HOMEWORK 4

Due Wednesday, April 5th, start of class

This homework covers Chapters 9.1-9.3, and 9.5-9.9. You should be working on your homework throughout these two weeks. If you can't solve some of the problems, please come to office hours. Email is fine only for very short questions.

THEORETICAL PORTION

The theoretical problems should be **neatly** numbered, written out, and solved. Do not turn in messy work.

1. In your own words, describe Type I and Type II error. Can they both be minimized? Which is worse?

2. (a) A random sample of 150 recent donations at a certain blood bank reveals that 72 were type A blood. Does this suggest that the actual percentage of type A donations differs from 39%, the percentage of the population having type A blood? Carry out a test of the appropriate hypotheses using a significance level of .01.

   (b) Would your conclusion have been different if a significance level of .05 had been used?

3. Under specific conditions, the drying time of a certain type of paint is normally distributed with a mean value of 55 minutes, and a standard deviation of 7 minutes. Use only the values given to you to solve this problem:

$$\Phi(0.385) = 0.65, \Phi(0.428) = 0.67, \Phi(0.71) = 0.762, \Phi(1.43) = 0.924, \Phi(1.92) = 0.973.$$

$$\Phi^{-1}(0.972) = 1.917, \Phi^{-1}(0.740) = 0.643, \Phi^{-1}(0.95) = 1.645$$

$$z_{0.0017} = 2.923, z_{0.016} = 2.14, z_{0.027} = 1.93.$$

   (a) You do an experiment, measuring the paint drying time for $n = 20$ samples. You measure an average drying time equal to 52 minutes, with a standard deviation equal to 7. Carry out a hypothesis test with $\alpha = 0.05$ to see if there is evidence that the average paint drying time is actually less than 55 minutes. Use your p-value to decide whether to accept or reject the null hypothesis.

   (b) What is the probability of a type I error in this experiment?

   (c) If the sample actually comes from a normal distribution with a mean drying time equal to 53 minutes (and a standard deviation of 7 minutes), what is the type II error of your test?

4. Let $X \sim P(\lambda)$. We want to test the null hypothesis that $\lambda = 15$, and the alternative hypothesis is that $\lambda > 15$. Our sample size for this test is equal to 1.

   (a) We will reject the null hypothesis if X is greater than a certain value c. What value should we choose for c such that the probability of a type I error is equal to 0.10?

   (b) If λ is actually equal to 25, what is the probability of making a type 2 error using the cut-off point from (a)?

5. Let $X_1, \ldots, X_n \sim i.i.d.Exp(\lambda)$. A company claims that the lifetime of their lightbulbs is longer than the lifetime of the leading brand, which is three years. You are going to test the hypothesis:

$$H_o : \mu = 5, H_a : \mu > 5.$$

   (a) Assume $n > 30$. We know from the CLT that $\bar{X} \sim \mathcal{N}\left(\mu, \sigma^2/n\right)$. Show that $n\bar{X} \sim \mathcal{N}(n\mu, n\sigma^2)$.

   (b) You are going to perform a hypothesis test using $\sum_i X_i$ instead of $\bar{X}$. What is your test statistic and decision rule for determining whether the null hypothesis should be rejected?

   (c) Perform the test of hypothesis with $\alpha = 0.10$ using your test procedure, with $n = 40, \sum_i X_i = 251.04$, and $s = 5.69$.

   (d) What is the p-value of your test? Provide an interpretation of this p-value.

   (e) Calculate the power of your test, assuming the true mean is equal to 6, 7, and 8.

6. **APPM 5570 only:** Refer back to the previous problem.

   (a) If $X_1, \ldots, X_n \sim i.i.d.Exp(\lambda)$, then $\sum X_i \sim Gamma(n, \lambda)$. Under the null hypothesis, write the equation for the cut-off point using the Gamma distribution instead of the normal distribution. Fix the probability of a Type I error at α.

(b) Using your answer from part(6a), derive an equation for calculating the probability of a Type II error of your test, assuming the true μ is equal to μ_a.

