# Open Quantum Systems   Answers to Problems 6.1 & 7.1 - JPBK

Jake Muff
jake.muff@helsinki.fi
Student number: 015361763
28/11/2020

# 1 Problem 6.1

I didn't quite understand this question and how to answer it. From my understanding, the circuit in the problem would have a bias current applied to it through one of the resisitors probably $T_1$. The two capacitors would shield the noise from the other resistor at low frequencies but at high frequencies I am not sure. I can see how to answer the question for a 1 capacitor 2 resisitor circuit but not sure how to answer with 2 capacitors as I'm not sure what effect they would have together on the noise.

# 2 Problem 7.1

*Derive the master equation for $\rho_{ge}$ Eq. (18) in the similar way as for $\rho_{gg}$ in the lecture.*
We want to evaluate

$$\dot{\rho}_{ge} = \langle g|\dot{\rho}|e\rangle$$

As in the lecture we have 4 equations

$$\mathrm{I} = -\gamma^2 \rho e^{iH_0 t'/\hbar}(a-a^\dagger)e^{iH_0(t-t')/\hbar}(a-a^\dagger)e^{-iH_0 t/\hbar}\langle \nu_n(t')\nu_n(t)\rangle$$

$$\mathrm{II} = \gamma^2 e^{iH_0 t'/\hbar}(a-a^\dagger)e^{-iH_0 t'/\hbar}\rho e^{iH_0 t/\hbar}(a-a^\dagger)e^{-iH_0 t'/\hbar}\langle \nu_n(t)\nu_n(t')\rangle$$

$$\mathrm{III} = \gamma^2 e^{iH_0 t/\hbar}(a-a^\dagger)e^{-iH_0 t/\hbar}\rho e^{iH_0 t'/\hbar})a-a^\dagger)e^{-iH_0 t'/\hbar}\langle \nu_n(t')\nu_n(t)\rangle$$

$$\mathrm{IV} = -\gamma^2 e^{iH_0 t/\hbar}(a-a^\dagger)e^{iH_0(t'-t)/\hbar}(a-a^\dagger)e^{-iH_0 t'/\hbar}\langle \nu_n(t)\nu_n(t')\rangle$$

We also have the follow facts which will prove useful

$$a^*|g\rangle = |e\rangle, a^*|e\rangle = |g\rangle$$

$$a|g\rangle = 0, a^*|g\rangle = 0$$

So now we evaluate $\langle g|\mathrm{I}\ldots\mathrm{IV}|e\rangle$ to get

$$\langle g|\mathrm{I}|e\rangle = -\gamma^2 \sum \langle g|\rho|i\rangle\langle i|e^{iH_0 t'/\hbar}(a-a^\dagger)e^{iH_0(t-t')/\hbar}(a-a^\dagger)e^{-iH_0 t/\hbar}|e\rangle\langle \nu_n(t')\nu_n(t)\rangle$$

$$= \gamma^2 \langle g|\rho|i\rangle\langle i|e^{-iH_0 t'/\hbar}(a-a^\dagger)e^{iH_0(t-t')/\hbar}|e\rangle e^{-iE_g t/\hbar}\langle \nu_n(t')\nu_n(t)\rangle$$

$$= \underbrace{\gamma^2 \langle g|\rho|i\rangle\langle i|e\rangle}_{\rho_{ge}} e^{iE_e t'/\hbar}e^{iE_g(t-t')/\hbar}e^{-iE_e t/\hbar}\langle \nu_n(t')\nu_n(t)\rangle$$

$$\langle g|\mathrm{I}|e\rangle = \gamma^2 \rho_{ge} e^{i\omega_0(t-t')}\langle \nu_n(t')\nu_n(t)\rangle$$
$$\langle g|\mathrm{II}|e\rangle = -\gamma^2 \rho_{ge} e^{i\omega_0(t-t')}\langle \nu_n(t)\nu_n(t')\rangle$$
$$\langle g|\mathrm{III}|e\rangle = -\gamma^2 \rho_{ge} e^{i\omega_0(t-t')}\langle \nu_n(t')\nu_n(t)\rangle$$
$$\langle g|\mathrm{IV}|e\rangle = \gamma^2 \rho_{ge} e^{i\omega_0(t'-t)}\langle \nu_n(t)\nu_n(t')\rangle$$

Now to get $\dot{\rho_{ge}}$ we use the following formula

$$\dot{\rho_{ge}}(t) = -\frac{1}{\hbar}^2 \int_{-\infty}^{t} dt' \Big( \sum_{k=\mathrm{I}\ldots\mathrm{IV}} \langle g|k|e\rangle \Big)$$

$$\dot{\rho_{ge}} = \frac{-\gamma^2}{\hbar}\rho_{ge}\int_{-\infty}^{t} dt' e^{i\omega_0(t-t')}\langle \nu_n(t')\nu_n(t)\rangle + e^{i\omega_0(t'-t)}\langle \nu_n(t)\nu_n(t')\rangle$$
$$-\frac{\gamma^2}{\hbar^2}\rho_{ge}\int_{-\infty}^{t} dt' e^{-i\omega_0(t-t')}\langle \nu_n(t')\nu_n(t)\rangle + e^{-i\omega_0(t'-t)}\langle \nu_n(t)\nu_n(t')\rangle$$

Double change of variables $u = t' - t$, $v = t - t'$

$$\dot{\rho_{ge}} = \frac{-2\gamma^2}{\hbar^2}\rho_{ge}\int_{-\infty}^{0} du e^{i\omega_0 u} e^{-i\omega_0 u} \underbrace{\langle \nu_n(u+t)\nu_n(t)\rangle}_{=\langle \nu_n(u)\nu_n(0)\rangle}$$
$$+ \int_{0}^{\infty} dv e^{-i\omega_0 v} e^{i\omega_0 v}\langle \nu_n(t)\nu_n(t-v)\rangle$$

Combining the integrals

$$\dot{\rho_{ge}} = -\frac{1}{2}\Big(\frac{\gamma^2}{\hbar^2}\int_{-\infty}^{\infty} du e^{-i\omega_0 u}\langle \nu_n(u)\nu_n(0)\rangle + \frac{\gamma^2}{\hbar^2}\int_{-\infty}^{\infty} du e^{-i\omega_0 u}\langle \nu_n(u)\nu_n(0)\rangle\Big)\rho_{ge}$$
$$= -\frac{1}{2}\Big(\frac{\gamma^2}{\hbar^2}S_\nu(\omega_0) + \frac{\gamma^2}{\hbar^2}S_\nu(-\omega_0)\Big)\rho_{ge}$$
$$= -\frac{1}{2}\big(\Gamma_\downarrow + \Gamma_\uparrow\big)\rho_{ge}$$

Where I have used the following facts

$$\omega_0 = \frac{E_e - E_g}{\hbar}$$
$$S_\nu(\omega_0) = \int_{-\infty}^{\infty} dt e^{i\omega_0 t}\langle \nu(t)\nu(0)\rangle$$
$$S_\nu(-\omega_0) = \int_{-\infty}^{\infty} dt e^{-i\omega_0 t}\langle \nu(t)\nu(0)\rangle$$
$$\Gamma_\downarrow = \frac{\gamma^2}{\hbar^2}S_\nu(\omega_0)$$
$$\Gamma_\uparrow = \frac{\gamma^2}{\hbar^2}S_\nu(-\omega_0)$$