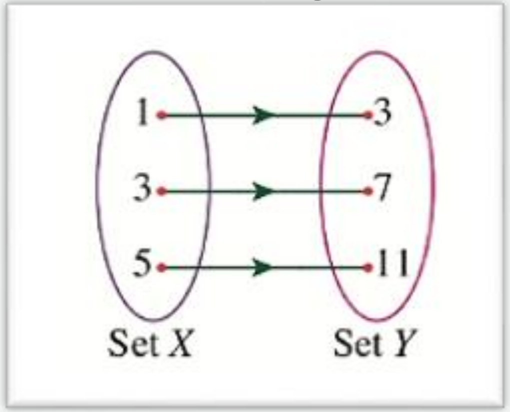
Function

Functions

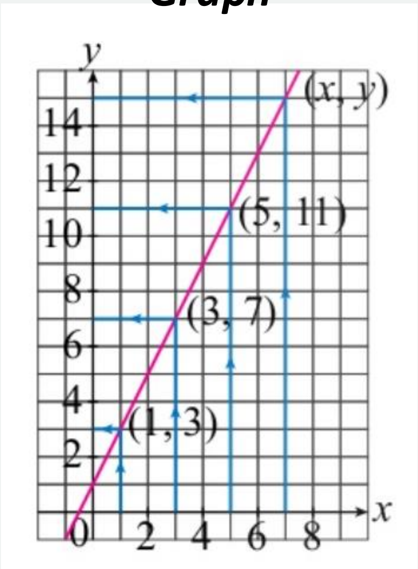
- Function here actually refers a relationship between two groups/parties of numbers

Arrow Diagram



$y = 2x + 1$

Graph



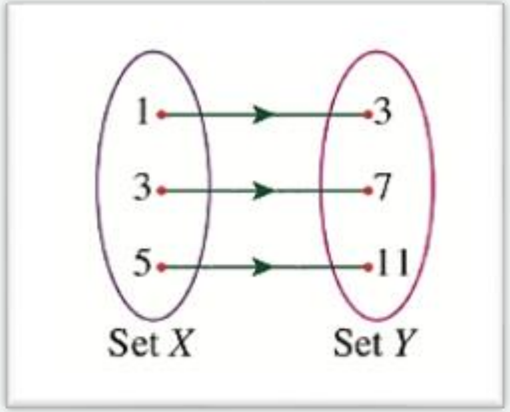
Ordered Pairs:

$\{(1,3)\ ,\ (3,7)\ ,\ (5,11)\}$

Function relating set X to set Y is a special relation where each element $x \in$ X is mapped to **one and only one** element $y \in$ Y.

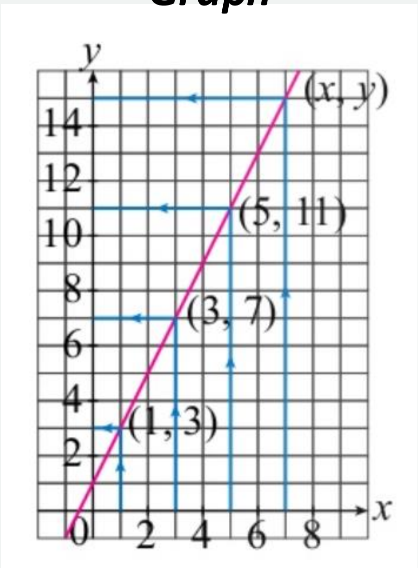
Functions

Arrow Diagram



$y = 2x + 1$

Graph



Ordered Pairs:

{(1,3) , (3,7) , (5,11)}

Function notation should be written as below:

