CSC615M MP1: Equivalence of Two States

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ABSTRACT

Finite State Machines are the simplest formal machines. They represent a set of transitions between a set of states producing a particular output or recognizing a particular regular language. In implementing these actual machines in hardware, it may be desirable to eliminate equivalent states. This is done by using a Depth-First Search approach through all pairs of states to find a pair that is distinguishable. Once a pair cannot be found, the two states can be concluded as equivalent. The algorithm was coded in Python and tested using equivalence class black box testing and passed all tests. This project can then be said to be successful in determining equivalent states and can be used in future projects regarding machine equivalence and modelling a Turing Machine.

I. INTRODUCTION

Finite State Automata are the simplest formal machines taught in undergraduate Theory of Computation classes. Their operation simply consists of a finite number of states, stimuli, and responses, transitions from states on different stimuli, and output mappings (Hopcroft, J.E. et al, 2006). There are three basic types of finite state machines: Mealy machines, where output is assigned to transitions; Moore machines, where output is assigned to states; and Finite State Accepters, where the output is only produced after the entire input string is read and the output is either acceptance or rejection.

In implementing FSM's, it may be necessary to reduce the number of states to minimize costs (Mano, M.M., 2007). This is done by combining equivalent states. In order to do this, one must first determine which states are equivalent in a given finite state machine.

Two states are equivalent if and only if, given that the machine starts at each of the states in consideration, any valid input string produces the same output. This is easily determined for small machines with less than ten states, but for larger, more complicated machines, it may be desirable to automate this process.

This project aims to automate the process of determining which states are equivalent. This project will only handle finite state machines of a definite type. This project will not cover hybrid accepters/Mealy machines/Moore machines. For finite state accepters, this project can only handle the deterministic type.

II. DESIGN

The algorithm used was an extension of the algorithm found in Hopcroft, J.E. et al (2006). In this textbook, to prove two states are equivalent, they must both be either accepting or rejecting and any state the two states transition into must be equivalent as well. To extend this into Mealy and Moore machines, one can consider a finite state accepter as a Moore machine with R = {0,1}, 0 meaning rejecting state and 1 meaning accepting state. One can further extend this to Mealy machines by considering all output mappings on all stimuli symbols.

Proving two states equivalent by the basic definition is impossible, however, since it is impossible to test all possible input strings as there are an infinite number of strings. What can be done instead is to prove that certain paths in the FSM would lead to distinguishable (non-equivalent) states.

Given that pairs of states in a single automaton can be represented as vertices and transitions can be represented as edges, this becomes a simple graph search problem. The goal of the search is to locate a vertex that contains two distinguishable nodes. If the entire graph is exhausted and a goal vertex is not found, it can be concluded that the nodes are distinguishable.

Table 2.1. is a depth-first search algorithm adapted from Russel, S. & Norvig, P. (2010).

Table 2.1. – DFS Algorithm for State Equivalence

```
DFS Search
DFS-equiv(currNode,explored,stimuli) returns TRUE if currNode's
   states are equivalent and FALSE otherwise

   currNode – has s1 and s2, the states to compare
   explored – list of explored nodes
   stimuli – list of input symbols

   # check all transition outputs for Mealy Machine, and static state
output for Moore and FSA
   if currNode's state's outputs are different
      return FALSE
   else
      add currNode to explored

      for each input symbol s in stimuli
         # expand gets the two states that the states in currNode
transition into
         newStates = expand(currNode,s)
         if newStates not in explored
            if DFS-equiv(newStates,explored,stimuli) returns FALSE
               return FALSE

      # if all target states are visited but none are distinguishable
      return TRUE
```

This algorithm is simply run on all pairs of states to determine which groups of states are equivalent or not.

III. IMPLEMENTATION

The system was implemented in Python. The project had four modules: FSM.py for modelling the actual machine, State.py to model the states, FSMReader.py to read the file, and Main.py to act as the driver.

The State class used a Python dictionary to store the mappings from input symbols to states and outputs. The FSM class used a dictionary to map the names of the states to the states themselves. Each state, when added to the FSM, has the FSM's name prepended to the state name. The FSM adapts the name of the file it originated from.

IV. TESTING

Four major test cases were written. This is in three sets of two cases, one where there are equivalent states present and one where there are none. The three sets are if there is one input symbol, if there are two input symbols, and if there are more than two.

