Asian Journal of Control, Vol. 19, No. 3, pp. 1239–1249, May 2017
Published online 28 November 2016 in Wiley Online Library (wileyonlinelibrary.com) DOI: 10.1002/asjc.1411

–Brief Paper–

ROBUST STABILIZATION OF NONLINEAR PVTOL AIRCRAFT WITH PARAMETER UNCERTAINTIES

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ABSTRACT

In this work, the stabilization control problem is investigated for the planar vertical take-off and landing (PVTOL) aircraft with unknown model parameters. To cope with the challenges caused by non-minimum phase and parametric uncertainties, a new appropriate output, different from the general centroid position output, is carefully constructed to ensure the zero dynamics asymptotically stable, then the sliding-mode technique is applied to design a state feedback control law. The proposed robust control law is proved able to asymptotically stabilize the PVTOL aircraft to the desired fixed position with null velocities and roll angle despite the unknown model parameters. Simulation examples illustrate the effectiveness of the proposed control algorithm.

Key Words: PVTOL aircraft, robust stabilization, parameter uncertainties, input-output linearization, sliding-mode control.

I. INTRODUCTION

Over the past few years, the problem of designing controllers for the PVTOL aircraft system has received increasing attention in the control community. The PVTOL aircraft is the natural restriction of a vertical/short take-off and landing aircraft to jet-borne operation (*e.g.* hover) in the vertical-lateral plane [1,2], and is also extensively used to develop and approximate the models of flying vehicles. The main difficulty of the control of PVTOL aircraft lies in its highly nonlinear, underactuated and non-minimum phase properties [2].

Numerous effective approaches have been developed to solve the stabilization control problem of PVTOL aircraft. Firstly, for making controller design easier, some effort has focused on the simplified PVTOL aircraft model without input coupling (*i.e.*, uncoupled aircraft) [3–6], because this coupling coefficient is generally small. In [3], by employing linear saturation functions, a relatively simple bounded control algorithm was derived achieving global asymptotic stabilization of uncoupled aircraft, which was further proved by [7] also effective for the weakly coupled aircraft. An observer-based control law was proposed in [4] with unmeasurable roll angle. Applying Lyapunov direct method and LaSalle invariance principle, Turker proposed two asymptotically stable stabilization control strategies, respectively, for the uncoupled [5] and the coupled [8] PVTOL aircraft systems. Considering unmeasurable velocities and bounded external disturbances, Cardenas [6] designed an output feedback control scheme by sliding-mode technique, which guaranteed local finite time stability of the closed-loop PVTOL aircraft system. Secondly, some researchers were aware that the coupling coefficient should be taken into account (hence viewed as nonzero) for gaining a better control performance [8–19]. In [9], the receding horizon control method was used to design a globally asymptotical stabilization controller with a low computational cost. Reference [10] extended the scheme developed in [9] to discrete-time case. A nonlinear predictive method and an optimal control technology were exploited, respectively, in [11] and [12] to solve the stabilization problem of coupled aircraft. On the other hand, for this case of strong input coupling, Olfati-Saber [13] constructed a novel coordinate transformation that decoupled the aircraft system, and furthermore provided a smooth static state feedback stabilization control law. The input transformation in [13], by contrast with those in [11,12], was valid for both the nonzero and the zero coupling coefficients. Then, many schemes, dependent on the converted aircraft model proposed by Olfati-Saber, were developed, see [14–19] for details. Wood modified the control law in [13], and gave a new controller [14] by minimizing the interconnection term between closed-loop subsystems.

Manuscript received 29 January 2016; revised 6 August 2016; accepted 2 September 2016.
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This work was supported by National Natural Science Foundation of China (No.61573034, No.61327807) and Sub-Project of 973 Program (No.2012CB821202).

In [15], a simple control scheme was developed by Lyapunov method, which could steer the aircraft asymptotically. In [16], an output feedback control law was constructed ensuring the global finite time stabilization of aircraft. Inspired by [3,16], references [17,18] presented finite time observer-based schemes, both achieving global stabilization with bounded inputs. In [19], a vertical position stabilizer and a horizontal/angular position stabilizer were deduced respectively via feedback linearization and back-stepping techniques, obtaining an asymptotic convergence performance of aircraft system. However, all the aforementioned control strategies will fail to work for unknown model parameters, which means that the stabilization control problem of uncertain PVTOL aircraft has not been completely investigated, and hence remains open.

Motivated by the previous observations, this paper is concerned with the robust stabilization control problem of PVTOL aircraft system with unknown model parameters. Firstly, based on invertible coordinate transformations, a new output containing all the states of system is smartly constructed ensuring the zero dynamics asymptotically stable and guaranteeing the input-output decoupling matrix globally invertible. Then, the sliding-mode control law is derived to drive the constructed output to zero in finite time. The stability analysis shows that, under the action of the proposed control scheme, the PVTOL aircraft asymptotically converges to the desired fixed position with vanishing roll angle and velocities. Furthermore, the robust property is also presented in our control strategy that not only obviates the need for knowing the real values of model parameters, but also keeps valid for both the uncoupled and the coupled cases.

The reminder of this work is organized as follows. The stabilization control problem of PVTOL aircraft is formulated in Section II, and is solved by designing an appropriate control law in Section III. Several numerical simulations are carried out to verify the effectiveness of the proposed control strategy in Section IV. Finally, Section V concludes the work.

II. PROBLEM FORMULATION

Consider the PVTOL aircraft described by [20]:

$$\begin{cases} \ddot{x} = -\frac{1}{M}T\sin\theta + \frac{2\sin\alpha}{M}F\cos\theta, \\ \ddot{y} = \frac{1}{M}T\cos\theta + \frac{2\sin\alpha}{M}F\sin\theta - g, \\ \ddot{\theta} = \frac{2l\cos\alpha}{J}F, \end{cases} \tag{1}$$

where (x, y) is the position of the center of mass of the aircraft, θ is the roll angle of aircraft with respect to the horizon, T and F are two control inputs, respectively standing for the thrust directed out the bottom of the aircraft and the rolling moment produced by a couple of equal forces, whose direction is not perpendicular to the horizontal body axis, but tilted by some fixed angle $\alpha \in (-\pi/2, \pi/2)$, M represents the mass of the aircraft, J the moment of inertia about the center of mass, $2l$ the distance between the wing-tips, and g the gravitational acceleration (Fig. 1).

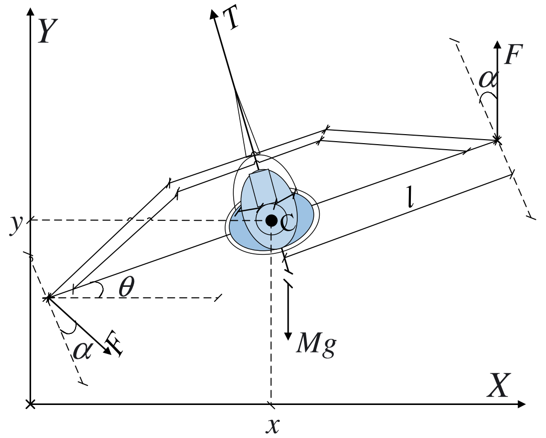

Fig. 1. The PVTOL aircraft. [Color figure can be viewed at wileyonlinelibrary.com]

Assumption 1. The modeling parameters (M, J, l, α) and the gravitational acceleration g are unknown constants with each one owning known upper/lower bounds, denoted by subscripts M/m, that is $0 < M_m \leq M \leq M_M$, $0 < J_m \leq J \leq J_M$, $0 < l_m \leq l \leq l_M$, $|\alpha| \leq \alpha_M < \pi/2$, $0 < g_m \leq g \leq g_M$.