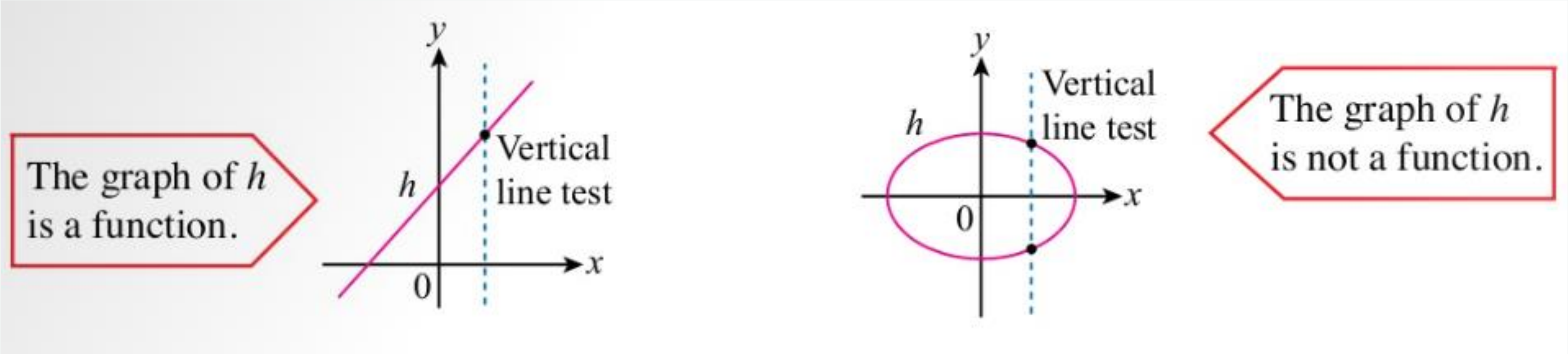
$f : x \rightarrow y$ or $f(x) = y$

$f : x \rightarrow 2x + 1$ or $f(x) = 2x + 1$

where x is the object and $2x + 1$ is the image

Function (Vertical Line Test)

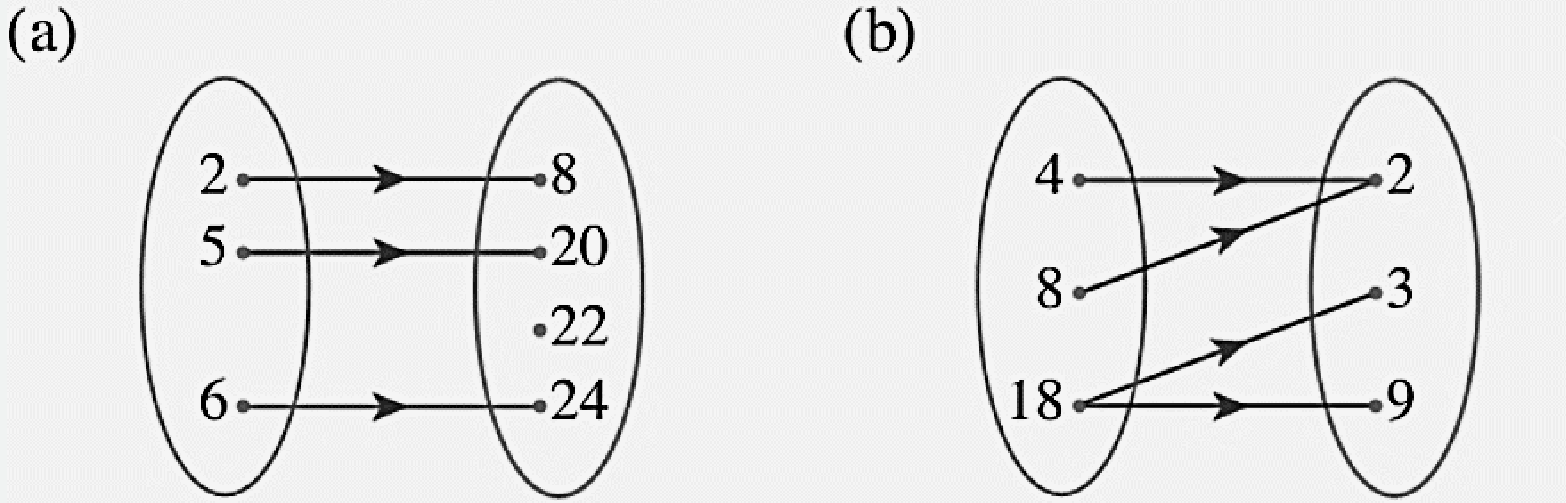
A Vertical Line Test can be used to determine whether a graph of a relation is a function:



If the vertical line cuts only at one point on the graph, then the relation is a function.

