Buffon's Needle

@1 Basics of Monte Carlo 2021 E1

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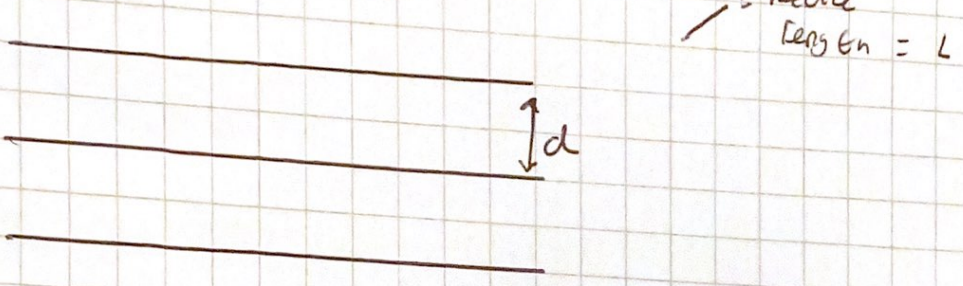
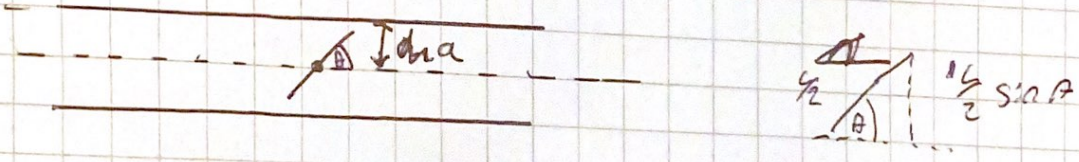
How to find $P_i = \pi_i$ from Buffon's needle when $L < d$

Show that the Probability of needle crossing a line is $P_{hit} = \frac{2L}{\pi d}$



The needle could fall in different ~~Vertical~~ vertical positions or angles

Focussing on the Midpoint of the needle



The Midpoint is always $\frac{L}{2}$ from either end.

a is the distance from the Midpoint line to one Horizontal line and can be at most $d/2$

i.e

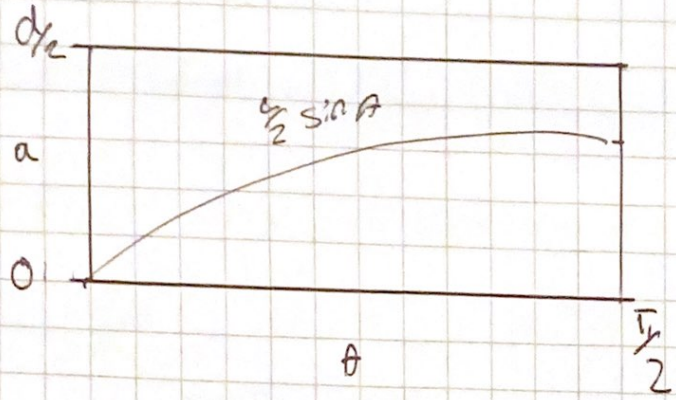
$$0 \leq a \leq d/2$$

Where the needle lands will always create an angle θ, between the needle and the midpoint line. This angle can at most be 90° or $\pi/2$ rad

$$0 < \theta < \pi/2$$

The needle will cross one horizontal lines if

$$a \leq \frac{L}{2} \sin\theta$$



$$\text{Probability} = \frac{\text{area under curve}}{\text{area under rectangle}}$$

$$A^{\text{area}}_{\text{under curve}} = \int_0^{\pi/2} \frac{L}{2} \sin\theta \, d\theta = \frac{L}{2}$$

$$A^{\text{area}}_{\text{rectangle}} = \frac{d}{2} \times \frac{\pi}{2} = \frac{d\pi}{4}$$

$$\frac{L}{2} \div \frac{d\pi}{4} = \frac{2L}{\pi d}$$

$$P_{\text{HIT}} = \frac{2L}{\pi d}$$ so now we can approximate π.

Note: Exp. The number of times a needle is thrown divided by the number of times the needle cross a line

$$\frac{N}{x} \to \frac{\pi d}{2L}$$