

# Backstepping-based control methodology for aircraft roll dynamics

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## Abstract

This article investigates a backstepping-based control method for aircraft roll dynamics. The research starts with a formulation of backstepping control law for a general class of a strict-feedback form of nonlinear dynamic systems. The backstepping control law is formulated by introducing a normal tracking error. Then, control and virtual control inputs are selected by addressing each layer of the design process with a chosen corresponding control Lyapunov function. The parameter assignment in each design layer is selected to ensure the stability of the entire system. Next, a backstepping-based control algorithm with online-gain schedule or variable gains is provided for the standard strict-feedback system. In order to validate the proposed method, application of roll dynamics of aircraft is implemented. Dynamic equations of free-to-roll aircraft model is restructured in a standard strict-feedback model for formulating the backstepping control. Then, a backstepping control-based control strategy is provided for aircraft free-to-roll dynamics. Indoor experimental and simulation studies of roll angle control for the L-59 free-to-roll aircraft model at NASA Langley Research Center are implemented to verify and validate the proposed approach.

## Keywords

Nonlinear control, backstepping control, flight control

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## Introduction

In the next generation, flight control design is expected to be done with reduced or no human intervention,<sup>1</sup> to optimize flight over multiple regimes,<sup>2</sup> or even to learn to fly in the presence of varying components such as wing damages, loading variations, and engine failure.<sup>3</sup> Many researchers have addressed these aspects to improve the reliability and flexibility in designing control systems. The conceptual design of an aircraft with morphing wings or flexible wings are proposed in, for example, the studies by Soflaa et al.,<sup>4</sup> Orlowski and Girard,<sup>5</sup> and Leylek and Costello,<sup>6</sup> which could provide major benefits for aircraft when it comes to optimizing lift, fuel efficiency, load alleviation, pitch, yaw, and roll control. The morphing wings are key factors to make aircraft to behave more similar to birds, which improve their performances in different flight conditions. In this approach, the wings of the aircraft can change their external shapes significantly in order to fulfill different mission requirements in flight. However, this approach is only capable of small lightweight aircrafts but not for heavyweight ones. From these obstacles, other approaches for heavyweight and fixed-wing aircraft are addressed by many researchers.<sup>7–11</sup> The neural

network-based flight control of online learning capability<sup>7</sup> is proposed to compensate for inversion error due to the imperfect modeling, approximate inversion, and sudden change in aircraft dynamics. Similar objectives are sought by the use of a direct neural network-based adaptive control architecture that compensates for unknown plant nonlinearities in a feedback linearizing control framework as proposed in the study by Calise and Rysdyk,<sup>8</sup> a tuning scheme is proposed which can guarantee the boundedness of tracking error and weight updates,<sup>9</sup> and an online training is accomplished to account for the inversion and modeling error.<sup>10</sup> A self-repairing control system<sup>11</sup> is proposed to implement the reconfiguration after control effect failures. This allows aerodynamic forces and

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moments to be produced by the other control effects to keep aircraft in normal flight.

Requirements, on smartness and flexibility, in future flight vehicle design which makes aircraft capable of adapting or self-adjusting (or leaning) after sudden changes in different operating conditions have led researchers to advanced control methods.<sup>12–18</sup> In particular, a framework of modified control design is presented for preventing the presence of input saturation from destroying learning capabilities and memory of an online approximator in feedback control systems.<sup>12</sup> In the study by Kaliyari et al.,<sup>13</sup> flight data identification of a high-performance aircraft with relaxed static stability is presented to highlight the impact of the dynamic factors on aerodynamic database update. The capability of aerodynamic data identification and model update is critical to develop a controller-based online learning method for flight vehicles,<sup>14,15</sup> in which an aircraft is capable of learning to fly by using on-board measurements and available control surfaces. An initial work of this direction was presented in the study by Grauer and Morelli,<sup>16</sup> where a linear quadratic (LQ)-based proportional–integral–derivative (PID) controller utilized the identified aerodynamic model to design an optimal PID controller for updating control gains at each sampling instant. In this article, a backstepping control (BSC)-based control algorithm is investigated for aircraft free-to-roll (FTR) dynamics. The proposed method plays a key role in developing a learn-to-fly algorithm of aircraft roll dynamics and further with full dynamics of aircraft. The contributions of the proposed control method consist of the following three points: (1) the article is trying to answer if there exists a feedback control law that can update simultaneously the dynamics model with the process of aerodynamic system identification in the studies by Morelli;<sup>14,15</sup> (2) a systematic procedure is provided for formulating a nonlinear BSC law with the capability of online update of new dynamic models in previous step; and (3) a new approach is formulated for an online gain-scheduling strategy for the achieved update model and commands for the entire envelope. Finally, software design for simulation and experimental studies are implemented for validating the proposed method.

