

THEORETICAL PORTION

PROBLEM #1.

- a. The max that can be drawn is \$200, while the minimum could be \$30. So for the interval of X, it represents $30 \leq X \leq 200$. This also satisfies that $f(x)$ is greater than zero for all x.

Total Win Amount	Bill Configuration (18 total)	PDF	CDF
\$30	\$10 \$10 \$10	0.01225	0.01980
\$40	\$10 \$10 \$20	0.01225	0.03960
	\$10 \$20 \$10		0.03960
	\$20 \$10 \$10		0.03960
\$50	\$10 \$20 \$20	0.00613	0.04950
	\$20 \$20 \$10		0.04950
\$60	\$20 \$20 \$20	0.00123	0.05149
	\$50 \$10	0.13072	0.26271
	\$10 \$50		0.26271
\$70	\$50 \$20	0.07843	0.38944
	\$20 \$50		0.38944
	\$10 \$10 \$50	0.03268	0.44224
\$80	\$10 \$20 \$50	0.02451	0.48185
	\$20 \$10 \$50		0.48185
\$90	\$20 \$20 \$50	0.00980	0.49769
\$100	\$50 \$50	0.18301	0.79340
\$110	\$100 \$10	0.03268	0.84620
	\$10 \$100		0.84620
\$120	\$10 \$10 \$100	0.00817	0.85941
	\$100 \$20	0.01961	0.89109
\$130	\$10 \$20 \$100	0.00613	0.90099
	\$20 \$10 \$100		0.90099
\$140	\$20 \$20 \$100	0.00245	0.90495
\$150	\$100 \$50	0.05229	0.98944
	\$50 \$100		0.98944
\$160	None		0.98944
\$170	None		0.98944
\$180	None		0.98944
\$190	None		0.98944
\$200	\$100 \$100	0.00654	1.00000
		0.61887	
	MEAN	55.22059	

- b. The mean of $X = E(X) = (30 \cdot 0.01225) + (40 \cdot 0.01225) + (50 \cdot 0.00613) + \dots + (200 \cdot 0.00654) =$
while the variance of $X =$

PROBLEM #2.

- a. $E(X + 3Y + 3) = E(X) + 3E(Y) + 3 = 2 + 3(4) + 3 = 17$
- b. $sd(X + 3Y + 3) = sd(X) + 3sd(Y) + 3 = 2 + 3(2) + 3 = 11$
- c. $E(3X - 2Y - 5) = 3E(X) - 2E(Y) - 5 = 3(2) - 2(4) - 5 = -7$
- d. $sd(3X - 2Y - 5) = 3sd(X) - 2sd(Y) - 5 = 3(2) - 2(2) - 5 = -3$

PROBLEM #3.

- a.

PROBLEM #4.

- a.

PROBLEM #5.

- a. $Cov(X, Y)$ Yes, the inversely related random variables can go below -1, even to negative infinity.
- b. Yes, if the covariance is negative then the correlation must also be negative
- c. $Cov(100X, Y)$ when $Cov(X, Y) = 0.3 \rightarrow 100 * Cov(X, Y) = 100 * 0.3 = 30$
- d. $Corr(100X, Y)$ would take the sign of the 100, which is +, so this equals +0.1
- e. $Cov(X, X) = E(X^2) - (E(X))^2 = Var(X)$
- f. $Corr(X, X) = Cov(X, X) / \sqrt{Var(X)} = Var(X) / Var(X) = 1$
- g. $Cov(100X, 10X) = 100 * 10 * Cov(X, X) = 1000Var(X)$
- h. $Corr(100X, 10X) = 1$, since we're only taking the sign of 100 and 10 (which is +)

COMPUTATIONAL PORTION

Code and graphs included with each part, not indexed.

PROBLEM #1 - COMPUTATIONAL

#1, part a.

code:

```
dgeom(7,.20)
```

output:

```
0.04194304
```

#1, part b.

code:

```
1 - pgeom(7,.20) - dgeom(7,.20)    #P<=7 for pgeom, then subtract what's at 7 (dgeom)
```

output:

```
0.1258291
```

#1, part c.

PROBLEM #2 - COMPUTATIONAL

#2, part a.

code:

```
fx=function(x){
  value=((x)/((x^2)+1))
  return(value)
}
integrate(fx,0,2)
resultsg=integrate(fx,0,2)
c=1/resultsg$value
c
```

```
gx=function(x){
  fx(x)*c
}
```

output:

```
0.804719 with absolute error < 3e-10
1.24267
```

#2, part b.

code:

```
Gx=function(x){
  value=integrate(gx,0,x)
  return(value)
}
```

```
calculate.Info=function(x){
  cat(gx(x), "\n")
  cat(Gx(x)$value)
}
```

```
calculate.Info(.5)
```

PROBLEM #3 - COMPUTATIONAL

#3, part a.

