FYMM/MMP IIIa 2020 Problem Set 5

Please submit your solutions for grading by **Monday 5.10.** in Moodle.

1. Show that the set of up to *nth* order complex polynomials,

$$P_n \equiv \{a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n | a_0, a_1, \ldots, a_n \in \mathbb{C}^n\}$$

   is a vector space. What is its (complex) dimension?

2. Find a faithful representation of $\mathbb{Z}_6$ in $\mathbb{R}^2$, thinking of group elements generated by anticlockwise 60 degree rotations.

3. Show that $SL(n, \mathbb{R})$ is a normal subgroup of $GL(n, \mathbb{R})$, and identify the quotient group $GL(n, \mathbb{R})/SL(n, \mathbb{R})$. Hint: consider the determinant map $\det : GL(n, \mathbb{R}) \to \mathbb{R} \setminus \{0\}$.

4. Show that all group elements belonging to the same conjugacy class have the same order of element.

5. Show that a linear map $L : V \to V$ is an automorphism if and only if $\mathrm{Ker}\, L = \{0\}$.