This article is organized as follows: in section “BSC law formulation,” a systematic procedure is presented for formulating the BSC law for the second-order single-input single-output (SISO) strict-feedback form of nonlinear dynamic systems; in section “Online gain-scheduling formulation,” formulation of online gain-scheduling of the BSC for the system is derived via the representation of gains as a function of performance specifications and state feedback; in section “BSC-based control algorithm for aircraft roll dynamics,” a BSC-based control algorithm for aircraft roll dynamics is provided for simulation and experimental studies for verifying and validating the algorithm; in section “Study of experiments,” experimental study of L-59

FTR aircraft model at NASA Langley Research Center is implemented; and in section “Conclusion,” conclusion and discussions are presented.

## BSC law formulation

Backstepping design is considered as a recursive design process in the studies by Tran and Newman,<sup>19</sup> Krstic et al.,<sup>20</sup> Choi et al.,<sup>21,22</sup> Tran and Choi<sup>23</sup> which breaks a design problem on the full system down to a sequence of sub-problems on lower order systems. Considering each lower order system with a Lyapunov function and paying attention to the interaction between two subsystems make a process to be modular and facilitate the design of a stabilizing controller. A second-order SISO strict-feedback system is selected to formulate the stabilizing control law. This procedure can be generalized for an  $n$ th-order strict-feedback form of a nonlinear dynamic system.

Consider a second-order SISO strict-feedback form of a nonlinear dynamic system as

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2, u)\end{aligned}\quad (1)$$

where  $x \in R^2$  is the vector of state variables,  $f(x)$  is a two-dimensional vector of scalar-valued functions, and  $u \in R$  is the scalar control input.

Assume that the function  $f_i (i = 1, 2)$  is continuous and differentiable with respect to the variables, and the variables  $x_2, u$  in equation (1) are solvable explicitly in terms of the other variables in the first and second equations, respectively. In our application,  $f_1$  and  $f_2$  are known functions of system parameters, states, and inputs. System parameters are estimated via algorithms in the study by Morelli.<sup>14</sup> Uncertainties in the system parameters can be improved by adding an integrator in the BSC design.<sup>19</sup> The objective is to design a control law for the strict-feedback form of the nonlinear dynamic system (1) such that the output  $y = h(x) = x_1 \rightarrow x_{ref}$  asymptotically, where  $x_{ref}$  is a constant, and global asymptotic stability is achieved with zero or acceptably small overshoot in the system response.

The BSC law for a strict-feedback structure is formulated by dividing a whole system into  $n$  subsystems such that the  $i$ th subsystem consists of the  $(i - 1)$ th subsystem plus an extra state and the  $n$ th subsystem is the original  $n$ th-order system via coordinate transformation and state feedback. By consecutively applying coordinate transformation and choosing a feedback control law via the control Lyapunov function (CLF) for each subsystem from the lowest to highest order and rewriting the feedback law in the original coordinates, the resulting controllers make the original deficient system a well-tracking command and asymptotically stable. The following steps are used for formulating the BSC law for the system (1).

### Step 1

The  $x_2$  variable is regarded as a control input of the first relation in equation (1), which is considered as the first subsystem. Thus,  $x_2$  is chosen to make the first subsystem globally asymptotically stable. The chosen function is called a virtual control law. By introducing  $\xi_1$  (an error signal) as

$$\xi_1 = x_1 - x_{ref} \quad (2)$$

and taking time derivative both sides of equation (2) and combining with the first subsystem of equation (1), one gets

$$\dot{\xi}_1 = \dot{x}_1 = f_1(\xi_1 + x_{ref}, x_2) \quad (3)$$

For the system (3), a CLF can be chosen such that a virtual control law is applied, its time derivative becomes negatively definite. The function is chosen as

$$V_1(\xi_1) = \frac{1}{2} \xi_1^2 \quad (4)$$

By taking time derivative of equation (4) and combining with equation (3), one finds the result

$$\dot{V}_1(\xi_1) = \xi_1 \dot{\xi}_1 = \xi_1 f_1(\xi_1 + x_{ref}, x_2) \quad (5)$$