Define the position error as

$$\bar{x} = x - x_d, \quad \bar{y} = y - y_d, \tag{2}$$

where (x_d, y_d) is the desired constant position.

The stabilization control problem of PVTOL aircraft studied in this work is stated as: under Assumption 1, find a control law $(T(\cdot), F(\cdot))$ to steer the position error, the roll angle and the corresponding velocities to zero, *i.e.*, $\lim_{t\to\infty}(\bar{x}, \bar{y}, \theta, \dot{\bar{x}}, \dot{\bar{y}}, \dot{\theta}) = 0$.

Remark 1. A state transformation [19, (1)], containing model parameters, is defined to obtain a simplified model without model parameters. Although this simplified model is actually easy to control and is widely exploited in [7–19], the resulting stabilization control schemes have to include the new defined states and hence include the model parameters, which are inapplicable to the case of unknown model parameters. Consequently, we turn to the original model (1) in order to develop a robust control scheme.

III. CONTROLLER DESIGN

In this section, first we construct the output with stable internal dynamics, and then design the sliding-mode control law to stabilize the constructed output.

3.1 Design of minimum phase output

Differentiating $(\bar{x}, \bar{y})$ in (2), one has the error dynamics

$$\begin{cases} \ddot{\bar{x}} = -aT\sin\theta + bF\cos\theta, \\ \ddot{\bar{y}} = aT\cos\theta + bF\sin\theta - g, \\ \ddot{\theta} = cF, \end{cases} \tag{3}$$

where $a = \frac{1}{M} > 0$, $b = \frac{2\sin\alpha}{M}$, $c = \frac{2l\cos\alpha}{J} > 0$. System (3) can be shown non-minimum phase for the nature output $(\bar{x}, \bar{y})$ [2]. Consequently, it is necessary to find a new output such that system (3) becomes minimum phase. To this end, we redefine the following output:

$$\begin{aligned} s_1 &= \dot{\bar{y}} + k_1\bar{y} + k_4\dot{\theta}\sin\theta, \\ s_2 &= \dot{\bar{x}} + k_2\bar{x} + k_3\theta + k_4\dot{\theta}\cos\theta, \end{aligned} \tag{4}$$

where $k_i (i = 1, 2, 3, 4)$ are design parameters to be determined such that system (3) on the manifold $s_1 \equiv s_2 \equiv 0$ is asymptotically stable (*i.e.*, system (3) with output (s_1, s_2) is minimum phase).

Differentiating (4) along (3), the error dynamics (3) can be written in the new form

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & \bar{b} \end{bmatrix} \begin{bmatrix} T \\ F \end{bmatrix} + \begin{bmatrix} f_1 - g \\ f_2 \end{bmatrix}, \tag{5a}$$

$$\begin{cases} \dot{\bar{y}} = -k_1\bar{y} - k_4\dot{\theta}\sin\theta + s_1, \\ \dot{\bar{x}} = -k_2\bar{x} - k_3\theta - k_4\dot{\theta}\cos\theta + s_2, \\ \ddot{\theta} = \frac{c}{\bar{b}}\left[\left(\dot{s}_1 - k_1\dot{\bar{y}} + g\right)\sin\theta + \left(\dot{s}_2 - k_2\dot{\bar{x}} - k_3\dot{\theta}\right)\cos\theta\right], \end{cases} \tag{5b}$$

where

$$\begin{aligned} \bar{b} &= b + k_4 c, \\ f_1 &= k_1\dot{\bar{y}} + k_4\dot{\theta}^2\cos\theta \\ &= k_1\left(s_1 - k_1\bar{y} - k_4\dot{\theta}\sin\theta\right) + k_4\dot{\theta}^2\cos\theta, \\ f_2 &= k_2\dot{\bar{x}} + k_3\dot{\theta} - k_4\dot{\theta}^2\sin\theta \\ &= k_2\left(s_2 - k_2\bar{x} - k_3\theta - k_4\dot{\theta}\cos\theta\right) + k_3\dot{\theta} - k_4\dot{\theta}^2\sin\theta. \end{aligned}$$

Lemma 1. System (3) on the invariant manifold $s_1 \equiv s_2 \equiv 0$ is regionally asymptotically stable provided that the design parameters $k_i (i = 1, 2, 3, 4)$ are selected satisfying

$$k_1 > 0, k_2 > 0, k_4 < -\frac{b}{c} = -\frac{J}{Ml}\tan\alpha, k_3 < k_2 k_4. \tag{6}$$

Proof. System (5a) implies that the relative degree of (3) with respect to output (4) is well defined and equal to (1, 1) if $a\bar{b} \neq 0$, which always holds as $a = \frac{1}{M} > 0, \bar{b} = b + k_4 c < 0$. On $s_1 \equiv s_2 \equiv 0$, one has $\dot{s}_1 \equiv \dot{s}_2 \equiv 0$, and hence can directly obtain the zero dynamics from (5b) as

$$\begin{cases} \dot{\bar{y}} = -k_1\bar{y} + H_1, \\ \dot{\bar{x}} = -k_2\bar{x} - k_3\theta - k_4\dot{\theta} + H_2, \\ \ddot{\theta} = \frac{c}{\bar{b}}\left[k_2^2\bar{x} + (g + k_2k_3)\theta + (k_2k_4 - k_3)\dot{\theta} + H_3\right], \end{cases} \tag{7}$$

where (H_1, H_2, H_3) are second- or higher-order terms with respect to $(\bar{x}, \bar{y}, \theta, \dot{\theta})$, respectively expressed as

$$\begin{aligned} H_1 &= -k_4\dot{\theta}\sin\theta, \\ H_2 &= -k_4\dot{\theta}\left(\cos\theta - 1\right), \\ H_3 &= k_1^2\bar{y}\sin\theta + \left(k_2^2\bar{x} + k_2k_3\theta - k_3\dot{\theta}\right)\left(\cos\theta - 1\right) \\ &\quad + g\left(\sin\theta - \theta\right) + k_4\left(k_1 - k_2\right)\dot{\theta}\sin^2\theta. \end{aligned}$$

Define the coordinate transformation

$$\begin{aligned} z_1 &= \frac{k_2}{g}\bar{x} + \frac{k_3}{g}\theta + \frac{\bar{b}}{cg}\dot{\theta}, \qquad z_2 = \theta, \\ z_3 &= l_1 z_1 + \left(d_3 - l_2\right) z_2 + \dot{\theta} \\ &= l_1\frac{k_2}{g}\bar{x} + \left(d_3 - l_2 + l_1\frac{k_3}{g}\right)\theta + \left(1 + l_1\frac{\bar{b}}{cg}\right)\dot{\theta}, \end{aligned} \tag{8}$$

where the two parameters (l_1, l_2) satisfy $l_1 = d_1/l_2, l_2 = \left(d_3 l_2 - d_2 + l_1\right)/l_2$ with

$$\begin{aligned} d_1 &= -\frac{c}{\bar{b}}gk_2 > 0, \qquad d_2 = -\frac{c}{\bar{b}}g > 0, \\ d_3 &= k_2 - \frac{c}{\bar{b}}\left(k_2k_4 - k_3\right) = \frac{1}{\bar{b}}\left(bk_2 + ck_3\right) > 0. \end{aligned} \tag{9}$$

