

# The CMA-ES

**Input:**  $\mathbf{m} \in \mathbb{R}^n$ ,  $\sigma \in \mathbb{R}_+$ ,  $\lambda$

**Initialize:**  $\mathbf{C} = \mathbf{I}$ , and  $\mathbf{p}_c = \mathbf{0}$ ,  $\mathbf{p}_\sigma = \mathbf{0}$ ,

**Set:**  $c_c \approx 4/n$ ,  $c_\sigma \approx 4/n$ ,  $c_1 \approx 2/n^2$ ,  $c_\mu \approx \mu_w/n^2$ ,  $c_1 + c_\mu \leq 1$ ,  $d_\sigma \approx 1 + \sqrt{\frac{\mu_w}{n}}$ ,  
and  $w_{i=1 \dots \lambda}$  such that  $\mu_w = \frac{1}{\sum_{i=1}^{\mu} w_i^2} \approx 0.3 \lambda$

**While not terminate**

$\mathbf{x}_i = \mathbf{m} + \sigma \mathbf{y}_i$ ,  $\mathbf{y}_i \sim \mathcal{N}_i(\mathbf{0}, \mathbf{C})$ , for  $i = 1, \dots, \lambda$  sampling

$\mathbf{m} \leftarrow \sum_{i=1}^{\mu} w_i \mathbf{x}_{i:\lambda} = \mathbf{m} + \sigma \mathbf{y}_w$  where  $\mathbf{y}_w = \sum_{i=1}^{\mu} w_i \mathbf{y}_{i:\lambda}$  update mean

$\mathbf{p}_c \leftarrow (1 - c_c) \mathbf{p}_c + \mathbb{1}_{\{\|\mathbf{p}_\sigma\| < 1.5\sqrt{n}\}} \sqrt{1 - (1 - c_c)^2} \sqrt{\mu_w} \mathbf{y}_w$  cumulation for  $\mathbf{C}$

$\mathbf{p}_\sigma \leftarrow (1 - c_\sigma) \mathbf{p}_\sigma + \sqrt{1 - (1 - c_\sigma)^2} \sqrt{\mu_w} \mathbf{C}^{-\frac{1}{2}} \mathbf{y}_w$  cumulation for  $\sigma$

$\mathbf{C} \leftarrow (1 - c_1 - c_\mu) \mathbf{C} + c_1 \mathbf{p}_c \mathbf{p}_c^T + c_\mu \sum_{i=1}^{\mu} w_i \mathbf{y}_{i:\lambda} \mathbf{y}_{i:\lambda}^T$  update  $\mathbf{C}$

$\sigma \leftarrow \sigma \times \exp\left(\frac{c_\sigma}{d_\sigma} \left(\frac{\|\mathbf{p}_\sigma\|}{\mathbb{E}\|\mathcal{N}(\mathbf{0}, \mathbf{I})\|} - 1\right)\right)$  update of  $\sigma$

**Not covered** on this slide: termination, restarts, useful output, boundaries and encoding