Problem Set 7 Statistical Methods

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1 Question 1

1. A poisson distributed variable has pmf

$$f(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where λ is the expectation value. In our case $\lambda = v$ so we have

$$f(n,v) = \frac{v^n e^{-v}}{n!}$$

For m observations we have (log = natural logarithm here)

$$\log L(v) = \sum_{i=1}^{m} \log f(n,v)$$

$$= \sum_{i=1}^{m} \log(v) n_i - mv - \log(n_1!)$$

For 1 observation we will have

$$\log L(v) = \log(v)n - v - \log(n!)$$

We find the maximum by taking the dervative w.r.t v

$$\frac{\partial}{\partial v} \log L(v) = \frac{n}{v} - 1 = 0$$

$$\hat{v} = n$$

2. To find if $\hat{v}$ is unbiased we need to show that the expectation value of $\hat{v}$ is equal to $\hat{v}$

$$E[\hat{v}] = E[n] = n$$

The variance is

$$E[\hat{v}^2] - E[\hat{v}]^2 = n^2 - n^2 = 0$$

3. The RCF bound is (V[] is variance)

$$V[\hat{v}] \geq (1 + \frac{\partial b}{\partial v})^2 / E[-\frac{\partial^2 logL(v)}{\partial v^2}]$$

Where b is

$$b = E[\hat{v}] - v$$

$$b = n - v \to \frac{\partial b}{\partial v} = -1$$

So

$$(1 + \frac{\partial b}{\partial v})^2 = (1 + (-1))^2 = 0$$

Top partial of the fraction is 0 so RCF is 0.

$$RCF \; V[\hat{v}] = 0$$

There is equality so $\hat{v}$ is efficient.

4. In the case of m observations we have $n_1, n_2 \ldots n_m$ independent and identicially distributed poisson variables, so we have

$$\log L(v) = \sum_{i=1}^{m} (n_i \log(v) - v - log(n_i!))$$

$$= \log v \sum_{i=1}^{m} n_i - mv - \sum_{i=1}^{m} \log(n_i!)$$

Maximising by taking the derivative w.r.t v and setting equal to 0

$$\frac{\partial}{\partial v} \log L(v) = \frac{1}{v} \sum_{i=1}^{m} n_i - m = 0$$

So $\hat{v}$ is

$$\hat{v}_m = \frac{1}{m} \sum_{i=1}^{m} n_i$$

Which is essentially the mean so

$$\hat{v}_m = mean(n) = \overline{n}$$