If the vertical line cuts more than one point on the graph, then the graph is not a function

Examples



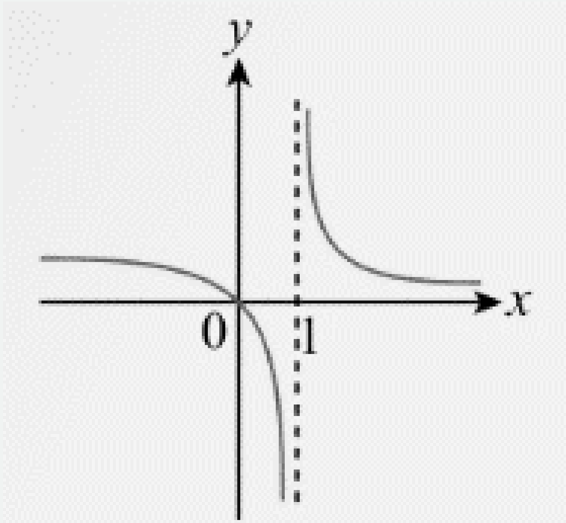
(a) This relation is a function because each object has only one image even though element 22 has no object.

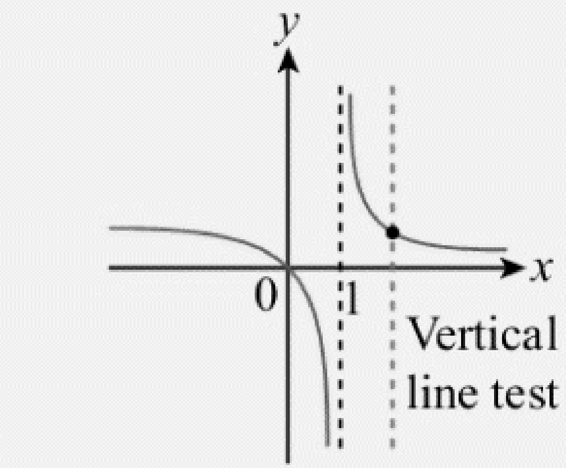
(b) This relation is not a function because it does not satisfy the condition of being a function, that is each object has only one image. Note that 18 has two images, that are $18 \rightarrow 3$ and $18 \rightarrow 9$.

Examples

Which of the following graph represents a function?

a)



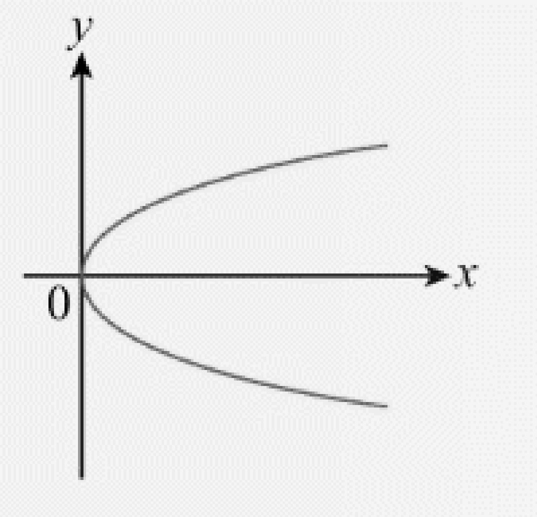


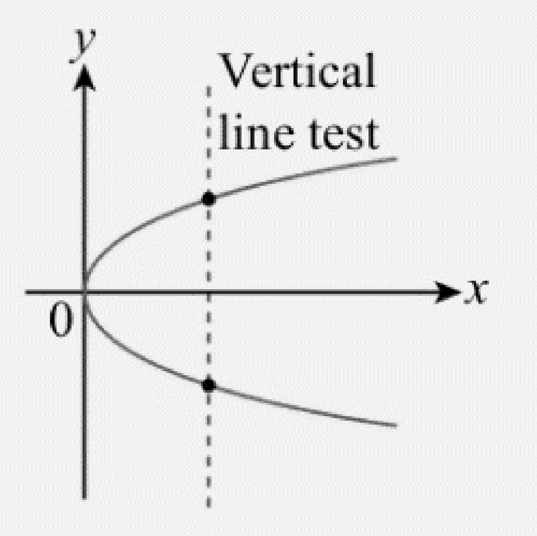
This graph is a function because when the vertical line test is carried out, the line cuts the graph at only one point, except when $x = 1$ where the line does not cut any point on the graph.

