Analyzing the Ringelmann Effect with the Repeated Measures ANOVA

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MAGA: The Package to make ANOVA great again

- The package bundles functionalities around the grand topic of one-way repeated measures ANOVA.
- Some of the functionalities have not been implemented in R yet. This package aims to fill this void.
- Each core functionality of the package represents a quantlet.
- After presenting the theory and code examples from the package, we will give a short overview of the technical implementation.

Outline

The Ringelmann Effect

- Maximilian Ringelmann (1861-1931):
  - French professor of agricultural engineering
- Findings:
  - Work performance depends on group size
  - Decreasing individual performance with increasing group size
  - Example: Pulling weights in differently sized groups

The Ringelmann Effect

- The Ringelmann Effect can be investigated with an experimental design
  - Dependent Variable: Individual performance
  - Independent Variable / Factor: Group size
  - Realization of different factor levels
- For our purpose: Data simulation

Quantlet 1: Data Simulation

- Simulation function:

```
rma_data = sim_rma_data(n = 30, k = 5, means =
    means, poly_order = NULL, noise_sd = c(155, 65,
    75, 15, 40), between_subject_sd = 60, NAs = 2)
```

- Simulate deviation between subjects:

```
mean_deviation = rnorm(n, mean = 0,
    sd = between_subject_sd)
ow_rma_data[, 2:(k + 1)] = ow_rma_data[, 2:(k + 1)]
    + mean_deviation
```

sim_rma_data

Quantlet 1: Data Simulation

- Simulate noise:

```
noise = matrix(NA, nrow = n, ncol = k)
    for (i in 1:k) {noise[, i] = rnorm(n,
    mean = 0, sd = noise_sd[i])}
ow_rma_data[, 2:(k + 1)] = ow_rma_data[, 2:(k + 1)]
    + noise
```

sim_rma_data

The Ringelmann Effect

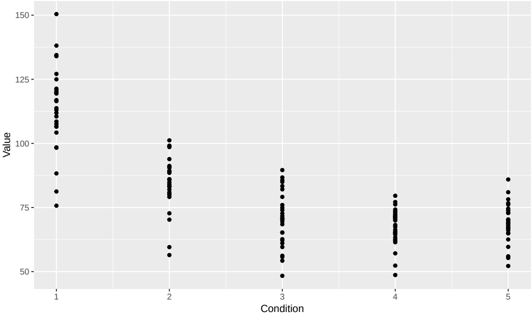

Figure 1: Simulated data.

sim_rma_data

The Repeated Measures ANOVA: Based on the ANOVA Model

- ANOVA: Analysis of Variance
- Comparison of the k factor level means
- Hypotheses:

$$
\begin{aligned}
H_0 &: \mu_1 = \mu_2 = \ldots = \mu_k \\
H_1 &: \exists i \neq j : \mu_i \neq \mu_j
\end{aligned}
$$

- Test is accomplished by decomposition of variance components
- For dependent data: Repeated Measures ANOVA

The Repeated Measures ANOVA

Subject	Factor.1	Factor.2	Factor.3	Factor.4	Factor.5
1	218.25	147.13	69.18	74.96	80.11
2	173.77	119.62	114.15	94.04	87.57
3	177.49	116.17	97.97	72.91	69.28
4	126.58	110.36	123.45	90.82	75.07
5	146.61	108.26	86.91	76.62	61.94
6	167.03	95.48	72.13	93.29	102.31

Table 1: Our simulated data consists of dependent data.

sim_rma_data

Quantlet 2: Repeated Measures ANOVA

- Repeated Measures ANOVA function: rma

```
rma(rma_data, id = 1)
```

- Define basic components:

```
grand_mean = mean(dependent_variable)
baseline_components = matrix(grand_mean, nrow = n,
    ncol = k)
conditional_means = colMeans(dependent_variable)
factor_level_components = matrix(conditional_means -
    grand_mean, nrow = n, ncol = k, byrow = TRUE)
subject_means = rowMeans(dependent_variable)
subject_components = matrix(subject_means -
    grand_mean, nrow = n, ncol = k)
```

