PAP334 – Exercises 1 – Model answers

Problem 1

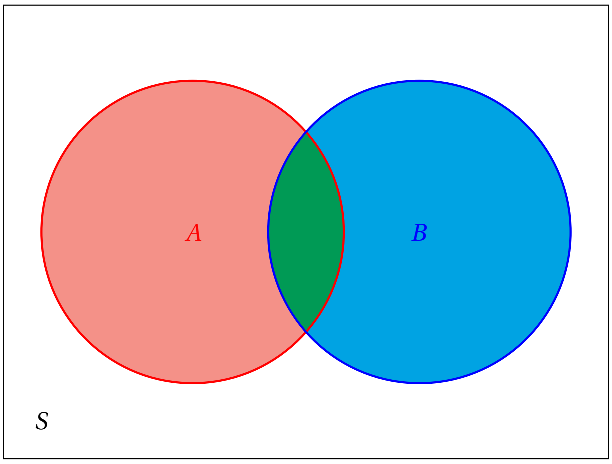
(i) Sets theory

Reminder. *Kolmogorov's axioms:*

1. *For every subset* $A \in S$, $P(A) \geq 0$,
2. *If* $A \cap B = \emptyset$, *then* $P(A \cup B) = P(A) + P(B)$,
3. $P(S) = 1$.

To show $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we may express the earlier as a union of three disjoint sets, e.g.

$$A \cup B = [A \setminus (A \cap B)] \cup [B \setminus (A \cap B)] \cup (A \cap B).$$



Then, using Kolmogorov's axiom 2, one can find

$$\begin{aligned} P(A \cup B) &= P(A \setminus (A \cap B)) + P(B \setminus (A \cap B)) + P(A \cap B) \\ &= P(A) - P(A \cap B) + P(B) - P(A \cap B) + P(A \cap B) \\ &= P(A) + P(B) - P(A \cap B) \ \square \end{aligned}$$

(ii) Bayes' theorem

We will use the following abbreviations for the various operating systems OS_i: W is Windows, M is MacOS, L is Linux, and C is ChromeOS. Furthermore, V is the set of computers infected by a virus. The following probabilities and conditional probabilities are given by the problem data: $P(OS_i)$, and $P(V|OS_i)$:

	W	M	L	C
$P(OS_i)$	0.80	0.12	0.05	0.03
$P(V\|OS_i)$	0.08	0.03	0.01	0.05

As a reminder, Bayes' theorem states

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The probability of a random user infected by a virus to be a MacOS user is $P(M|V)$, which can be rewritten as $P(V|M)P(M)/P(V)$ using the latter. Therefore, we need to compute

$$P(V) = \sum_i P(V|OS_i)P(OS_i) = 0.0696.$$

Hence, $P(M|V) = 0.0517$, or 5.2%. Equivalently, the probability of an infected user not to be the owner of a Windows machine can be expressed as

$$P(\bar{W}) = 1 - P(W|V) = 1 - P(V|W)P(W)/P(V) = 8.1\%.$$

Problem 2

The givens of the problem are the strips inter-distance d. Here, the problem is simplified through a uniform, one-dimensional definition of the detector array (with a simple binary readout). Furthermore, we may assume all particles entering the sensitive volume have a trajectory perpendicular to the strips plane. We will then use the uniform distribution to model this particles arrival, with the following probability distribution function (PDF):

$$f(x|\alpha,\beta) = \begin{cases} (\beta-\alpha)^{-1} & \text{for } \alpha \le x \le \beta \\ 0 & \text{elsewhere.} \end{cases}$$

Here, the α and β parameters can be used to depict the boundaries/edges of the silicon sensors. The mean value of this PDF, or first order moment, can be computed using

$$\mu = \int_{-\infty}^{+\infty} \mathrm{dx}\, \mathrm{x} \cdot \mathrm{f(x}|\alpha,\beta) = \int_{\alpha}^{\beta} \mathrm{dx} \frac{\mathrm{x}}{\beta-\alpha} = \frac{\alpha+\beta}{2}.$$

Equivalently, the variance may be computed as the second order moment around the mean value we just obtained:

$$\sigma^2 = \int_{-\infty}^{+\infty} \mathrm{dx} \left(\mathrm{x} - \frac{\alpha+\beta}{2}\right)^2 \cdot \mathrm{f(x}|\alpha,\beta) = \frac{1}{12}(\beta-\alpha)^2.$$

For two adjacent strips, for instance at a distance nd and $(n+1)d$ of the full detector edge, this variance takes the form

$$\sigma^2 = \frac{1}{12}\left((n+1)d - nd\right)^2 = \frac{d^2}{12},$$

which is independent from the strip position (expected from the detector geometry). Therefore, the uncertainty on the detected particle position can be expressed as the standard deviation $\sigma = d/\sqrt{12}$.