Open Quantum Systems Answers to Problems 5.1 & 5.2 - JPBK

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15/11/2020

1 Answers

1. Problem 5.1. If the microscopic energy conservation is assumed then

$$H_0 = H_e + H_p = 0$$

Therefore

$$\dot{H}_e = -\dot{H}_p$$

So we can see

$$(\varepsilon_k - \varepsilon_{k-q}) = \hbar\omega_q$$

Because if substituted into $\dot{H}_e$ we get $\dot{H}_p$ as

$$\omega_q^{1/2} \times \omega_q = \omega_q^{3/2}$$

And the extra $\hbar$ factor is canceled through the prefactor before the sum

$$\frac{-i\gamma}{\hbar}$$

So $\dot{H}_e$ is

$$\dot{H}_e = \frac{-i\gamma}{\hbar} \sum_{k,q} \omega_q^{1/2} (\varepsilon_k - \varepsilon_{k-q})(a_k^\dagger a_{k-q} c_q - a_{k-q}^\dagger a_k c_q^\dagger)$$

N.B Is equation 4 in the lecture notes correct? In the reference in the lecture notes it gives a different equation for $\dot{H}_p$.

2. Problem 5.2. The classical Jospheson junction current is given by

$$F_J = E_J(1 - cos(\phi))$$

With

$$E_J = \frac{I_c}{2e}$$

Apply a bias current which is just E_J at a non critical current so we get

$$F_J = E_J(1 - \cos\phi) - \frac{\hbar I}{2e}\phi$$

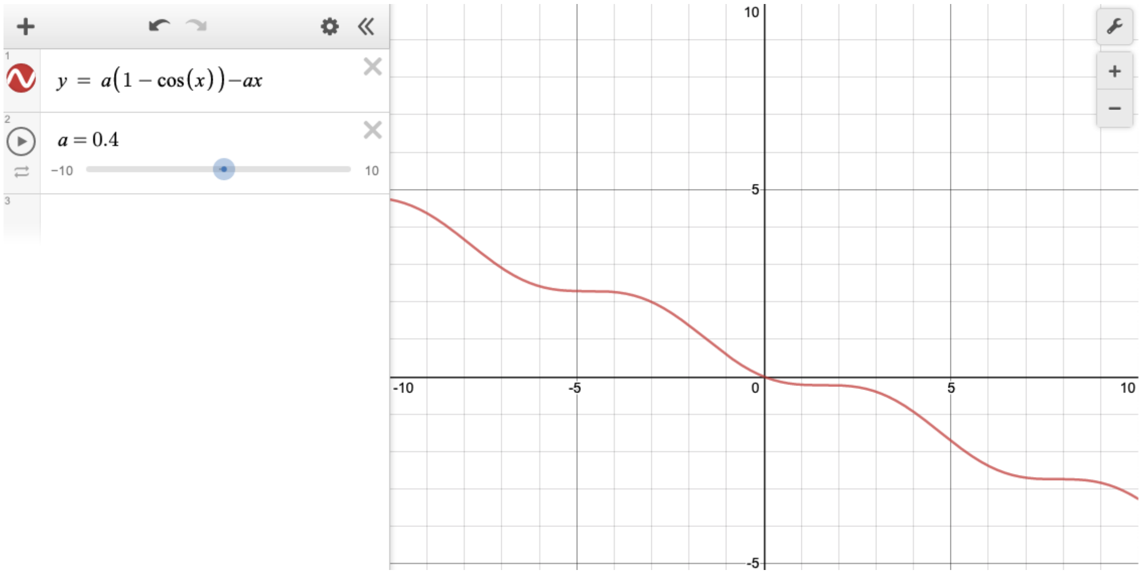

Figure 1: Plot of the Potential as a function of ϕ. The angle of tilt is given by $\frac{I}{I_c}$

This forms a washboard potential

$$U(\phi) = E_J(1 - cos\phi) - \phi\frac{I}{I_c}$$

Where $\frac{I}{I_c}$ is the normalized bias current. The first Josphenson relation describes the AC Josphenson effect

$$\hbar\dot{\phi} = 2eV$$

$$\hbar\frac{d\phi}{dt} = 2eV$$

From this we see that the critical current of the junction is when this is maximised so we can rearrange E_J so that

$$I_c = \frac{E_J 2e}{\hbar} = \frac{2e}{\hbar}E_J$$