Quantlet 2: Repeated Measures ANOVA

- Define basic components:

```
error_components = dependent_variable -
    baseline_components - factor_level_components -
    subject_components
```

rma

The Repeated Measures ANOVA

	Source	Sum of squares	Degrees of freedom	Mean squares	F-value	p-value
1	Baseline	1769610.20	1.00	1769610.20	1430.36	<0.001
2	Factor	174706.07	4.00	43676.52	142.23	<0.001
3	Subject	33403.82	27.00	1237.18		
4	Error	33166.05	108.00	307.09		
5	Total	2010886.14	140.00			
6	Corrected total	241275.94	139.00	1735.80		

Table 2: ANOVA-table for our Repeated Measures ANOVA.

The Repeated Measures ANOVA: An Advantageous Model

- Problem of ANOVA: In case of large variance between different subjects
  $\Rightarrow$ High error variance $\Rightarrow$ Loss of power in F-Test
- Repeated Measures ANOVA considers the between subject variance separately
  $\Rightarrow$ Relatively low error variance $\Rightarrow$ Gain of power in F-Test

Quantlet 3: ANOVA and SSE Reduction

- Reduction of sum of squares error function:

```
rma_sse_reduct(rma_data, id = 1,
     plot_type = "pie", return_anova_table = FALSE)
```

- ANOVA function:

```
ow_a(rma_data, id)
```

rma_sse_reduct

Quantlet 3: ANOVA and SSE Reduction

- ANOVA-tables of Repeated Measures ANOVA and ANOVA

```
ow_a_results = ow_a(rma_data, id)[[1]]
rma_results = rma(rma_data, id)[[1]]

sse_anova = ow_a_results[3, 2]
ss_subject_anova = 0

sse_rma = rma_results[4, 2]
ss_subject_rma = rma_results[3, 2]
```

rma_sse_reduct

The Repeated Measures ANOVA: An Advantageous Model

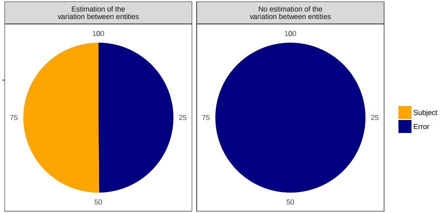

Figure 2: Pie chart on the reduction of sum of squares (SSE) in percentages.

rma_sse_reduct

The Repeated Measures ANOVA: Confidence Intervals

- The computation of the confidence intervals has to be adjusted in the Repeated Measures ANOVA rma_ci

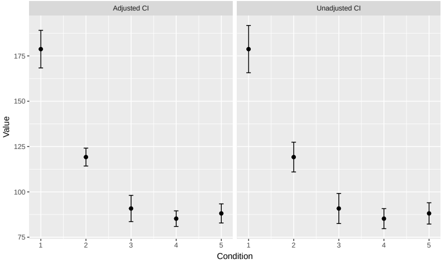

Figure 3: Unadjusted and adjusted confidence intervals.

Quantlet 4: Confidence Intervals

- Confidence interval function:

```
rma_ci(rma_data, C_level = 0.95, id = 1, print_plot
  = TRUE)
```

- Computation of adjusted standard errors:

```
cf = sqrt(k/(k - 1))

AdjVal = data.frame(Adj = (cf * ((
  rma_data_long$value - EEmlong$Em + Gm) -
  MeFlmlong$Flm)) + MeFlmlong$Flm)
```

rma_ci

The Repeated Measures ANOVA: Effect Size Measures

- Two measures of effect size:
  - η^2
  - η_p^2

	Source	eta squared	partial eta squared
1	Factor	0.72	0.84

Table 3: Effect size measures for our simulated data.

rma_effect_size

Quantlet 5: Effect Size Measures

- Effect size measures function:

```
rma_eta(rma_data, id = 1, append = FALSE)
```