By satisfying the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for equation (5), a virtual control law denoted as  $\alpha_1$  for  $x_2$  can be chosen as

$$\begin{aligned} -c_1 \xi_1 &= f_1(\xi_1 + x_{ref}, x_2) \\ \Rightarrow x_2 &\equiv \alpha_1(c_1, x_1, x_{ref}) \end{aligned} \quad (6)$$

where  $c_1$  is a positive gain. By doing so, the CLF derivative is negatively definite, or

$$\dot{V}_1(\xi_1) = \xi_1 \dot{\xi}_1 = -c_1 \xi_1^2 < 0 \quad \forall \xi_1 \neq 0 \quad (7)$$

### Step 2

By choosing the state feedback in equation (6) and a change in coordinate indicated below

$$\xi_2 = x_2 - \alpha_1(c_1, x_1, x_{ref}) \quad (8)$$

the subsystem can be rewritten as follows

$$\begin{aligned} \dot{\xi}_1 &= -c_1 \xi_1 \\ \dot{\xi}_2 &= f_2(\xi_1 + x_{ref}, \xi_2 + \alpha_1, x_3) - \dot{\alpha}_1 \end{aligned} \quad (9)$$

where  $\dot{\alpha}_1$  is calculated from time derivative of equation (6) or  $\dot{\alpha}_1 = (\partial \alpha_1(c_1, x_1, x_{ref}) / \partial x_1) f_1(x_1, x_2)$ .

A CLF  $V_2(\xi_1, \xi_2)$  can be chosen such that it makes the subsystem in equation (9) asymptotically stable with the virtual control law, that is

$$V_2(\xi_1, \xi_2) = V_1(\xi_1) + \frac{1}{2} \xi_2^2 \quad (10)$$

By taking time derivative of equation (10) and combining with equation (9), one finds the result

$$\dot{V}_2(\xi_1, \xi_2) = -c_1 \xi_1^2 + \xi_2 \{f_2(\xi_1 + x_{ref}, \xi_2 + \alpha_1, x_3) - \dot{\alpha}_1\} \quad (11)$$

To meet the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for equation (11), a real control law  $u$  can be chosen as

$$\begin{aligned} -c_2 \xi_2 &= f_2(\xi_1 + x_{ref}, \xi_2 + \alpha_1, u) - \dot{\alpha}_1 \\ \Rightarrow u &\equiv \alpha_2(c_1, c_2, x_1, x_2, x_{ref}) \end{aligned} \quad (12)$$

where  $c_2$  is a positive gain. By doing so, the CLF derivative is again negatively definite, or

$$\dot{V}_2(\xi_1, \xi_2) = -c_1 \xi_1^2 - c_2 \xi_2^2 < 0 \quad \forall \xi_1, \xi_2 \neq 0 \quad (13)$$

Thus, there exists a CLF for equation (10), state-feedback laws in equations (6) and (12) and a change of state transformations in equations (2) and (8) such that the system (1) is transformed into the following form

$$\begin{aligned} \dot{\xi}_1 &= -c_1 \xi_1 \\ \dot{\xi}_2 &= -c_2 \xi_2 \end{aligned} \quad (14)$$

The solutions of the decoupled system (14) are asymptotically stable around the origin. The desired settling time of the system is obtained by gain selection  $c_i (i = 1, 2)$ . Thus, stability and performance specifications on the system (1) are achieved with the proposed BSC law. A block diagram of the proposed BSC logic is found similarly in the study by Tran<sup>18</sup> for the second-order strict-feedback structure of nonlinear dynamic systems.

In summary, analysis of stability and performance of the proposed control method are addressed in detail in which an extended analysis on to which extend the proposed method for concepts of learn-to-fly vehicles is presented. In Lyapunov-based approach, the BSC design requires control laws to be included a part or overall of system parameters. As a result, some steady-state error or even a poor performance occurs due to the system errors. This prevents BSC-based control from being popular in real applications. However, this obstacle is addressed as an advantage in terms of control design with the capability of online learning from new estimated model in the presence of sudden changes and different operating conditions. The combined use of capability of the BSC-based model update and online-gain scheduling in next section provides a basic concept for control ability of flight vehicles.

