FYMM/MMP IIIa 2020 Problem Set 6

Please submit your solutions for grading by **Monday 12.10.** in Moodle.

1. **Mattress flipping.** Bed mattress manufacturers recommend rotating a mattress twice a year. Let us consider the mathematics of mattress flipping. In Figure 1 are depicted the three ways of rotating a mattress by 180 degrees (*i.e.*, "flipping it") around the three symmetry axis, denoted by **R, P, Y**, plus the identity transformation **I** (doing nothing). These operations form a group. (Assume that the mattress has patterns so that you can identify its upperside and underside, front and rear. Hint: you may use a sheet of paper.)

   i) Construct the Cayley (multiplication) table for the 4 operations **I,R,P,Y**.

   ii) Can you identify the group of mattress flipping operations?

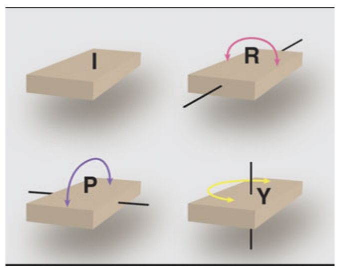

Figure 1: Mattress flipping.

2. Construct the character table of $\mathbb{Z}_2 \times \mathbb{Z}_2$.

3. In the Example in the updated lecture notes, it was suggested that the number of conjugacy classes of a permutation group is equal to the number of distinct "cycle types". Prove this statement for the permutation group S_4. How many inequivalent irreducible unitary representations does S_4 have?

4. Given a vector space V, prove that every $\omega \in (V^*)^*$ can be uniquely associated with a vector $\vec{v} \in V$ such that $\omega(f) = \langle f, \vec{v} \rangle$.

5. Let the (1,0)-tensor R have the components

   $$R^1 = a \, ; R^2 = a^2 \, ; R^3 = a^4$$

   and the (0,1)-tensor S have the components

   $$S_1 = -b \, ; S_2 = c \, ; S_3 = -d \, .$$

   Calculate all the components T^μ_ν of the (1,1)-tensor $T = R \otimes S$.