# Statistical Methods Fall 2020 Answers to Problem Set 1

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1. Exercise 1

   (a) Proof of $P(A \cup B)$
   For this we mainly use Kolmogorov's third axiom as well as the nature of associativity:
   Suppose there is
   $$A \cup (B \cap A^c)$$
   Where $A^c$ is the complementary to A such that $P(A^c) = 1 - P(A)$. Then we can express $A \cup B$ as the union of two disjoint sets:
   $$A \cup (B \cap A^c) = (A \cup B) \cap (A \cup A^c) = A \cup B$$
   Which can also be written in terms of B as the union of two disjoint sets:
   $$B = B \cap (A \cup A^c) = (B \cap A) \cup (B \cap A^c)$$
   Using Axiom 3 we can write:
   $$P(A \cup B) = P(A \cup (B \cap A^c)) = P(A) + P(B \cap A^c) \tag{1}$$
   And for the $B$ part
   $$P(B) = P(B \cap A) + P(B \cap A^c)$$
   Rearranged gives:
   $$P(B \cap A^c) = P(B) - P(B \cap A)$$
   Subsituting this into (1) gives
   $$P(A \cup B) = P(A \cap (B \cap A^c)) = P(A) + P(B \cap A^c)$$
   $$= P(A) + P(B) - P(B \cap A)$$

   (b) Probability that he/she is a MacOS user given that their computer has been infected with a virus $P(MacOS|Virus)$, which, using Bayes Theorem gives
   $$P(MacOS|Virus) = \frac{P(Virus|MacOS)P(MacOS)}{P(V)}$$

Now we know $P(Virus|MacOS) = 3\%$ and the probability of having MacOS $P(MacOS) = 14\%$ and from the 2nd axiom we can figure out $P(Virus)$ from

$$P(Virus) = \sum_i P(OS_i \cup Virus) + P(OS_i \cup NoVirus) = 1$$

Where $i = 1, 2, \ldots$ is each OS. Probability virus is then $P(Virus) = 6.96\%$. So we have

$$P(MacOS|Virus) = \frac{3\% \times 12\%}{6.96\%} = 5.172\%$$

(c) Probability that he/she is not a Windows user given that they have the virus will be

$$P(NotWindows|Virus) = \frac{P(Virus|Notwindows)P(Notwindows)}{P(Virus)}$$

With

$$P(Virus|Notwindows) = P(Virus|MacOS) + P(Virus|Linux) + P(Virus|Chrome) = 9\%$$

$$P(Notwindows) = P(MacOS) + P(Linux) + P(Chrome) = 1 - P(Windows) = 20\%$$

So we have

$$P(NotWindows|Virus) = \frac{9\% \times 20\%}{6.96\%} = 25.86\%$$

2. Exercise 2

For this question we have a uniform distribution:

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

$$E[x] = \int_\alpha^\beta \frac{x}{(\beta - \alpha)} dx$$

$$V[x] = \int_\alpha^\beta \frac{[x - \frac{1}{2}(\alpha + \beta)^2]}{(\beta - \alpha)} dx$$

In our case we can take the halfway point between each strip and integrate over. In that case $\beta = \frac{d}{2}$ and $\alpha = \frac{-d}{2}$. We therefore have

$$V[x] = \int_{\frac{-d}{2}}^{\frac{d}{2}} \frac{[x - \frac{1}{2}(\frac{-d}{2} + \frac{d}{2})^2]}{(\frac{d}{2} - \frac{-d}{2})} dx$$

$$= \int_{\frac{-d}{2}}^{\frac{d}{2}} \frac{x^2}{d} dx$$

$$= \frac{d^2}{12} = \frac{d}{\sqrt{12}}$$