### Online gain-scheduling formulation

The design of state-feedback law in equations (6) and (12) that the gains  $c_i (i = 1, 2, \dots)$  must be selected. From equation (14), it is clear that these gains are the reciprocals of the time constant of the resulting decoupled first-order system. If a 1% setting time  $t_s$  is specified, then the gains would meet the specification if they are selected to be the constants  $c_1 = c_2 = (4.6/t_s)$ . Since

initial conditions when step changes are commanded may differ, there might be value in incorporating values, resulting in what is called here semi-value gains. Then, the gains are calculated as a function of the settling time, initial conditions, and command.

Taking integral both sides of the first relation of equation (14) and returning with the original variables, one gets

$$x_i - x_{ref} = (x_{i0} - x_{ref})e^{-c_i t} \quad (15)$$

where  $t$  is time variable and  $x_0$  is an initial value. The index  $i$  has values of 1 and 2 in this equation. Also, for  $i = 2$ ,  $x_{ref}$  should be replaced by  $\alpha_{1ref}$  in equation (15).

Equation (15) can be rewritten as

$$c_i = -\frac{1}{t} \ln \frac{x_i - x_{ref}}{x_{i0} - x_{ref}} \quad (16)$$

For a specific 1% settling time  $t_s$ , the gain  $c_i$  can be calculated from equation (16) as

$$c_i = -\frac{1}{t_s} \ln \frac{-0.01x_{ref}}{x_{i0} - x_{ref}} \quad (17)$$

One advantage of semi-variable gains is that the settling time can be assigned specifically for the output response. The values of  $c_1, c_2$  in equation (17), respectively, require positive values. Then, the following relation must be hold

$$-0.01x_{ref}/(x_{i0} - x_{ref}) < 1, \quad (18)$$

Note that the condition in equation (18) limits the operational range of the method. Thus, a combined use of constant gains or a modified version will provide a better control strategy.

The aforementioned gains are used for experimental studies of rolling angle control for an L-59 aircraft model. The BSC-based control method with the proposed gains shows the limitations and advantages in different operational conditions. To show the applicability during real implementation, an analysis of experimental results when applied to the actual physical model is emphasized in section ‘‘Study of experiments’’ of this article.

### BSC-based control algorithm for aircraft roll dynamics

In this article, implementation of the backstepping-based control is generated in the following three steps: (1) a real-time system identification of flight aerodynamic model is implemented, (2) a backstepping-based flight control algorithm which is capable of utilizing the identified new models is provided to reschedule the control strategy to achieve the stability and performance, and (3) an online gain-scheduling strategy is used for the achieved update model and commands for the entire envelope. The first step was developed and tested by a group of scientists at the NASA Langley Research Center using a multivariate orthogonal function (MOF)



Figure 1. Experimental model of L-59 aircraft.

method.<sup>14,15,17</sup> For the contribution of the article, a semi-variable gain BSC is formulated and evaluated for the second step. Then, the combined use of identification algorithm, backstepping flight control, and semi-variable online gain scheduling are applied to achieve the proposed control implementation of aircraft.

### Roll dynamics of the L-59 aircraft model

Consider the L-59 model airplane in FTR experiment shown in Figure 1.

The mathematical model of the FTR L-59 aircraft model is written as a multi-input single-output (MISO) strict-feedback form of nonlinear dynamic systems

$$\begin{aligned} \dot{\phi} &= p \\ \dot{p} &= \frac{1}{J_{xx}} (mgz_c g \sin(\phi(t)) + \bar{q} S b C_l(E)) \end{aligned} \quad (19)$$

where  $E$  is a vector of explanatory variables. The parameters and variables are defined in Tables 1 and 2, respectively.  $\alpha$  is the attack angle,  $\beta$  is the sideslip angle,  $p$  is the roll rate,  $\delta_{a_l}$  is the left aileron,  $\delta_{a_r}$  is the right aileron,  $\delta_e$  is the elevator, and  $\delta_r$  is the rudder.