It is worth mentioning that the coordinate transformation (8) is globally invertible due to $g > 0$ and $k_2 > 0$. Then, the rescaled zero dynamics becomes:

$$
\begin{aligned}
\dot{\bar{y}} &= -k_1\bar{y} + H_1, \\
\dot{z}_1 &= \frac{k_2}{g}\left(-k_2\bar{x} - k_3\theta - k_4\dot{\theta} + H_2\right) + \frac{k_3}{g}\dot{\theta} + \\
&\quad \frac{\bar{b}}{cg}\frac{c}{\bar{b}}\left[k_2^2\bar{x} + \left(g + k_2k_3\right)\theta + \left(k_2k_4 - k_3\right)\dot{\theta} + H_3\right] \\
&= \theta + \underbrace{\frac{k_2}{g}H_2 + \frac{1}{g}H_3}_{\triangleq \bar{H}_1} = z_2 + \bar{H}_1, \\
\dot{z}_2 &= \dot{\theta} = z_3 - l_1z_1 - \left(d_3 - l_2\right)z_2, \\
\dot{z}_3 &= l_1\left(\theta + \bar{H}_1\right) + \left(d_3 - l_2\right)\dot{\theta} + \frac{c}{\bar{b}}\Big[k_2^2\bar{x} \\
&\quad + \left(g + k_2k_3\right)\theta + \left(k_2k_4 - k_3\right)\dot{\theta} + H_3\Big] \\
&= l_1\theta + \left(d_3 - l_2\right)\dot{\theta} + \frac{c}{\bar{b}}gk_2\left[\frac{k_2}{g}\bar{x} + \frac{k_3}{g}\theta + \frac{\bar{b}}{cg}\dot{\theta}\right] + \\
&\quad \frac{c}{\bar{b}}g\theta + \underbrace{\left[\frac{c}{\bar{b}}\left(k_2k_4 - k_3\right) - k_2\right]}_{=-d_3}\dot{\theta} + \underbrace{l_1\bar{H}_1 + \frac{c}{\bar{b}}H_3}_{\triangleq \bar{H}_2} \\
&= l_1z_2 - l_2\dot{\theta} - d_1z_1 - d_2z_2 + \bar{H}_2 \\
&= -l_2\left[\frac{d_1}{l_2}z_1 + \left(d_3 - \frac{d_3l_2 - d_2 + l_1}{l_2}\right)z_2 + \dot{\theta}\right] + \bar{H}_2 \\
&= -l_2\left[l_1z_1 + \left(d_3 - l_2\right)z_2 + \dot{\theta}\right] + \bar{H}_2 = -l_2z_3 + \bar{H}_2.
\end{aligned} \tag{10}
$$

Now, since $k_1 > 0$ and $(H_1, \bar{H}_1, \bar{H}_2)$ are all second- or higher-order terms of the states $(\bar{y}, z_1, z_2, z_3)$, we can check the linear parts, concluding that system (10) is asymptotically stable if $l_1 > 0, d_3 - l_2 > 0$ and $l_2 > 0$ [21]. From $l_1 = d_1/l_2, l_2 = \left(d_3l_2 - d_2 + l_1\right)/l_2$, it follows that $l_2^2 = d_3l_2 - d_2 + d_1/l_2$, which means that l_2 is a solution of equation $f(l_2) = 0$, where $f\left(l_2\right) \triangleq l_2^3 - d_3l_2^2 + d_2l_2 - d_1$. Since

$$
\begin{aligned}
f\left(\frac{d_1}{d_2}\right) &= \left(\frac{d_1}{d_2}\right)^2\left(\frac{d_1}{d_2} - d_3\right) = k_2^2\frac{c}{\bar{b}}\left(k_2k_4 - k_3\right) < 0, \\
f\left(d_3\right) &= d_2d_3 - d_1 = -\frac{c}{\bar{b}}g\left[k_2 - \frac{c}{\bar{b}}\left(k_2k_4 - k_3\right)\right] + \frac{c}{\bar{b}}gk_2 \\
&= \frac{1}{\bar{b}^2}c^2g\left(k_2k_4 - k_3\right) > 0,
\end{aligned}
$$

using intermediate value theorem yields that there exists a solution, denoted by $l_2^* \in \left(\frac{d_1}{d_2}, d_3\right)$, for $f(l_2) = 0$. Consequently, we let $l_2 = l_2^*$, and naturally $l_1^* = d_1/l_2^*$, which are both positive due to $l_2^* > \frac{d_1}{d_2} > 0$. Furthermore, $l_3^* \triangleq d_3 - l_2^* > 0$ follows from $l_2^* < d_3$. As a result, (l_1^*, l_2^*, l_3^*) can be substituted for $(l_1, l_2, d_3 - l_2)$ in (10), and accordingly, system (10) is turned into the following asymptotically stable form

$$
\begin{cases}
\dot{\bar{y}} = -k_1\bar{y} + H_1, \\
\dot{z}_1 = z_2 + \bar{H}_1, \\
\dot{z}_2 = z_3 - l_1^*z_1 - l_3^*z_2, \\
\dot{z}_3 = -l_2^*z_3 + \bar{H}_2.
\end{cases} \tag{11}
$$

In the following, we will give an estimate of the attractive basin of system (11) using Lyapunov method. For this purpose, we consider the following Lyapunov function:

$$
\begin{aligned}
V_1 &= 0.5\left(\bar{y}^2 + l_1^*z_1^2 + 2\gamma_1z_1z_2 + z_2^2 + \gamma_2z_3^2\right) \\
&= 0.5Z^T\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & l_1^* & \gamma_1 & 0 \\ 0 & \gamma_1 & 1 & 0 \\ 0 & 0 & 0 & \gamma_2 \end{bmatrix}}_{\triangleq P}Z,
\end{aligned} \tag{12}
$$

where $Z = \left[\bar{y}, z_1, z_2, z_3\right]^T$, γ_1 and γ_2 satisfy

$$
0 < \gamma_1 < \min\left(\frac{l_1^*l_3^*}{l_1^* + 0.25l_3^{*2}}, \sqrt{l_1^*}\right), \tag{13a}
$$

$$
\frac{0.25\left(l_1^* - \gamma_1^2\right)}{l_2^*\left(l_1^*l_3^* - l_1^*\gamma_1 - 0.25l_3^{*2}\gamma_1\right)} < \gamma_2. \tag{13b}
$$

From (13a), we can find the numerator and denominator in (13b) are both positive, so $\gamma_2 > 0$, which, together (13a), guarantees $V_1 > 0$. Differentiating V_1 yields

$$
\begin{aligned}
\dot{V}_1 &= \bar{y}\left(-k_1\bar{y} + H_1\right) + l_1^*z_1\left(z_2 + \bar{H}_1\right) \\
&\quad + \gamma_1z_2\left(z_2 + \bar{H}_1\right) + \gamma_1z_1\left(z_3 - l_1^*z_1 - l_3^*z_2\right) \\
&\quad + z_2(z_3 - l_1^*z_1 - l_3^*z_2) + \gamma_2z_3\left(-l_2^*z_3 + \bar{H}_2\right) \\
&= -Z^T\underbrace{\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & \gamma_1l_1^* & 0.5\gamma_1l_3^* & -0.5\gamma_1 \\ 0 & 0.5\gamma_1l_3^* & l_3^* - \gamma_1 & -0.5 \\ 0 & -0.5\gamma_1 & -0.5 & \gamma_2l_2^* \end{bmatrix}}_{\triangleq Q}Z \\
&\quad \underbrace{+ Z^TP\left[H_1 \ \bar{H}_1 \ 0 \ \bar{H}_2\right]^T}_{\triangleq R}.
\end{aligned}
$$