Examples

Which of the following graph represents a function?

b)





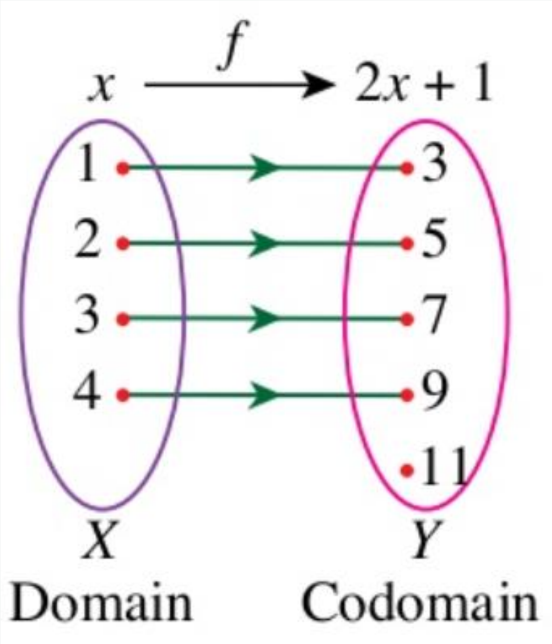
This graph is not a function because when the vertical line test is carried out, the line cuts the graph at two points

Domain and Range of Function

- Domain:
  The set of possible values of $\boldsymbol{x}$ which defines a function
- Range:
  The set of values of $\boldsymbol{y}$ that are obtained by substituting all the possible values of $\boldsymbol{x}$

Domain and Range of Function

Arrow Diagram:



The elements in set x: **Domain**
The possible elements in set y: **Codomain**
The actual obtained values of functions f in set y: **Range**

Domain = {1, 2, 3, 4}
Codomain = {3, 5, 7, 9, 11}
Range = {3, 5, 7, 9}

Composite Functions

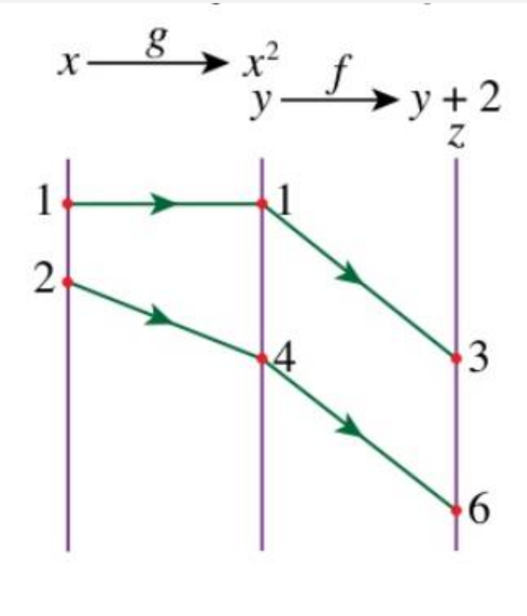
The process of combination by replacing two functions f and g to generate $f[g(x)]$ or $g[f(x)]$ is known as the composition of two functions and is written as $fg(x)$ or $gf(x)$.

$fg(x)$ is read as "f composed with g and x" and is defined by $fg(x) = f[g(x)]$.

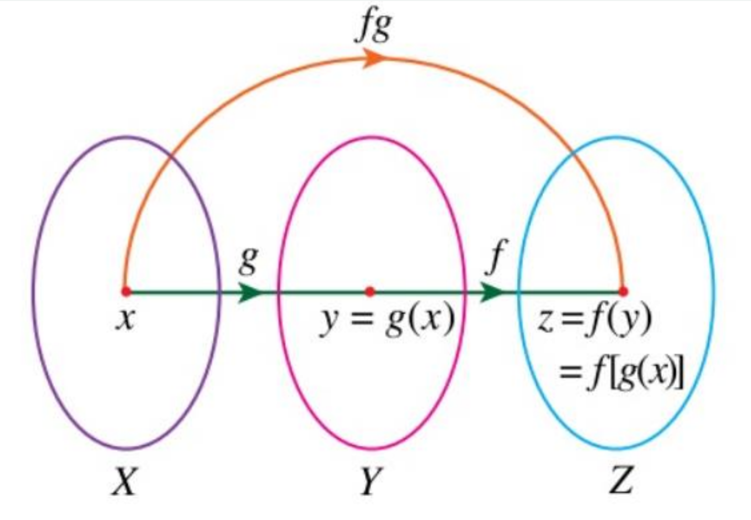
Given two functions $f(x)$ and $g(x)$, the product of combination of two functions that written as $fg(x)$ or $gf(x)$ are defined by $fg(x) = f[g(x)]$ or $gf(x) = g[f(x)]$.