By introducing the new control input as

$$u = C_l(\alpha, \beta, p, \delta_{a_l}, \delta_{a_r}, \delta_e, \delta_r) \quad (20)$$

where  $C_l$  is determined by table lookup databases that are equivalent with specified variables:  $\alpha$  is the attack angle,  $\beta$  is the sideslip angle,  $p$  is the roll rate,  $\delta_{a_l}$  is the left aileron,  $\delta_{a_r}$  is the right aileron,  $\delta_e$  is the elevator, and  $\delta_r$  is the rudder. The system (19) can be rewritten as a SISO strict-feedback system (1) or

$$\begin{aligned} \dot{\phi} &= p \\ \dot{p} &= \frac{1}{J_{xx}} \{mgz_c g \sin(\phi(t)) + \bar{q} S b u\} \end{aligned} \quad (21)$$

The aircraft roll dynamics in equation (21) is exactly in the form of the strict-feedback system. Thus, the theoretical development in section ‘‘BSC law formulation’’ is applied for formulating the control law. The achieved control input  $u$  is substituted in equation (20) to calculate the real inputs  $(\delta_e, \delta_f, \delta_a, \delta_r)$  by solving the optimization problems.

**Table 1.** Definition of experimental variables.

Experimental parameters			
Variables	Description	Range	Unit
$\phi$	Roll or bank angle	$[-90, 90]$	$^\circ$
$S$	Wing reference area	3.14	$\text{ft}^2$
$B$	Wing reference span	3.937	ft
$z_{cg}$	Center of gravity position	0.1464	inch
$mg$	Test model weight	21.72	lbs

### Controller design

The objective is to design a control law  $u$  for aircraft roll dynamics (equation (21)) such that roll angle  $p(t) \Rightarrow p_{ref}$  (roll command) asymptotically, where  $p_{ref}$  is a constant. Furthermore, time response of the system can meet design requirement with zero or acceptably small overshoots that are approximated to be 2% and a 1.2s settling time in the presence of model parameter errors and varying sting pitch and yaw angles. With the achieved new control input, a follow on problem is to allocate the new input  $u$  to the real inputs  $\delta_{a_l}$ -left aileron and  $\delta_{a_r}$ -right aileron to achieve the performance specifications.

Note that the system (21) is exactly in the form of the SISO strict-feedback system (1). Thus, a similar procedure in section “BSC law formulation” is applied for the system (21) to generate the BSC for aircraft roll dynamics, as shown in equation (22)

$$u = \frac{I_{yy}}{qSb} \left( -c_2(p - \alpha_1) + \dot{\alpha}_1 - \frac{1}{I_{xx}} mgz_{cg} \sin(\phi) \right) \quad (22)$$

The terms  $\alpha_1$  and  $\dot{\alpha}_1$  are determined by equations (23) and (24), respectively

$$\alpha_1 = -c_1(\phi - \phi_{ref}) \quad (23)$$

$$\dot{\alpha}_1 = -c_1 p \quad (24)$$

where  $c_1, c_2$  are the positive gains. The achieved control input (equation (22)) is substituted in equation (20) to calculate the real control inputs ( $\delta_e, \delta_{a_l}, \delta_{a_r}, \delta_f, \delta_r$ ) by solving control allocation problems in the next section.

### Control allocation

By introducing the new control input  $u = C_l(E)$ , the MISO system (19) can be rewritten in equation (21) or in a SISO form that is convenient for the backstepping approach. The next step is to allocate the  $u$  control input on the right and left aileron deflections. With the achieved  $u$  in equation (22) and  $C_l(E)$  in terms of other state variables, one problem is to find the solutions for  $\delta_e, \delta_{a_l}, \delta_{a_r}, \delta_f, \delta_r$  from the equation  $u = C_l(E)$  with a given current set of state variables in order to make the output track the command. In this research, the solutions  $\delta_e, \delta_f, \delta_r$  are specified for solving the inputs  $\delta_{a_l}, \delta_{a_r}$ . A full version for allocating to all control inputs can be found in the study by Mekky and Gonzalez.<sup>17</sup>

**Table 2.** Definition of measured variables.

Measured parameters			
Variables	Description	Value	Unit
$p$	Roll rate	$[-200, 200]$	deg/s
$I_{xx}$	Moment of Inertia	0.0805	slug ft <sup>2</sup>
$\dot{p}$	Roll acceleration	$[-200, 200]$	deg/s <sup>2</sup>
$\delta_{a_l}$	Left aileron deflect	$[-25, 25]$	$^\circ$
$\delta_{a_r}$	Right aileron deflect	$[-25, 25]$	$^\circ$
$\alpha$	Angle of attack	$[0, 40]$	$^\circ$
$\beta$	Sideslip angle	$[-40, 40]$	$^\circ$
$\bar{q}$	Dynamic pressure	2	lbf/ft <sup>2</sup>
$C_l(E)$	Aerodynamic moment	$[-]$	(-)
$\theta$	Sting pitch angle	$[0, 25]$	$^\circ$
$\psi$	Sting yaw angle	$[-20, 20]$	$^\circ$

**Iteration methods.** This approach is proposed to find the solutions of one algebraic equation of two variables. The values of the initial guess are very close to the real solutions, only then may the method provide global solutions. For the simulation and experimental studies, the values of the initial guess for the iteration are selected as previous values of the iteration or the equilibrium values. The advantage of the method is a simple structure, which may speed up the iteration process for the solution but may not provide robust behavior in general.

