ENTROPY 2019 - QUALIFICATION ROUND

Calculus, Optimization and Linear Algebra

The questions will be multiple choice of 4 choices, 1 or more CORRECT ANSWER for each question.

If you have any questions, please contact ENTROPY's hotline: (028) 3724 6560 or 0937 367 366

Question 12

The the maximum directional derivatives of a function f at a given point P is?	
A	1
B	undefined
C	$\nabla f(P)$
D	$\|\nabla f(P)\|$

- ○ A
- ○ B
- ◉ C
- ○ D

Question 13

Which of the following statements is TRUE?	
A	$\nabla f(a,b)$ is parallel to the graph of $z = f(x,y)$ at (a,b)
B	$\nabla f(a,b)$ is parallel to the level curve of $z = f(x,y)$ at (a,b)
C	$\nabla f(a,b)$ is perpendicular to the graph of $z = f(x,y)$ at (a,b)
D	$\nabla f(a,b)$ is perpendicular to the level curve of $z = f(x,y)$ at (a,b)

- ○ A
- ○ B
- ○ C
- ◉ D

Question 14

Which of the below constraints can be reformulated as a linear constraint in a linear optimization problem? Here, x and y denote the decision variables.	
A	$y + \cos x \leq 0.7$
B	$xy \leq 1$, where y can be positive or negative
C	$x/y \leq 1$, where y is known as positive
D	$x^y \leq 1$, where y is known as positive

- ○ A
- ○ B
- ◉ C
- ○ D

Question 15

Consider the following linear programming problem:

Maximize: $3x_1 - 2x_2$

subject to: $x_1 - x_2 = 5$

$x_1, x_2 \geq 0$

Which of the following statements is true?

A	The optimal value of the problem is 15.
B	The problem is unbounded.
C	The optimal value of the problem is 0.
D	The optimal value of the problem is 5.

- ( ) A
- (•) B
- ( ) C
- ( ) D

Question 16

Could you indicate which of the following loss functions are convex?

A	Misclassification loss
B	Logistic loss
C	Hinge loss
D	Exponential Loss with the formula $e^{-yf(x)}$

- [ ] A
- [ ] B
- [x] C
- [x] D

Question 17

Consider the following linear programming problem: Maximize: $x_1 + 2x_2$ subject to: $x_1 + x_2 \leq 5$ $x_1 + 3x_2 \leq 9$ Which is an infeasible solution for the above linear programming problem?	
A	$(x_1, x_2) = (1, 3)$
B	$(x_1, x_2) = (3, 1)$
C	$(x_1, x_2) = (1, 1)$
D	$(x_1, x_2) = (3, 2)$

- (●) A
- ( ) B
- ( ) C
- ( ) D

Question 18

Given f, f', f'' continuous function on R. Which of the following statements is TRUE?	
A	$f(x - \alpha f'(x)) \leq f(x)$ for some $\alpha > 0$ small enough
B	$f(x + \alpha f'(x)) \leq f(x)$ for all $\alpha > 0$
C	$f(x - \alpha f'(x)) = f(x)$ for all $\alpha > 0$
D	All hold TRUE

- ( ) A
- ( ) B
- ( ) C
- (●) D

Question 19

Given the sigmoid function defined as follow: $$\sigma(x) = \frac{1}{1+e^{-x}}$$ what is the derivative of sigmoid activation function?	
A	$1-\sigma(x)$
B	$\sigma(x)^2$
C	$\sigma(x)[\,1-\sigma(x)]$
D	All of the above

- ○ A
- ○ B
- ◉ C
- ○ D

Question 20

Given a multivariate optimization problem as follow: $$\min_x f(x),\ (x\ is\ \ a\ vector).$$ Which of the following statements hold true	
A	When the Hessian is negative definite, critical point is local maximizer
B	Solving this minimization problem is exactly equivalent to finding some x such that $\nabla f(x) = 0$
C	If f is continuous and twice differentiable, then the Hessian is guaranteed not to be singular
D	Steepest Descent performs poorly when the Hessian is poorly conditioned

- ○ A
- ◉ B
- ○ C

Question 21

Suppose that we have f and g be the convex functions. Which of the following statements is TRUE?	
A	f - g is convex
B	fg is convex
C	$h = \max\{f, g\}$ is convex
D	$af + bg$ for all a, b

- (●) A
- ( ) B
- ( ) C
- ( ) D

Question 22

Assume that A is a 3×3 matrix with the property that $A^2 = A$. Which of the following statements MUST be true:	
A	$A = I_3$
B	$det(A) \neq 0$
C	$det(A^3) = det(A)$
D	None of the above

- (●) A
- ( ) B
- ( ) C
- ( ) D

Question 23

Which of the following statement is TRUE about the Hessian matrix of a function f?	
A	If the Hessian is positive definite (e.g., all eigenvalues are positive) at the point x, then x is a local minimum of f
B	If the Hessian is negative definite (e.g., all eigenvalues are negative), then x is a local maximum of f
C	If the Hessian has a mix of positive and negative eigenvalues, then x is a saddle point of f
D	All of the above

- ○ A
- ○ B
- ◉ C
- ○ D

Question 24

Given a square matrix $A \in R^{n\times n}$, let x be an eigenvector of A with corresponding eigenvalue λ. Which of the following statements is not TRUE?	
A	x is an eigenvector of $A + aI$ with eigenvalue $\lambda + a$
B	$A^k x = \lambda^k x$ for any integer k
C	If A is invertible, then x is an eigenvector of $A-1$ with eigenvalue λ
D	If A is invertible, then x is an eigenvector of $A-1$ with eigenvalue $\lambda-1$

- ○ A
- ○ B
- ○ C
- ◉ D