Open Quantum Systems Answers to Exercise Set 2 & 3 - JPBK

Jake Muff
jake.muff@helsinki.fi
Student number: 015361763
10/10/2020

1 Answers

1. Problem 2.1. Obtain heat in a nearly quasi-static drive in a two level system. This question troubled me, in particular, the meaning behind "nearly quasi-static". I'm not sure what this means as a quasi-static process is one in which it is almost or nearly static or stationary so the term "nearly" here doesn't seem to have any meaning. I read the paper [4], mentioning quasistatic drive and I will assume that you wish us to study the Quasistatic limit in which W is close to the ideal limit of $-K_B Tln(2)$ or the case where there is equality $W = Q$

   There is also the case from where the Hamiltonian simply reads [10]

   $$H = E_C(n - n_g)^2$$

   For a single electron box where $E_C = \frac{e^2}{2C}$. The cited paper mentions 'sweeping' out the quasi-static drive to $n_g = 1/2$ allowing for the increase in the entropy of the charge system giving

   $$\Delta S = K_B ln(2)$$

   And due to the process being quasi-static, ΔS is equal to the increase in the entropy of the bath so

   $$\Delta Q = T\Delta S = K_B Tln(2)$$

   Maxwell's demon allows the system to be a "nearly" quasistatic driven system as the degeneracy can be 'ramped', turning the process cyclic and extracting heat of $K_B Tln(2)$ from the bath. Experimentally, Jukka finds that the average heat dissapated is

   $$Q_{avg} = -0.75 K_B Tln(2)$$

2. Problem 2.2: Proof of Landauer's Principle
   Consider a two level system with thermodynamic entropy $K_B ln(2)$, which was proven by Von Neumann in 1949. The process of the erasure of the bit gives us a starting point of a microstate where the entropy is zero.
   According to the 2nd law of thermodynamics, entropy cannot decrease - it must go somewhere-, so it is 'transferred' to the environment with temperature T giving us an energy cost of atleast $K_B Tln(2)$ so

   $$E \geq K_B Tln(2)$$

This is a very simple version where the proof as laid out by Von Neumann was not explicitely stated. Thermodynamic entropy cannot be so easily related to the information entropy. Jukka Pekola wrote a very interesting paper [3] which related the reversal to Maxwell's limit of thermodynamic efficiency and shows that it is consistent with the 2nd law of thermodynamics.
A more rigourous mathematical proof is the following:
The work extracted during isothermal quasistatic expansion is given by

$$\Delta Q = \Delta W = \int_{\frac{V}{2}}^{V} \rho dV$$

$$= \int_{\frac{V}{2}}^{V} \frac{K_B T}{V} dV$$

$$= K_B T ln(2)$$

The second law of thermodyanmics states that the entropy of an isolated system can never decrease with time. Applying this to the above fixes ΔQ

$$\Delta Q \geq K_B T ln(2)$$

$$E \geq K_B T ln(2)$$

This can also be proven from isothermal compression as well

$$\Delta Q = -\Delta W$$

$$= -\int_{v}^{\frac{V}{2}} \rho dV = -\int_{V}^{\frac{V}{2}} \frac{K_B T}{V} dV$$

$$= K_b T ln(2)$$

3. Problem 3.1.

   (a) Why CBT in the universal regime where $E_C << K_B T$ can be considered a primary thermometer?
   In a 'Primary' thermometer the measuredd property of matter is known so well that temperature can be calculated without any unknown quantities. Which is CBT we have

$$\frac{G}{G_T} = 1 - \frac{2E_C}{K_B T} g\Big(\frac{eV}{2K_B T}\Big)$$

$$g(x) = \frac{x sinh(x) - 4 sinh^2(x/2)}{8 sinh^4(x/2)}$$

   So

$$\frac{G}{G_T} = 1 - \frac{e^2/C}{K_B T} \frac{x sinh(x) - 4 sinh^2(x/2)}{8 sinh^4(x/2)}$$

