FYMM/MMP IIIa 2020 Problem Set 4

Please submit your solutions for grading by **Monday 28.9.** in Moodle.

1. As discussed in supplementary notes, the presentation for the dihedral group D_4 is $D_4 = \langle r, f | r^4, f^2, rfrf \rangle$, where r is a rotation by 90 degrees and f is a reflection about a line through the midpoints of the edges. Generalize this for other dihedral groups D_n with $n \geq 5$. (In other words, find and motivate the presentation for D_n).

2. Draw a picture of the braid (of 4 strands) $\sigma_3 \sigma_1^{-1} \sigma_2^{-1} \sigma_3 \sigma_1 \sigma_3$.

3. Consider the left action of $SO(3)$ on the sphere $S^2 \subset \mathbb{R}^3$ defined by the matrix-times-column-vector multiplication. Parameterize the isotropy group of

$$x = \begin{pmatrix} -9/39 \\ -60/65 \\ 4/13 \end{pmatrix} .$$

4. Consider the set of *Möbius transformations*

$$\text{Mob} = \left\{ f_A : \mathbb{C} \to \mathbb{C} | f_A(z) = \frac{az+b}{cz+d}; \, A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C}) \right\} \tag{1}$$

   (a) Show that Mob is a group, with composition of mappings as the product.

   (b) Show that the mapping

$$f : SL(2, \mathbb{C}) \to \text{Mob}; \, f(A) = f_A \tag{2}$$

   is a homomorphism.

   (c) Find a subgroup H of $\mathrm{SL}(2, \mathbb{C})$ such that the quotient group $\mathrm{SL}(2, \mathbb{C})/H$ is isomorphic to Mob. Give reasons why.

5. Let V_1, V_2 be vector spaces, $L : V_1 \to V_2$ a linear map. Show that ImL and $KerL$ are vector subspaces of V_1 and V_2.