- Computation of η^2 and η_p^2:

```
eta_sq = SS_Factor/SS_K_Total

eta_partial = SS_Factor/(SS_Factor + SS_Error)
```

rma_effect_size

Sphericity

- An important requirement: The variance of differences are equal for each pair of factor levels
- Test for sphericity: Mauchly test
- Measurement of sphericity ($\epsilon \in [0, 1]$):
  - Greenhouse & Geisser: ϵ_{GG}
  - Box: ϵ_B
  - Huynh & Feldt: ϵ_{HF}
- These can be used to correct the degrees of freedom and therefore adjust the p-values if sphericity is violated

Quantlet 6: Test and Adjustment for Sphericity

- Sphericity function:

```
rma_spheri(rma_data, id = 1, append = TRUE)
```

	Source	Mauchly's W	Chi square	df	p
1	Factor	0.20	41.48	9.00	0.00

Table 4: Result of the Mauchly test.

rm_spheri

Sphericity

	Source	G. - G.	Box	Huynh-Feldt
1	Epsilon	0.25	0.55	0.60
2	Adj. p-Value	0.05	$<$0.001	$<$0.001

Table 5: Adjusted p-values.

rma_spheri

Quantlet 6: Test and Adjustment for Sphericity

- Choose recommended adjustment: rma_spheri

```
if (p_w < 0.05) {
if (p_factor_lb < 0.05) {
 anova_table[, "Recommended Lower-Bound corrected p-Value (Greenhouse & Geisser, 1959))"] = c(NA, p_factor_lb, NA, NA, NA, NA)}
 else {if (epsilon_gg < 0.75) {anova_table[, "Recommended Box corrected p-Value (Geisser & Greenhouse, 1958)"] = c(NA, p_factor_gg, NA, NA, NA, NA)}
 else {anova_table[, "Recommended Huynh-Feldt corrected p-Value (Huynh & Feldt, 1976)"] = c(NA, p_factor_hf, NA, NA, NA, NA)}}}
```

Orthogonal Polynomial Contrasts

- Further analysis of factor effect
- Requirement: Level of measurement at least interval
- Factor effect can be decomposed into polynomial trend components
- Polynomial trend components can be tested by polynomial contrasts
- If there shall be no redundant information in each trend component, the contrasts have to be orthogonal
  - Maximum of orthogonal contrasts: $k - 1$

Quantlet 7: Orthogonal Polynomial Contrasts

- Contrast function:

```
rma_opc(rma_data, id = 1, maxpoly = NA, print_plot =
   TRUE)
```

Source	SS	Contribution	t	p
.L	32414.14	0.74	-15.67	0.00
.Q	10898.57	0.25	10.68	0.00
.C	360.39	0.01	-3.40	0.00
^4	3.42	27.00	-0.27	0.40

rma_opc

Plotting the Orthogonal Polynomial Curves

```
poly_plot = ggplot(data = rma_data_long, aes(x = condition, y = value)) + geom_point() + labs(col = "Order of \npolynomial", x = "Condition", y = "Value", title = "Orthogonal polynomial contrasts") +
geom_path(data = poly_curve_data, aes(x, y, color = var), lwd = 1.2) + scale_color_discrete(labels = as.character(1:(k - 1)))
```

rma_opc

Orthogonal Polynomial Contrasts

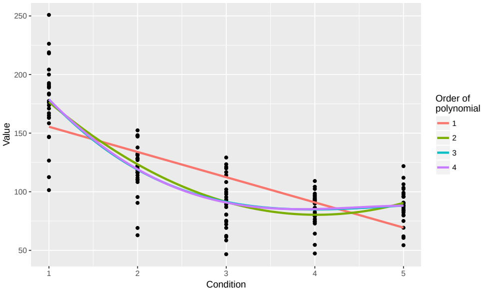

Figure 4: Orthogonal polynomial contrasts.

rma_opc

Our Package: Motivation for Making a Package

- A package bundles together code, data, documentation, and tests
- Makes it easy to share and publish code with others (CRAN, Github via Devtools)
- Loads all relevant functions into the namespace
- Automatically checks and installs dependency if necessary
- Packages allow to document functions, so that they easily be used by others (help function, argument list, etc.)

