Numerical methods in scientific computing 2021

Exercise 4
Return by Tuesday 16.2.2021 23:59 to Moodle
Exercise session: Thursday 18.2.2021

Problem 1. *(computer)* (8 points)
Assume the following function:

$$f(x)=\sin\left[3\pi\frac{x^3}{x^2-1}\right]+\frac{1}{2} \quad .$$

A) Write a function `bisect_f(a,b)` that implements the bisection method to find the solution of $f(x)=0$ in the interval `[a,b]` and returns the solution.
B) Write a function other `newton_f(x0)` which uses the Newton's method starting from a given point `x0` to find and return the same root. Put both your functions in a source file named "`roots`". You cannot use any library implementations of bisection or Newton methods.
C) Use both methods to obtain the two smallest values of x that satisfy $f(x)=0$ in the interval $[0,1]$. Explain your results and comment on the computational efficiency of the two methods (when the same accuracy is required).

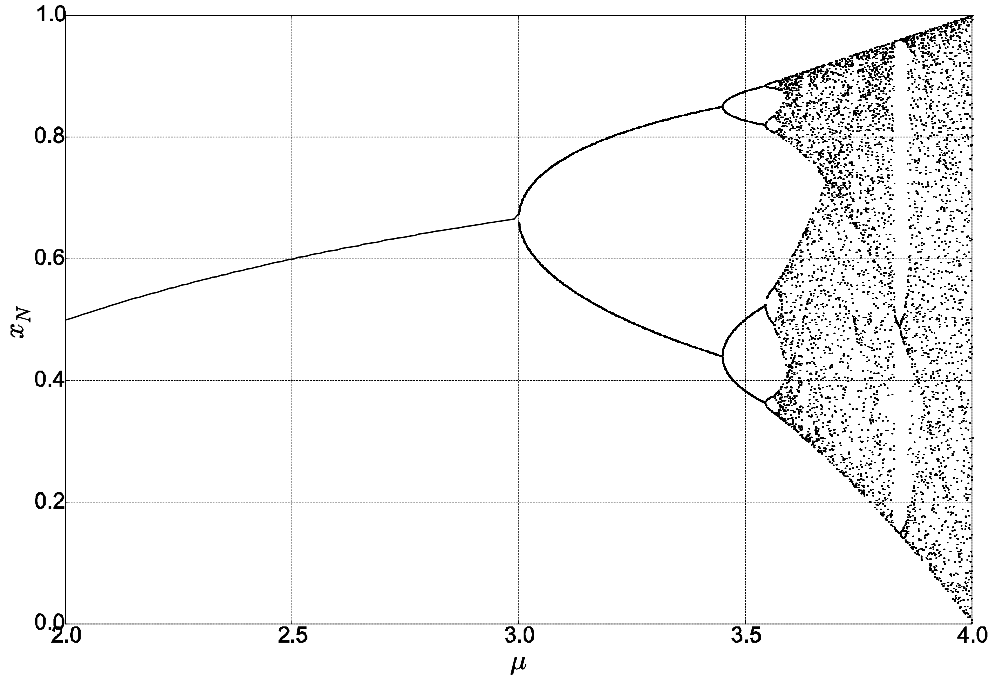
Problem 2. *(computer)* (4 points)
A) Modify your previous code and write a function `newton_g(x0,B)` which uses the Newton's method to calculate the unique zero of the function
$g(x)=x+e^{-Bx^2}\cos(x)$,
for a given parameter `B` and initial guess `x0`. Your function should return the solution and be written in the same file "`roots`" as in problem 1.
B) Calculate the zero for $B=0.1, 1, 10, 100$ and suitable initial guess. What happens if you start the iteration at $x_0=0$ and why?

Problem 3. *(pencil and paper, computer)* (6 points)

Consider the iteration

$$x_{i+1} = \mu x_i (1 - x_i) \qquad (1)$$

If we plot x_N (where *N* is a very large integer) as a function of μ, we obtain the following fractal-like figure:



Iteration (1) can also be interpreted as an application of Newton's method to find a zero of a function:

$$f(x) = 0; \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

A) Find out what is the function that produces iteration (1) when applying Newton's method.

B) Plot the real part of the function in the interval $x \in (0.3, 1)$ for μ=2.5, 3.0, 3.2, 3.5. What happens to the function when $\mu = 3$? Use your findings to explain the bifurcation behavior exhibited at that point in the above figure.

Problem 4. *(computer)* (6 points)

A) Write a function `myroots(N,p)` which calculates all the complex roots r_i of a real polynomial $P(x) = \sum_{n=0}^{N-1} p_n x^n$ for an input array `p` of size `N` by using the eigenvalue method presented in the lecture notes. For the calculation of the eigenvalues of a matrix you can use a library function. Your function shall return a complex array containing the N roots.

B) Show that your function gives correct results.