# FYMM/MMP IIIb 2020 Problem Set 1

Please submit your solutions for grading by Monday 2.11. in Moodle (there is a link where you can do this after the exercise sheet).

1. Check whether the following $(X, \tau)$ is a topological space or not: $X = \{0, 1, 2\}$ and $\tau = \{\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$?

2. Let $\tau_{triv}$ be the trivial topology and $\tau_{disc}$ be the discrete topology. Then consider the topological spaces $X_1 = (\mathbb{R}, \tau_{disc})$ and $X_2 = (\mathbb{R}, \tau_{triv})$. Show that the identity map $id : X_1 \to X_2, x \mapsto x$ is not a homeomorphism.

3. Show that $\mathbb{R}^n$ with the usual topology is Hausdorff

4. Let $f : M \to N$ be a homeomorphism. Define a map $f_* : \pi_1(M, x_0) \to \pi_1(N, f(x_0))$ such that $f_*([\gamma]) = [f \circ \gamma])$. Show that $f_*$ is an isomorphism (i.e. $\pi_1(M, x_0) \cong \pi_1(N, f(x_0)))$. In particular, you will need to check that the map is well-defined.

5. **Examples of Homotopy groups**.

   (a) Suppose $M = \mathbb{R}^3 \setminus \{\text{point}\}$. Identify $\pi_1(M)$.

   (b) Suppose $M = S^2 \setminus \{2 \text{ points}\}$. Identify $\pi_1(M)$.

   (c) Suppose $M = \mathbb{R}^3 \setminus \{$ 2 different parallel lines $\}$. Identify $\pi_1(M)$. (*Hint:* recall the definition of groups with generators.)

   It is helpful to use deformation retracts in the reasoning. A heuristic motivation is enough, you do not need to include proofs of continuity.