October 14, 2020

0.1 Question 2: Kolmogorov-Smirnov Test

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[204]: import numpy as np
       import pandas as pd
       import matplotlib.pyplot as plt
       from scipy import stats
       from scipy.stats import norm
       from scipy.interpolate import interp1d
       from scipy import optimize
       from scipy.special import kolmogorov
       from scipy.stats import kstwobign
       import matplotlib.patches as mpatches
       Data1 = [80, 44, 56, 65, 33, 52, 42, 104]
       Data2 = [18, 40, 108, 70, 135, 68, 32, 30, 38, 122, 49, 91]
       Data3 = Data1+Data2
       test = []
       for x in range(181):
           test.append(x)
       Data1_sorted = np.sort(Data1)
       Data2_sorted = np.sort(Data2)
       Data3_sorted = np.sort(Data3)
       #stats.kstest(Data1, 'norm')
       kst = stats.ks_2samp(Data1, Data2)
       print(kst)
       #print(np.sort(Data1))
       #DataY1 = np.cumsum(np.sort(Data1))/ np.sum(Data1)
       #DataY2 = np.cumsum(np.sort(Data2))/np.sum(Data2)
       Datay1 = np.linspace(1/len(Data1), 1, len(Data1))
       Datay2 = np.linspace(1/len(Data2), 1, len(Data2))
       Datay3 = np.linspace(1/len(Data3), 1, len(Data3))
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F1 = interp1d(np.sort(Data1), Datay1, kind="previous", bounds_error = False, ___
\rightarrowfill_value = (0,1)) #interpolation for Data set 1
F2 = interp1d(np.sort(Data2), Datay2, kind="previous", bounds_error = False, ___
\rightarrowfill_value = (0,1)) #interpolation for data set 2
F3 = interp1d(np.sort(Data3), Datay3, kind="previous", bounds_error = False,
\rightarrowfill_value = (0,1))
diffmax = 0
for deg in Data1+Data2:
    diff = np.abs(F1(deg)-F2(deg))
    diffmax= diff if diff > diffmax else diffmax
print('KS Distance: %f' %diffmax) #i
S = np.sqrt((len(Data1)*len(Data2)) / (len(Data1)+len(Data2))) #critical values_{\square}
\rightarrow (i.e big sqrt)
P_KS = kolmogorov(diffmax*S) #here we dont use 1- F_k because kolmogorov from
→scipy gives the complementary cum dist
print('P_KS: %f' %P_KS) #ii
gamma = 0.1
thetaD = np.linspace(0,180,180)
thetaR = thetaD*(np.pi/180)
\#E = (qamma*np.sin(theta) + theta)*(180/np.pi) #integral of dist over theta
def angular_cdf(theta, gamma):
    return -0.5*(1.+gamma/2.)*(np.cos(theta) - 1.)
def angular(theta, gamma):
    return 1 + gamma*np.cos(theta)
#calculating diff_max for expected vs data3=data1+data2
KSD_A = 0
for angle in Data3:
    diff_A = np.abs(angular_cdf(angle, gamma) - F3(angle))
    KSD_A = diff_A if diff_A > KSD_A else KSD_A
print('KS Distance for angular dist: %f' %KSD_A)
S_A = np.sqrt((len(Data1)+len(Data2)))
P_KS_A = kolmogorov(KSD_A*S_A) # 0.35456057524597623
```

```
print('P_KS for angular dist: %f' %(P_KS_A))

x = np.linspace(0,180,180)
plt.plot(x, F1(x))
plt.plot(x,F2(x))
plt.plot(x, angular_cdf(thetaR,gamma))
blue_patch = mpatches.Patch(color='blue', label='Data1')
orange_patch = mpatches.Patch(color='orange', label='Data2')
green_patch = mpatches.Patch(color='green', label='CDF for Angular Dist')
plt.legend(handles=[blue_patch, orange_patch, green_patch])
plt.xlabel('Angles 0-180 deg')
plt.ylabel('Probability')
```

Ks_2sampResult(statistic=0.2916666666666667, pvalue=0.7432404540763674)

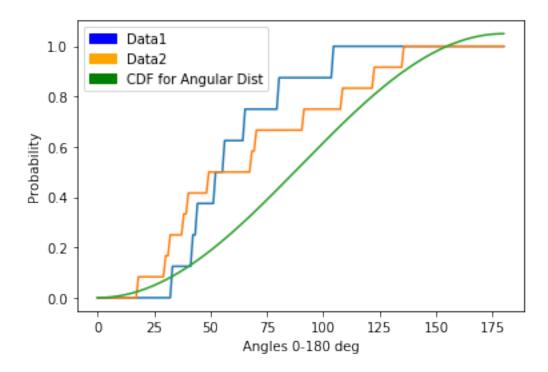
KS Distance: 0.291667

P_KS: 0.808819

KS Distance for angular dist: 0.575142

P_KS for angular dist: 0.000004

[204]: Text(0, 0.5, 'Probability')



As we can see the two angle distributions (Data1 & Data2) out calculations indicate that they do indded originate from the same source as the P value is fairly high. The null hypothesis (H_0) is that the sample follows the distribution and we accept it. Interestingly the in built stats.ks_2samp

gives a more accurate result but still similar. The test statistic is also less than the critical value up to a signifiance level of 0.001 which correlates our results.

For the combined data set vs the expected distribution we find that the two samples were from populations with different distributions i.e originate from different sources and they are not compatible.

Even though my results lead me to what I presume is the correct answer I believe my actual numbers for the P-Value and D-test statistic are off but I was unable to find out why.

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