Problem Set 9 Statistical Methods

Jake Muff

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1 Question 1

1. Covariance of y_1 and y_2. Note that y_2 can be rewritten

$$y_2 = \frac{1}{m}\sum_{j=n-c+1}^{m+n-c} x_j = \frac{1}{m}\sum_{j=1}^{m} x_j$$

So we have

$$cov[y_1, y_2] = E\big[(\frac{1}{n}\sum_{i=1}^{n} x_i)(\frac{1}{m}\frac{1}{m}\sum_{j=1}^{m} x_j)\big] - \mu^2$$

$$= \frac{1}{nm}\Big[(nm - n - m)\mu^2 + \mu^2(n+m) + c\sigma^2\big] - \mu^2$$

$$= \frac{c\sigma^2}{nm}$$

This method is adapted from Glen cowan's book equation 5.13. The difference arises from the c term which is introduced as an overlap between the observations. I was unsure how to calculate this for the given summation in the question, I just transformed them into more manageable ones. I haven't shown the equality between the different summations for y_2 however this is trivial.
From this we can also see that the correlation coefficient is

$$\rho = \frac{c}{\sqrt{nm}}$$

$$\sigma_1 = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_2 = \frac{\sigma}{\sqrt{m}}$$

2. The weighted average for correlated results (section 7.6 GC) is

$$\hat{\lambda} = wy_1 + (1-w)y_2$$

Where the weights w is

$$w = \frac{\sigma^2 - \rho\sigma_1\sigma_2}{2\sigma_2 - 2\rho\sigma_1\sigma_2} = \frac{1}{2}$$

So we have

$$\hat{\lambda} = \frac{1}{2}y_1 + \frac{1}{2}y_2 = \frac{y1 + y_2}{2}$$

The variance is given by

$$V[\hat{\lambda}] = \frac{1}{\frac{1}{V[y_1]} + \frac{1}{V[y_2]}} = \frac{\sigma^2}{2}$$

3. The weighted w_i are given by

$$\frac{\sum_{i,j=1}^{N} (V^{-1})_{ij}}{\sum_{k,l=1}^{N} (V^{-1})_{kl}}$$

The weights will always be positive because

$$\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{m}{n}}$$

So the correlation coefficient will always be less than or equal to the difference between the variances. The average will always be between y_1 and y_2.