**Optimization methods.** In general, if the number of variables in an algebraic equation set is more than the number of equations, then optimization subject to the constraints is a good candidate for finding the solutions. Instead of finding the solutions  $\delta_{a_l}, \delta_{a_r}$  of the equation  $u = C_l(E)$ , the optimization formulation leads to finding the solution to minimize the tracking error sum, or

$$\underset{\delta_{a_l}, \delta_{a_r}}{\text{minimize}} \quad J = \int_0^t (\phi - \phi_{ref})^2 dt \quad (25)$$

subject to the constraints

$$\begin{aligned} \delta_{a_l} \delta_{a_r} &\leq 0 \\ u - C_l(E) &= 0 \end{aligned} \quad (26)$$

where the first equation of equation (26) shows the opposite directions of right and left ailerons. If the right aileron is up, then the left aileron must be down. The second equation of equation (26) makes sure that achieved gains must satisfy equation (20) in the article. Both approaches are used in experiments in this research.

### Gain-schedule strategy

As we discussed in the previous section, the conditions on states and command must satisfy equation (18). Thus, a combined strategy of optimal constant and semi-variable gains provides a better control-based

control strategy. The approach to find optimal gains leads to finding optimal solutions for  $c_1, c_2$  to minimize the tracking error sum and aileron deflections

$$\text{minimize}_{c_1, c_2} J = \int_0^t \left[ w_e (\phi - \phi_{ref})^2 + w_u (\delta_{a_r}^2 + \delta_{a_l}^2) \right] dt \quad (27)$$

subject to the constraints

$$\begin{aligned} \delta_{a_l} \delta_{a_r} &\leq 0 \\ -25^\circ &\leq \delta_{a_l}, \delta_{a_r} \leq 25^\circ \\ u - C_l(E) &= 0 \end{aligned} \quad (28)$$

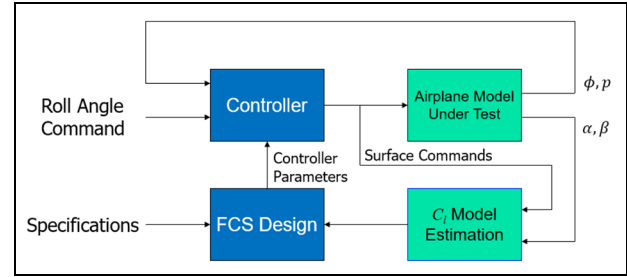
where  $w_e, w_u$  are the weights, which are determined via the requirements of the performance specifications. Those values are selected by a trial and error technique. The new input  $u$  is determined by equation (22). By using the modified genetic algorithm,<sup>24</sup> the optimal gains for the BSC law are achieved as  $c_1 = 4.13s^{-1}, c_2 = 4.28s^{-1}$ . For experiments, a combined use of semi-variable gains in equation (17) and achieved optimal gains is implemented for the control method of aircraft free-to-roll (F2R) dynamics.

Figure 2 shows a block diagram of the BSC-based control algorithm for aircraft FTR dynamics. Implementation of this method will follow the following two steps: (1) a system identification procedure of flight aerodynamic model is proposed to provide a new model due the sudden change or external effects for update strategy after a certain time and (2) the achieved new model is used for the BSC-based control to generate new inputs to adapt the different variation of the system. The former is implemented by the MOF method,<sup>14,15</sup> and the latter is a main contribution of the article.

