QM IIa 2021 Problem Set 3

Solutions are due in 2 pm on Wednesday Feb 10, as a pdf file into the return box in the course Moodle page. 1.
All problems are from Sakurai's "Modern Quantum Mechanics".

1. **Periodically driven harmonic oscillator.** Consider a one-dimensional simple harmonic oscillator whose classical angular frequency is ω_0. For $t < 0$ it is known to be in the ground state. For $t > 0$ there is also a time-dependent potential
$$V(x,t) = F_0 x \cos \omega t \tag{1}$$
where F_0 is a constant in both space and time. Obtain an expression for the expectation value $\langle x \rangle$ as a function of time using time-dependent perturbation theory to lowest nonvanishing order. Is this procedure valid for $\omega = \omega_0$? You may use
$$\langle n|'|x|n|=\rangle \sqrt{(\hbar/2m\omega_0)}(\sqrt{n+1}\delta_{n',n+1} + \sqrt{n}\delta_{n',n-1}) \ . \tag{2}$$

2. Consider a particle bound in a simple harmonic oscillator potential. Initially ($t < 0$), it is in the ground state. At $t = 0$ a perturbation of the form
$$V(x,t) = Ax^2 e^{-t/\tau} \tag{3}$$
is switched on. Using time-dependent perturbation theory, calculate the probability that, after a sufficiently long time ($t \gg \tau$), the system will have made a transition to a given excited state. Consider all final states.

3. A hydrogen atom in its ground state $[(n,l,m) = (1,0,0)]$ is placed between the plates of a capacitor. A time-dependent but spatial uniform electric field (not potential!) is applied as follows:
$$\vec{E} = \begin{cases} 0 & \text{for } t < 0, \\ \vec{E}_0 e^{-t/\tau} & \text{for } t > 0 \ , \end{cases} \tag{4}$$
where $\vec{E}_0$ is in the positive z-direction. Using first-order time-dependent pertubation theory, compute the probability for the atom to be found at $t \gg \tau$ in each of the three $2p$ states: $(n,l,m) = (2,1,\pm 1 \ or \ 0)$. Repeat the problem for the $2s$ state: $(n,l,m) = (2,0,0)$. You need not attempt to evaluate radial integrals, but perform all other integrations (with respect to angles and time).

4. Consider a composite system made up of two spin $\frac{1}{2}$ objects. For $t < 0$, the Hamiltonian does not depend on spin and can be taken to be zero by suitably adjusting the energy scale. For $t > 0$, the Hamiltonian is given by
$$H = \left(\frac{4\Delta}{\hbar^2}\right) \vec{S}_1 \cdot \vec{S}_2 \ . \tag{5}$$
Suppose the system is in $|+-\rangle$ for $t \leq 0$. Find, as a function of time, the probability for being found in each of the following states $|++\rangle, |+-\rangle, |-+\rangle$, and $|--\rangle$:

(a) By solving the problem exactly.

(b) By solving the problem assuming the validity of first-order time-dependent perturbation theory with H as a perturbation switched on at $t = 0$. Under what condition does (b) give the correct results?

Clebsch Gordan Coefficients