SHERLOCK: Scheme using Hidden markov models for Establishing Reiteratively a List Of Candidate period-spacings with liKelihood

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Introduction

Recent space-borne missions such as the NASA Kepler and TESS mission have provided light curves of many pulsating stars, and future missions like ESA's PLATO mission are expected to increase this number even more. The so-called gravity (g)-mode pulsators are particularly interesting to study the internal stellar structure, as the periods of g~modes excited in core-hydrogen burning stars are mostly sensitive to the physics in the deep interior. These g-mode pulsators are typically modelled with a forward-modelling approach to infer fundamental stellar properties such as the mass, age, and rotation velocity Mombarg et al. (2021). Starting from a processed light curve, frequencies are extracted via iterative prewhitening. This yields a list of frequencies and their amplitudes of the g-mode pulsations, but also combination frequencies and possibly spurious ones. The stellar oscillations are described by spherical harmonics with three quantum numbers; the spherical degree, azimuthal order, and radial order. In order to compare the observed frequencies with models, the pulsation modes and the corresponding quantum numbers need to be identified. For g-mode pulsators, this is typically done by exploiting a relation between the pulsations period of consecutive radial order Miglio et al. (2008), and equal (ℓ, m). In period space, the so-called period-spacings of g-mode pulsations should follow a pattern. The next step is the search for period-spacing patterns in the frequency list is often non-trivial and is often done by a trained asteroseismologist. Yet, this means that the found period-spacing patterns are somewhat subjective, which can have consequences for the fundamental stellar parameters in the modelling. Automated routines to search for period-spacing patterns exist Garcia et al. (2022), but these do not account for any shifts in the pulsation periods due to the effect of a gradient in the mean molecular weight building up as the star evolves, or mode coupling. This can result in missing pulsation modes that are particularly interesting to test the theory of stellar structure. SHERLOCK is an open-source Python routine designed to extract to most probable period-spacing pattern based on predictions of a grid of stellar pulsation models. Moreover, it provides a metric of confidence how likely a selected period is actually part of the pattern. This way in the modelling a lower priority can be assigned to pulsation periods that are more likely to have been mistaken.

Methology

The tool presented here is based on Hidden Markov Models (HMMs). A HMM is defined by a sequence of hidden states h_i, and visible states, where the index i indicates the time step or iteration. HMM are for example used to correct typos make by the accidental pressing of an adjacent key instead. In this analogy, the hidden states h_i are the intended letters and the visible states v_i are the pressed keys. Given state h_i, the probability for a state h_{i+1} is given by, $P(h_{i+1}|h_i)$, and a referred to as the transition probability. This transition probability in this analogy can be computed by training on a dictionary to determine the probability of

₄₂ the next letter given the previous one. At each time step, the HMM emits a visible state v_i
₄₃ with probability, $P(v_i|h_i)$, which is referred to as the emission probability. This probability
₄₄ determines how likely it is to press a specific key, given an intended letter. Translating this
₄₅ example to the search for period-spacing patterns, we can think of the transmission probability
₄₆ as the probability of a period-spacing ΔP_i belonging to the pattern according to theory, given
₄₇ a previous period-spacing ΔP_{i-1}. This probability is determined from a grid of stellar pulsation
₄₈ models that provides probability distributions. The SHERLOCK algorithm works with differences
₄₉ in period-spacings,

$$\delta(\Delta P)_i = \Delta P_i - \Delta P_{i+1} = (P_{i+1} - P_i) - (P_{i+2} - P_{i+1}), \tag{1}$$

₅₀ where $P_i < P_{i+1} < P_{i+2}$. For each radial order in the grid, we compute $ (P)_1 $ and $
₅₁ (P)2 $. For an observed $ (P)\{1, obs\} $, we select the 500 values of $\delta(\Delta P)_1$ that are
₅₂ closest to the observed one. Next, we compute distribution of the values of $\delta(\Delta P)_2$, that is,
₅₃ the expected next expected difference in period-spacing. This distribution is interpolated to
₅₄ make a PDF from which the transition probability is computed. The emission probability
₅₅ is based on the number of periods in the list that falls within a 95% confidence interval,
₅₆ where periods with higher amplitudes are favoured over lower amplitude periods, as lower
₅₇ amplitude periods are more likely to be spurious. This probability is defined as,

$$P_{\text{emis},i} = \frac{A_i}{\sum_j A_j}, \tag{2}$$

₅₈ where A_i is the amplitude of period P_i, and the index j runs over all potential periods
₅₉ that fall within the search window.

₆₀ The algorithm is initiated with the periods of three consecutive radial orders, provided
₆₁ by the user. SHERLOCK will then compute a search window defined as the minimum and
₆₂ maximum period for which the probability is larger than 0.001. Then, for each candidate
₆₃ in the search window a total probability is computed,

$$P_{\text{total},i} = P_{\text{emis},i} P_{\text{trans},i}, \tag{3}$$

₆₄ and is normalised such that $\sum_i P_{\text{total},i} = 1$. The period with the highest total probability
₆₅ is then selected and the next iteration starts.

Stellar models

₆₇ In order to calculate the transmission probability, a grid of GYRE pulsation models was
₆₈ constructed to compute the periods of the stellar eigenmodes, specifically for dipole ($\ell = 1$)
₆₉ modes with $m \in [-1, 0, 1]$ and for radial orders $n_{\text{pg}} = -100$ to -1. Since the probabilites
₇₀ are based on differences in periods-spacing, period-spacing patterns of higher spherical
₇₁ degree can also be found without needing the extent the grid, as these modes behave
₇₂ the same way as dipole modes with the same sign for m. A uniform rotation profile is
₇₃ assumed, where the a randomly picked rotation rate between 0 and 0.6 times the critical
₇₄ rotation rate is picked for each stellar equilibrium model. The stellar equilibrium models
₇₅ are computed with MESA, where the mass is sampled according to a Sobol sequence between
₇₆ 3 and $9\,M_\odot$. The chemical diffusion coefficient due to core boundary mixing (overshoot) is
₇₇ described by an exponential decaying function where coefficient related to the width of the
₇₈ zone, f_{ov}, is sampled in a similar way between values of 0.005 and 0.035. The chemical
₇₉ mixing in the radiative envelope is modelled by mixing induced due to internal gravity
₈₀ waves (IGWs).

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