Findings

Feasibility

- The polyhedral question-design method can design questions in real time for a realistic number of parameters.
- The analytic center estimation heuristic yields real time partworth estimates that provide reasonable accuracy with little or no bias.

Monte Carlo Experiments

- Polyhedral question design shows the most promise for lower numbers of questions where it does well in all tested heterogeneity and response-error domains.
- Fixed question design remains best for larger numbers of questions, but poly/random does well for homogeneous populations and high response errors.
- AC estimation shows promise for heterogeneous populations and low response errors; HB performs well when the population is homogeneous and response errors are high.
- When SE data are available and relatively noisy:
  - Polyhedral question design continues to perform well.
  - Purebred estimation methods may be preferred.
- When SE data are available and relatively accurate:
  - Question design is relatively less important.
  - For homogeneous populations, HBSE is the best estimation method.
  - For heterogeneous populations, WHSE is the best estimation method.

External-Validity Experiment

- The field test domain had high heterogeneity and PC response errors, and moderately low SE response errors. The findings are consistent with the Monte Carlo results in this domain.
- Polyhedral question design shows promise irrespective of the availability of SE data. This is true for all tested estimation methods.
- When SE data are unavailable, analytic center estimation is a viable alternative to hierarchical Bayes estimation, especially when paired with polyhedral question design.
- When SE data are available, existing hybrid estimation methods (HBSE and WHSE) appear to outperform the ACSE hybrid.

Product Launch

- The laptop bags were launched to the market, but we must be cautious when evaluating predictive accuracy because there were many differences between the empirical experiment and the market launch. In addition, there were insufficient data for relative comparisons.
- With these caveats, the conjoint analyses correlate well with the marketplace outcome.