(c) Perform the test of hypothesis with $\alpha = 0.10$ using both test procedures, with $n = 40, \sum_i X_i = 251.04$, and $s = 5.69$. Do you come to the same conclusions using both your test from this problem and the test from the previous problem?

(d) What is the p-value for each of your tests?

(e) Calculate the power for both of your tests, assuming the true mean is equal to 6, 7, and 8. Is one test more powerful?

COMPUTATIONAL PORTION

The computational portion of your homework should be neatly done and include all graphs, code, and comments, labeled and in order based on the problem you are addressing. Do *not* put graphs in at the end, stick code in random locations, or do anything else that will make this homework difficult to read and grade. **LABELS ARE YOUR FRIEND, USE THEM.** If you turn in something that is messy or out of order, it will be returned to you with a zero. All computations should be done using R, which can be downloaded for free at https://cran.r-project.org/.

1. When sample sizes are small, our confidence intervals and test statistics require the normality assumption. It can be difficult to tell by just looking at a histogram whether the underlying distribution is normal, particularly when we do not have many observations. In this case, we must examine histograms as well as use logic and rationale to assess whether the normality assumption is valid.

   The data set "IsItNormal.txt" contains 6 columns of data, each with 25 observations. The data from each column were sampled as follows:

   - Column 1: 25 scores on the final for Calculus I.
   - Column 2: For 25 days we track the number of people who walk into a store (right as it opens) before someone buys something.
   - Column 3: From Feb 1-25, the number of people in the engineering building who walked to work that day.
   - Column 4: The amount of time (in minutes) it took 25 students to finish their answer for the first problem on a test.
   - Column 5: 25 error measurements (in mm) of a well-calibrated device.
   - Column 6: The amount of rainfall (in inches) for 25 days in September.

   Investigate the normality assumption of each data sample through histograms, boxplots, normality plots, and by using what you know about how the data was collected. For each column, make a conclusion on whether the data come from a normal distribution. If the assumption does not seem to hold, provide another reasonable distribution the data may come from. Be sure to include evidence and work to support your conclusions.

2. Automatic identification of the boundaries of significant structures within a medical image is an area of ongoing research. The paper "Automatic Segmentation of Medical Images Using Image Registration: Diagnostic and Simulation Applications" (*J. of Medical Engr. and Tech.*, 2005:53-63) discussed a new technique for such identification. A measure of the accuracy of the automatic region is the average linear displacement (ALD). ALD observations for a sample of 49 kidneys (units of pixel dimensions) are given in the data file labeled "HW4ALDdata.txt".

   (a) Plot a histogram of the ALD observations. Is it plausible that ALD is approximately normally distributed? Must normality be assumed prior to hypothesis about true average ALD? Explain.

   (b) The authors commented that in most cases the ALD is on the order of 1.0. Does the data provide strong evidence for concluding that the true average ALD under these circumstances is different than 1.0 at a 5% significance level? Carry out an appropriate test of hypotheses.

   (c) Calculate the p-value of the data under these hypotheses.

3. A sample of 12 radon detectors of a certain type was selected, and each was exposed to 100 pCi/L of radon. The resulting readings can be found in the "HW4Radondata.txt" file.

   (a) Plot a histogram of the data.

(b) Does this data suggest that the population mean reading under these condition differs from 100? State and test the appropriate hypothesis using $\alpha = 0.05$. What assumptions about the data did you make?

(c) Calculate the P-value of this data under these hypotheses.

4. You have developed a new type of superglue that you think is stronger than the current leading brand, which has a mean bond strength of 5 and a standard deviation of 4. You want to patent your new glue, however, you first need to actually test if your glue is better. One company is interested in funding this study, but first they want to know the following things:

   - How many samples you will need (i.e. n, your sample size).
   - The power of your test, based on n.

   Assume your glue's bond strength has a standard deviation of 4 (same as the leading brand).

   (a) Use simulations to calculate a 3-dimensional power curve for values of n ranging from 50 to 150 (in increments of 10) and for mean bond strengths of 5.2 to 6 (in increments of 0.2).

   (b) Plot your power curve using a 3-dimensional plot in R. (You may make your own, or use something that already exists.)