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CS682
Computer Vision
Quiz 5

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Q1) Derive an expression for epipolar matrix E for this system.

Solution:- In our given system, the two cameras are said to form a rectified pair as their camera coordinates system differ only by a translation of their origins.

It is given by $T = [t_x, t_y, 0]$ as the translation between the two points is only along x and y axes.

Now inorder to derive epipolar matrix E,

$$E = [T_x] R$$

$R = I$ where I is the identity matrix of size 3×3.

$$[T_x] = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

Therefore,

$$E = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

Q2.) Prove that the epipolar lines are all parallel to the direction of translation.

Solution:-

We need to prove that $T^T \cdot l' = 0$ for all epipolar line $l' = Ex$.

Let us consider the point x to be on the epipolar line. Let the point x be (u', v')

$$l' = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$l' = \begin{bmatrix} t_y \\ -t_x \\ -t_y u' + t_x v' \end{bmatrix}$$

Now we find $T^T \cdot l'$ by putting the l' obtained above

We know $T^T = [t_x \ t_y \ 0]$

$$T^T \cdot l' = [t_x \ t_y \ 0] \begin{bmatrix} t_y \\ -t_x \\ -t_y u' + t_x v' \end{bmatrix}$$

$$= t_x t_y - t_x t_y$$

$$= 0$$

Hence proved

Q3) Consider three images I1, I2, I3 that have been captured by a system of three cameras and suppose fundamental matrices F_{13} and F_{23} are known. Given a point x_1 in I_1 and a corresponding point x_2 in I_2, the corresponding point in x_3 in I_3 is uniquely determined by the fundamental matrices F_{13} and F_{23}.

Write an expression for x_3 in terms of x_1, x_2, F_{13} and F_{23}

Solution :-

F_{13} relates the points x_1 and x_3 of the images I_1 and I_3

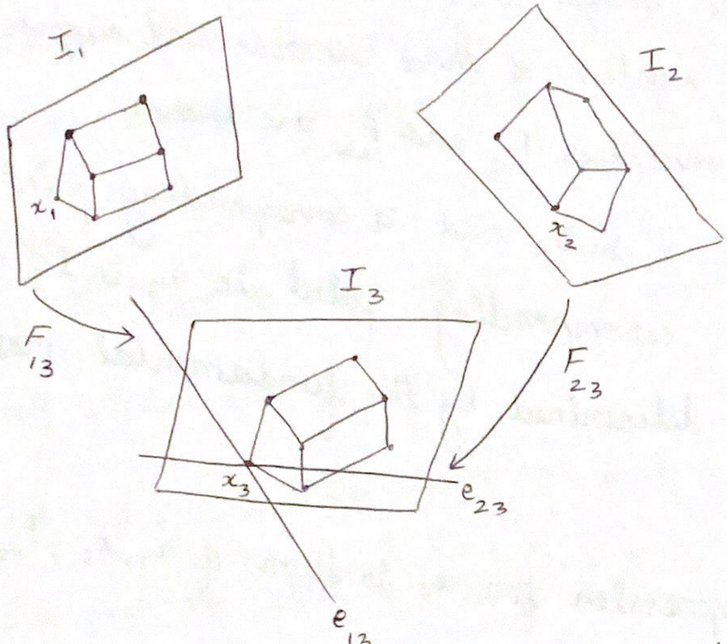
F_{23} relates the points x_2 and x_3 of the images I_2 and I_3

To get x_3,

We know that point x_3 matches point x_1 in the image I_1 and cosequently must lie on the epipolar line correspondi-ng to x_1. Since we know F_{13}, this epipolar line may be computed and is equal to $F_{13}x_1$.

By a similar argument, x_3 must lie on the epipolar line $F_{23}x_2$. Taking the intersection of epipolar lines gives

$$\boxed{x_3 = (F_{13}x_1) \times (F_{23}x_2)}$$

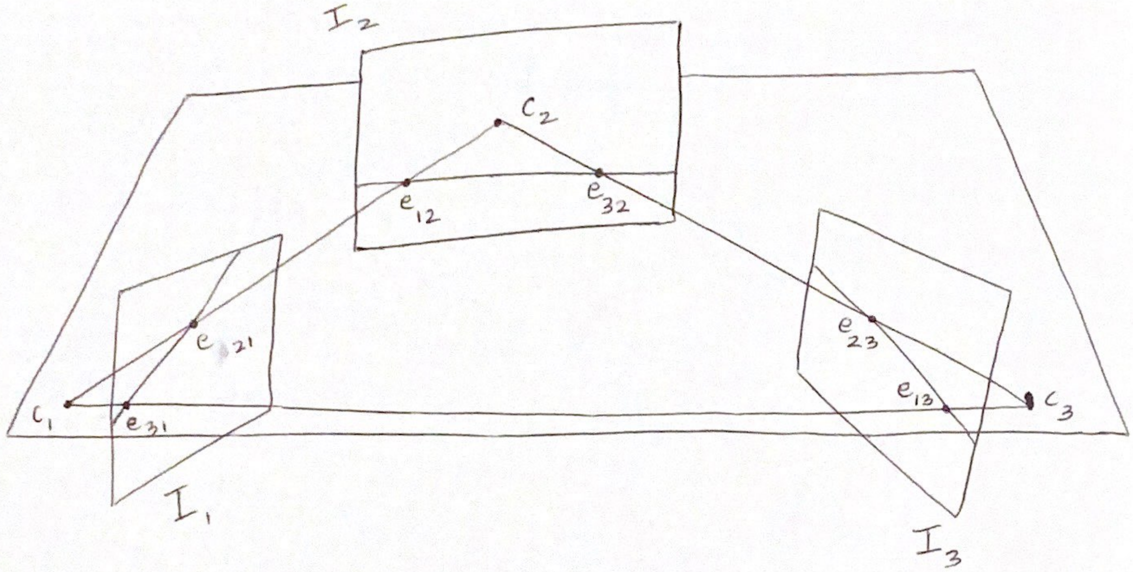


Point x_3 is computed as the intersection of epipolar lines passing through the two epipoles e_{13} and e_{23}. However if x_3 lies on the line through the two epipoles, then its position cannot be determined. Points close to the line through the epipoles will be estimated with poor precision.

The degeneracy condition that x_3, e_{13} and e_{23} are collinear in the third image means that the camera centers C_1 and C_2 and the 3D point X lie in a plane through the center C_3 of the third camera. Thus X lies on the trifocal plane defined by the three camera centers.

Epipolar transfer will fail for points X lying on the trifocal plane and will be inaccurate for points lying near that plane. The trifocal plane is not uniquely defined as the three camera centers are collinear. In this case

$$e_{13} = e_{23}$$



I_2
C_2
e_{12}
e_{32}
e_{21}
C_1
e_{31}
I_1
e_{23}
e_{13}
C_3
I_3