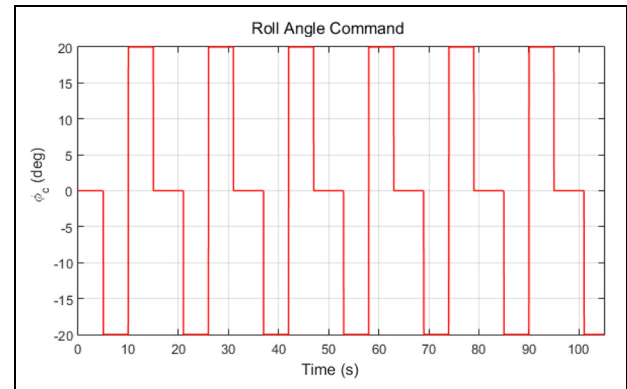
## Study of experiments

Experiments are implemented for a one-degree-of-freedom setup for L-59 aircraft inside a 2-foot Low-Speed Tunnel, NASA Langley Research Center, as shown in Figure 1. The test aircraft is a commercially available, remotely controlled L-59 model mounted on a sting that controls both pitch and yaw angles while keeping the aircraft F2R about its  $x$ -axis. The model has been instrumented to independently control the deflections of its seven control surfaces: ailerons (2), flaps (2), elevators (2), and rudder. For the purpose of the experimental study, a block diagram in Figure 2 is used for designing control software for simulation and experimental studies.

The experiments are tested with two different roll angle and pitch angle profiles. The roll angle profile consists of  $\pm 20^\circ$  doublets shown in Figure 3. For pitch angle, several cases are considered with a constant small angle of  $5^\circ$ . Also, Case 2 considers variations in pitch angle profiles. In this second profile, multiple changes in pitch angle are applied for the study in order to verify the robustness of the proposed method for different



**Figure 2.** Block diagram of the BSC-based control algorithm for aircraft FTR dynamics.



**Figure 3.** Roll angle command in time.

flight conditions. Also, the experimental results were generated with different types of gains that are constant gains, semi-variable gains, and variable gains. The values of constant gains were computed from an optimization process in equations (27) and (28) and expressions for the semi-variable gains are represented in equation (17).

Experimental study is implemented with different conditions. To implement the BSC-based control strategy, a PD or proportional derivative-based control using available aerodynamic coefficients was implemented for initial operation of the experimental system. After certain time, a BSC-based control is replaced for further experiments. In this time window, system identification was turned on for estimating new aerodynamic coefficients. With the achieved aerodynamic coefficients, the BSC-based control was applied for the remaining period in the experiments.

Figure 4 shows time responses of roll angle and aileron deflections with a constant pitch of  $5^\circ$  and zero yaw degrees. In this experiment, a PD-based control by using available aerodynamic coefficients was implemented for the first 80 s. In the next 100 s, a real-time system identification is initiated for the first estimation of new aerodynamic coefficients. Then, a BSC-based control algorithm is applied for the achieved model of flight aerodynamics during the next 100 s. At this time, the real-time system identification is again initiated for estimating new aerodynamic coefficients. Then, the



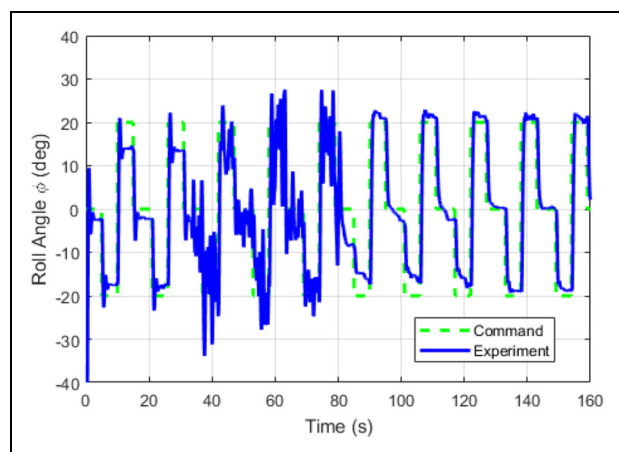


Figure 4. Experimental time responses.

BSC-based control algorithm is again applied for the achieved model in the second estimation for the last 100 s. The dashed line (or green line) and solid line (or blue line) in Figure 4 show the roll angle command and measured roll angle response, respectively. The blue line in Figure 4 shows time response of roll angle. By observing the performance of roll angle response in different model estimations, the experiment results indicate that the performance is improved considerably as a better model estimation is applied. The BSC-based algorithm provides also a learning-like ability and improves the performance and stability. With the same experimental conditions, the study<sup>17</sup> by another research group at NASA Langley center shows that the linear quadratic regulator (LQR)-based control is not providing a good response. The reason is that this method is not able to adapt the change in the aerodynamic model. Experimental results by using PID and LQR algorithms can be seen further in the study by Mekky and Gonzalez.<sup>17</sup>