Under (13), for the 3×3 submatrix of Q that is obtained by removing the first row and the first column of Q, its first, second, and third leading principal minors are

(1) $\gamma_1 l_1^* > 0$,
(2) $\gamma_1 l_1^* \left(l_3^* - \gamma_1\right) - 0.25 l_3^{*2} \gamma_1^2 = \gamma_1 l_1^* l_3^* - \gamma_1^2 \left(l_1^* + 0.25 l_3^{*2}\right) > \gamma_1^2 \left(l_1^* + 0.25 l_3^{*2}\right) - \gamma_1^2 \left(l_1^* + 0.25 l_3^{*2}\right) = 0$,
(3) $l_1^* l_2^* \gamma_1 \gamma_2 (l_3^* - \gamma_1) + 0.25 l_3^* \gamma_1^2 - 0.25 \gamma_1^2 (l_3^* - \gamma_1) - 0.25 l_1^* \gamma_1 - 0.25 l_2^* l_3^{*2} \gamma_1^2 \gamma_2 = l_2^* \gamma_1 \gamma_2 \left(l_1^* \left(l_3^* - \gamma_1\right) - 0.25 l_3^{*2} \gamma_1\right) + 0.25 \gamma_1 \left(\gamma_1^2 - l_1^*\right) > 0.25 \gamma_1 \left(l_1^* - \gamma_1^2\right) + 0.25 \gamma_1 \left(\gamma_1^2 - l_1^*\right) = 0$.

Combining $k_1 > 0$ and the above three inequalities, we have $Q > 0$ [22]. For R in the expression of $\dot{V}_1$, using the higher-order property of $(H_1, \bar{H}_1, \bar{H}_2)$ gives that $|R| \leq \rho \|Z\|^3$ holds for some positive constant ρ in the small neighborhood of origin $D = \left\{Z \in \mathbb{R}^4 \,|\, \|Z\| \leq r\right\}$, $r > 0$, where $\|\cdot\|$ denotes the Euclidean norm of '$\cdot$'. Thus, in the set D, $\dot{V}_1$ satisfies

$$\begin{aligned} \dot{V}_1 &\leq -\lambda_{\min}(Q) \|Z\|^2 + \rho \|Z\|^3 \\ &= -(1-\eta)\lambda_{\min}(Q)\|Z\|^2 + \|Z\|^2 \left(\rho\|Z\| - \eta\lambda_{\min}(Q)\right) \\ &\leq -(1-\eta)\lambda_{\min}(Q)\|Z\|^2, \forall \|Z\| \leq \frac{\eta\lambda_{\min}(Q)}{\rho}, 0 < \eta < 1, \end{aligned}$$

where $\lambda_{\min}(Q) > 0$ is the minimum eigenvalue of Q. Therefore, the estimate of the attractive basin of system (11) can be expressed as

$$\Omega_{c^*} = \left\{Z \in \mathbb{R}^4 \,\middle|\, V_1 \leq c^*\right\}, \tag{14}$$

where $c^* < \min\limits_{\|Z\| = r^*} V_1 = 0.5\lambda_{\min}(P) r^{*2}$ and $r^* = \min\left\{r, \eta\lambda_{\min}(Q)/\rho\right\}$. See Section 8.2 in [23] for details.

In conclusion, under the condition (6), the origin of nonlinear zero dynamics (7) (*i.e.*, system (3) on the manifold $s_1 \equiv s_2 \equiv 0$) is regionally asymptotically stable [23,24] with the estimate of the basin of attraction Ω_{c^*}. This ends the proof of Lemma 1.

Remark 2. Although the real values of model parameters are unknown and thus are inapplicable for the selection of k_4 in (6), utilizing their upper/lower bounds can give rise to a conservative but practicable condition for (6), that is

$$k_1 > 0,\ k_2 > 0,\ k_4 < k_{4M},\ k_3 < k_2 k_4, \tag{15}$$

where $k_{4M} = -\frac{J_M}{M_m l_m} \tan\left|\alpha_M\right| \leq -\frac{J}{Ml} \tan\alpha$.

Remark 3. To the best knowledge of the authors, the minimum phase output (4) is for the first time designed to stabilize PVTOL aircraft. With the aid of this output, the stabilization problem is reduced to the regulation problem of (s_1, s_2). Furthermore, the output design is quite ingenious, shown as follows. First, the terms $(\dot{\bar{y}} + k_1\bar{y}, \dot{\bar{x}} + k_2\bar{x})$ are in the filtered form. Second, the derivatives of those filtered terms plus $(k_4\dot{\theta}\sin\theta, k_4\dot{\theta}\cos\theta)$ (see (5b)) guarantee that, the coefficient matrix of control input $[T, F]^T$ can be written as the product of the known invertible matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and the unknown part $\mathrm{diag}\{a, \bar{b}\}$, which makes the sequent robust controller design easier. Third, the exclusion of θ in the expression of s_1 allows the linear part of the first equation of (7) to only contain $\bar{y}$, and the inclusion of θ in the expression of s_2 provides a possibility of making the linear parts of the last two equations of (7) asymptotically stable.

3.2 Controller design

After constructing the minimum phase output, our control objective is then reduced as: design a control law to steer the output (4) to zero. However, due to the existence of unknown parameters $(a, \bar{b}, g)$, the stabilization problem of (4) cannot be solved by traditional feedback linearization approach. To overcome this difficulty, an output transformation is firstly constructed to realize input-output decoupling, and the sliding-mode technique is then applied to stabilize the new output.

We construct a globally invertible output transformation as

$$\begin{bmatrix} \bar{s}_1 \\ \bar{s}_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \tag{16}$$

whose derivative along (5a) is computed as

$$\begin{aligned} \dot{\bar{s}}_1 &= aT + \left(f_1 - g\right)\cos\theta - f_2\sin\theta - \bar{s}_2\dot{\theta} \\ &= a\Bigg[T + \underbrace{\frac{1}{a}}_{\triangleq p_1} \underbrace{\left(f_1\cos\theta - f_2\sin\theta - \bar{s}_2\dot{\theta}\right)}_{\triangleq \delta_{11}} \\ &\quad - \underbrace{\frac{g}{a}}_{\triangleq p_2} \underbrace{\cos\theta}_{\triangleq -\delta_{12}}\Bigg] = \frac{1}{p_1}\left(T + p_1\delta_{11} + p_2\delta_{12}\right), \\ \dot{\bar{s}}_2 &= \bar{b}F + \left(f_1 - g\right)\sin\theta + f_2\cos\theta + \bar{s}_1\dot{\theta} \\ &= \bar{b}\Bigg[F + \underbrace{\frac{1}{\bar{b}}}_{\triangleq -p_3} \underbrace{\left(f_1\sin\theta + f_2\cos\theta + \bar{s}_1\dot{\theta}\right)}_{\triangleq \delta_{21}} \\ &\quad - \underbrace{\frac{g}{\bar{b}}}_{\triangleq -p_4} \underbrace{\sin\theta}_{\triangleq -\delta_{22}}\Bigg] = \frac{1}{p_3}\left(\bar{F} + p_3\delta_{21} + p_4\delta_{22}\right), \end{aligned} \tag{17}$$

where $\bar{F} = -F$ is the new input. Note that the new combination parameters $p_j (j = 1, 2, 3, 4)$ are all positive due to the facts that $M > 0, g > 0$ and $\bar{b} < 0$.

