FYMM/MMP IIIb 2020 Problem Set 2

Please submit your solutions for grading by **Monday 9.11.** in Moodle.

1. In this (perhaps a bit more challenging?) problem you need the following theorem (you do not need to prove it). Let $f : X \to Y$ be a homeomorphism, p a point $p \in X$. Then $f : X \setminus \{p\} \to Y \setminus \{f(p)\}$ is also a homeomorphism. Your task is to show that:

   (a) $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$

   (b) $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$ (when $n \geq 3$).

   (Hint: Use convenient topological invariants for help.)

2. What is $\pi_2(M)$ when i) $M = \mathbb{R}^3 \setminus \{1 \text{ point}\}$, ii) $M = \mathbb{T}^2$? (Hint: $\mathbb{T}^2 = S^1 \times S^1$)

3. Construct the coordinate neighbourhoods and coordinate functions for S^1 following the example in the lecture notes. Explain also in pictures. (You may also wish to check the Wikipedia entry, `https://en.wikipedia.org/wiki/Manifold`)

4. Express the vector field
$$V = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$
in *paraboloidal* coordinates (u, v, φ), the coordinate transformation is
$$x = uv\cos\varphi \ , \ y = uv\sin\varphi \ , \ z = \frac{1}{2}(u^2 - v^2).$$