Numerical methods in scientific computing 2021

Exercise 2
Submit your solution to Moodle no later than Tuesday 2.2.2021 23:59
Exercise session: Thursday 4.2.2021

Problem 1. *(pencil and paper)* (6 points)
Show that for a vector $\mathbf{x}$ of length n

$$\lim_{p\to\infty}\left[\sum_{i=1}^{n}\left|x_i{}^{p}\right|\right]^{1/p}=\max_{1\le i\le n}(|x_i|),$$

which justifies the notation $\|\mathbf{x}\|_\infty$ for this norm.

Problem 2. *(computer)* (6 points)
In the Kahan summation algorithm the error due to the finite precision (`e`) is calculated in every iteration of the summation loop in addition to the sum itself (`s`). The algorithm can be written in C as below:

```
float s,x,y,t,e;
. . .
s= ...the first term in the sum...;
e=0.0;
for (i=2;i<=imax;i++) {
      x= ...whatever we have to sum...;
      y=x-e;
      t=s+y;
      e=(t-s)-y;
      s=t;
}
```

A) Write a function `harmonic_kahan(N)` that uses the above algorithm to calculate and return the first N terms of the harmonic sum of Exercise 1, problem 2. Put your function in a file named `harmonic_kahan`. Use single precision.
B) By using your function, check if the value of the sum saturates to a finite value as in Exercise 1. Explain your findings.

Problem 3. *(computer; C, C++ or Fortran)* (6 points)
A) Download the attached package `ex2_p3.zip`. The package contains a small program that reads in the a system and solves it using the LAPACK library. Compile (either the Fortran or C version) and solve the systems given in the files `matrix6` and `matrix100`. In your answer give and explain the compilation and run commands you used and the corresponding solution vectors $\mathbf{x}$.
B) Edit `matrix6` in such a way that it become singular. Return the matrix itself and the program output. Explain your findings.

Problem 4. *(computer; C, C++ or Fortran)* (6 points)
A) Write a function `residual(N,A,x,b,m)` that calculates and returns the

norm-*m* of the residual $(\|\mathbf{A}\,\mathbf{x}-\mathbf{b}\|_m)$ of the solution of an *N*×*N* linear system $\mathbf{A}\,\mathbf{x}=\mathbf{b}$. Use the convention that for `m=0`, the norm- ∞ is calculated. Put your function in a source file named `residual`. You can use library implementations for matrix multiplication but not for norm calculation.

B) Use your program to calculate the residual for the systems of Problem 3 for $m=1,2,\infty$.