Based on (16), the output dynamics (where s_1 and s_2 are affected by both T and F) is converted as (17), which has an input-output decoupling form [21] in the sense that $\bar{s}_1$ is steered only by T, and $\bar{s}_2$ steered only by $\bar{F}$. For the more acceptable form (17), we consider non-negative functions $V_2 = 0.5p_1\bar{s}_1^2$ and $V_3 = 0.5p_3\bar{s}_2^2$, and calculate their derivatives along (17) as

$$\begin{aligned} \dot{V}_2 &= p_1\bar{s}_1\dot{\bar{s}}_1 = \bar{s}_1\left(T + p_1\delta_{11} + p_2\delta_{12}\right), \\ \dot{V}_3 &= p_3\bar{s}_2\dot{\bar{s}}_2 = \bar{s}_2\left(\bar{F} + p_3\delta_{21} + p_4\delta_{22}\right). \end{aligned} \tag{18}$$

Observing (18), we design the following control law as

$$\begin{aligned} T &= -\delta_{11}\hat{p}_1 - \delta_{12}\hat{p}_2 + T^r, \\ \bar{F} &= -\delta_{21}\hat{p}_3 - \delta_{22}\hat{p}_4 + \bar{F}^r, \end{aligned} \tag{19}$$

where

$$\hat{p}_j = 0.5(p_{jM} + p_{jm})(j = 1, 2, 3, 4) \tag{20}$$

is used to estimate the real value p_j by algebraic average of its maximum value p_{jM} and minimum value p_{jm}, and $\left(T^r, \bar{F}^r\right)$ are additional control inputs to be determined later.

Plugging (19) into (18) results in

$$\begin{aligned} \dot{V}_2 &= \bar{s}_1\left(T^r + \delta_{11}\bar{p}_1 + \delta_{12}\bar{p}_2\right), \\ \dot{V}_3 &= \bar{s}_2\left(\bar{F}^r + \delta_{21}\bar{p}_3 + \delta_{22}\bar{p}_4\right), \end{aligned} \tag{21}$$

where $\bar{p}_j = p_j - \hat{p}_j (j = 1, 2, 3, 4)$. Then, the effect of uncertain terms in $(\dot{V}_2, \dot{V}_3)$ can be canceled by choosing $(T^r, \bar{F}^r)$ as

$$\begin{aligned} T^r &= -\text{sign}\left(\bar{s}_1\right)\left(\bar{p}_{1M}\left|\delta_{11}\right| + \bar{p}_{2M}\left|\delta_{12}\right| + \varepsilon_1\right), \\ \bar{F}^r &= -\text{sign}\left(\bar{s}_2\right)\left(\bar{p}_{3M}\left|\delta_{21}\right| + \bar{p}_{4M}\left|\delta_{22}\right| + \varepsilon_2\right), \end{aligned} \tag{22}$$

where $(\varepsilon_1, \varepsilon_2)$ are both positive constants, and

$$\bar{p}_{jM} = 0.5(p_{jM} - p_{jm}) \geq 0 (j = 1, 2, 3, 4) \tag{23}$$

implies

$$\begin{aligned} \left|\bar{p}_j\right| &= \left|p_j - \hat{p}_j\right| = \left|p_j - 0.5\left(p_{jM} + p_{jm}\right)\right| \\ &= \left|0.5\left(p_j - p_{jm} - \left(p_{jM} - p_j\right)\right)\right| \\ &\leq 0.5\max\{p_j - p_{jm}, p_{jM} - p_j\} \\ &\leq 0.5\max\{p_{jM} - p_{jm}, p_{jM} - p_{jm}\} = \bar{p}_{jM}. \end{aligned}$$

With the above inequality $\left|\bar{p}_j\right| \leq \bar{p}_{jM} (j = 1, 2, 3, 4)$ and the control law (22), we can make the uncertain terms in the expression of $(\dot{V}_2, \dot{V}_3)$ non-positive, and so do $(\dot{V}_2, \dot{V}_3)$, that is

$$\begin{aligned} \dot{V}_2 &= \bar{s}_1\left(\delta_{11}\bar{p}_1 + \delta_{12}\bar{p}_2\right) - \left|\bar{s}_1\right|\left(\bar{p}_{1M}\left|\delta_{11}\right| + \bar{p}_{2M}\left|\delta_{12}\right| + \varepsilon_1\right) \\ &\leq \left|\bar{s}_1\right|\left[\left(\left|\bar{p}_1\right| - \bar{p}_{1M}\right)\left|\delta_{11}\right| + \left(\left|\bar{p}_2\right| - \bar{p}_{2M}\right)\left|\delta_{12}\right| - \varepsilon_1\right] \\ &\leq -\varepsilon_1\left|\bar{s}_1\right|, \\ \dot{V}_3 &\leq -\varepsilon_2\left|\bar{s}_2\right|. \end{aligned} \tag{24}$$

Hence, $W_1 = \sqrt{2V_2/p_1} = \left|\bar{s}_1\right|$ and $W_2 = \sqrt{2V_3/p_3} = \left|\bar{s}_2\right|$ satisfy

$$\begin{aligned} D^+W_1 &= \frac{2\dot{V}_2/p_1}{2\sqrt{2V_2/p_1}} = \frac{\dot{V}_2/p_1}{\left|\bar{s}_1\right|} \leq -\frac{\varepsilon_1}{p_1}, \\ D^+W_2 &= \frac{2\dot{V}_3/p_3}{2\sqrt{2V_3/p_3}} = \frac{\dot{V}_3/p_3}{\left|\bar{s}_2\right|} \leq -\frac{\varepsilon_2}{p_3}. \end{aligned} \tag{25}$$

Applying comparison lemma [23] to (25) shows that $W_1(t) \leq W_1(0) - \frac{\varepsilon_1}{p_1}t$, $W_2(t) \leq W_2(0) - \frac{\varepsilon_2}{p_3}t$ and $|\bar{s}_1(t)| \leq |\bar{s}_1(0)| - \frac{\varepsilon_1}{p_1}t$, $|\bar{s}_2(t)| \leq |\bar{s}_2(0)| - \frac{\varepsilon_2}{p_3}t$ for all $t \geq 0$. Therefore, the system trajectory reaches the manifold $\bar{s}_1 \equiv \bar{s}_2 \equiv 0$, and so reaches $s_1 \equiv s_2 \equiv 0$ in finite time, then remains on it thereafter.

The above obtained results can be summarized as the following lemma.

Lemma 2. The control laws (19) and (22) ensure that output (4) converges to zero in finite time provided $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$.

