QM IIa 2021 Problem Set 4

Solutions are due in 2 pm on Wednesday Feb 17, as a pdf file into the return box in the course Moodle page.
The problems 1-4 are from Richard L. Liboff's "Introductory Quantum Mechanics", the problem 5 is from Sakurai.

1. The scattering amplitude for a certain interaction is given by

$$f(\theta) = \frac{1}{k}(e^{ika}\sin(ka) + 3ie^{i2ka}\cos\theta)$$

where a is a characteristic length of the interaction potential and k is the wave-number of incident particles. What is the s-wave differential cross section for this interaction?

2. Analysis of the scattering of particles of mass m and energy E from a fixed scattering center with characteristic length a finds the phase shifts

$$\delta_l = \sin^{-1}\left[\frac{(iak)^l}{\sqrt{(2l+1)l!}}\right] .$$

(a) Show that the closed expression for the total cross section as a function of incident energy E is

$$\sigma = \frac{4\pi\hbar^2}{2mE}\exp\left(\frac{-2mEa^2}{\hbar^2}\right) .$$

(b) At what values of E does s-wave scattering give a good estimate of σ?

3. Apply the Born approximation

$$f(\theta) = -\frac{2m}{\hbar^2 q}\int_0^\infty dr rV(r)\sin(qr)$$

where $q = 2k\sin(\theta/2)$ to the Yukawa potential (also called the shielded Coulomb potential)

$$V(r) = -\frac{Ze^2\exp(-r/a)}{r} .$$

Show that the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{(2mZe^2/\hbar^2)^2}{[q^2+(1/a)^2]} .$$

4. An important parameter is scattering theory is the scattering length a. This length is defined as the negative of the limiting value of the scattering amplitude as the energy of the incident particle goes to zero, in other words

$$a = -\lim_{k\to 0} f(\theta) \ .$$

For low-energy scattering and relatively small phase shift, show that

$$a = -\lim_{k\to} \frac{\delta_0}{k}$$

and the total cross section can be written as

$$\sigma = 4\pi a^2 \ .$$

(Hint: s-wave scattering.)

5. A spinless particle is scattered by a time-*dependent* potential

$$V(\vec{x}, t) = V(\vec{x}) \cos(\omega t).$$

Show that if the potential is treated to first order in the transition amplitude, the energy of the scattered particle is increased or decreased by $\hbar\omega$. Obtain $d\sigma/d\Omega$.