Statistical Methods Exercises 3 Autumn 2020

Return your solutions by 12.00 Finnish time on Thursday 1.10.2020 to Moodle course page: `https://moodle.helsinki.fi/course/view.php?id=30207`

1. **Random numbers & CLT**. (a) Using an implementation of the multiplicative linear congruential method (see either lectures notes or article by *L'Ecuyer*), write a short computer program that generates 10000 random values uniformly distributed between 0 and 1, and display the result as a histogram.

   (i) Is the distribution truly uniform? Devise some good method to test the uniformity of the generated random numbers.

   (ii) Calculate the mean and variance of the random numbers. Compare them to the expectated mean and variance of a uniform distribution.

   2 random numbers independent (n, n+1)

   (iii) Calculate the correlation coefficient of each random number pair n and $n+1$ generated after each other. Any significant correlation?

   (b) Show qualitatively that the Central Limit Theorem (CLT) works by generating 1000×12 random numbers with values uniformly distributed between 0 and 1. Display the sum of each group of 12 random numbers in a histogram. Compare the distribution to a Gaussian with the expected mean and standard deviation. Does the CLT work ?

2. **Inverse transform method & Breit-Wigner distribution**. A Breit-Wigner distribution describes the energy distribution of an unstable quantum state in i.e. atomic, molecular, nuclear or particle physics

   $$dN/dE \;\propto\; \frac{(\Gamma_s)^2}{(E-E_s)^2 + (\Gamma_s/2)^2}\,,$$

   where E_s and Γ_s are the mean energy and the uncertainty on the mean energy of the state. The latter is related to the Heisenberg uncertainty principle and inversely proportional to the lifetime of the state.

   (i) Describe how to generate a Breit-Wigner (BW) distribution using the inverse transform method.

   (ii) Make a computer program that generates random numbers according to a BW distribution with $E_s = 25$ (a.u.) and $\Gamma_s = 5$ (a.u.) and plots them into a histogram. a.u. = arbitrary units.

   (iii) Fit the central part of the generated distribution with a Gaussian and compare the obtained mean μ with the original E_s and the obtained standard deviation σ with the original Γ_s.