The actual control inputs (T, F) can be obtained by $F = -\bar{F}$, (19) and (22) as

$$\begin{aligned} T &= -\delta_{11}\hat{p}_1 - \delta_{12}\hat{p}_2 - \varepsilon_1\text{sign}\left(\bar{s}_1\right) - \\ &\quad \text{sign}\left(\bar{s}_1\right)\left(\bar{p}_{1M}\left|\delta_{11}\right| + \bar{p}_{2M}\left|\delta_{12}\right|\right), \\ F &= \delta_{21}\hat{p}_3 + \delta_{22}\hat{p}_4 + \varepsilon_2\text{sign}\left(\bar{s}_2\right) + \\ &\quad \text{sign}\left(\bar{s}_2\right)\left(\bar{p}_{3M}\left|\delta_{21}\right| + \bar{p}_{4M}\left|\delta_{22}\right|\right), \end{aligned} \tag{26}$$

where $(\hat{p}_j, \bar{p}_{jM}, j = 1, 2, 3, 4)$ are defined in (20) and (23), respectively, whose values can be calculated based on

$$\begin{aligned} p_{1M} &= M_M, & p_{1m} &= M_m, \\ p_{2M} &= M_M g_M, & p_{2m} &= M_m g_m, \\ p_{3M} &= -1/\bar{b}_M, & p_{3m} &= -1/\bar{b}_m, \\ p_{4M} &= -g_M/\bar{b}_M, & p_{4m} &= -g_m/\bar{b}_m, \end{aligned}$$
$$\bar{b}_M = 2J_M^{-1}l_m\cos\left(\alpha_M\right)\left(k_4 - k_{4M}\right),$$
$$\bar{b}_m = -2\sqrt{\left(M_m^{-1}\right)^2 + \left(k_4 l_M J_m^{-1}\right)^2}.$$

Up to now, we obtain the main result of this contribution as following.

Theorem 1. Suppose the control parameters $k_i (i = 1, 2, 3, 4)$ satisfy (15), $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, then the sliding-mode control law (26) guarantees that the position error, the roll angle and the velocity variables $(\bar{x}, \bar{y}, \theta, \dot{\bar{x}}, \dot{\bar{y}}, \dot{\theta})$ converge to zero asymptotically.

Remark 4. Compared to the existing works [12,19], where the control strategies rely on completely known model parameters, our control scheme is robust to all unknown model parameters, thanks to the smartly constructed minimum phase output (4), the input decoupling transformation (16) and the application of sliding-model technique.

IV. SIMULATION

In this section, the effectiveness and the robust characteristic of the proposed control law are demonstrated via several numerical simulation examples. Consider a PVTOL aircraft with model parameters in [20] as

$$M = 5.0 \times 10^4 \text{kg}, \quad J = 1.25 \times 10^4 \text{kg} \cdot \text{m}^2,$$
$$l = 5.0\text{m}, \quad \alpha = \pi/6, \quad g = 9.81\text{m} \cdot \text{s}^{-2}.$$

For simulation purposes, we assume that the upper and lower bounds of model parameters are known as $M_m = 4.0\times10^4$kg, $M_M = 6.0\times10^4$kg, J_m=1.0×10^4kg$\cdot$m^2, J_M= 1.5×10^4kg $\cdot$ m^2, $l_m = 4.0$m, $l_M = 6.0$m, $\alpha_M = \pi/4$, g_m=9.7633m$\cdot$s^{-2} and g_M=9.8619m$\cdot$s^{-2}. In turn, one can get $k_{4M} = -0.0937$ according to Remark 2. Next, we calculate $p_{1m} = 4.0\times10^4$, $p_{1M} = 6.0\times10^4$, $p_{2m} = 3.9053\times10^5$, $p_{2M} = 5.9171 \times 10^5$, $p_{3m} = 208.322$, $p_{3M} = 678.8138$, $p_{4m} = 2.0339 \times 10^3$ and $p_{4M} = 6.6944 \times 10^3$, leading to

$$\left(\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4\right) = (50000,\ 491120, 443.576, 4364.2),$$
$$\left(\bar{p}_{1M}, \bar{p}_{2M}, \bar{p}_{3M}, \bar{p}_{4M}\right) = (10000, 100590, 235.246, 2330.2),$$

based on (20) and (23), respectively.

The simulation is based on the following procedure: (i) by Theorem 1, select $k_1 = 0.2$, $k_2 = 0.1$, $\varepsilon_1 = 1.0 \times 10^4$, $\varepsilon_2 = 15$, $k_4 = -4$ and $k_3 = -6.5 < k_2 k_4 = -0.4$; (ii) set the initial states and the desired position as $(x, \dot{x}, y, \dot{y}, \theta, \dot{\theta})(0) = (0, 0, 0, 0, 0, 0)$ and $(x_d, y_d) = (20, 20)$; (iii) compute the control signal (26) online and apply the resulting control signals to the PVTOL aircraft system. Furthermore, to make the computer program run easier, the symbolic function 'sign(w)' is replaced by the high-slope hyperbolic tangent function 'tanh(w/0.1)'. The simulation results are obtained in Fig. 2, showing the aircraft's geometric path in the XY plane, the time responses of the states $(x - x_d, y - y_d, \theta, \dot{x}, \dot{y}, \dot{\theta})$ and the control inputs (T, F). It is clear that, under the proposed control scheme, the aircraft can move to the desired fixed position, the states $(x - x_d, y - y_d, \theta, \dot{x}, \dot{y}, \dot{\theta})$ are all bounded and convergent to zero, and the control inputs (T, F) keep bounded and respectively approach $(Mg, 0)$ with $Mg = 4.905 \times 10^5$N. It can be also observed that the proposed control strategy is robust to model parameters, in the sense that the control input (26) relies on the upper/lower bounds of model parameters instead of their accurate values.

In the following, we present the simulation results for our control strategy in case of accurate model parameters, and the results for the strategy in [19]. For both cases, the initial states and the desired position are set as above.

When having access to the knowledge of model parameters, the control law (26) can be improved by using accurate model parameters, that is

$$\begin{aligned} T &= -\delta_{11} p_1 - \delta_{12} p_2 - \varepsilon_1 \bar{s}_1, \\ F &= \delta_{21} p_3 + \delta_{22} p_4 + \varepsilon_2 \bar{s}_2. \end{aligned} \tag{27}$$

Applying (27) and selecting the control gains as above, we have the simulation results plotted in Fig. 3. Comparing Fig. 2 with Fig. 3 shows the influence of parametric uncertainties on the resulting control actions. Clearly, the time plots of control input (27) are more flat than those of control signal (26). This is because that the additional input $(T^r, \bar{F}^r)$ in (26) uses the upper bounds of uncertain terms to address the unknown parameters, leading to a big control signal.

In [19], the proposed control algorithm can guarantee the accurate PVTOL aircraft system locally asymptotically stable. For testing the performance of controller given by [19], we present two simulation examples. In the first example, we set the control gains as $m = 0.75$, $m' = 0.049$, $a = 0.35$, $b = 0.65$, $c = 0.5$, $k_2 = 50$, $k_3 = 1$ according to the condition (26) in [19], and obtain the simulation results in Fig. 4, showing that the proposed controller in [19] can achieve stabilization control in this case. In the second example, we assume that the mass of PVTOL is decreased (due to, for instance, fuel consumption) to 90% of the original value. In this situation, we keep the control gains and mass parameters used in controller unchanged, and get new results in Fig. 5, showing that the position error $y - y_d$ is not convergent to zero but divergent with time under small mass uncertainty. Combination of the two examples indicates that the control scheme provided in [19] is not robust against the model parameters.