The first test case is a single input FSA that accepts odd numbers with no redundant states as shown in Figure 4.1. This was stored in file odd.txt.

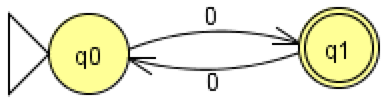


Figure 4.1. – FSA accepting odd number of 0's

The second test case was the same FSA with redundant states as seen in Figure 4.2. This was stored in file oddRedundant.txt.

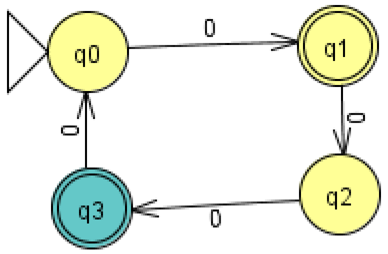
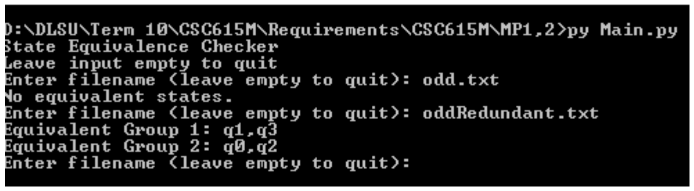


Figure 4.2. – FSA accepting odd number of 0's with redundant states

The outputs for these two cases are shown in Figure 4.3.



```
D:\DLSU\Term 10\CSC615M\Requirements\CSC615M\MP1,2>py Main.py
State Equivalence Checker
Leave input empty to quit
Enter filename (leave empty to quit): odd.txt
No equivalent states.
Enter filename (leave empty to quit): oddRedundant.txt
Equivalent Group 1: q1,q3
Equivalent Group 2: q0,q2
Enter filename (leave empty to quit):
```

Figure 4.3. – Outputs for first Test Cases

For the first set, the program outputted the expected output.

The second set of test cases had Moore Machines. The machine outputs A if there have been no two symbols that are adjacent and equal yet and B if there has been. Figure 4.4. shows the minimized version and Figure 4.5. shows the redundant version.

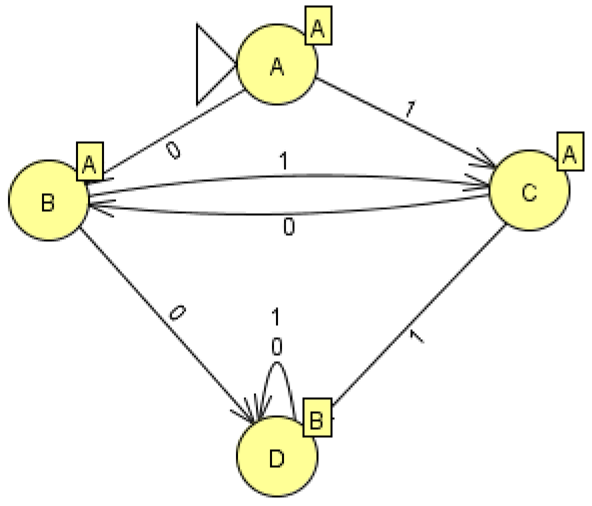


Figure 4.4. – Consecutive Symbol Detector Moore Machine

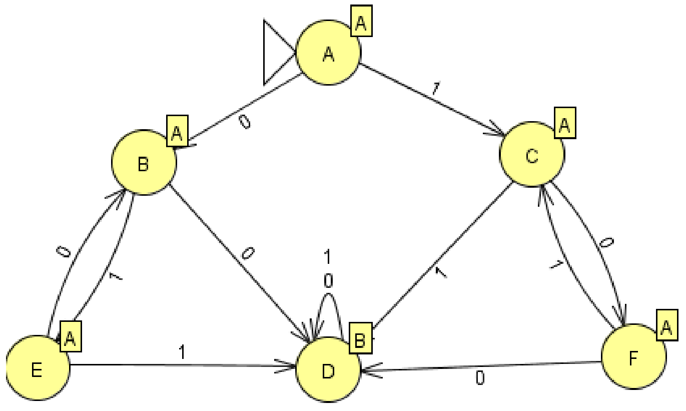
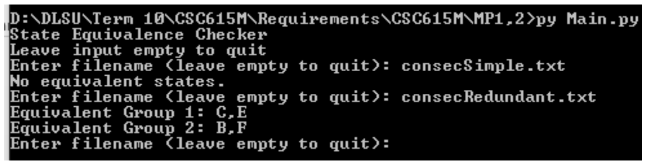