Figure 5 shows time responses of roll angle and aileron deflections with constant pitch of  $10^\circ$  and constant yaw angle of  $5^\circ$ . With the same experimental condition as in the previous case for the experiment, a PD-based control and real-time system identification was implemented simultaneously for the first 140 s. As a good model estimation is achieved, a BSC-based control algorithm is applied for the remaining of the experiment. The simulation and experimental results of roll angle responses in solid line (or red line) and dashed line (or green line), respectively, in the top portion of Figure 5 indicates that (1) simulation and experimental results show the reliability and validation of the proposed control design methodology and (2) the response behavior meets the requirements of the proposed design with well-tracking command without overshoot. This is verified in the theoretical development

For the purpose of validating the robustness of the proposed algorithm, the same experiment conditions as in previous cases but a further multiple changes in pitch angle are applied for the study. Figure 6 shows

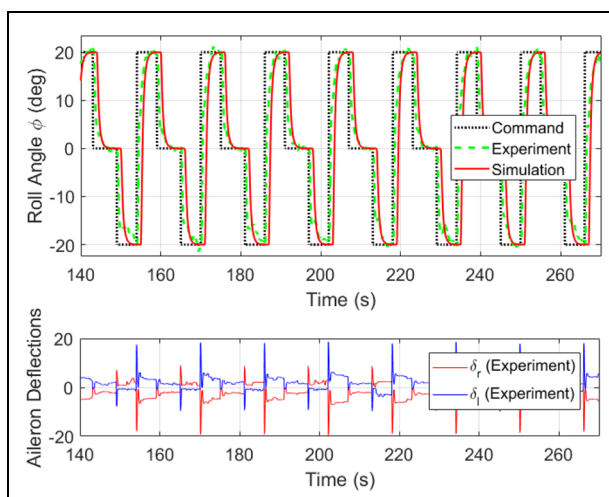


Figure 5. Experimental time responses with constant pitch and yaw angles.

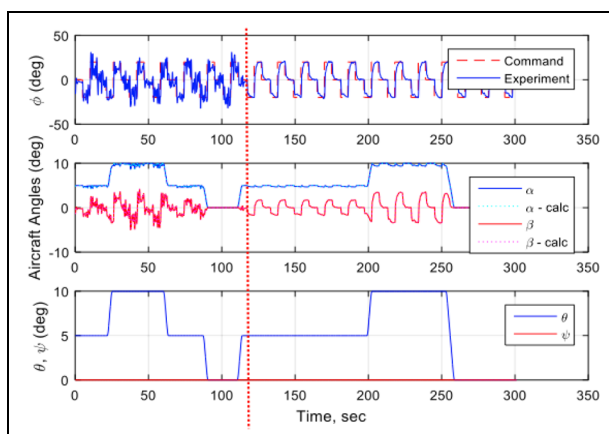


Figure 6. Experimental time responses with varying pitch and yaw angles.

responses of state variables and aileron deflections with varying pitches. Similarly, solid line (or blue line) in the top portion of Figure 6 indicates the response behavior has a well-tracking command (red or dashed line) with multiple changes in pitch angles, as shown in blue line in the bottom portion of Figure 6. The experimental results verify that the BSC-based control algorithm is reliable and robust.

## Conclusion

The article investigates a backstepping-based control methodology for nonlinear roll flight dynamics of aircraft. The article starts with a BSC formulation and control strategy for the strict-feedback form of the nonlinear dynamic system. For a real application, a roll dynamics of the L-59 aircraft model is considered for simulation and experimental study. In this approach, roll dynamic equations for aircraft are derived for

analysis and design purposes. A BSC-based control algorithm is provided for aircraft FTR dynamics via model update capacity of the BSC law. Study of simulation and experiments on roll dynamics of aircraft with different operating conditions validates the proposed method. Experimental results also show that the performance and stability are improved considerably as a better model estimation is replaced. Thus, the BSC-based method can be considered as a learning-like ability to adapt the variations in system parameters due to the change in the model.

Although simulation results indicate that the BSC-based control design provides a well performance with different flight conditions, the research is only applied for a simple case of F2R aircraft dynamics. Thus, a backstepping-based control strategy for the full nonlinear aircraft model where both longitudinal and lateral-directional dynamics are present simultaneously needs to be addressed may provide a better and more robust control system.


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