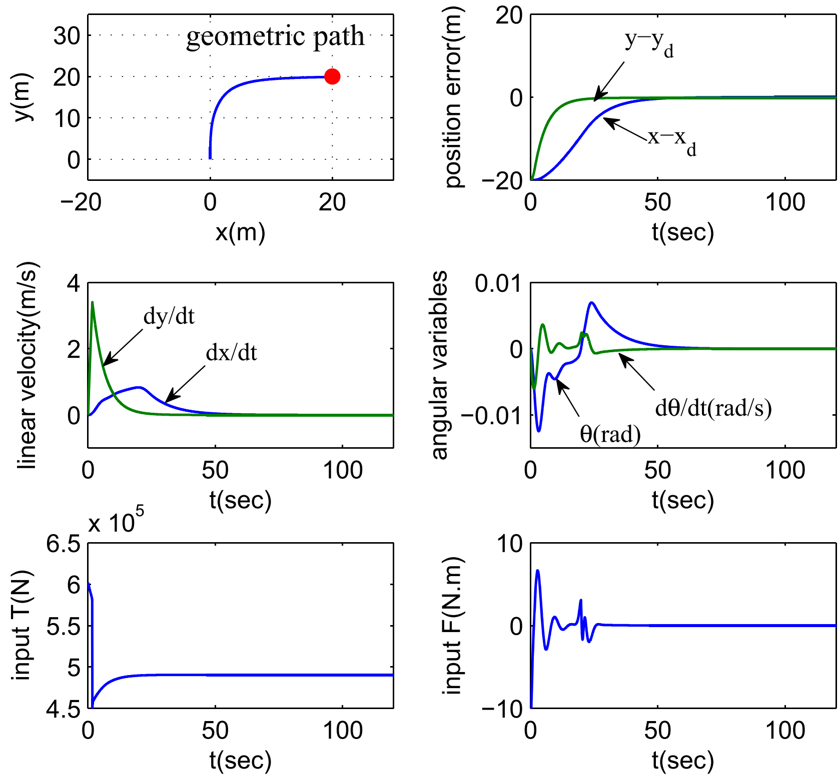

Fig. 2. Closed-loop response of a PVTOL aircraft under the control law (26). [Color figure can be viewed at wileyonlinelibrary.com]

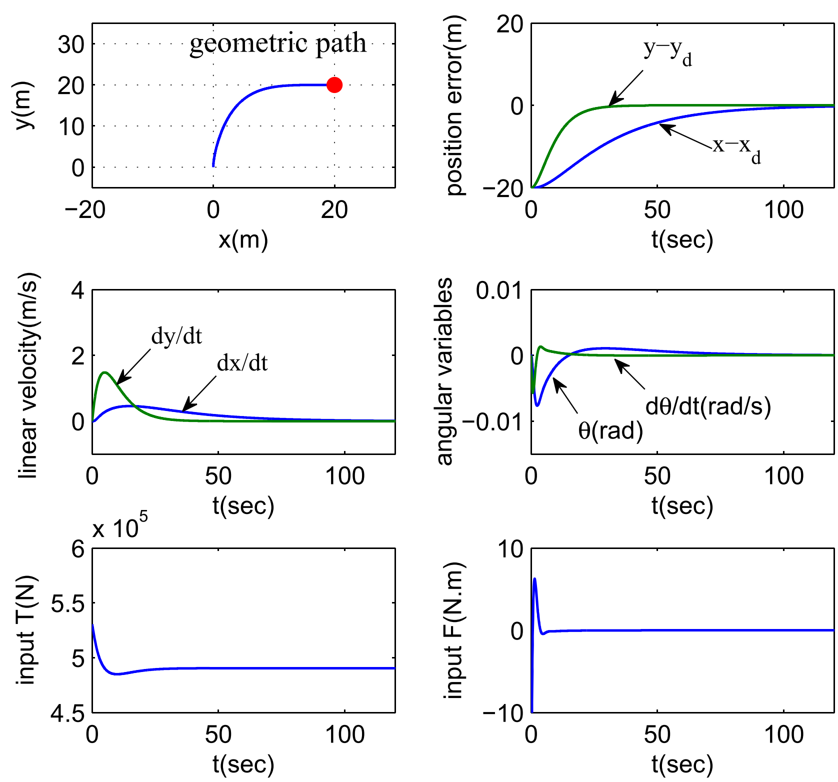

Fig. 3. Closed-loop response of a PVTOL aircraft under the control law (27). [Color figure can be viewed at wileyonlinelibrary.com]

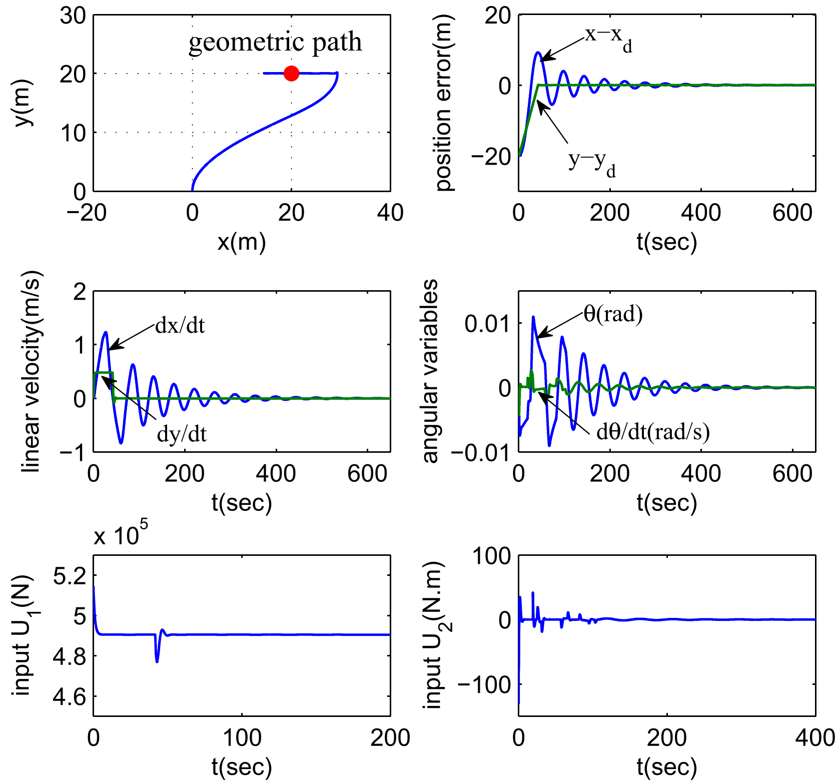

Fig. 4. Closed-loop response of a PVTOL aircraft under the control strategy in [19]. [Color figure can be viewed at wileyonlinelibrary.com]

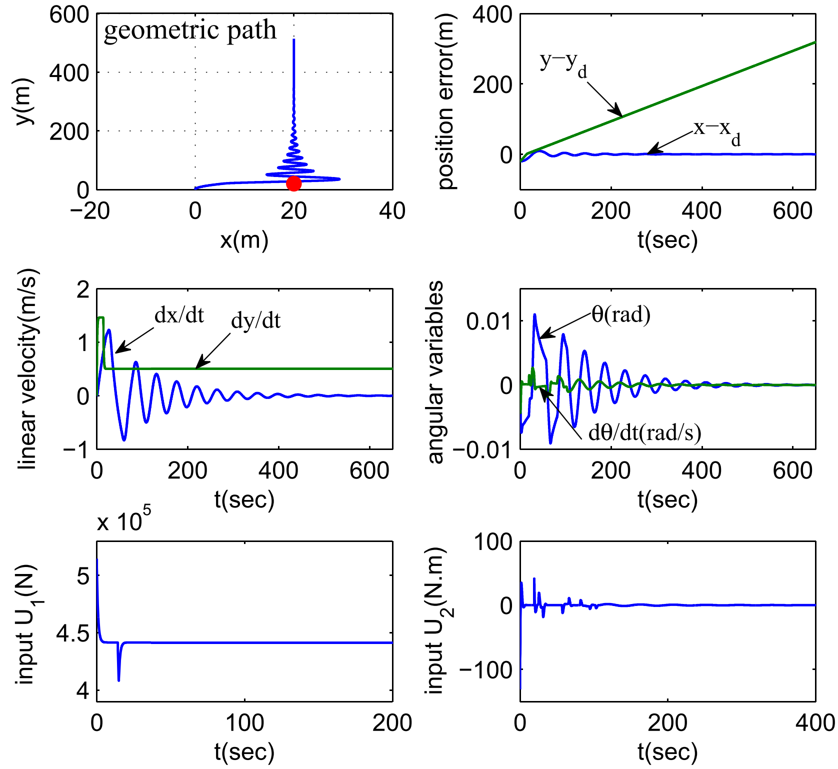

Fig. 5. Closed-loop response of a PVTOL aircraft with mass parameter uncertainty under the control strategy in [19]. [Color figure can be viewed at wileyonlinelibrary.com]