Figure 4.5. – Consecutive Symbol Detector Moore Machine with Redundant States

The program's outputs for the two are in Figure 4.6.



```
D:\DLSU\Term 10\CSC615M\Requirements\CSC615M\MP1,2>py Main.py
State Equivalence Checker
Leave input empty to quit
Enter filename (leave empty to quit): consecSimple.txt
No equivalent states.
Enter filename (leave empty to quit): consecRedundant.txt
Equivalent Group 1: C,E
Equivalent Group 2: B,F
Enter filename (leave empty to quit):
```

Figure 4.6. – Output for Second Set of Test Cases

For the second set, the program outputted the expected output.

For the last set, a Mealy machine with inputs 0, 1, and X was created. The machine would start out echoing the input until an X is encountered. It outputs the X but will then complement the input until another X is encountered. Figure 4.7. shows this machine minimized and Figure 4.8. shows it with redundant states.

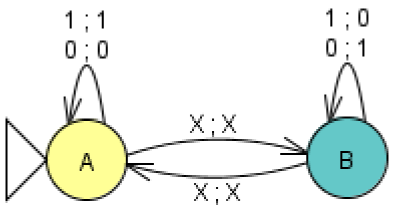


Figure 4.7. – Echo/Complement Machine

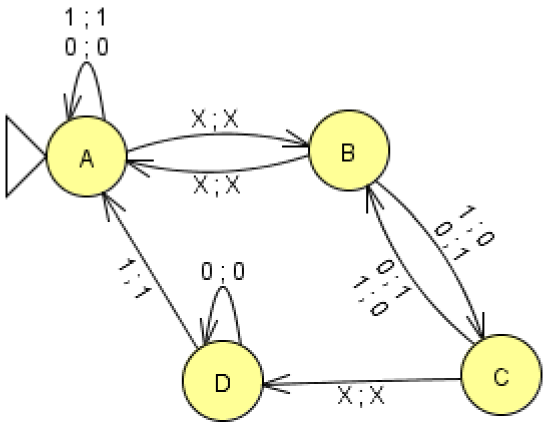
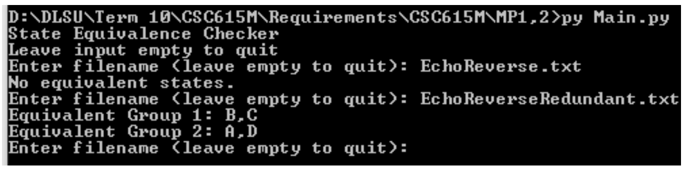


Figure 4.8. – Echo/Complement Machine with Redundant States

Figure 4.9 shows the program’s output, which is the expected output.



```
D:\DLSU\Term 10\CSC615M\Requirements\CSC615M\MP1,2>py Main.py
State Equivalence Checker
Leave input empty to quit
Enter filename (leave empty to quit): EchoReverse.txt
No equivalent states.
Enter filename (leave empty to quit): EchoReverseRedundant.txt
Equivalent Group 1: B,C
Equivalent Group 2: A,D
Enter filename (leave empty to quit):
```

Figure 4.9. – Output for Third Set of Test Cases

By equivalence class testing, these test cases should prove that the program is in an acceptable state.

V. CONCLUSION

In conclusion, the project was successfully implemented. The program can successfully determine the equivalent states of a finite state automaton, whether it be an accepter, Mealy, or Moore Machine.

This program will also be a useful step for the future projects. Determining whether two FSM’s are equivalent is simply calling the DFS method on a specific pair. The machine models can also be used to model a Turing Machine with slight modifications

REFERENCES

[1] Hopcroft, J.E., Motwani, R., and Ullman, J. D., (2006), Introduction to Automata Theory, Languages and Computation 3rd edition, Addison-Wesley Publishing Company. Reprinted by Jemma Inc.

[2] Mano, M. M. , Digital Design, 4th Ed., Peason/Prentice Hall, 2007

[3] Russell, S. & Norvig, P. (2010). *Artificial Intelligence: A Modern Approach*, 3rd Ed. New Jersey: Prentice-Hall.