Where

$$x = \frac{eV}{2K_BT}$$

And for $N = 2$ case

$$E_C = \frac{e^2}{2C}$$

In the case where $E_C << K_BT$ the normalized conductance does not depend on the CBT's dimensions and can be considered a primary thermometer (As $2E_c/K_BT \to 0$)

(b) If we consider the full width half minimum of the normalized conductance so $V = V_{1/2}$ and set this to λ

$$\frac{eV_{1/2}}{NK_BT} = \lambda$$

$$V_{1/2} = \frac{\lambda N K_B T}{e}$$

The numerical number was calculated to be

$$\lambda = 5.434$$

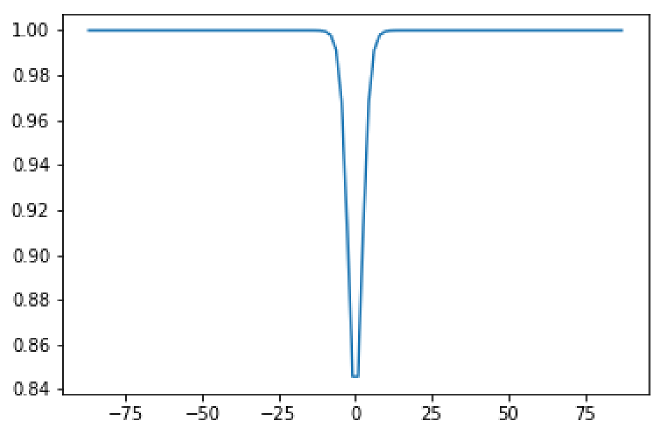

Figure 1: Plot of the normalized conductance using matplotlib. Axis values aren't quite correct but this shows an inverse bell curve. A full width half minimum approach can be used to calcuate $V_{1/2}$

2 Note

I found these exercises particularly hard, mainly due to the nature of the questions, however I believe this was due to getting use to the particular thermodynamic jargon used of which I am not used to. I found an abundance of papers and research completed online (mostly by Jukka Pekola) which taught me a great deal and helped supplement what was taught in the lectures, however did not help me to complete the questions theoretically. I have attached references in my bibliography at the bottom to works which I heavily relied on.

References

[1] Anna Feshchenko.
Electron Themometry, refrigeration and heat transport in nanostructures at sub-kelvin temperatures Doctoral Thesis, Department of Applied Physics, Low Temperature Laboratory, Aalto University

[2] M. Tinkham Introduction to Superconductivity Dover Publications, 2004

[3] Averin, Dmitri V. and Pekola, Jukka P.
Reversing the Landauer's erasure: Single-electron Maxwell's demon operating at the limit of thermodynamic efficiency. Jan 2017.

[4] Jonne V. Koskia, Ville F. Maisia, Jukka P. Pekola, and Dmitri V. Averind. *Experimental realization of a Szilard engine with a single electron.* August 2014.

[5] J. V. Koski, V. F. Maisi , T. Sagawa and J. P. Pekola. *Experimental study of mutual information in a Maxwell Demon.* May 2014.

[6] Gavin E. Crooks. *Nonequilibrium Measurements of Free Energy Differences for Microscopically Reversible Markovian Systems.* October 1997.

[7] Rogério J. de Assis, José S. Sales, Udson C. Mendes, and Norton G. de Almeida. *Two-level quantum Otto heat engine operating with unit efficiency far from the quasi-static regime under a squeezed reservoir.* March 2020.

[8] Enrique Muñoz, Francisco J. Peña and Alejandro González. *Magnetically-Driven Quantum Heat Engines: The Quasi-Static Limit of Their Efficiency* May 2016.

[9] Robert Alicki. *Introduction to Quantum Thermodynamics: History and Prospects* May 2018.

[10] Vasco Cavina Andrea Mari and Vittorio Giovannetti. *Slow dynamics and thermodynamics of open quantum systems* August 2017.

[11] Pekola, Jukka P *Towards quantum thermodynamics in electronic circuits.* 2015.