Open Quantum Systems Answers to Exercise Set 1 - JPBK

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1. Problem 1.1

   (a) Obtain
   $$I = \frac{1}{eR_T}\int d\epsilon\ n_L(\epsilon)n_R(\epsilon+eV)[f_l(\epsilon)-f_R(\epsilon+eV)]$$
   To do this we subsititute in the forward and backward tunneling rates into
   $$I = e(\Gamma_f - \Gamma_b) \tag{1}$$
   Noticing that the "transparancy" value is equal[1] to
   $$|\tau|^2 = \frac{1}{e^2R_T}$$
   $$I = e(|\tau|^2\int d\epsilon\ n_L(\epsilon)f_L(\epsilon)n_R(\epsilon+eV)[1-f_R(\epsilon+eV)]$$
   $$-|\tau|^2\int d\epsilon\ n_R(\epsilon+eV)f_R(\epsilon+eV)n_L(\epsilon)[1-f_L(\epsilon)])$$
   $$I = e|\tau|^2\int n_L(\epsilon)f_L(\epsilon)n_R(\epsilon+eV) - n_L(\epsilon)n_R(\epsilon+eV)f_R(\epsilon+eV)d\epsilon$$
   $$= e|\tau|^2\int d\epsilon\ n_L(\epsilon)n_R(\epsilon+eV)[f_L(\epsilon)-f_R(\epsilon+eV)]$$
   $$= \frac{1}{eR_T}\int d\epsilon\ n_L(\epsilon)n_R(\epsilon+eV)[f_L(\epsilon)-f_R(\epsilon+eV)]$$

2. Problem 1.2
   The left and right electrode temperatures have occuptation of energy levels as describes by the fermi distribution
   $$f_{L,R}(\epsilon) = \frac{1}{1+\exp((\epsilon-\epsilon_{f,L(R)})/k_BT_{L,R})} \tag{2}$$
   So that we define
   $$\epsilon_{F,L(R)} = 0 \text{ and } \epsilon_{f,R} = \epsilon - eV$$
   Applying electron hole symmetry we get equation 18 in the lecture notes. The density of states is approximately constant close to the fermi energy so we have

$n_{L,R}(\epsilon) = 1$.
Using the fermi distribution above and equations 18 from the lecture notes for the forward and backward tunneling rates we get

$$\Gamma_{f,b}(V) = \frac{1}{e^2 R_T} \frac{\pm eV}{1 - \exp(\mp eV/k_B T)}$$

Which is the same as

$$\Gamma = \frac{1}{e^2 R_T} \frac{eV}{1 - \exp(-eV\beta)}$$

Assuming of course that $T_L = T_R = T = \frac{1}{k_B \beta}$
Subsituting the above equation with these assumptions into (1) gives a linear relationship such that $I(V) = \frac{V}{R_T}$ showing that the junction is ohmic.

3. Problem 1.3 For problem 1.3 we are looking at a NIS junction at low temperatures where $eV, k_B T << \Delta$. For this we can use the Bardeen-Cooper-Schrieffer theory of superconductors where electrons form a condensate of cooper pairs that occupy the ground state below the critical temperature. Single electron excitations in the superconductor are seperated from the ground state by a gap labelled Δ such that

$$n_s(\epsilon) = n_s(0) \begin{cases} \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2}} & \text{for } |\epsilon| > \Delta \\ 0 & |\epsilon| < \Delta \end{cases}$$

Where $n_s(0)$ is constant and equivalent to $n_R(\epsilon)$ in the notes.
To answer this lets change some notation and introduce [2] a new factor G_{NIN} to represent the conductance for the NIN junction that is independent of V.

$$I_{NIN} = \frac{1}{eR_T} \int d\epsilon \; n_L(\epsilon) n_R(\epsilon + eV)[f_L(\epsilon) - f_R(\epsilon + eV)]$$

$$I_{NIN} = G_{NIN} V$$

We can make this relation as it is ohmic. Such that we can now write the tunnelign current for NIS as

$$I_{NIS} = \frac{G_{NIN}}{e} \int d\epsilon \; n_s(\epsilon)[f_s(\epsilon) - f_N(\epsilon + eV)]$$

And our conductance G_{NIS} for the NIS junction is

$$G_{NIS} = \frac{dI_{NIS}}{dV} = G_{NIN} \int n_s(\epsilon) \Big[- \frac{\partial f_N(\epsilon + eV)}{\partial (eV)} \Big] d\epsilon$$

The conductance at $V = 0$ is related to the width of the gap Δ through evaluating the derivatives over each other. We can now introduce our low temperature conditions.
For $k_B T << \Delta$

$$\frac{G_{NIS}}{G_{NIN}}\Big|_{V=0} = \sqrt{\frac{2\pi\Delta}{k_B T}} e^{-\Delta/k_b T}$$

For voltages $eV << \Delta$ this changes to

$$\sqrt{2\pi\Delta k_B T} e^{-(\Delta - eV)/k_B T}$$

Where $I_0 = \sqrt{2\pi\Delta k_B T}$
We could take this further if we introduced the Dynes density of states, however I have kept to using the BCS DoS

References

[1] Anna Feshchenko *Electron Themometry, refrigeration and heat transport in nanostructures at sub-kelvin temperatures* Doctoral Thesis, Department of Applied Physics, Low Temperature Laboratory, Aalto University

[2] M. Tinkham Introduction to Superconductivity Dover Publications, 2004