Composite Functions

Given functions $f(x) = x + 2$ and $g(x) = x^2$. The diagram below shows part of mapping of function g followed by function f.



$$1 \xrightarrow{g} 1^2 = 1 \xrightarrow{f} 1 + 2 = 3$$
$$2 \xrightarrow{g} 2^2 = 4 \xrightarrow{f} 4 + 2 = 6$$
$$x \xrightarrow{g} x^2 = y \xrightarrow{f} y + 2 = z = x^2 + 2$$



Thus, **conclusion** is $\boxed{fg(x) = f[g(x)]}$

Examples

Example: If f(x)=x2-9 and g(x)=9-x2 , find out the domain of (fg) and the composition of (fg)(x)

Solution:

To find the composition of (fg)(x)

$$(f \circ g)(x) = f(g(x))$$
$$(f \circ g)(x) = f(\sqrt{9 - x^2})$$
$$(f \circ g)(x) = f(\sqrt{9 - x^2})^2 - 9$$
$$(f \circ g)(x) = \; - x^2$$

Examples

Example: If $f(x) = 2x + 3$ then find $(f \circ f)(x)$

Solution:

Since, $(f \circ f)(x) = f[f(x)]$

$= 2(2x + 3) + 3$

$= 4x + 9$

Example: Find $(g \circ f)(x)$ $if\ f(x) = 2x + 3\ and\ g(x) =- x^2 + 5$

Solution:

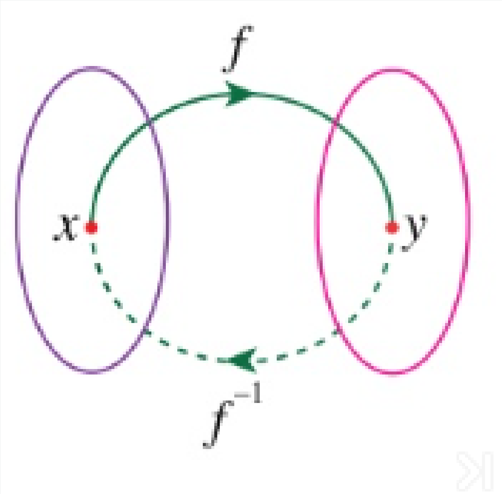
Since $(g \circ f)(x) = g[f(x)]$

$= (2x + 3)^2 + 5$

$=- 4x^2 + 12x - 9 + 5$

$=- 4x^2 - 12x - 4$

Inverse Functions



What is Inverse Function?

Referred to the graph at the left, it can be seen that every graph of the function and its graph of inverse function is symmetrical.

The graph of f^{-1} is the reflection of the graph of f in the line $y = x$

$$\boxed{f : x \to y \Leftrightarrow f^{-1} : y \to x \text{ or } y = f(x) \Leftrightarrow x = f^{-1}(y)}$$