The above simulation examples show that the control scheme proposed in [19] can only work for the PVTOL aircraft with known model parameters. However, our control scheme always works well for the PVTOL aircraft no matter whether the parameters are known or not.

V. CONCLUSION

In this work, we have developed a nonlinear control approach solving the stabilization problem of PVTOL aircraft with unknown model parameters. The approach involves the sliding-mode technique and the Lyapunov approach. Using novel coordinate transformations, we design the output as a function of error states, and obtain the resulting zero dynamics and its stability condition. Applying sliding-model control, we construct a control algorithm to eliminate the effect caused by uncertain parameters, and hence to achieve the asymptotic robust stabilization. The contribution of this work lies in the fact that the proposed control law is not only capable of asymptotically steering the aircraft to the desired stationary position but also is highly robust against the model parameters. Our future research topic may focus on extending the proposed approach to solve the trajectory tracking control problem of PVTOL aircraft.

REFERENCES

1. Sanchez, A., P. Garcia, P. Castillo, and R. Lozano, "Simple real-time stabilization of vertical take-off and landing aircraft with bounded signals," *J. Guid. Control Dyn.*, Vol. 31, No. 4, pp. 1166–1176 (2008).
2. Hauser, J., S. Sastry, and G. Meyer, "Nonlinear control design for slightly non-minimum phase systems: application to V/STOL aircraft," *Automatica*, Vol. 28, No. 4, pp. 665–679 (1992).
3. Zavala-Rio, A., I. Fantoni, and R. Lozano, "Global stabilization of a PVTOL aircraft model with bounded inputs," *Int. J. Control*, Vol. 76, No. 18, pp. 1833–1844 (2003).
4. Sanchez, A., I. Fantoni, and R. Lozano, "Observer-Based control of a PVTOL aircraft," *16th IFAC World Congress*, Prague, Czech Republic, pp. 1980–1980 (2005).
5. Turker, T., H. Gorgun, and G. Cansever, "Stabilization of uncoupled PVTOL aircraft based on a Lyapunov function," *Trans. Inst. Meas. Control*, Vol. 34, No. 5, pp. 578–584 (2012).
6. Cardenas, R. and L. T. Aguilar, "Output feedback sliding mode control of a PVTOL including actuators dynamics," *IEEE Int. Conf. Control Applicat.*, Denver CO, USA, pp. 1482–1486 (2011).
7. Lopez-Araujo, D. J., A. Zavala-Rio, I. Fantoni, S. Salazar, and R. Lozano, "Global stabilization of the PVTOL aircraft with lateral force coupling and bounded inputs," *Int. J. Control*, Vol. 83, No. 7, pp. 1427–1441 (2010).
8. Turker, T., T. Oflaz, H. Gorgun, and G. Cansever, "A stabilizing controller for PVTOL aircraft," *Amer. Control Conf., Montreal, Canada*, pp. 909–913 (2012).
9. Poulin, G., A. Chemori, and N. Marchand, "Minimum energy oriented global stabilizing control of the PVTOL aircraft," *Int. J. Control*, Vol. 80, No. 3, pp. 430–442 (2007).
10. Chemori, A. and N. Marchand, "Global discrete-time stabilization of the PVTOL aircraft based on fast predictive control," *17th IFAC World Congress*, Seoul, Korea, pp. 1747–1752 (2008).
11. Chemori, A. and N. Marchand, "A prediction-based nonlinear controller for stabilization of a non-minimum phase PVTOL aircraft," *Int. J. Robust Nonlinear Control*, Vol. 18, No. 8, pp. 876–889 (2008).
12. Xin, M. and H. J. Pan, "Robust Control of PVTOL aircraft with a nonlinear optimal control solution," *J. Aerosp. Eng.*, Vol. 23, No. 4, pp. 265–275 (2009).
13. Olfati-Saber, R., "Global configuration stabilization for the VTOL aircraft with strong input coupling," *IEEE Trans. Autom. Control*, Vol. 47, No. 11, pp. 1949–1952 (2002).
14. Wood, R. and B. Cazzolato, "An alternative nonlinear control law for the global stabilization of the PVTOL vehicle," *IEEE Trans. Autom. Control*, Vol. 52, No. 7, pp. 1282–1287 (2007).
15. Lozano, R., P. Castillo, and A. Dzul, "Global stabilization of the PVTOL: real-time application to a mini-aircraft," *Int. J. Control*, Vol. 77, No. 8, pp. 735–740 (2004).
16. Frye, M. T., S. Ding, C. Qian, and S. Li, "Global finite-time stabilization of a PVTOL aircraft by output feedback," *Joint 48th IEEE Conf. Decis. Control, 28th Chinese Control Conf.*, Shanghai, China, pp. 2831–2836 (2009).
17. Frye, M. T., S. Ding, C. Qian, and S. Li, "Fast convergent observer design for output feedback stabilization of a planar vertical takeoff and landing aircraft," *IET Contr. Theory Appl.*, Vol. 4, No. 4, pp. 690–700 (2010).
18. Zavala-Rio, A., I. Fantoni, and G. Sanahuja, "Finite-time observer-based output-feedback control for the global stabilization of the PVTOL aircraft with bounded inputs," *Int. J. Syst. Sci.*, Vol. 47, No. 7, pp. 1543–1562 (2016).

19. Aguilar-Ibanez, C., J. H. Sossa-Azuela, and M. S. Suarez-Castanon, "A backstepping-based procedure with saturation functions to control the PVTOL system," *Nonlinear Dyn.*, Vol. 83, No. 3, pp. 1247–1257 (2016).
20. Marconi, L., A. Isidori, and A. Serrani, "Autonomous vertical landing on an oscillating platform: an internal-model based approach," *Automatica*, Vol. 38, No. 1, pp. 21–32 (2002).
21. Jia, Y. M., "Robust control with decoupling performance for steering and traction of 4WS vehicles under velocity-varying motion," *IEEE Trans. Control Syst. Technol.*, Vol. 8, No. 3, pp. 554–569 (2000).
22. Li, Z. C., "The properties of the block diagonal matrix," *J. Sci. Teach. Coll. Univ.*, Vol. 31, No. 6, pp. 20–22 (2011).
23. Khalil, H. K., *Nonlinear Systems*, Third edition, Prentice Hall, Upper Saddle River, NJ (2002).
24. Isidori, A., "The zero dynamics of a nonlinear system: From the origin to the latest progresses of a long successful story," *Eur. J. Control*, Vol. 19, No. 5, pp. 369–378 (2013).