RadNet Decay Problems

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1 Decay

Given the number of isotopes as $\mathcal{N} = \mathcal{N}_0 e^{-\lambda t}$, t is in hours, and $\mathcal{A}$ as the time derivative of $\mathcal{N}$, we first show that the time derivative of $\mathcal{A}$ produces equation 1.

$$\frac{d\mathcal{A}}{dt} = -\lambda \mathcal{A} \tag{1}$$

$$\mathcal{N} = \mathcal{N}_0 e^{-\lambda t} \tag{2}$$
$$\frac{d\mathcal{N}}{dt} = \mathcal{A} = -\mathcal{N}_0 \lambda e^{-\lambda t} \tag{3}$$
$$\frac{d\mathcal{A}}{dt} = \lambda \mathcal{N}_0 \lambda e^{-\lambda t} = -\lambda \mathcal{A} \tag{4}$$

Now we solve the differential equation to get/check $\mathcal{A}$ again.

$$\frac{d\mathcal{A}}{dt} = -\lambda \mathcal{A} \tag{5}$$
$$\frac{d\mathcal{A}}{\mathcal{A}} = -\lambda dt \tag{6}$$
$$\ln \mathcal{A} = -\lambda t + C \tag{7}$$
$$\mathcal{A} = C e^{-\lambda t} \tag{8}$$

Using the data in Table 1 of the handout, we can now determine the constants. For α:

$$1125 = C e^{-\lambda(.25)} \tag{9}$$
$$633 = C e^{-\lambda(.75)} \tag{10}$$
$$C = \frac{1125}{e^{-\lambda(.25)}} = 1125 e^{\lambda(.25)} \tag{11}$$
$$633 = 1125 e^{\lambda(.25)} e^{-\lambda(.75)} \tag{12}$$
$$633 = 1125 e^{-\lambda(.5)} \tag{13}$$
$$\lambda = -2 \ln\left(\frac{633}{1125}\right) \tag{14}$$
$$\lambda \approx 1.1504 \tag{15}$$
$$C \approx 1125 e^{(1.1504)(.25)} \tag{16}$$
$$C \approx 1499.88 \tag{17}$$
$$\text{And because } C = \mathcal{N}_0 \lambda^2\text{:} \tag{18}$$
$$\mathcal{N}_0 = 1133.33 \tag{19}$$
$$\text{Thus for } \alpha\text{:} \tag{20}$$
$$A_\alpha \approx 1499.88 e^{-1.1504t} \tag{21}$$

Again, with the data from Table 1, we find $\mathcal{A}$ for β:

$$A_\beta = 5712.09 e^{-1.1201t} \tag{22}$$
$$\text{Where } \mathcal{N}_0 = 4552.83 \tag{23}$$

2 Determining the Rate

Given equation 24, we can solve the diffential equation to find the rate at which the particles are being gathered.

$$\frac{d\mathcal{A}}{dt} = \mathcal{R} - \lambda\mathcal{A} \tag{24}$$

$$\frac{d\mathcal{A}}{dt} + \lambda\mathcal{A} = \mathcal{R} \tag{25}$$

$$\mathcal{A} = e^{-I}\int \mathcal{R}e^{I}dt + Ce^{-I} \tag{26}$$

$$\text{Where } I = \int \lambda dt = \lambda t + c \tag{27}$$

$$\mathcal{A} = e^{-\lambda t + c}\int \mathcal{R}e^{\lambda t + c}dt + Ce^{-\lambda t + c} \tag{28}$$

$$= Ce^{-\lambda t}\left(\frac{\mathcal{R}e^{\lambda t}}{\lambda}\right) + Ce^{-\lambda t} \tag{29}$$

$$\mathcal{A} = C\left(\frac{\mathcal{R}}{\lambda} + e^{-\lambda t}\right) \tag{30}$$

Now using the data from the previous section and when $t_{\text{stop}} = 72.73$, we can solve for $\mathcal{R}$:

$$1499.88 = C\frac{\mathcal{R}}{1.504} + \cancelto{0}{Ce^{(-1.1504)(72.73)}} \tag{31}$$

$$\text{Assuming } C \text{ will be absorbed by } \mathcal{R}\text{:} \tag{32}$$

$$\mathcal{R}_\alpha = 1725.46 \tag{33}$$

$$\mathcal{R}_\beta = 6398.11 \tag{34}$$

We can now find the dose rate (in terms of pCi/m^3):

$$\mathcal{R}_\alpha/m^3 = \frac{1725.43}{60} = 28.76\frac{\text{pCi}}{m^3} \tag{35}$$

$$\mathcal{R}_\beta/m^3 = \frac{6398.11}{60} = 106.635\frac{\text{pCi}}{m^3} \tag{36}$$

We must stop here for now. This is due to the fact that we have not discussed what isotope(s) is causing the radiation. We need this information to convert the mean lifetime to rads or sieverts. We also have not discussed the amount of air breathed into the lungs.