Our Package: Tools to Create a Package in R

- roxygen2
  - Enables documentation to be written directly into the R script
- devtools
  - Load packages still under development e.g. from Github
- Github
  - A package can be handled like a repository, which enables colloboration
- RStudio
  - Provides many helpful functionalities for creating a package (create, build, check)

Create a Package in R-Studio

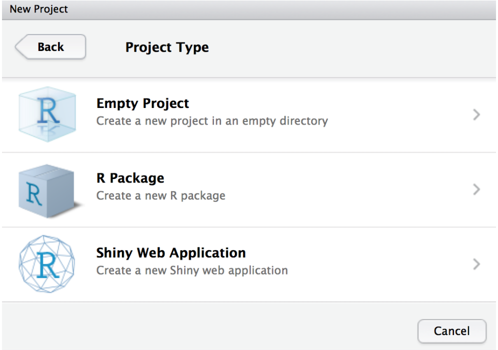

Figure 5: Select R-Package to directly create a Package

Helpfiles with roxygen2

```
#' Visualize reduction of SSE due to model selection
#'
#' Compare the sums of squared errors in regular one-way ANOVA and one-way RM ANOVA.
#'
#' @param ow_rma_data An object of type data.frame. Each row should represent one subject and each column one
#' @param id An integer specifying the column position of the subject ID. Default is 1. Set to "none" if the d
#' @param plot_type A character specifying the type of plot that is returned to visualize the error variance r
#' @param return_anova_table Logical. If TRUE, a regular oneway ANOVA table without repeated measurements is r
#'
#' @return Returns an object of type list.
#' \item{plot}{A ggplot object. The type of plot is determined by the argument plot_type}
#' \item{anova_table}{An object of type data.frame containing a regular oneway ANOVA table without repeated me
#' @author Joachim Munch, Frederik Schreck, Quang Nguyen Duc, Constantin Meyer-Grant, Nikolas Hoeft
#' @note Note that the one-way ANOVA without repeated measures is for illustration purposes only since the dat
#' @examples
#'
#'
#' @rdname ow_rma_sse_reduct
#' @export

ow_rma_sse_reduct = function(ow_rma_data){
```

Helpfiles with roxygen2

ow_rma_sse_reduct {MAGA} R Documentation

Visualize reduction of SSE due to model selection

Description

Compare the sums of squared errors in regular one-way ANOVA and one-way RM ANOVA.

Usage

```
ow_rma_sse_reduct(ow_rma_data)
```

Arguments

`ow_rma_data`	An object of type data.frame. Each row should represent one subject and each column one variable.
`id`	An integer specifying the column position of the subject ID. Default is 1. Set to "none" if the data does not contain an ID variable.
`plot_type`	A character specifying the type of plot that is returned to visualize the error variance reduction. Possible values are "pie" and "bar". Default is "pie".
`return_anova_table`	Logical. If TRUE, a regular oneway ANOVA table without repeated measurements is returned additionally. Default is FALSE.

Value

Returns an object of type list.

`plot`	A ggplot object. The type of plot is determined by the argument plot_type
`anova_table`	An object of type data.frame containing a regular oneway ANOVA table without repeated measurements

Note

Note that the one-way ANOVA without repeated measures is for illustration purposes only since the data structure is correlated across the factor levels because of the dependent measurements The ANOVA without

Package: Things to consider

- Use function names that speak for themselves and use them consistently.
  - "There are only two hard things in computer science: cache invalidation and naming things." Phil Karlton
- Error handling
  - Make sure that functions are robust regarding violation of the required input, e.g. character vector supplied although a numeric vector is needed. Use if-statements or try().
- Custom error and warning messages
  - stop() interrupts the code and returns an error message
  - warning() executes the code but returns a warning message

Thank you for your Attention!

Literature

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- Geisser, S. & Greenhouse, S. W. (1958). An extension of Box's results on the use of the F distribution in multivariate analysis. Annals of Mathematical Statistics, 29, 885-891.
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