Statistical Methods Fall 2020 Answers to Problem Set 4

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1 Question 2: Test Statistic

For this question we have two gaussian distributions. For the pions $\mu = 0$ as it is centered at 0 and $\sigma = 1$. For the kaons $mu = 3.0$ and $\sigma = 1$.
So for this question the hypothesis H_0 is for the pions with a probability distribution of

$$g(w|H_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-0)^2}$$

And the hypothesis H_1 with a pdf

$$g(w|H_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-3)^2}$$

So that

$$H_0 = \pi \ , \ H_1 = K$$

1. What is the kaon selection efficiency when requiring $w > 2.0$?

$$\begin{aligned}\epsilon_K &= \int_{-\infty}^{2} g(w|K) dw \\ &= \int_{-\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-3)^2} \\ &= 0.158655\end{aligned}$$

Or, alternatively calculate

$$\alpha = \int_{2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-3)^2} dw$$

$$= 0.841345$$

Where

$$\epsilon_K = 1 - \alpha$$

So

$$\epsilon_K = 0.158655$$

2. What is the probability that a pion will be accepted as a kaon when requiring $w > 2.0$?

$$\int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w)^2}$$
$$= 0.0227501$$

3. Suppose a sample of particles consists of 95 % pions and 5 % kaons. What is the purity of the kaon sample selected by $w > 2.0$?
Purity from GC can be defined as

$$p_K = \frac{\int_{-\infty}^{w} a_K \cdot g(w|K)dw}{\int_{-\infty}^{w}(a_K g(w|K) + (1 - a_K)g(w|\pi))dw}$$

With $a_K = 5\%$. This gives us

$$p_K = 0.00847$$
$$p_K = 0.847\%$$

4. Design a kaon selection for the same sample that gives real kaons in at least 4 cases out of 5. What will be the requirement on w?
Question is basically asking what should the cut value be at a minimum for a sample of kaons with a purity of 80%. To answer this use the equation from GC

$$\frac{g(w|K)}{g(w|\pi)} > 0.8$$

$$\frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(w-3)^2}}{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}w^2}} > 0.8$$

Which reduces down to

$$e^{\frac{6w-9}{2}} > 0.8$$

So

$$w > 1.4256$$