code:

```
par(mfrow=c(2,2))
```

```
# $\alpha=1$ ,  $\beta=1$ 
```

```
hist((rweibull(n=5000, 1, scale = 1)),freq=FALSE, col='lightblue',main="Weibull Distribution for  $\alpha=1$ ,  $\beta=1$ ", ylim=c(0,1), xlab = "Random Variable (x)")
curve(dweibull(x, 1, scale = 1), add=TRUE, col='red', lwd=1, text(3,.6,"Theoretical
Distribution", col='red'))
```

```
# $\alpha=3$ ,  $\beta=1$ 
```

```
hist((rweibull(n=5000, 3, scale = 1)),freq=FALSE, col='lightgreen',main="Weibull Distribution for  $\alpha=3$ ,  $\beta=1$ ", ylim=c(0,1.2),xlab = "Random Variable (x)")
curve(dweibull(x, 3, scale = 1), add=TRUE, col='red', lwd=1, text(1.7,.8,"Theoretical
Distribution", col='red'))
```

```
# $\alpha=1$ ,  $\beta=3$ 
```

```
hist((rweibull(n=5000, 1, scale = 3)),freq=FALSE, col='orange',main="Weibull Distribution for  $\alpha=1$ ,  $\beta=3$ ", ylim=c(0,.3),xlab = "Random Variable (x)")
curve(dweibull(x, 1, scale = 3), add=TRUE, col='red', lwd=1, text(10,.2,"Theoretical
Distribution", col='red'))
```

```
# $\alpha=3$ ,  $\beta=3$ 
```

```
hist((rweibull(n=5000, 3, scale = 3)),freq=FALSE, col='yellow',main="Weibull Distribution for  $\alpha=3$ ,  $\beta=3$ ", ylim=c(0,.5),xlab = "Random Variable (x)")
curve(dweibull(x, 3, scale = 3), add=TRUE, col='red', lwd=1, text(5,.3,"Theoretical
Distribution", col='red'))
```

output:

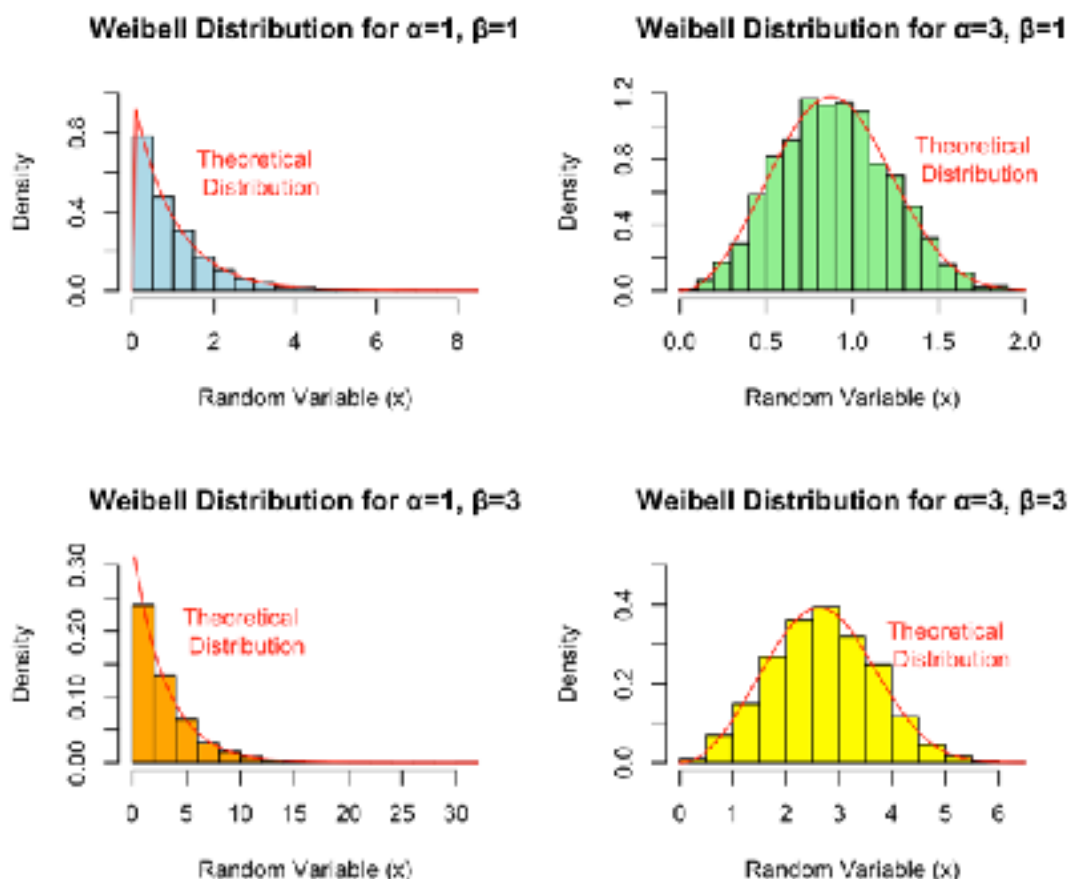
PROBLEM #4 - COMPUTATIONAL

#4, part

a.

code:

line 1



```
lightbulbs=rexp(7500,rate=.75)
mean(lightbulbs)
sd(lightbulbs)
```

output:

```
1.327306 #mean
1.343139 #standard deviation
```

theoretical:

$1/\lambda = 1/.75 = 1.333$ (repeating) or $4/3$

#4, part b.

code:

```
line 1 sum(lightbulbs<=2)/7500
```

output:

```
0.7792
```

theoretical:

#4, part c.

code:

```
line 1 sum(lightbulbs>2)-sum(lightbulbs>4))/7500
```

output:

```
0.1738667
```

Just over 17% of the light bulbs will fail in between 2 to 4 years. This is expected as some bulbs will last longer than the mean, though the mean is at 1.3 years. And with a standard deviation of 1.3, the upper end of that spectrum is 2.6 years. So to last up to 4 years is impressive.