FYMM/MMP IIIb 2020 Problem Set 6

Please submit your solutions for grading by **Monday 7.12.** in Moodle.

1. Calculate the Riemann tensor, the Ricci tensor, and the scalar curvature using the Levi-Civita connection of the standard metric g of the unit sphere S^2. In local spherical coordinates, the metric reads

$$g = d\theta \otimes d\theta + \sin^2\theta \; d\phi \otimes d\phi \; . \tag{1}$$

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2. **Symmetry of a sphere**. Consider the metric g given in the previous exercise. Show that

$$\begin{aligned} L_1 &= -\cos\phi \frac{\partial}{\partial\theta} + \cot\theta \sin\phi \frac{\partial}{\partial\phi} \\ L_2 &= \sin\phi \frac{\partial}{\partial\theta} + \cot\theta \cos\phi \frac{\partial}{\partial\phi} \\ L_3 &= \frac{\partial}{\partial\phi} \end{aligned} \tag{2}$$

are its Killing vectors. Calculate the commutators $[L_a, L_b]$ and identify the associated symmetry.

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3. Suppose T_a are $N \times N$ matrices satisfying the commutation relation

$$[T_a, T_b] = i f_{abc} T_c \tag{3}$$

and $b_i^\dagger$ (b_i), $i = 1, \ldots, N$ are a set of creation (annihilation) operators satisfying

$$[b_i, b_j^\dagger] = \delta_{ij} \; ; \; [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0 \; . \tag{4}$$

Show that the operators

$$\chi_a \equiv \sum_{i,j=1}^{N} (T_a)_{ij} b_i^\dagger b_j \tag{5}$$

satisfy

$$[\chi_a, \chi_b] = i f_{abc} \chi_c \; . \tag{6}$$

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4. Before beginning this exercise, you will need to read the previous exercise and the beginning of section 6.3 of the lecture notes to find all the necessary definitions.

   (a) Calculate the structure constants f_{147} and f_{458} in SU(3).

   (b) Show that the Gell-Mann matrices λ_2, λ_5 and λ_7 generate an SU(2) subalgebra of SU(3).