$$\left.\begin{aligned} &f(x) = y \\ &f^{-1}(y) = x \end{aligned}\right\}$$

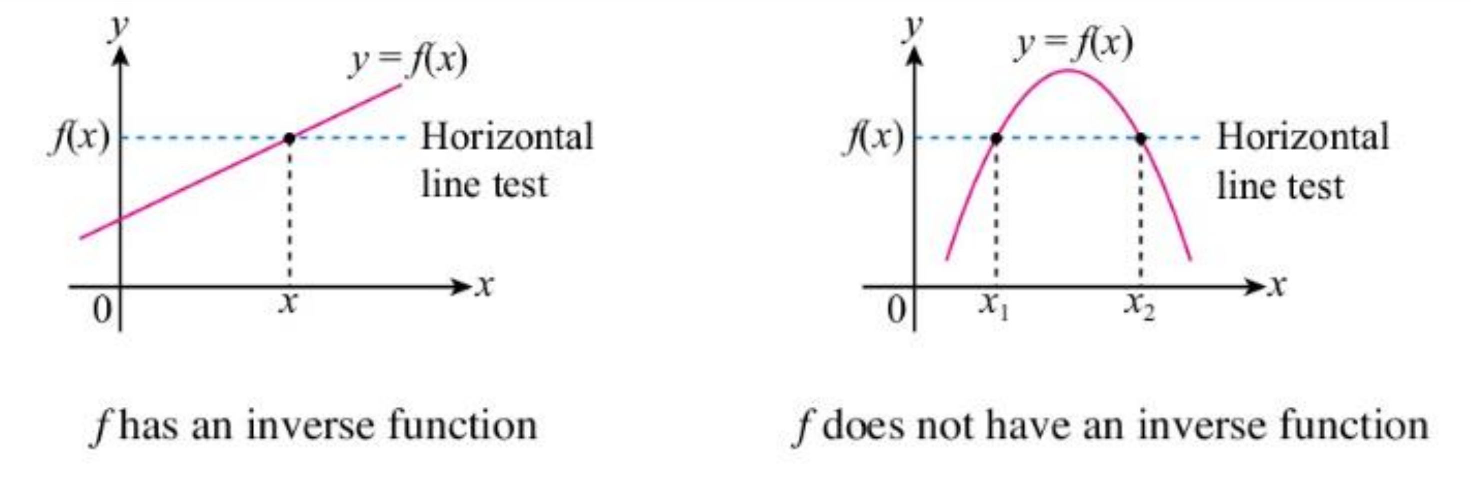
$$f(x) = y$$

$$x = \frac{y}{f^{1}}$$

$$x = f^{-1}(y)$$

Inverse Functions (Horizontal Line Test)

To determine whether this graph of a function has an inverse function, a **horizontal line test** can be carried out.



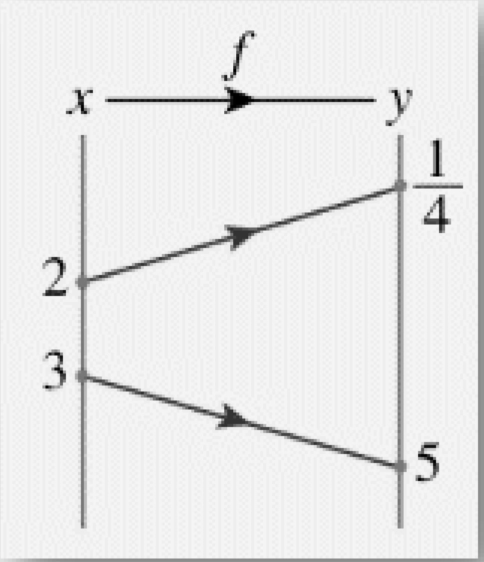
If horizontal line cuts at **only one point** of the graph, then this type of function **is a one-to-one function** and **has an inverse function**

If *more than one point*, then this function is *not one-to-one function* and *does not have an inverse function*

Examples

In the arrow diagram on the right, the function f maps x to y. Determine

(a) $f^{-1}\left(\frac{1}{4}\right)$ (b) $f^{-1}(5)$



(a) From the given arrow diagram, we obtain

$f(2) = \frac{1}{4}$, thus $f^{-1}\left(\frac{1}{4}\right) = 2$.

(b) By inverse mapping, $f^{-1} : 5 \to 3$.

Then, $f^{-1}(5) = 3$. $\leftarrow$ $\boxed{f : x \to y \Leftrightarrow f^{-1} : y \to x}$

Examples

A function is defined as $f(x) = \dfrac{x}{x-4}, x \neq 4$. Determine

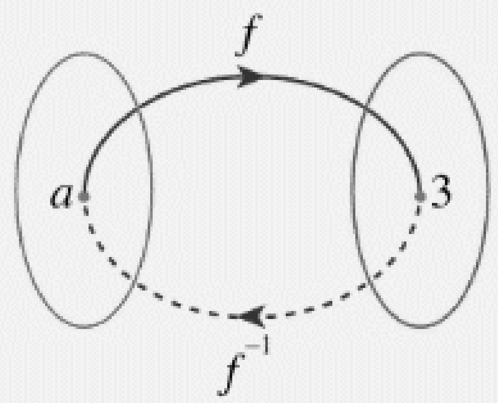
(a) the image of 2 under f, (b) $f^{-1}(3)$.

(a) The image of 2, $f(2) = \dfrac{2}{2-4} = -1$

(b) Let $a = f^{-1}(3)$,

$$f(a) = 3$$
$$\frac{a}{a-4} = 3$$
$$a = 3(a-4)$$
$$a = 3a - 12$$
$$2a = 12$$
$$a = 6$$

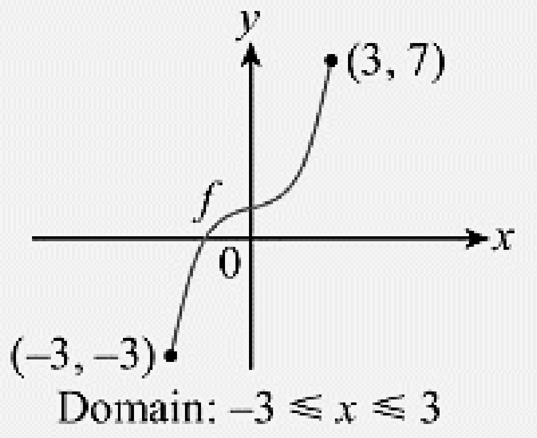
Thus, $f^{-1}(3) = a = 6$

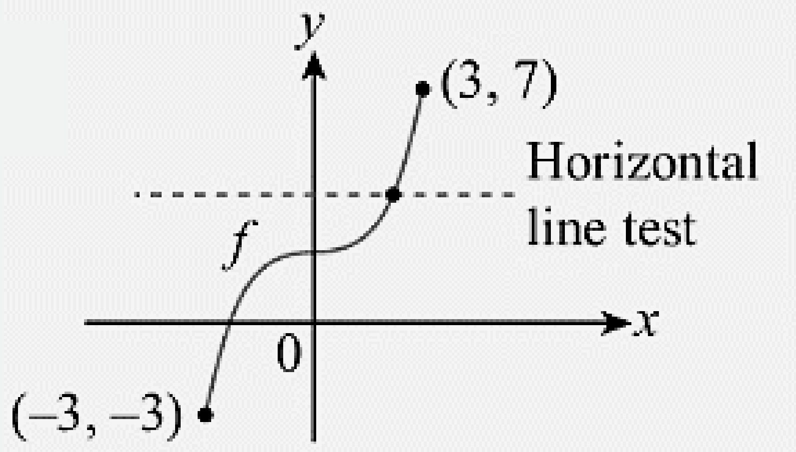


Examples

Which of the following graph represents an inverse function?

a)



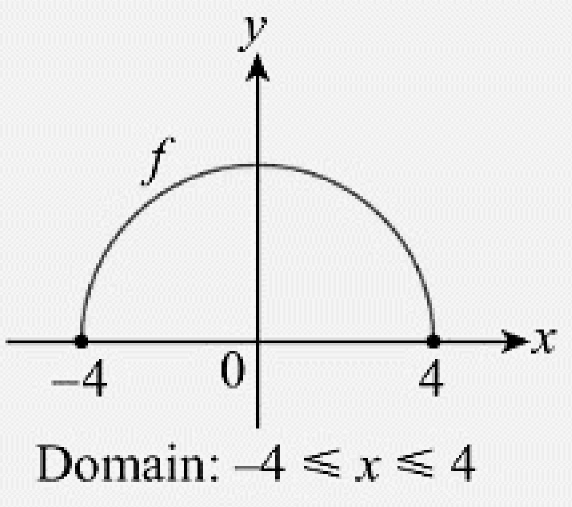


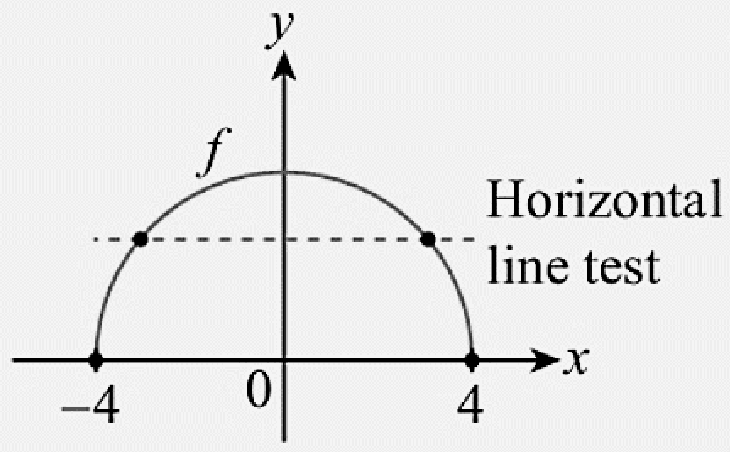
When the horizontal line test is carried out, the horizontal line cuts the graph of function *f* at only one point. This means that function *f* is a one-to-one function. Thus, function *f* has an inverse function.

Examples

Which of the following graph represents an inverse function?

b)





When the horizontal line test is carried out, the horizontal line cuts the graph of function f at two points. This means that function f is not a one-to-one function. Thus, function f has no inverse function.